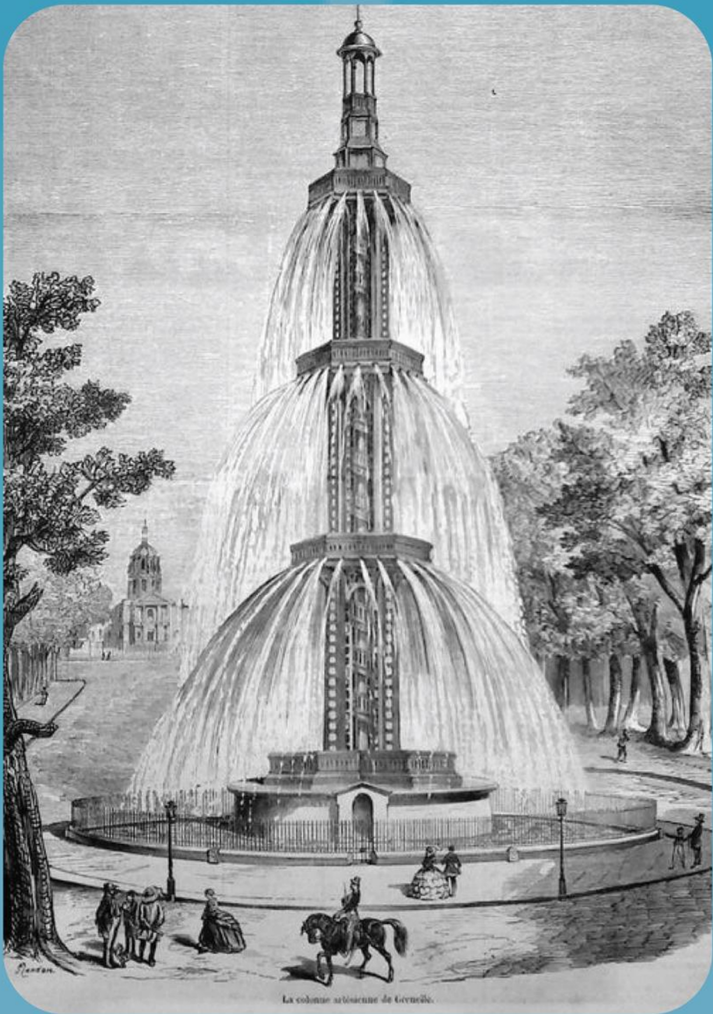


History and Hydraulics of Flowing Wells

Xiao-Wei Jiang and John Cherry

THE GROUNDWATER PROJECT

History and Hydraulics of Flowing Wells

The Groundwater Project

lease Consider Donating

The Groundwater Project relies on private funding for book production and management of the Project.

Please consider making a donation to the Groundwater Project so books will continue to be freely available.

Thank you.

Xiao-Wei Jiang
Professor, China University of Geosciences
Beijing, China

John Cherry
Adjunct Professor, University of Guelph
Guelph, Ontario, Canada
Professor Emeritus, University of Waterloo
Waterloo, Ontario, Canada

History and Hydraulics of Flowing Wells

The Groundwater Project
Guelph, Ontario, Canada

Published by The Groundwater Project, Guelph, Ontario, Canada, 2024.

History and Hydraulics of Flowing Wells / Xiao-Wei Jiang and John Cherry - Guelph, Ontario, Canada, 2024.

72 pages.

ISBN: 978-1-77470-061-7

DOI: https://doi.org/10.21083/CPET1503↗.

Please consider signing up to the GW-Project mailing list to stay informed about new book releases, events, and ways to participate in the GW-Project. When you sign up to our email list, it helps us build a global groundwater community. Sign up↗.

APA 7th Edition Citation:

Jiang, X.-W., & Cherry, J. (2024). *History and hydraulics of flowing wells*↗. The Groundwater Project. https://doi.org/10.21083/CPET1503.

THE GROUNDWATER PROJECT

Cover Image: (https://parisianfields.files.wordpress.com/2012/10/puits-grenelle-fontaine.png↗).

Dedication

Dedicated to the generous sharing of groundwater knowledge.

Table of Contents

COPYRIGHT .... **IV**

DEDICATION .... **V**

TABLE OF CONTENTS .... **VI**

THE GROUNDWATER PROJECT FOREWORD .... **VIII**

FOREWORD .... **IX**

PREFACE .... **XI**

ACKNOWLEDGMENTS .... **XII**

1 INTRODUCTION **1**

2 TERMINOLOGY OF FLOWING WELLS AND AQUIFERS **5**

- 2.1 Flowing Wells 5
- 2.2 Aquifers/Aquitards 5
- 2.3 Confined, Semiconfined, and Unconfined Water/Aquifers 6

3 MID-NINETEENTH CENTURY FLUID MECHANICS, HYDROLOGY, AND GROUNDWATER CIRCULATION **8**

- 3.1 Fluid Mechanics 8
- 3.2 Hydrological Cycle 8
- 3.3 Early Explanations of Flowing Wells and Groundwater Circulation 9
- 3.4 Darcy and Dupuit's Education and Working Experience 10

4 UNDERSTANDING OF CONDITIONS OF FLOWING WELLS IN SELECT COUNTRIES SINCE THE1800S **12**

- 4.1 France 12
- 4.2 Great Britain 15
- 4.3 The United States of America 17
- 4.4 Australia 21
- 4.5 Canada 22
- 4.6 China 24
- 4.7 Summary and Implications 25

5 GROUNDWATER FLOW PATTERNS AND CONDITIONS FOR FLOWING WELLS **27**

- 5.1 In Confined Aquifers 27
- 5.2 In Thick Unconfined Aquifers 29
- 5.3 In Semi-confined Aquifers 30

6 HYDRAULICS OF FLOWING WELLS IN A CONFINED AQUIFER WITH A FLAT INITIAL POTENTIOMETRIC SURFACE **37**

- 6.1 Steady-State Discharge Rate of Flowing Wells in Confined Aquifers 37
- 6.2 Transient Discharge Rate of Flowing Wells in Infinite Confined Aquifers 40

7 HYDRAULICS OF FLOWING WELLS IN SEMI-CONFINED AQUIFERS WITH A FLAT POTENTIOMETRIC SURFACE **42**

- 7.1 Transient Discharge Rate 42
- 7.2 Steady-state Discharge Rate 43

8 HYDRAULICS OF FLOWING WELLS IN BASINS WITH A BACKGROUND BASINAL FLOW FIELD **45**

- 8.1 Transient Discharge Rate and Water Sources in a Semi-Confined Aquifer of a Layered Basin 45
- 8.2 Vertical Profiles of Discharge Rate in Flowing Wells with Long Screens 48

9 WRAP-UP **52**

10 EXERCISES **54**

Exercise 1 - Analytical Analysis of a Zone with Flowing Wells in a Homogeneous Basin 54
11 REFERENCES 56
12 BOXES........ 65
Box 1 - Terms Used for Flowing Wells in 33 Textbooks 65
Box 2 - The Mathematical Model and Solution of a Transient Discharge Rate of Flowing Wells in Infinite Confined Aquifers 68
Box 3 - The Mathematical Model of a Transient Discharge Rate of Flowing Wells in Semi-Confined Aquifers with a Flat Potentiometric Surface 69
13 EXERCISE SOLUTIONS 70
Solution Exercise 1 70
14 ABOUT THE AUTHORS 71

The Groundwater Project Foreword

The UN-Water Summit on Groundwater, held on 7-8 December 2022, at the UNESCO Headquarters in Paris, France, concluded with a call for Government and other stakeholders to scale up efforts to better manage groundwater. The intent of the call to action was to inform relevant discussions at the UN 2023 Water Conference that was held on 22-24 March 2023 at UN Headquarters in New York City. One of the required actions is *strengthening human and institutional capacity*, to which groundwater education is fundamental.

The 2024 World Water Day theme is '*Water for Peace*', which focuses on the critical role water plays in the stability and prosperity of the world. The UN Water website states that *more than 3 billion people worldwide depend on water that crosses national borders*. There are 592 transboundary aquifers; yet most of these aquifers do not have an intergovernmental cooperation agreement in place for sharing and managing the aquifer. While groundwater plays a key role in global stability and prosperity, it also makes up 99% of all liquid freshwater, so it is at the heart of the freshwater crisis. *Groundwater is an invaluable resource*.

The Groundwater Project (GW-Project) is a registered Canadian charity founded in 2018, committed to the advancement of groundwater education as a means to accelerate action related to our essential groundwater resources. We are committed to *making groundwater understandable* and, with that, enable *building the human capacity for sustainable development and management of groundwater*. To that end, the GW-Project creates and publishes high-quality books about *all-things-groundwater*, for all who want to learn about groundwater. Our books are unique in that they synthesize knowledge, are rigorously peer reviewed, are translated in many languages, and are free of charge. An important tenet of GW-Project books is a strong emphasis on visualization with clear illustrations to stimulate spatial and critical thinking. The GW-Project started publishing books in August 2020, and, by the end of 2023 had published 44 original books and 58 translations. The books are available at gw-project.org↗.

The GW-Project embodies a new type of global educational endeavor made possible through the contributions of a dedicated international group of volunteer professionals from diverse disciplines. Academics, practitioners, and retirees contribute by writing and/or reviewing books aimed at diverse levels of readers including children, teenagers, undergraduate and graduate students, as well as professionals in groundwater fields and the general public. More than 1,000 dedicated volunteers from 70 countries and six continents are involved—and participation is growing. Revised editions of the books are published from time to time. Readers are invited to propose revisions.

We thank our sponsors for their ongoing financial support. Please consider donating to the GW-Project so we can continue the publication of books free of charge.

The GW-Project Board of Directors, January 2024

Foreword

Groundwater is unseen to the eye and, prior to the 1700s, the thinking about groundwater was shrouded in mystery and folklore. Modern groundwater science began in the mid-1800s, prompted by the vivid manifestations of groundwater seen by the public and water engineers in the form of water gushing high into the air from flowing wells.

Although flowing wells were regarded as impressive events as early as the twelfth century, the wells were not spectacular gushers because they resulted from shallow drilling. Spectacular flowing wells resulted from new technologies for deep drilling that appeared in France in the 1830s; this instigated the beginnings of quantitative groundwater science. However, simplified thinking about groundwater is evident in publications about this foundational work. This limited the advancement of groundwater science; unfortunately, some of this simplified thinking persists today.

This book examines the origins of modern groundwater science from flowing wells. Its objective is to explain how groundwater science evolved with its particular limitations in the context of aquifers, their confining beds, and evolution in thinking toward the broader concept of aquitards. For example, we explain how modern fluid mechanics and geology evolved from the 1700s with important advances in the early part of the 1800s. Then, for groundwater, the key advance was the arrival of technology for deep drilling in the early 1800s that resulted in abundant flowing wells in France and England.

Following that initial period of deep drilling, quantitative groundwater science—later to become known as hydrogeology—came from the work of two individuals who were schoolmates in engineering education in Paris: Henry Darcy (1803–1858) and Jules Dupuit (1804–1866). First, Darcy published his *empirical law* on the relationship between flow rate and head loss in 1856—an essential prerequisite for Dupuit to follow up on a few years later with the mathematical foundations for steady flow to a well. Dupuit also correctly illustrated potentiometric surfaces responsible for flowing wells. These set the stage for modern thinking in the two most basic themes in hydrogeology: groundwater flow systems and aquifer hydraulics. However, as is commonly the case in the evolution of science, their foundational work was so brilliant and impactful that it seemed to limit the thinking of those who followed as they encountered factors and complexities beyond those considered by Darcy and Dupuit.

Major advancements that followed Darcy and Dupuit occurred in the United States beginning in the late 1800s. The hydrogeologic circumstances of the areas initially studied in the United States mostly reflected thinking prevalent in Paris and London. Predictably, this gave the impression that these hydrogeologic conditions were generalizable, and resulted in the first terminologies and conceptualizations about aquifer systems and groundwater flow systems. Some of this became misleading and was partly incorrect—which is examined in this book that also expands on the hydraulics of flowing wells.

The lead author, Dr. Xiao-Wei Jiang, a professor of hydrogeology at the China University of Geosciences, Beijing, China, has—for more than one decade--extensively examined the historical origins of the key ideas that underpin modern hydrogeology. The co-author, Dr. John Cherry, took his first university course concerning groundwater science in 1960; over the past 60 years, he has also observed how early ideas considered to be true with their associated terminology often limited our thinking for decades.

John Cherry, The Groundwater Project Leader

Guelph, Ontario, Canada, November 2023

Preface

This book traces two themes running through the history of quantitative groundwater science beginning with the foundational works of Henry Darcy (Darcy, 1856) and Jules Dupuit (Dupuit, 1863). The first theme includes concepts of groundwater flow systems that initially were entirely focused on plug flow in strata bound aquifers. This persisted until M. K. Hubbert's seminal publication in 1940 on the theory of groundwater flow, which was founded on the principles of physics. The second theme is about the ideas of aquifer hydraulics that evolved conceptually from aquifers with "pipe flow" that are bounded above and below by strata known as confining layers. For nearly a century, aquifer hydraulics was founded on this idea that confining strata were impervious. Finally, in the 1940s groundwater scientists realized that inter-aquifer leakage should be considered in aquifer hydraulics.

The early concepts evolved from the ideas of Darcy (1856) and Dupuit (1863), then were reinforced and expanded upon in the United States—again with a strong influence of flowing wells. They persisted for over half a century with their inherent misconceptions until Hubbert's 1940 advance in theory. The long delay in the development of more refined concepts is partly attributable to the meanings ascribed to the word *artesian*, which were distorted by imprecise thinking. Even today, the term artesian has multiple meanings in the literature. Hubbert's (1940) theory made it known that a continuous low-permeability stratum overlying the aquifer was not required for wells to flow freely at the surface.

The existence of groundwater as a resource for drinking and agriculture has been known since antiquity. The term artesian is commonly used as a modifier for an aquifer in which water level is above the top of the aquifer (artesian well or artesian conditions). The term *artesian aquifer* is synonymous with the concept of the aquifer having a continuous impervious confining bed without acknowledgment of the confining bed transmitting water. However, modern hydrogeology now recognizes that, in aquifer systems, the confining beds (now known as aquitards) commonly have substantial permeability and are as important as aquifers. We now recognize that in many cases the aquitards govern long-term aquifer yield and hydrogeochemistry. However, it took nearly a century after the work of Darcy and Dupuit for groundwater science to become modern hydrogeology.

This book is a journey in groundwater ideas up to the end of the 1970s, how ideas originated and evolved, and how some of the ideas were corrected or abandoned. In the 1980s, there was an explosion in groundwater research around the world, which led to hydrogeology becoming a modern quantitative science. Current developments related to flowing wells are included in this book, however the broader evolution of hydrogeology during this modern period is left for future books.

Acknowledgments

We deeply appreciate the thorough and useful reviews of and contributions to this book by the following individuals:

- John. F. Hermance, Professor Emeritus, Brown University, Providence, Rhode Island, USA
- Garth van der Kamp, Research Scientist, University of Saskatchewan, Saskatoon, Saskatchewan, Canada
- John Wilson, Professor Emeritus, New Mexico Institute of Mining and Technology, Socorro, New Mexico, USA
- Brian Smerdon, Research Associate, Faculty of Science – Earth & Atmospheric Sciences, University of Alberta, Edmonton, Alberta, Canada
- Connie Bryson, Editor, The Groundwater Project, Canada.
- Andrew Cohen, Adjunct Professor, New Jersey Institute of Technology, New York, USA

Special thanks to former or current PhD students Jun-Zhi Wang, Zhi-Yuan Zhang, and Yi-Peng Zhang at China University of Geosciences (Beijing) who conducted research on hydraulics of flowing wells. We thank Yi-Peng Zhang and Tao-Tao Ji for drawing some figures for this book.

We are grateful for Amanda Sills and the Formatting Team of the Groundwater Project for their oversight and copyediting of this book. We thank Eileen Poeter (Colorado School of Mines, Golden, Colorado, USA) for reviewing, editing, and producing this book.

1 Introduction

The gushing or overflowing of water from flowing wells (Figure 1) has long attracted public attention as well as scientific curiosity. Flowing wells were widespread in sedimentary basins of many countries before intensive groundwater withdrawals began, and many fundamental concepts and classic studies of groundwater hydrology were inspired by the flowing wells.

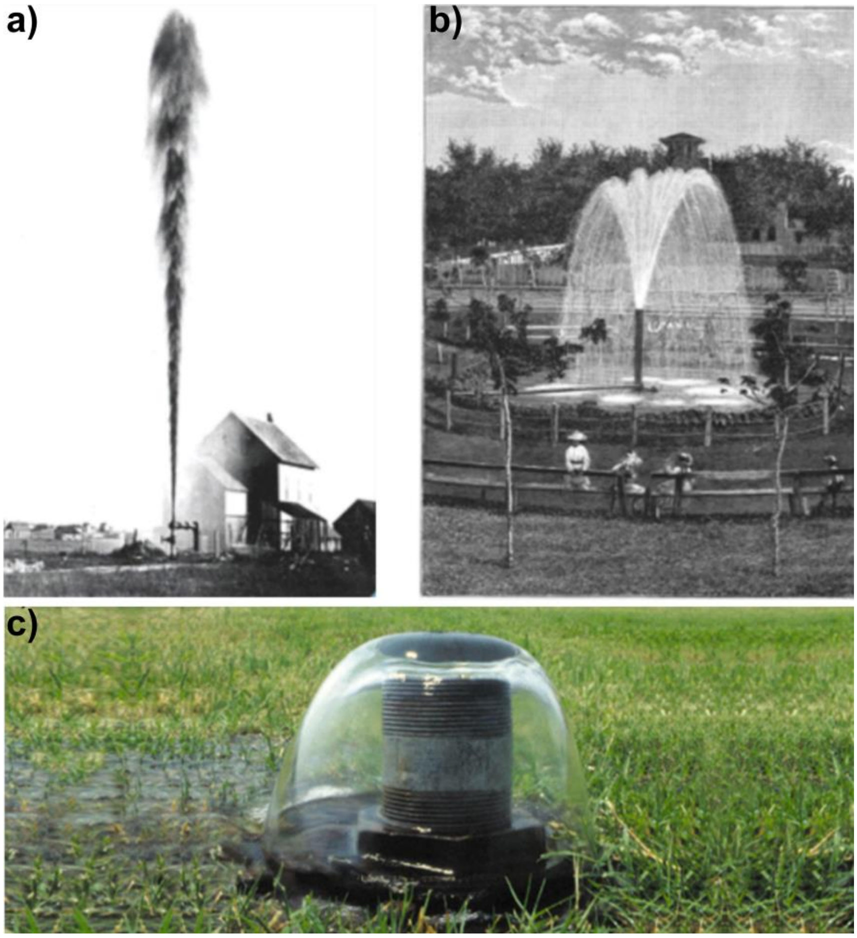


Figure 1 - Photos or drawings of flowing wells in the USA: a) Woonsocket, South Dakota (reproduced from Darton, 1909); b) Prairie du Chien, Wisconsin (reproduced from Chamberlin, 1885); c) Brunswick, Georgia (reproduced from Water Science School, 2019↗).

Groundwater as a quantitative science can be traced to the famous published works of Darcy (1856) and Dupuit (1863). Darcy and Dupuit were hydraulic engineers who reported on their research in the form of data tables, graphs, mathematical formulations, and simple figures (Jiang et al., 2020). Their work was followed up in the United States in the late 1800s when the need for groundwater supply became urgent as immigrant settlers migrated westward in search of land, especially in the northern Plains regions where agriculture required a source of water.

In America, pioneer researchers were educated in geology rather than hydraulic engineering as had been the case in France. Most of the seminal works occurred within the US Geological Survey (USGS), established in 1879. Advances in groundwater science resulted from geological thinking founded on the creation of conceptual models that are usually expressed as cross-sectional representations of subsurface conditions. These representations are applied to either a particular study area or to represent conditions believed to occur more generally.

The generalized representations are now known as *conceptual models*; an assembly of such models within a theme constitutes a *paradigm*. A paradigm is a conceptual scheme or general theory that helps to provide scientists in a particular field with a broad theoretical framework. According to Kuhn (1962), who originated the paradigm concept, a paradigm is an essential framework for progress during a period of time in a scientific endeavor; however, as paradigms promote stability and entrench thinking, so too they can limit progress and innovation.

This book about the early pillars of groundwater science traces the origins and evolution of two of the earliest paradigms in groundwater science: one is concerned with gravity-driven flow systems and the other with aquifer hydraulics. Although this book is based primarily on older literature of historic significance, it also includes recent advances in the understanding of the hydraulics of flowing wells.

The conceptualization of a groundwater flow system as shown in Figure 2 has been and continues to be the most commonly published introductory representation of a groundwater system, with only minor differences. As shown later in this book (Sections 4.1 through 4.3), cross-sectional diagrams with many of key features of Figure 2 concerning confined aquifers, outcrop recharge area, and flowing wells had already appeared in the 1800s. Figure 2 represents how aquifers, their confining beds, groundwater recharge, and groundwater flow in a system of aquifers and confining beds are generally perceived. Unfortunately, Figure 2 also embodies most of the misconceptions and erroneous views about groundwater systems that persist in common communications aimed at introductory readers. It is likely that such representations do more harm than good in that they represent the thinking of the old paradigm for groundwater flow, aquifers, and aquifer hydraulics.

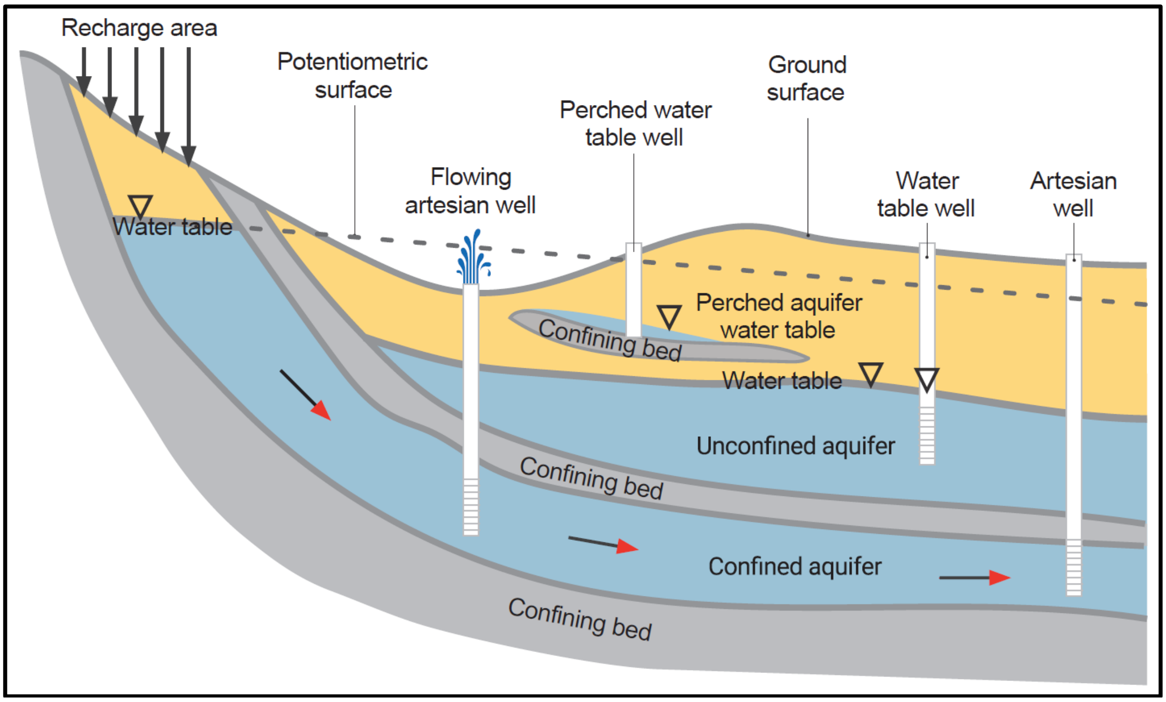


Figure 2 - A commonly used but potentially misleading plot showing three types of aquifers and three types of wells, including a flowing artesian well tapping the confined aquifer (modified from United Nations, 2022).

This book examines how the old paradigm came about, as triggered by flowing wells, and describes the transition to the new paradigm on which modern hydrogeology now rests. The emphasis in this book is on the presentation of conceptual figures and simulations that show how these ideas evolved.

By the mid-1800s, the mother sciences of groundwater hydrology—including fluid mechanics, hydrology, and geology—matured (Section 3). By this time, the drilling of many flowing wells in France had triggered the emergence of groundwater hydrology (hydrogeology) as a separate discipline. The widespread occurrence of regional-scale confined aquifers accompanying flowing wells in many sedimentary basins in France and the USA (Sections 4.1 and 4.2) led to the common misconception that an overlying aquitard was an indispensable condition for flowing wells (Section 5.1). However, by plotting how groundwater flows from topographic highs to lows, Hubbert (1940) explicitly pointed out that an overlying aquitard is *not* a necessary condition of flowing wells at all (Section 5.2). Flowing wells in unconfined aquifers were later identified in Canada and China (Sections 4.3 and 4.4).

In the classic textbook by Freeze and Cherry (1979), flowing wells were divided into two groups (Figure 3):

1. Geologically controlled flowing wells, in which water flows from wells in confined aquifers; and
2. Topographically controlled flowing wells, in which water flows from wells in unconfined aquifers.

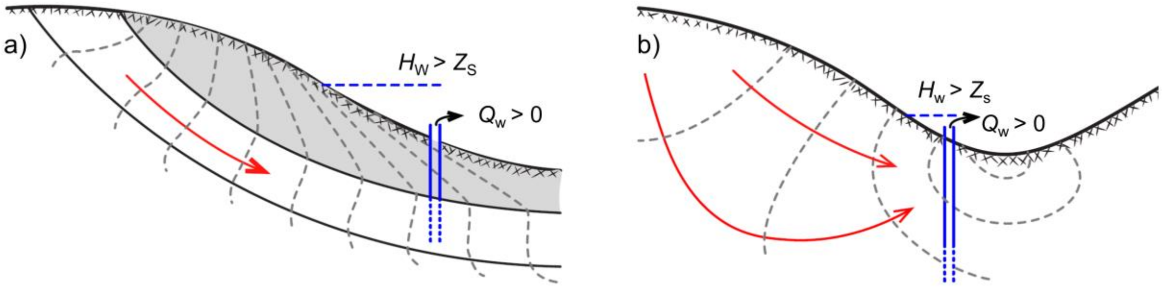


Figure 3 - Cross sections showing two endmember conditions of flowing wells: a) a geologically controlled flowing well in the confined aquifer of a basin; b) a topographically controlled flowing well in a homogeneous basin (modified from Freeze & Cherry, 1979). Q_w represents the discharge rate of the flowing well, H_w represents the hydraulic head near the flowing well, and Z_s represents the elevation of the land surface near the flowing well.

As pointed out by Johnson and others (2020), geologically controlled flowing wells are the most well-known and are widely acknowledged as typical conditions needed for flowing wells. Conversely, topographically controlled flowing wells are often neglected in the literature and, indeed, often are not depicted in standard hydrogeology textbooks.

In modern hydrogeology, the role of aquitards in cross-formational flow is widely acknowledged. In fact, many geologically controlled flowing wells were drilled near large rivers, implying these wells are also controlled by topography (i.e., associated with upward groundwater discharge toward rivers). In this book, after introducing conditions of geologically controlled flowing wells in purely confined aquifers (Section 5.1) and topographically controlled flowing wells in purely homogeneous basins (Section 5.2), we introduce a numerical model of the simultaneous controls of topographic undulation, recharge/discharge, and aquitards on flowing wells (Section 5.3).

Quantification of the relationship between discharge rate in wells and head loss (drawdown), which is one of the main topics of well hydraulics, can be traced to Dupuit (1863). Although Dupuit acknowledged that the potentiometric surface near a flowing well is influenced by the regional hydraulic gradient (Section 4.1), it was assumed in Dupuit's model of well discharge that the initial potentiometric surface is flat, thus the potentiometric surface is radially symmetrical. This assumption was inherited by almost all models of steady-state or transient well hydraulics; its limitation was not realized until the phenomenon of wellbore flow (intraborehole flow) was identified in the late 1900s (Reilly et al., 1989; Molz et al., 1994).

In this book, we begin with classic models for the hydraulics of flowing wells in purely confined aquifers and in semi-confined (leaky) aquifers with a flat initial potentiometric surface (Section 6 and Section 7). We then proceed to introduce models for flowing wells in basins—including layered basins and homogeneous basins—with a topographically controlled flow system (Section 8).

2 Terminology of Flowing Wells and Aquifers

2.1 Flowing Wells

In Europe, one of the first flowing wells was identified in 1126 at Lillers in Artois, an ancient province in northern France (Margat et al., 2013). The higher plateaus of the province of Artois constitute the recharge area of many flowing wells (Davis & De West, 1966). Because *Artesia* is the historical Latin name of Artois, flowing wells in this region were so famous that flowing wells were termed *artesian wells*. It was the phenomenon of water overflow at the surface that attracted people's attention to the wells of Artois (Fuller, 1906; Norton, 1897).

In 1805, the term *artesian fountain* was applied in the French scientific literature (Lionnais, 1805). In the 1820s and 1830s, the term artesian well was widely used in France, Britain, and the USA (Arago, 1835; Buckland, 1836; Garnier, 1822; Héricart de Thury, 1830; Storrow, 1835). In Chamberlin's (1885) classic report, the term *artesian well* was used interchangeably with flowing well. Probably because many artesian wells changed from flowing wells to non-flowing wells during groundwater exploitation, the term artesian well embraced both flowing and non-flowing artesian wells in the 1890s (Norton, 1897).

Today, the term artesian well means flowing well in some textbooks. However, in many others, it simply means a well penetrating a confined aquifer. Currently, synonyms of flowing wells include overflowing wells, free flowing wells, flowing artesian wells, artesian flowing wells, overflowing artesian wells, and free flowing artesian wells. Details of the terms related to flowing wells found in textbooks are introduced in Box 1↴.

2.2 Aquifers/Aquitards

Groundwater occurs in permeable geologic formations known as aquifers (Todd, 1959). Various versions of the definitions of aquifers can be found in Table 1 of van der Gun (2022) and are not repeated here. According to Meinzer (1928), the word *aquifer* was first introduced by Norton (1897) into the English literature from the French word *aquifère*. The words aquifer and aquifère can be traced to their Latin origins: *aqui-* is a combining form of *aqua*, meaning water, and *–fer* (*-fère*) comes from *ferre*, to bear. Before that, the terms *porous stratum* and *permeable formation* were used in the UK to represent aquifers (Bond, 1865; Whitaker, 1888), whereas the terms *water-bearing formation (bed, stratum,* or *deposit)* were used as synonyms of aquifer in the USA (Carpenter, 1891; Chamberlin, 1885).

Chamberlin (1885) pointed out that no stratum is entirely impervious and introduced the term *confining bed* to refer to impervious or semi-impervious beds overlying/underlying aquifers. According to Fetter (1980), confining beds (layers, units) can be further divided into aquitards, aquicludes, and aquifuges.

The latter two terms *aquiclude* and *aquifuge* were introduced in two pre-1960 textbooks (Todd, 1959; Tolman, 1937). An aquiclude is an impermeable formation that may contain water but is incapable of transmitting significant water quantities; an aquifuge is an impermeable formation that neither contains nor transmits water. Clay is an example of an aquiclude; solid granite is an example of an aquifuge.

In textbooks published since the 1960s (Bouwer, 1978; Davis & De Wiest, 1966; De Weist, 1965; Fetter, 1980; Freeze & Cherry, 1979), the term *aquitard* has been used with increasing frequency. According to Davis and De Wiest (1966), an aquitard stores water and transmits enough water to be significant in the study of regional migration of groundwater but not enough to supply individual wells. Bouwer (1978) pointed out that aquitards are sufficiently permeable to transmit water vertically to or from a confined aquifer but are not permeable enough to laterally transport water like an aquifer. The term aquifuge was dropped in some textbooks (De Wiest, 1965; Freeze & Cherry, 1979; Hiscock, 2005).

Because aquifers separated by aquitards are usually hydraulically connected, the term *aquifer system* was proposed in the early 1970s. According to Domenico (1972), an aquifer system comprises an aquifer, a confining unit, or a combination of aquifers and confining units that contain a reasonably distinct source of groundwater. According to Poland and others (1972), an aquifer system is a heterogeneous body of intercalated permeable and poorly permeable material that functions regionally as a water-yielding hydraulic unit; it comprises two or more permeable beds separated at least locally by aquitards that impede groundwater movement but do not greatly affect the regional hydraulic continuity of the system. In Freeze and Cherry (1979), the term *aquifer-aquitard system* was proposed, which is a synonym of *aquifer system*.

2.3 Confined, Semiconfined, and Unconfined Water/Aquifers

In the USA, the adjective artesian was popularized by Chamberlin (1885) but was extended to describe water where the pressure head is higher than the bottom of the overlying confining bed. In the following decades, such terms as *artesian pressure*, *artesian water*, *artesian aquifer,* and *artesian basin* were proposed (Fuller, 1906; Meinzer, 1928). According to Meinzer (1928, p.39), artesian water is *groundwater that is under sufficient pressure to rise above the zone of saturation*, and *an artesian aquifer is one that contains artesian water*.

Because the water in an artesian aquifer is confined under artesian pressure, the terms *confined water* and *confined aquifers* emerged in Tolman (1937) and were widely used in later textbooks. Here, we reproduce the definition of confined aquifer given in Todd (1959) and Freeze and Cherry (1979). Confined aquifers—also known as artesian or pressure aquifers—occur where groundwater is confined under pressure greater than atmospheric by overlying relatively impermeable strata (Todd, 1959). A confined aquifer is an aquifer that is confined between two aquitards (Freeze & Cherry, 1979).

When the confining beds overlying or underlying an aquifer are semipermeable, the aquifer could receive a significant inflow through the semipermeable confining beds, especially under pumping conditions. In such a case, the aquifer is a *semiconfined aquifer* (Todd, 1959; Hiscock, 2005) *or leaky aquifer* (Freeze & Cherry, 1979; Hantush, 1959; Hantush & Jacob, 1955; Hiscock, 2005). In the earliest journal paper on hydraulics of leaky aquifers, the term *leaky artesian aquifer* was used by Jacob (1946).

A term accompanying *confined water* in Fuller (1906) is *unconfined water*. Todd (1959) defined an unconfined aquifer to be an aquifer in which a water table serves as the upper surface of the zone of saturation. Synonyms of unconfined water found in textbooks include *phreatic water (*Daubrée, 1887*)*, *free water* (Tolman, 1937); synonyms of unconfined aquifer include *phreatic aquifer* and *water-table aquifer*.

3 Mid-Nineteenth Century Fluid Mechanics, Hydrology, and Groundwater Circulation

Hydrogeology, which is an offspring of hydrology and geology, developed as an organized science only after the maturation of the two mother sciences in the eighteenth and nineteenth centuries (Meyer et al., 1988). Here we summarize the status of fluid mechanics, hydrological cycle, and groundwater circulation by the middle of the nineteenth century. Note that geological structure is critical to groundwater circulation; however, a summary of the status of geology at that time is beyond the scope of this book. The geological structures of selected artesian basins are introduced in Section 4.

3.1 Fluid Mechanics

The hydraulics of groundwater in the subsurface stemmed from fluid mechanics in pipes or open channels. In the eighteenth century, Daniel Bernoulli (1700–1782) showed that energy was conserved along a streamline in steady, incompressible, and inviscid flow. However, in real fluids, there is head loss along a streamline. According to Brown (2002) and Ritzi and Bobec (2008), the most accepted relationship in the early nineteenth century for head loss in pipe flow was described by de Prony's equation as shown by Equation (1).

$$\Delta h = \frac{L}{D}(aV + bV^2) \tag{1}$$

where:

Δh = head loss (L)

L = flow distance (L)

D = pipe diameter (L)

a and b = empirical friction coefficients associated with laminar and turbulent flow, respectively (-)

V = velocity (LT^{-1})

At high flow velocities, the first-order term can be dropped for computational convenience: a = 0. In this case, Equation (1) is equivalent to the Chézy equation that describes turbulent flow in open channels.

By measuring friction losses in small diameter (0.029 to 0.142 mm or 0.001 to 0.006 in) capillary tubes over a range of conditions, Poiseuille (1841) developed an empirical linear relationship between flow rate and head loss when b in Equation (1) equals 0.

3.2 Hydrological Cycle

Quantitative measurements of hydrologic phenomena were made in the late seventeenth century by Pierre Perrault (1608–1680) and Edme Mariotte (1620–1684). By measuring the average annual rainfall over a small part of the upper Seine basin and the annual discharge of the river from that catchment, Perrault (1674) estimated that runoff was only one-sixth of total precipitation, thus proving that precipitation was sufficient to supply the stream water in the Seine. Perrault also studied evaporation. However, he thought there was little infiltration of rainwater into the subsurface and springs were fed by river water (Biswas, 1970).

By measuring the flow of the Seine at Paris, Mariotte confirmed Perrault's work that the source of river water in the Seine River is rainfall. Based on the observations that the flow of springs increases in rainy weather and diminishes in times of drought, he believed that springs are fed by rainwater that infiltrated the subsurface (Fetter, 2004). Mariotte's important contributions were published in Paris in 1690 and were later published as collected works in Leiden in 1717 (Mariotte, 1717, after De Wiest, 1965). De Wiest (1965) mentioned that Mariotte also discussed the properties of fluids and the origin of flowing wells, but the authors of this book do not know the details.

3.3 Early Explanations of Flowing Wells and Groundwater Circulation

Groundwater was produced from shallow flowing wells in the Western Desert of Egypt in the first millennium BCE (Commander, 2005). The first correct explanation of the cause of flowing wells was given by the Persian philosopher, polymath, and scientist Sheikh Abu Rayhan al-Biruni, who lived from 973 to 1048 (Davis & De Wiest, 1966).

In Europe, based on observations of flowing wells in Modena, Italy, Bernardino Ramazzini (1633–1714) and Antonio Vallisnieri (1661–1730) connected flowing wells to topography, confined aquifers, and/or precipitation. Ramazzini plotted a geological section (Figure 4) showing flowing wells penetrating a confined aquifer and receiving their water from an underground reservoir with a higher water level in a nearby mountain (Biswas, 1970). Therefore, Ramazzini already suspected the role of topography (i.e., hydraulic gradient) between the underground reservoir in the nearby mountains and the well in the occurrence of flowing wells (de Vries, 2007). However, he thought the source of water in the underground reservoir in the surrounding mountain was more likely to be from the sea.

In 1715, based on observations in the Apennine Mountains in Italy, Vallisnieri argued that the source of water in the flowing wells of Modena, Italy, must be rainfall and snowmelt in the adjacent Apennine Mountains (De Wiest, 1965). This was the start of thinking about the simultaneous control exercised on flowing wells by topography and recharge from precipitation.

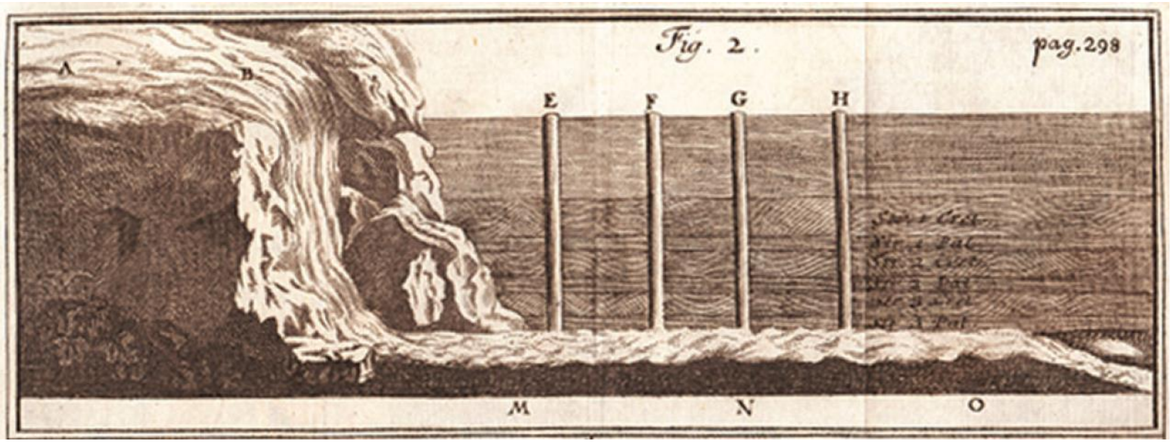


Figure 4 - Bernardino Ramazzini's conceptual drawing shows the cause of flowing wells in a confined aquifer (reproduced from Duffy, 2017).

In the early nineteenth century, development of the technique of cable-tool drilling (also called percussion drilling) resulted in the drilling of many flowing wells in France. Garnier (1822) published the first technical guidebook on drilling *artesian* wells. He received a prize from the Society for the Encouragement of Industry due to the publication of this book, which reflects the interest of the French in flowing wells. The French government was strongly supportive of the Society. Garnier (1822) provided a schematic plot showing the occurrence of flowing wells in a confined aquifer, with groundwater recharge in the outcrop (Figure 5). A limitation of the plot is that the head loss along the flow path in the confined aquifer was not considered in the potentiometric surface.

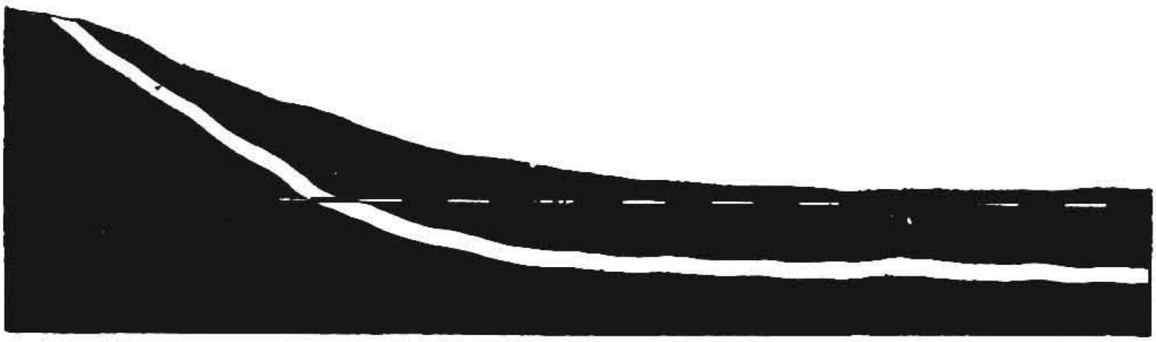


Figure 5 - A plot showing the control of a confined aquifer on flowing wells given by Garnier (1822; reproduced from Versluys, 1930).

3.4 Darcy and Dupuit's Education and Working Experience

Henry Darcy and Jules Dupuit were classmates during their undergraduate and graduate education in Paris. Darcy entered L'École Polytechnique [Polytechnical School] in 1821 and L'École des Ponts et Chaussées [School of Bridges and Roads] in 1823. Dupuit was admitted one year later. Darcy and Dupuit finished their studies in 1826 and 1827, respectively. According to Garnier (1822), during the time of their studies, the recharge of groundwater in the outcrops of topographic highs and the flow of water from topographic highs through confined aquifers to flowing wells had been widely accepted. As students of civil engineering, they were taught about fluid mechanics, the hydrological cycle, and groundwater circulation based on what was known in the eighteenth century.

After graduation, both entered the Corps des Ponts et Chaussées [Corps of Bridges and Roads]. Darcy's work was related to groundwater throughout his career, while Dupuit's work only turned to groundwater when he succeeded Darcy as Chief Director for Water and Pavements in Paris in 1850; as such, this marked his entrance to the field of groundwater hydrology.

4 Understanding of Conditions of Flowing Wells in Select Countries Since the1800s

4.1 France

Because of improved drilling technology, there were already many flowing wells in France for water supply in the early nineteenth century. Based on flowing wells in France, Héricart de Thury (1829) published a theoretical book on the causes of flowing wells. He tried to use the potentiometric surfaces shown in Figure 6 to explain the causes of flowing wells. He also showed the geological formations and the potentiometric surfaces in the Paris Basin in a separate plot (Figure 7). Similar to Garnier (1822), the head loss was not considered in the potentiometric surfaces shown by Héricart de Thury (1829).

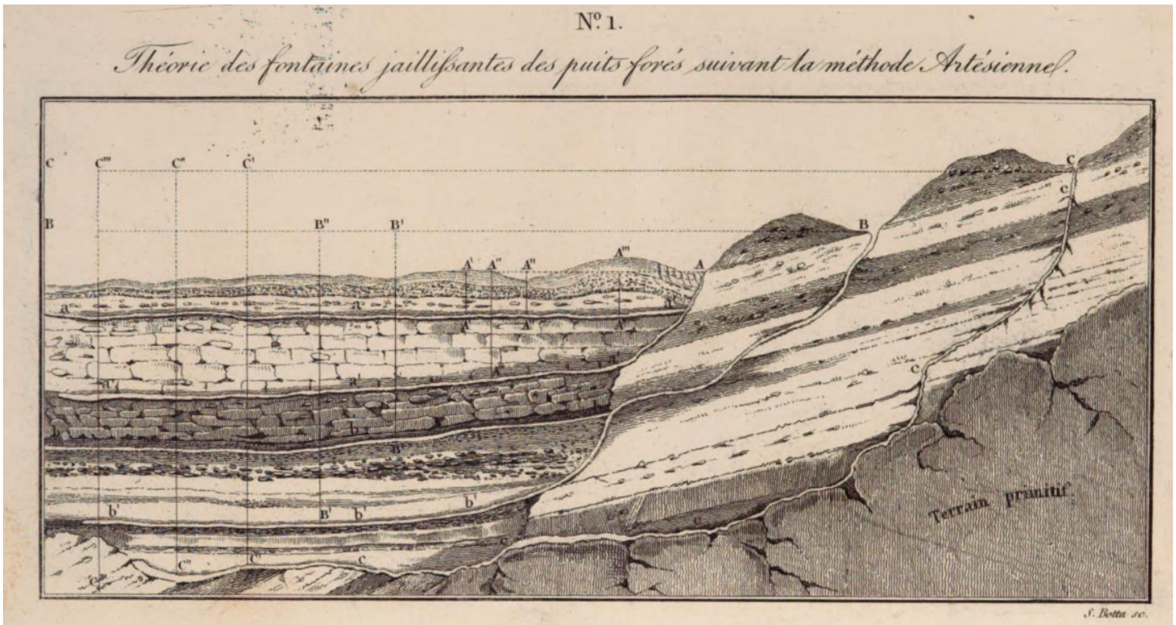


Figure 6 - A schematic plot showing potentiometric surfaces of three different conduits (starting from A, B, and C) between formations. A limitation is the neglect of head loss through the conduits (reproduced from Héricart de Thury, 1829).

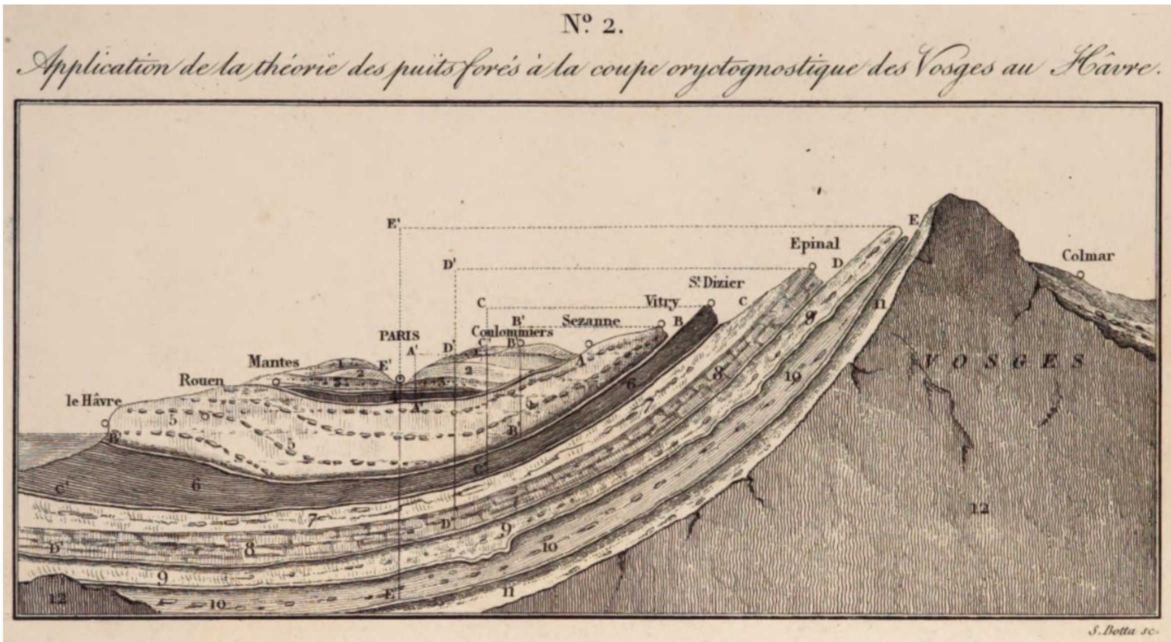


Figure 7 - A cross section showing the geology and the potentiometric surfaces in the Paris Basin. A limitation is the neglect of head loss through the conduits (reproduced from Héricart de Thury, 1829).

In 1832, a murderous wave of cholera struck Paris. Hospitals were unable to keep pace with the volume of new patients and fear of cholera became a driving force behind urban planning. Because the geology of the Paris Basin was already known (Figure 7), the City Council (François Arago, a geological engineer, was the mayor of Paris) decided to drill the first artesian well in Paris to supply its inhabitants with uncontaminated water. The well was in Grenelle, 1.7 km (≅1 mi) away from the Seine River (Figure 8).

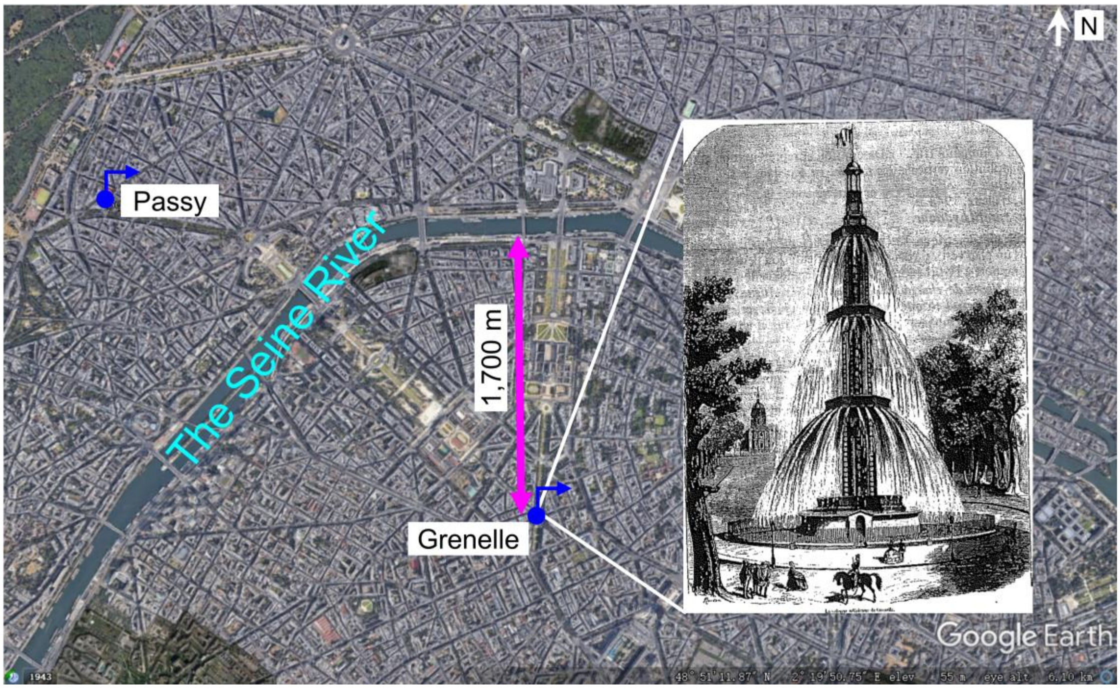


Figure 8 - The location of the Grenelle Well and the Passy Well, Paris, France. The inset is an illustration from 1860 showing the Grenelle Well functioning as a fountain (reproduced from Botham, 2021; 1,700 m ≅ 1 miles).

Between 1833 and 1841, the Grenelle Artesian Well was drilled by Louis-Georges Mulot, who had successfully drilled other artesian wells in France. This well had a depth of 548 m (1,798 ft) below the land surface, which was much deeper than existing flowing wells at that time—the existing wells ranged in depth from 36 to 177 m (118 to 581 ft; data from Arago, 1835). Because the water level could rise to a height of 33 m (108 ft) above ground surface, the well was encased in a 13-story (42 m (138 ft)) iron tower (the inset in Figure 8). The tower—the highest of the city at that time—became a major tourist attraction and, when the well was closed, the public could ascend the spiral staircase to view the surroundings from above.

Unfortunately, water ceased flowing from the Grenelle Artesian Well in 1903 due to decreased hydraulic head, and the tower was later demolished. It is interesting that the tower was replaced by a statue of Louis Pasteur (1822–1895), who made remarkable breakthroughs in the understanding of the causes and preventions of diseases and contributed to the foundations of hygiene, public health, and much of modern medicine.

In the mid-1800s, Darcy (1856) also provided a geological cross section of the Paris Basin, showing aquifers, aquitards, and two flowing wells tapping two different confined aquifers (Figure 9). He acknowledged that recharge occurs where aquifers are exposed at topographic highs around the basin rim. Dupuit (1863) then showed the regional potentiometric gradient of the Paris Basin and three flowing wells tapping the confined aquifer (Figure 10). Compared with de Thury's potentiometric surfaces, a significant improvement in Dupuit's is the consideration of head loss through aquifers.

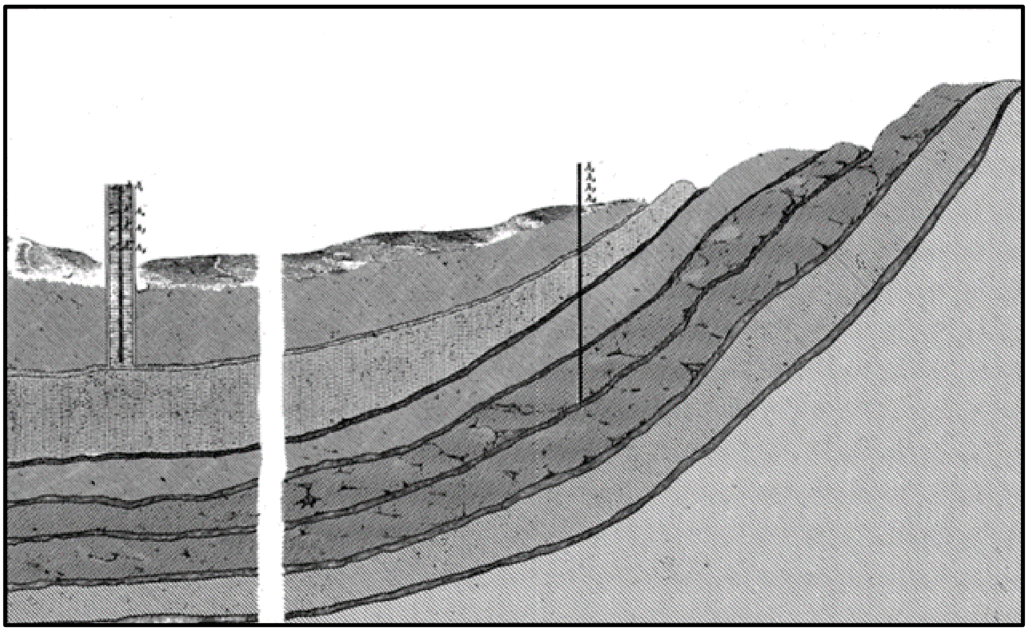


Figure 9 - Stratigraphy and structural geology of the Paris Basin (from Darcy, 1856; reproduced from Ritzi & Bobec, 2008). The well on the left (representing the famous Grenelle Artesian Well) taps a Cretaceous sandstone (the *Greensand* in Darcy's time) that is interstratified with thicker chalk strata.

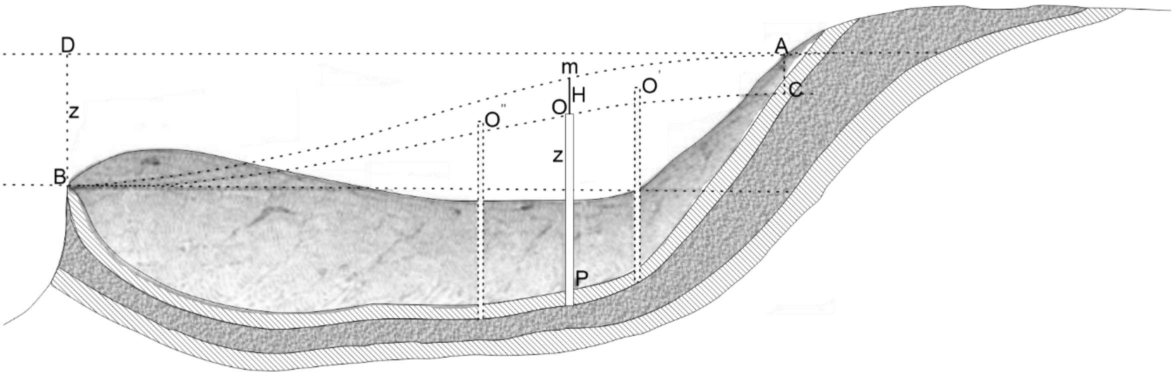


Figure 10 - Flowing wells (O, O', and O'') tapping the confined aquifer in the topographic lows of the Paris Basin. Also shown are the potentiometric surfaces (AB and CB) at two states (from Dupuit, 1863; modified from Ritzi & Bobec, 2008).

Figure 9 and Figure 10 demonstrate that hydrogeologists in the mid-1800s already knew how groundwater circulates in confined aquifers. Darcy's solid background in geology and hydrology, coupled with his knowledge of fluid mechanics led to his thinking on flow rates measured at different orifices in deep flowing wells (Section 4.1), including the Grenelle Artesian Well. The relationship between flow rates and elevations of different orifices also inspired Dupuit (1863), who became the first hydrogeologist to derive a formula for steady-state well discharge (Section 6.1). In 1861, another flowing well was finished in Passy—only 3 km (1.86 miles) away from the Grenelle well (Figure 8). The newly drilled well also led Dupuit to think about well interference and capture zone geometries, but this topic is beyond the scope of this book.

4.2 Great Britain

By the end of the 1700s, some shallow flowing wells had been dug in Great Britain. For example, a flowing well (artificial spring) with a depth of ≈3.7 m (≈12 ft) was dug ≈91 m (≈299 ft) away from the Darwent River in the city of Derby, Derbyshire, England, in 1785 (Darwin, 1785), and one of the first flowing wells near London was completed in 1794 (Buckland, 1836).

The successful experiences of water supply from flowing wells led to construction of more flowing wells. According to Macintosh (1827), James Ryan obtained a patent on boring for minerals and water in 1805, while John Goode obtained a patent on boring for the purpose of obtaining and raising water in 1823, indicating that drilling was active in Great Britain in the early nineteenth century. According to an article in *Monthly Magazine and British Register* (The Social Economist, 1822), two flowing wells with depths of 32 m (105 ft) and 37 m (121 ft) were drilled in the town of Tottenham (north London, England) in 1821 (one of them is shown in Figure 11). By 1822, many flowing wells had existed for a period of time in various parts of the country.

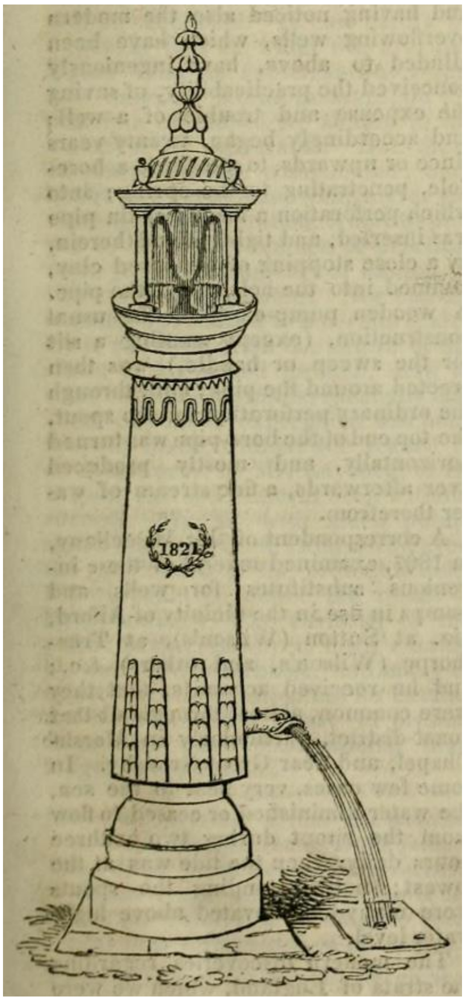


Figure 11 - A flowing well drilled in the town of Tottenham, north London, England, in 1821 (reproduced from The Social Economist, 1822).

The publication of Garnier's handbook (1822) in France aroused further interest in flowing wells in Great Britain (Farey, 1823). Over the next decade, Garnier (1822) and the more theoretical work of Héricart de Thury (1829) were widely used in Great Britain because of the absence of similar books in English. According to Mather (2013), the theory behind the occurrence of flowing artesian conditions was well understood by William Buckland (1784–1856), who produced excellent explanatory cross sections as shown in Figure 12. However, head loss was not considered as shown by the horizontal line A to B in Figure 12. By 1849, a section across the London Basin illustrating the origin of London's artesian wells was widely circulated (Figure 13).

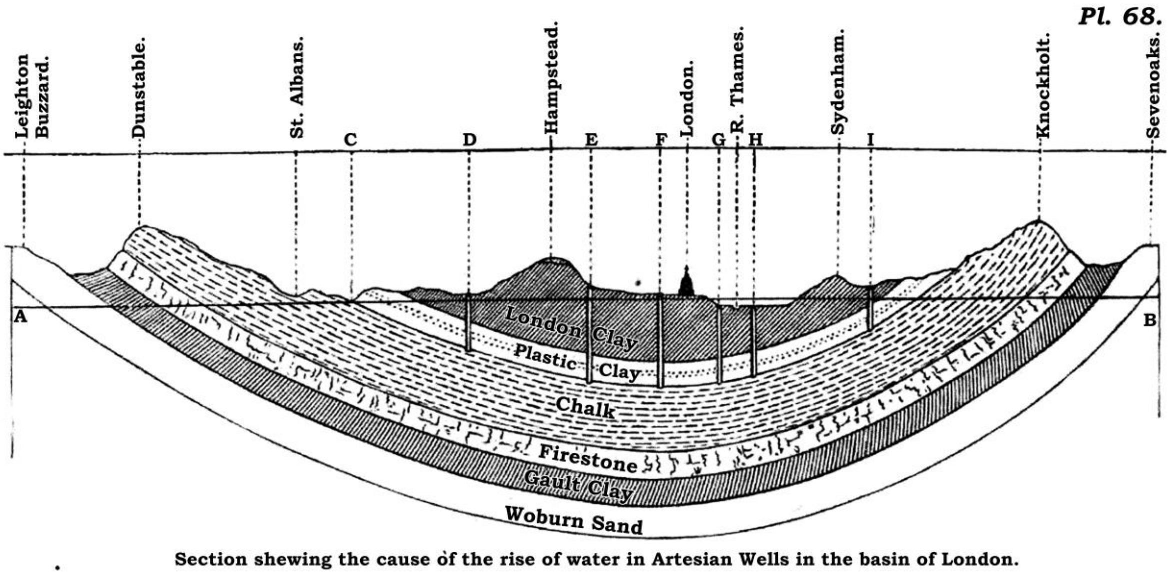


Figure 12 - Cross section from Buckland (1836) showing the basin-shaped disposition of strata in the London Basin and illustrating the rise of water in artesian wells. The horizontal line A-B represents the level to which water would rise by hydrostatic pressure in any perforations through the London Clay into sands of the Plastic Clay or into the Chalk, such as those represented at D, E, F, G, H, and I. If the perforation is made at G or H, where the land surface is below the line A-B, the water will rise in a flowing artesian fountain (reproduced from Mather, 2013).

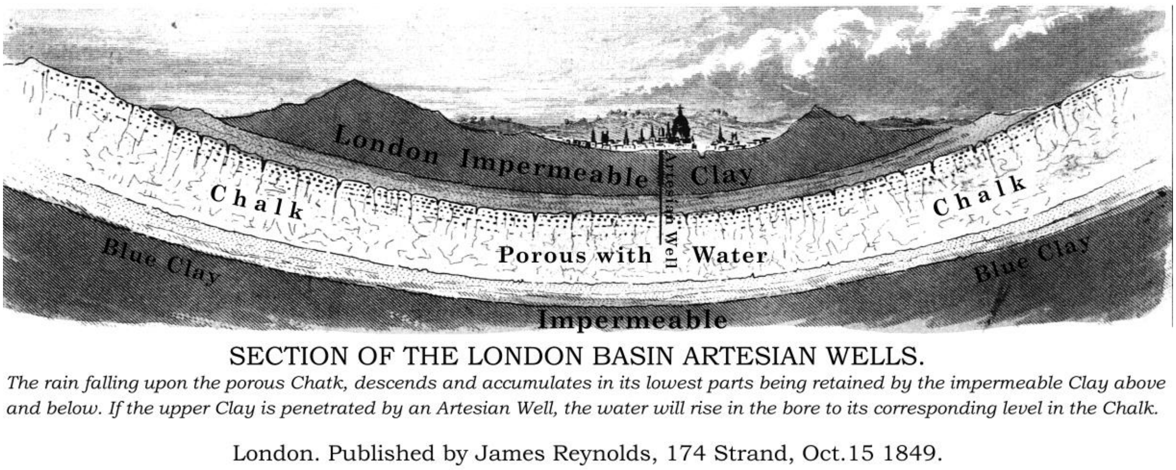


Figure 13 - A cross section showing the artesian wells of the London Basin compiled by Morris (1849). A sentence explaining the causes of flowing wells is shown in the plot (reproduced from Mather, 2013).

4.3 The United States of America

In the USA, increased demand for water resources for drinking and agriculture was driven by population increase due to immigration and Western expansion. Aided by the development of drilling technology since the nineteenth century, deep groundwater was accessed by drilling numerous wells, many of which were flowing wells in their initial stages. Under this circumstance, several distinguished geologists turned their talents to hydrogeology in efforts to identify the fundamental controls exercised by lithology and structure on flowing wells (Chamberlin, 1885; Darton, 1897).

Development of the Cambrian - Ordovician sandstone aquifer system in Illinois - Wisconsin can be traced to 1864 when a flowing well with a depth of 217 m (711.42 ft) was drilled in Chicago, Illinois (Konikow, 2013). In 1876, a flowing well with a depth of 293 m (961.29 ft) was drilled in Prairie du Chien, Wisconsin—only 1.6 km (1 mile) away from the Mississippi River. The initial flow rate was as high as 3,270 m^3/d (10,728^3 ft/d), thus this well was named the Greatest Artesian Fountain in America (Meiter, 2019). The photo of this great flowing well (the inset in Figure 14) was used as the frontispiece or cover image of several reports and books (Chamberlin, 1885; Deming, 2002; Freeze & Back, 1983).

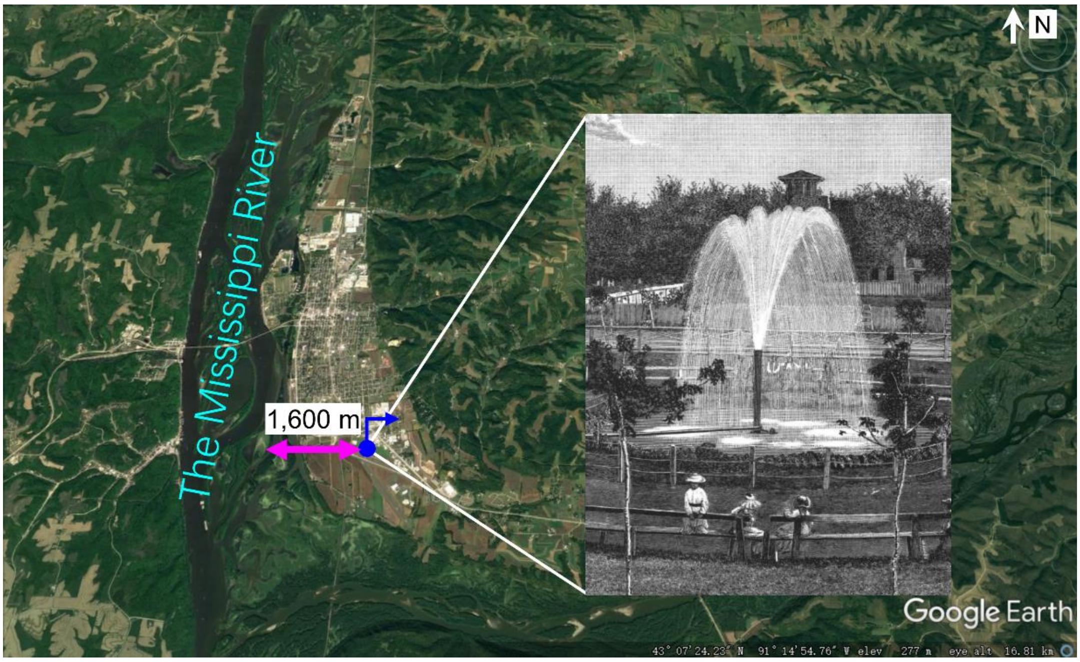
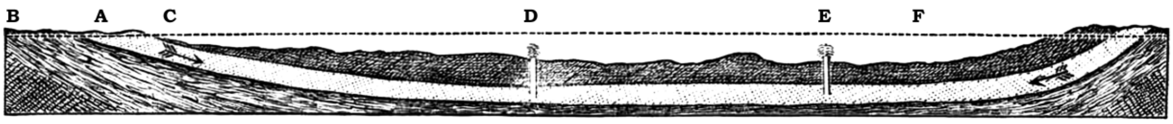


Figure 14 - The location of “the Greatest Artesian Well in America”, Prairie du Chien, Wisconsin. The inset is a photo of the artesian well (from https://www.wisconsinhistory.org/Records/Image/IM8832).

The occurrence of numerous flowing wells in Illinois and Wisconsin directly contributed to Chamberlin’s (1885) classic report on conditions of flowing wells. Chamberlin (1885) provided several cross sections showing the conditions of flowing wells in stratified aquifers (Figure 15), although he did not consider the head loss in the aquifer. As mentioned by Tolman (1937), Chamberlin’s cross sections were reproduced in every treatise on groundwater in the late 1800s and early 1900s, including Meinzer’s (1923) USGS report.

a)



Ideal section illustrating the chief requisite conditions of artesian wells. A, a porous stratum; B and C, impervious beds below and above A, acting as confining strata; F, the height of the water-level in the porous bed A, or, in other words, the height of the reservoir or fountain-head; D and E flowing wells springing from the porous water-filled bed A.

b)

Section illustrating the transition of a porous water-bearing bed, A, into a close-textured impervious one. Being inclo- -sed between the impervious beds B and C, it furnishes the conditions for an artesian fountain. D.

Figure 15 - Two cross sections from Chamberlin showing the conditions of flowing wells (reproduced from Chamberlin, 1885).

In the Great Plains of the USA, interest in groundwater emerged in the 1880s due to the irrigation demands caused by widespread drought. In South and North Dakota within the Great Plains, some 400 deep wells were drilled into the Dakota sandstone by 1896—over 350 of them were flowing wells (Darton, 1897). Figure 16 is a cross sectional model of the Dakota aquifer system drawn by Darton indicating that the Dakota sandstone is a confined aquifer. Measurements of hydraulic head in numerous wells tapping the Dakota sandstone aquifer clearly shows the hydraulic gradient (Figure 17). Therefore, the simple apparatus as shown in Figure 18 was widely used to illustrate the hydrogeological conditions of a confined aquifer (Darton, 1905; Slichter, 1902).

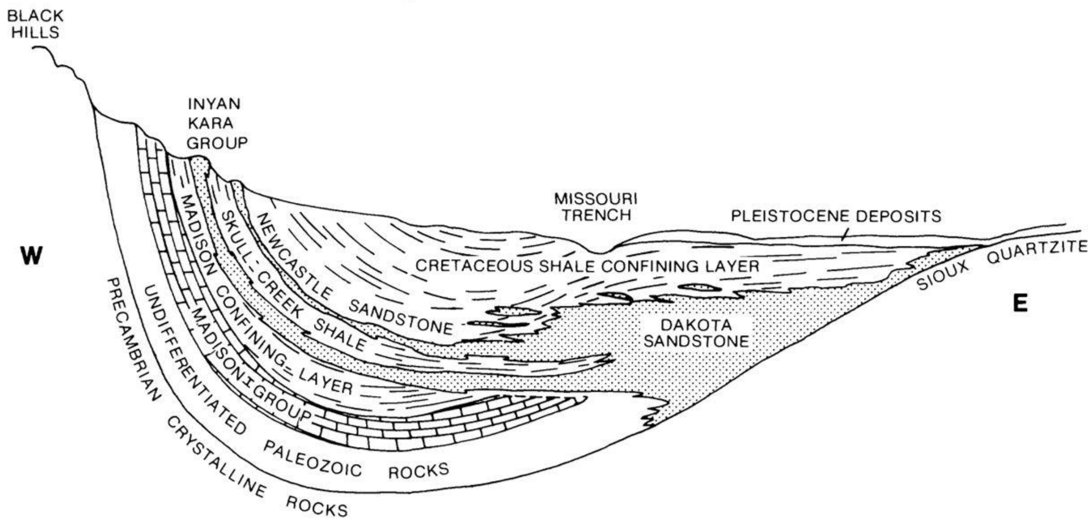


Figure 16 - West to east cross section showing the Dakota aquifer system (reproduced from Bredehoeft et al., 1983).

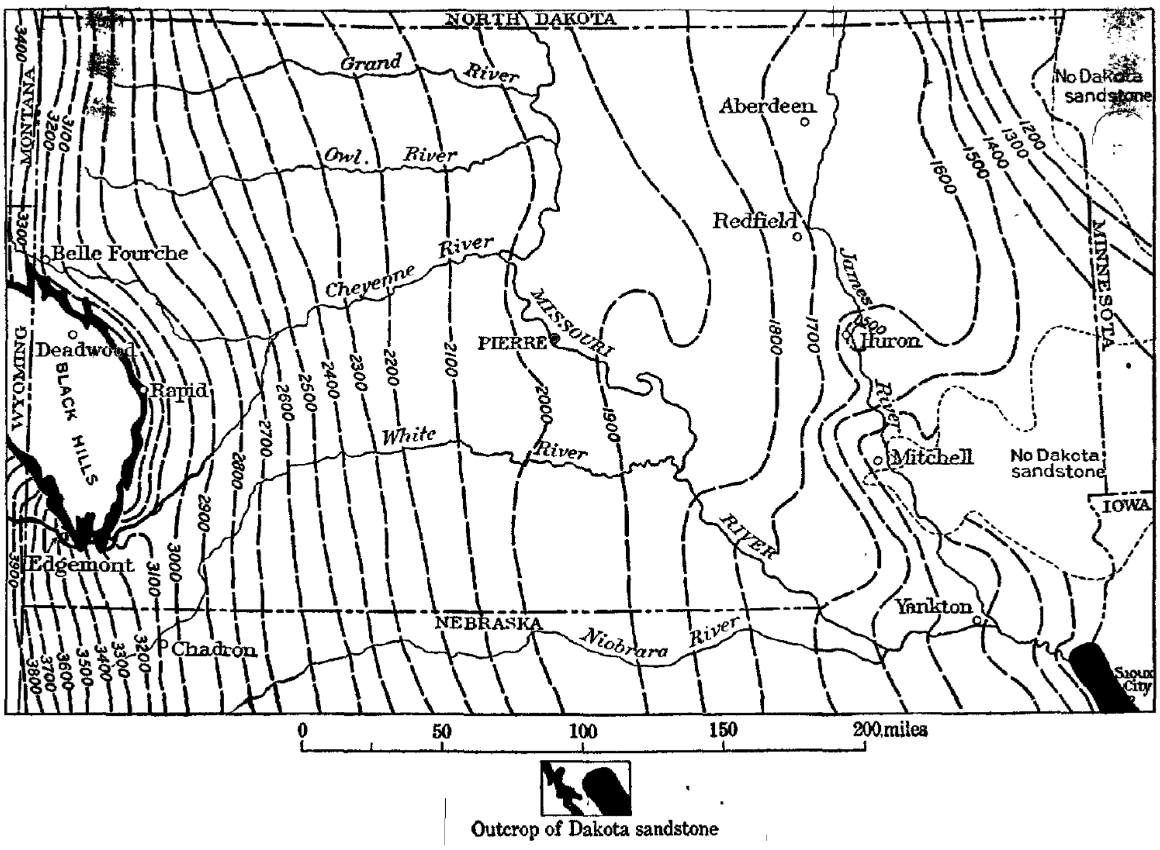


Figure 17 - Contours of hydraulic head showing the hydraulic gradient of the Dakota sandstone aquifer in South Dakota (reproduced from Darton, 1909; 50 miles = 80.47 km).

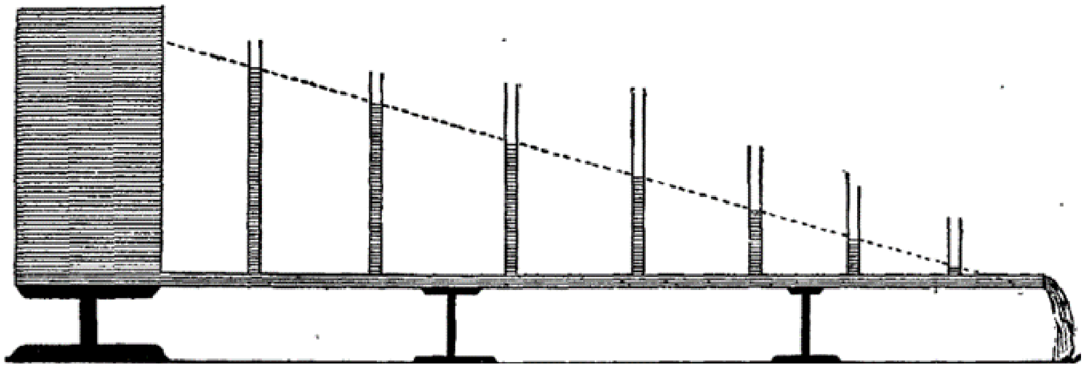


Figure 18 - An apparatus used to illustrate the flow pattern in a purely confined aquifer (reproduced from Darton, 1905). The left reservoir represents groundwater in the recharge area, which flows toward and discharges on the right side. The vertical pipes represent piezometers, and the dashed line represents the potentiometric surface with linearly decreasing hydraulic head.

Tolman (1937) used the Cambrian - Ordovician sandstone aquifer system in Illinois - Wisconsin and the Dakota sandstone aquifer in the Great Plains as two typical examples of artesian aquifers. He pointed out Chamberlin's limitation of neglecting head loss on the potentiometric surfaces and provided a modified cross section showing flowing artesian conditions of the Cambrian - Ordovician sandstone aquifer system (Figure 19). The upward leakage through the confining stratum in the discharge area is clearly shown in this figure—the first time that leakage through aquitards was illustrated.

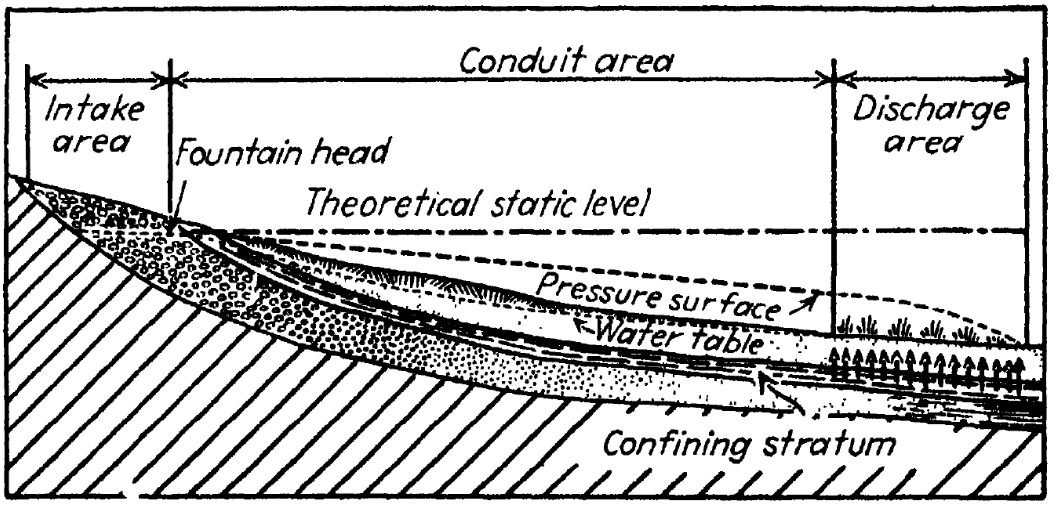


FIG. 133.—Artesian conditions in stratiform aquifer.

Figure 19 - A cross section showing flowing artesian conditions based on the conditions of the Cambrian - Ordovician sandstone aquifer system in Illinois - Wisconsin (reproduced from Tolman, 1937).

Due to the introduction of the jetting method of drilling around 1900, thousands of small-diameter wells were drilled to the Dakota sandstone during the following two decades. There were about 10,000 deep wells in South Dakota in 1915 and between 6,000 to 8,000 deep wells in North Dakota in 1923 (Meinzer & Hard, 1925). Increased withdrawal of deep groundwater caused many flowing wells to become non-flowing wells, and decreased the flow rates in the remaining flowing wells. Decreased production rates led to the birth of the concept of *compressibility* and the role compressibility plays in production of groundwater from confined aquifers (Meinzer, 1928; Meinzer & Hard, 1925).

In the flowing wells of the Dakota aquifer, it was noted that *"the pressure increases for several hours or even days after the flow is shut off, and when opened the flow decreases in the same way until the normal flow is reached"* (Meinzer, 1928, p. 277). Several decades later, two models were proposed to obtain hydraulic parameters by using decreasing discharge rate with time in flowing wells: 1) based on observations in the Grand Junction artesian basin in Colorado, Jacob and Lohman (1952) proposed a model for a constant-drawdown aquifer test in purely confined aquifers (Section 6.2); and 2) based on field observations in the Roswell artesian basin in New Mexico, Hantush (1959) proposed a model for a constant-drawdown aquifer test in semi-confined aquifers (Section 7.1).

4.4 Australia

The Great Artesian Basin covers one-fifth of the total area of Australia and is one of the largest and best-known groundwater basins in the world (Ordens et al., 2020). The first shallow flowing well was dug in 1878 to a depth of 43 m (141 ft) by using an auger near a spring in New South Wales, while the first deep machine-drilled flowing well was completed in 1887 at a depth of 393 m (1,289 ft) near Cunnamulla, Queensland (Williamson, 2013). By the end of the nineteenth century, there were already around 1,000 flowing wells on the continent (van der Gun, 2019). This exploitation of artesian water from flowing wells contributed to the emergence of hydrogeology as a discipline in Australia (Williamson, 2013), and the development of such sources has played a vital role in the pastoral industry in the arid and semi-arid regions of Australia (Habermehl, 2020).

Due to the occurrence of intervening aquifers and aquitards, the Great Artesian Basin is a multi-layered confined aquifer system (Figure 20), and flowing wells in this basin are geologically controlled. Although head drawdowns of up to 100 m (328 ft) have been recorded in highly developed areas, hydraulic heads in the Jurassic and Lower Cretaceous aquifers are still above ground surface throughout most parts of the basin (Habermehl, 2020). A comprehensive review of the history and recent research status of the basin is provided by Ordens and others (2020).

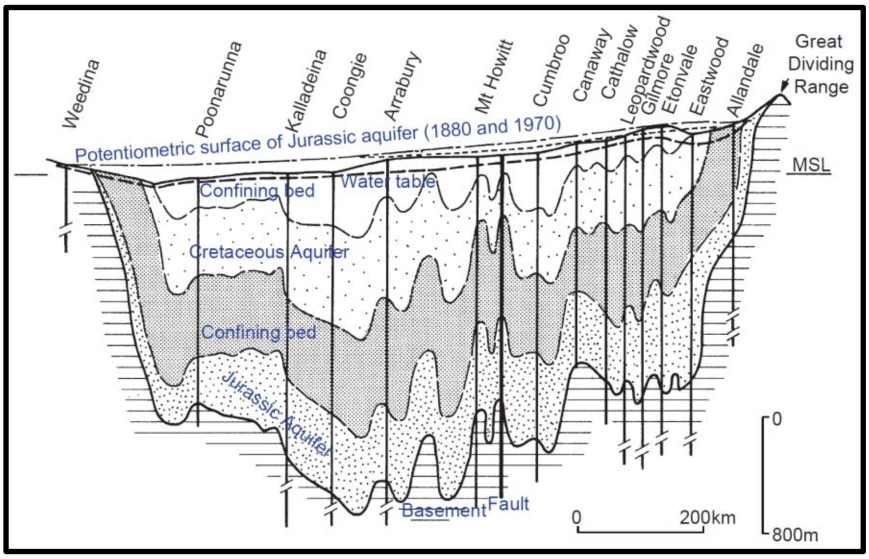


Figure 20 - Simplified cross section of the Great Artesian Basin, Australia, showing the major aquifers and confining units (modified from Habermehl 1980) [200 km ≅ 124 miles, 800 m ≅ 2,625 ft].

4.5 Canada

The Canadian Prairies—occupying the southern parts of the provinces of Alberta, Saskatchewan, and Manitoba—comprise a major region for grain production. The hydrogeology of the Canadian Prairies has been studied since the beginning of the twentieth century; greater demand for groundwater after World War II led to more detailed research. Groundwater in this region is largely obtained from surficial Pleistocene glacial drift and the underlying sandstone of Tertiary or Cretaceous age. Due to the large number of flowing wells, either in glacial drift or bedrock, great attention was paid to the relation between topography, geology, and areas with flowing wells (McKay et al., 1936; Meyboom, 1966; Tóth, 1966).

In the drift aquifer, a similarity between the water table and local topographic undulations in many parts of the Canadian Prairies—as well as decreasing head with depth in the recharge area and increasing head with depth in the discharge area—as indicated in Hubbert's (1940) classic sketch (Section 5.2) of regional groundwater flow were widely observed (Farvolden, 1961; Jones, 1962; Meyboom, 1962; Tóth, 1962). In the summer of 1962, Peter Meyboom and József Tóth conducted a hydrogeologic survey in the Trochu area in central Alberta, which is representative of the hydrogeology of the Canadian Prairies. This region included ten shallow flowing wells ranging in depth from 9 to 27 m (30 to 89 ft) in topographically low areas (Tóth, 1966). Applying intuitive thinking based on field observations in the Trochu area, Meyboom (1962) modified Hubbert's classic plot of the regional flow pattern by considering the permeability difference between the shallow and deep aquifers as well as the role of evaporation , which he termed the *Prairies Profile* (Figure 21). The distributions of areas of shallow and deep flowing wells were explicitly labeled.

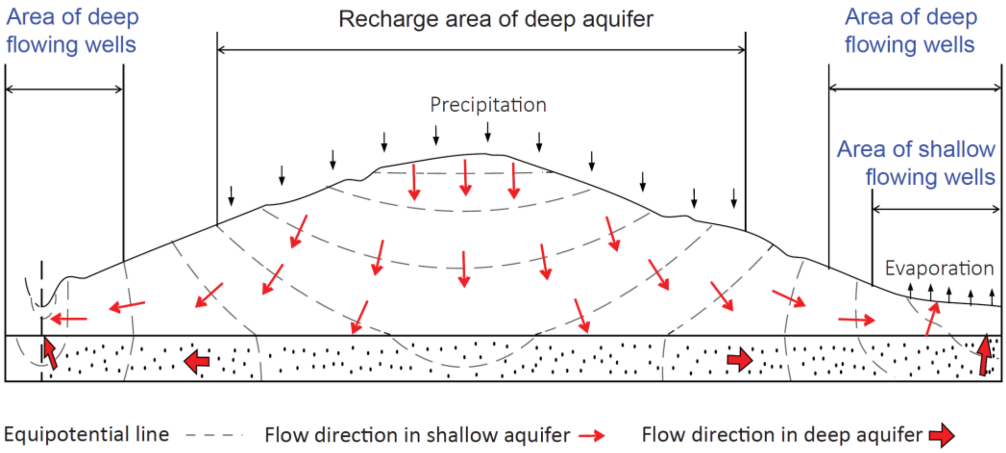


Figure 21 - The Prairies Profile showing topographically driven flow systems in a heterogeneous basin with a higher permeability layer in the bottom (modified from Meyboom, 1962). The solid line represents the topography.

Based on observations of flowing wells in the study area, Tóth (1966) realized that these flowing wells are topographically controlled flowing wells and are typical manifestations of groundwater discharge. Moreover, Tóth (1962) thought that the discharge area should not be limited to the river; he developed an analytical model to quantify the pattern of groundwater flow from water table highs to lows in a homogeneous basin, which led to the theory of *regional groundwater flow*. In order to combine Tóth's flow pattern in homogeneous basins and Meyboom's flow pattern in a heterogeneous basin, Tóth's mathematical model was further developed by Alan Freeze (Freeze & Witherspoon, 1967). The details of the theory of regional groundwater flow are discussed in other GW-Project books.

4.6 China

As early as in the eleventh century, deep drilling using bamboo pipes was employed in Sichuan Province, China, to reach brines from 100 m (328 ft) deep boreholes (Vogel, 1993). Due to the success of developing flowing wells for brines in the seventeenth century, *Ziliujing* [flowing well] became the name of a town in Sichuan Province. In 1835, a 1,001 m (3,284 ft) deep flowing well at Shenhai—frequently mistaken as Xinhai in the English literature—was constructed for producing brines and gases (Vogel, 1993).

After the foundation of the People's Republic of China in 1949, many deep wells were drilled to produce groundwater for agriculture. In the late 1950s, the success of drilling flowing wells led to a campaign to find more flowing wells in many basins of the country. Unfortunately, due to overexploitation, flowing wells have disappeared in many regions, including the North China Plain.

One of the most intensively studied groundwater basins is the Ordos Plateau in northwestern China, composed mainly of a thick sandstone aquifer of Cretaceous age that is up to 1,000 m (3,280 ft) in thickness (Figure 22). Because the overlying thin Quaternary deposits have much higher permeability, and because there are no continuous aquitards within the Cretaceous sandstone, the Ordos Plateau can be conceptualized as a typical thick unconfined aquifer (Hou et al., 2008; Jiang et al., 2018). The suitable hydrogeological conditions controlled by undulating topography, recharge from precipitation, and hydraulic conductivity lead to the occurrence of topography-driven hierarchically nested flow systems (Jiang et al., 2014). As early as the 1950s, several deep boreholes drilled into the Cretaceous sandstone in topographic lows became flowing wells; subsequently, numerous flowing wells were drilled for agricultural purposes. Because there is no continuous aquitard in the Ordos Plateau, these flowing wells are topographically controlled.

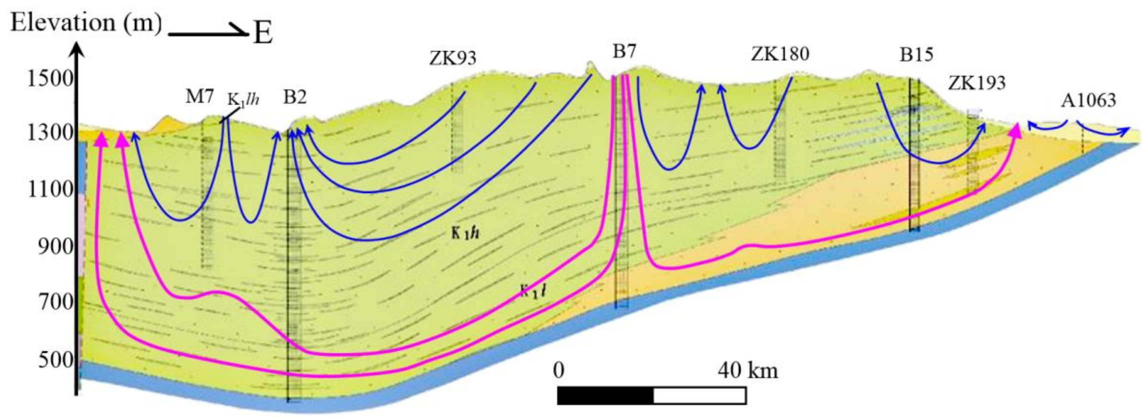


Figure 22 - A geological cross section in the Ordos Plateau, China, showing topographically controlled groundwater flow in the thick Cretaceous sandstone aquifer with no continuous aquitards (modified from Hou et al., 2008) [conversions: 40 km = 25 miles; 500 m = 1,640 ft; 1,500 m = 4,921 ft].

The Wudu Lake catchment is one of the numerous small catchments in the Ordos Plateau, covering an area of approximately 200 km^2 (124.3 $miles^2$). In 2015, there were 15 flowing wells in this catchment (Wang et al., 2015). In a recent study, 13 flowing wells were sampled to examine the hydrochemical evolution and behavior of lithium and its isotopes (Ji et al., 2022). As shown in Figure 23, all these flowing wells are located in topographic lows near the lake. Although most flowing wells drilled into the Cretaceous sandstone are cased only in the very shallow part that corresponds to the Quaternary deposits, groundwater samples from these flowing wells with long-screens are representative of deep groundwater that has undergone long travel times and travel distances (Wang et al., 2015; Zhang et al., 2018; Ji et al., 2022). The characteristics of the hydrochemistry of these long-screen wells inspired one of the authors of this book (Xiao-Wei Jiang) to study the hydraulics of long-screen flowing wells in a macroscopically homogeneous basin (Section 8.2).

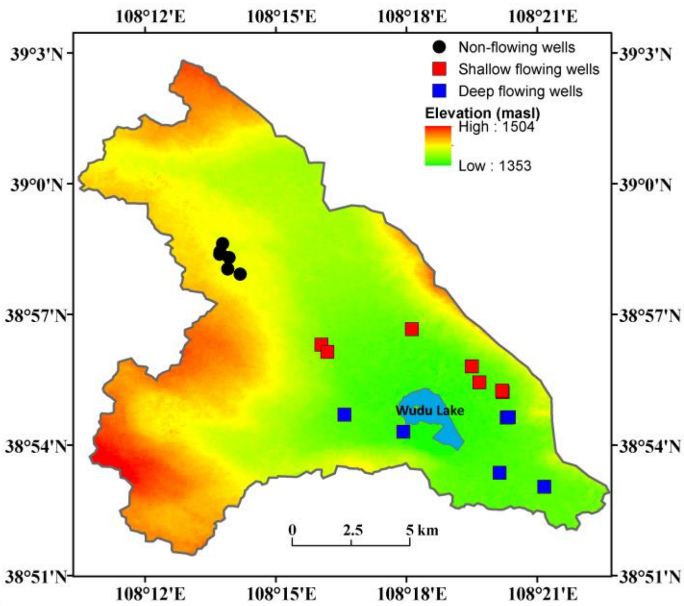


Figure 23 - Locations of non-flowing wells in topographic highs and flowing wells in topographic lows in the Wudu Lake catchment, Ordos Plateau, China (modified from Ji et al., 2022) [1,353 masl = 4,438 ft above sea level (fasl); 1,504 masl = 4,934 fasl].

4.7 Summary and Implications

As shown in the geological cross sections given in Section 4.1 to Section 4.4, the confined aquifers are all overlain by aquicludes or aquitards. As a result of the widespread occurrence of regional-scale confined aquifers in sedimentary basins with flowing wells in France, USA, Australia, and Canada, the understanding of confined flow bounded by two confining beds became a broadly useful conceptualization. The pattern of confined flow is introduced in Section 5.1. The interpretation of steady-state or transient discharge rate of flowing wells is introduced in Section 6.

As shown in the geological cross section given in Section 4.6, there is no continuous aquitard in the Ordos Plateau in China. The numerous flowing wells in topographic lows provided a unique opportunity to confirm topographically controlled flowing wells and led to a rethinking of the hydraulics of flowing wells. The pattern of groundwater flow in thick unconfined aquifers is introduced in Section 5.2 and interpretation of vertical profiles of steady-state discharge rate is introduced in Section 8.2.

Historically, many well-known geologically controlled flowing wells were very close to large rivers in the topographic lows of a basin. However, until it was realized that aquitards can significantly transmit groundwater vertically, hydrogeologists did not realize that geologically controlled flowing wells were also closely related to upward groundwater discharge to the large rivers. The pattern of cross-formational flow is introduced in Section 5.3. Interpretation of discharge rate without considering the background flow field is introduced in Section 7, whereas interpretation of discharge rate by considering the background flow field is introduced in Section 8.1.

5 Groundwater Flow Patterns and Conditions for Flowing Wells

5.1 In Confined Aquifers

In the evolution of thinking about the source of water in flowing wells, by the early nineteenth century, it was universally accepted that groundwater came from rainfall and found its way through the pores or fractures of a permeable stratum sandwiched between two water-tight strata at depth (Garnier, 1822). The conditions of flowing wells were considered to be approximately the same by several hydrogeologists in the nineteenth century (Bond, 1865; Chamberlin, 1885; Héricart de Thury, 1830). In a classic report, *The Requisite and Qualifying Conditions of Artesian Wells*, Chamberlain (1885) described seven conditions he deemed necessary for the occurrence of a flowing well:

1. a pervious stratum to permit the entrance and the passage of the water;
2. a water-tight bed below to prevent the escape of the water downward;
3. a similar impervious bed above to prevent escape upward, for the water, being under pressure from the fountain-head, would otherwise find relief in that direction;
4. an inclination of these beds, so that the edge at which the water enters will be higher than the surface at the well;
5. a suitable exposure of the edge of the porous stratum, so that it may take in a sufficient supply of water;
6. an adequate rainfall to furnish this supply; and,
7. an absence of any escape for the water at a lower level than the surface at the well.

The conditions of flowing wells in many classic confined aquifers in France, UK, USA, and Australia illustrated in Sections 4.1 to 4.4 strengthened the conceptual model of piston flow in confined aquifers (Figure 24).

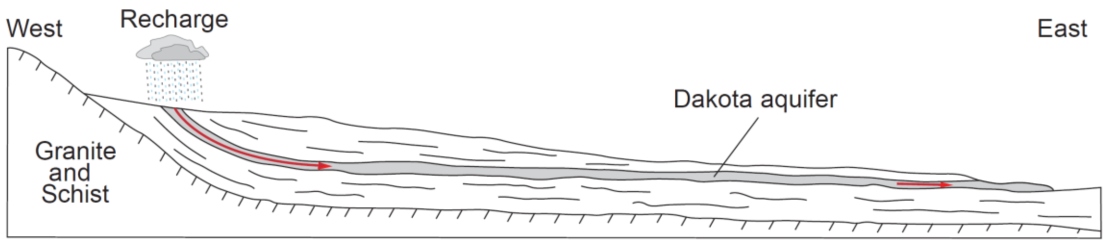


Figure 24 - The profile of the Dakota, USA, confined aquifer and its confined flow bounded by aquitards (modified from Fetter, 2001). The red lines with arrows represent the direction of confined flow.

Figure 2 in Section 1 showed a commonly used diagram depicting groundwater recharge and flow direction in a confined aquifer bounded by two confining beds and a flowing well tapping the confined aquifer. Here, we reproduce two earlier versions of similar plots: one from Tolman (1937; Figure 25) and the other from Todd (1959; Figure 26).

Many subsequent textbooks reproduced or slightly modified Todd's schematic plot to show aquifers and flowing wells.

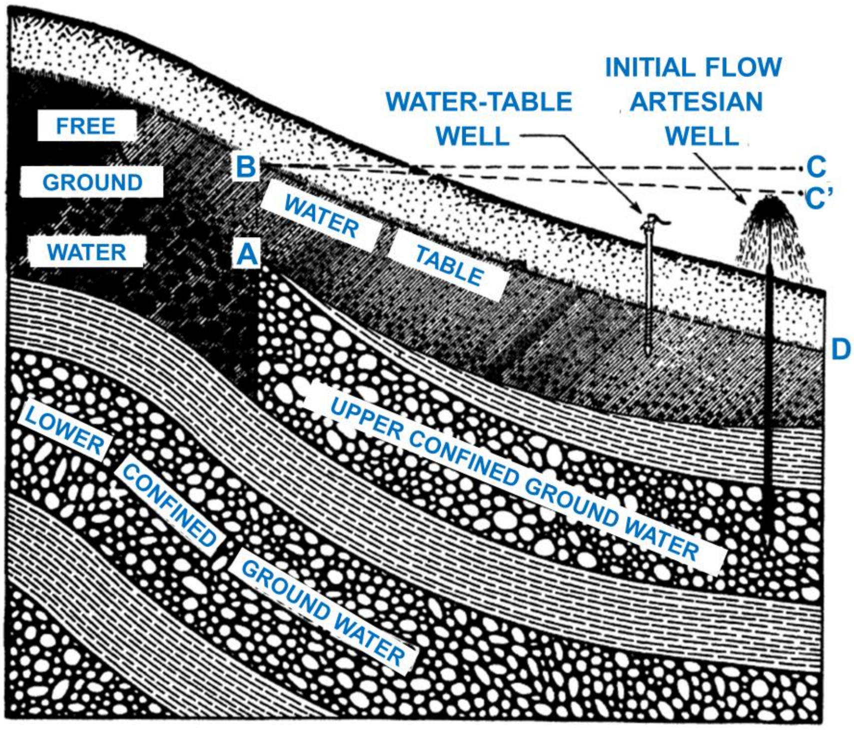


Figure 25 - A schematic showing free groundwater, confined groundwater and a flowing well (reproduced from Tolman, 1937). The label "A" indicates the upper edge of the upper confined aquifer; BC and BC' are the theoretical and actual static levels of the upper confined aquifer; BD is the water table.

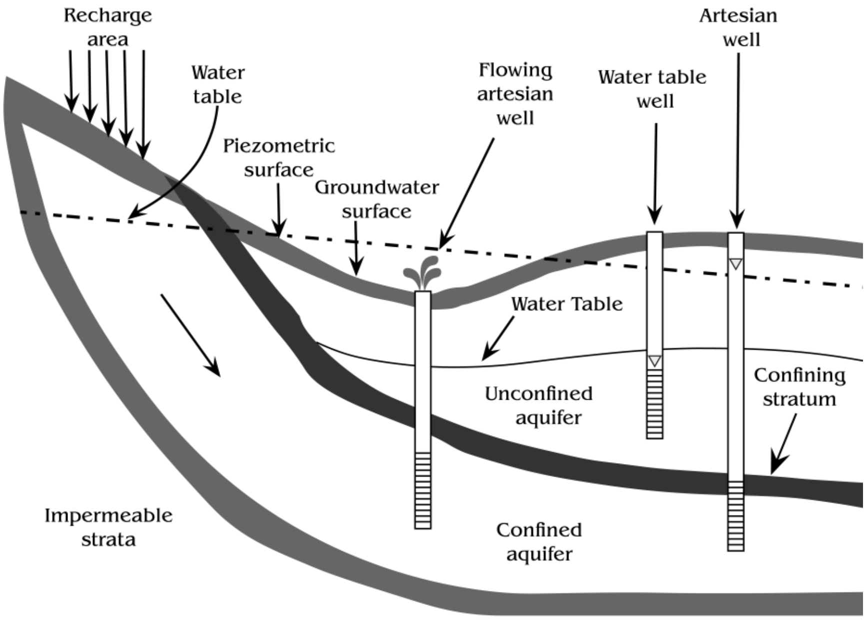


Figure 26 - A schematic showing unconfined and confined aquifers, with a flowing well and an artesian well tapping the confined aquifer (modified from Todd, 1959).

5.2 In Thick Unconfined Aquifers

Based on the principle of the conservation of mass and the laws of thermodynamics, Hubbert (1940) defined the potential of subsurface water and obtained graphical solutions of regional groundwater flow in a homogeneous and isotropic aquifer with a symmetrical topography between two streams (Figure 27). The point with the highest elevation of topography corresponds to the divide. A component of downward flow away from the water table occurs in topographic highs and a component of upward flow toward the water table occurs in topographic lows.

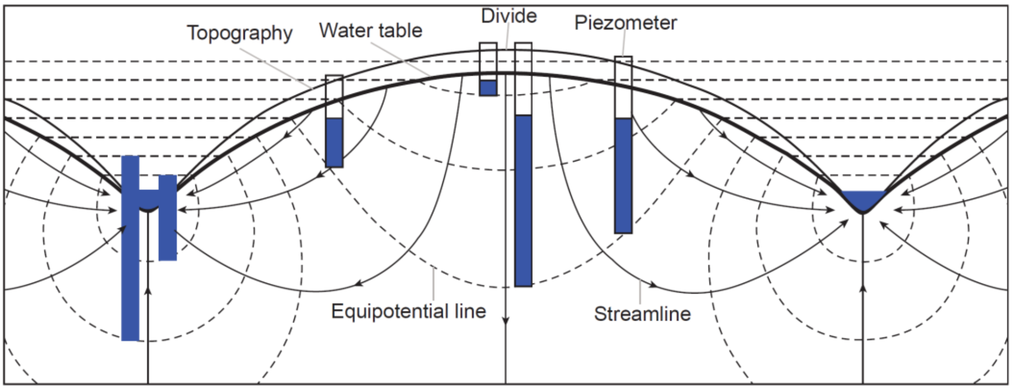


Figure 27 - The flow net of groundwater flow between two streams obtained by Hubbert (1940) (modified from Fetter, 1994). The blue bars represent pressure head in selected piezometers.

In the topographic lows near the streams, hydraulic head increases with depth and could be higher than the land surface. Therefore, even if there is no confining bed either above or below the aquifer, vertical boreholes in topographic lows could be flowing wells, implying that the second and third conditions given by Chamberlin (1885) are not necessary conditions for flowing wells. Such flowing wells in unconfined aquifers belong to the category of topographically controlled flowing wells as defined by Freeze and Cherry (1979).

Hubbert (1940) and Freeze and Cherry (1979) pointed out that conceptual models such as the Dakota sandstone configuration have been overused in the literature as a model of the regional groundwater recharge process, and this overuse obscured a more complete understanding of flowing wells. Hubbert also pointed out, "*If it ever becomes practicable to drill inclined or horizontal wells, artesian water can be obtained at many places where it cannot be obtained from vertical drill holes*" (1940, p. 915). In fact, the karez↗ well system widely used in Central Asia can be considered as a horizontal flowing well.

Tóth (1962, 1963) proposed a mathematical model to quantify topography-driven groundwater flow in homogeneous basins; this was considered to be a paradigm shift in hydrogeology (Anderson, 2008; Bredehoeft, 2018; Mádl-Szőnyi, 2008). In Tóth's (1962, 1963) model, the top of the basin has a prescribed-head boundary condition to characterize the undulating water table, and all other boundaries have a no-flow boundary condition. Exercise 1 ↴ gives the analytical solution for a flow pattern induced by the undulating water table and the zone with flowing wells. A Groundwater Project author, Paul Hsieh, created an online interactive tool for experimenting with topographically driven flow↗ in heterogeneous basins of various aspect ratio as discussed in Poeter and Hsieh (2020).

5.3 In Semi-confined Aquifers

As was shown in Figure 19, cross-formational flow through aquitards in the discharge area had been illustrated by Tolman (1937). In fact, cross-formational flow is not restricted to the discharge area. Freeze and Witherspoon (1967) extended Tóth's homogeneous basin model into layered heterogeneous basins and demonstrated numerically that buried aquifers need not outcrop to produce flowing artesian conditions (Figure 28); that is, the water level can rise above the ground surface. Therefore, the fourth and fifth conditions given by Chamberlin (1885) are not necessary conditions for flowing wells.

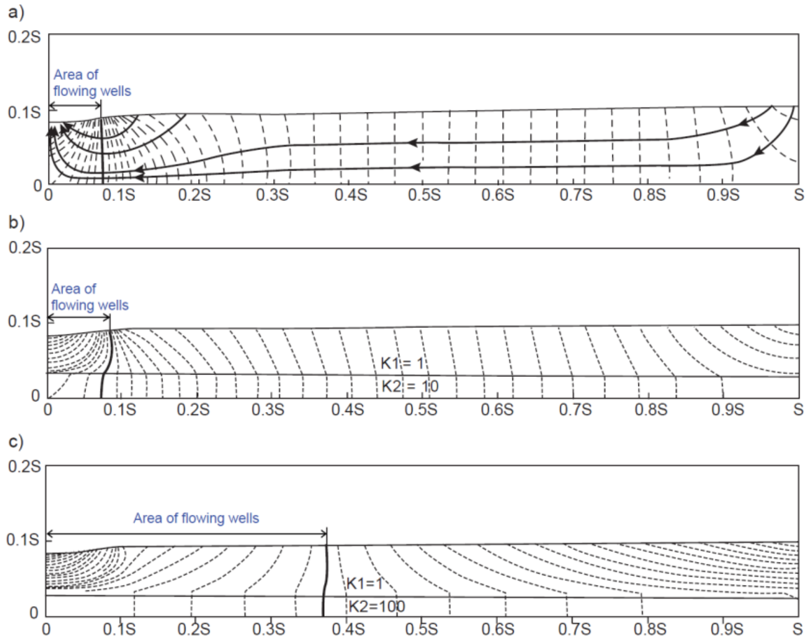


Figure 28 - a) Distributions of equipotential lines, streamlines, and area with flowing wells in a homogeneous basin. b) and c) The enlargement of area of flowing wells in two-layer basins with different K_2 (modified from Freeze & Witherspoon, 1967). As K_2 increases, the area of flowing wells expands significantly.

Freeze and Witherspoon (1967) also found that the zone with flowing wells increases with the increasing hydraulic conductivity of the confined aquifer. However, a limitation of their model is that the water table is prescribed, not determined by hydraulic conductivity and groundwater recharge/discharge rates. Moreover, the degree of confinement is small (with $K_2/K_1 = 10$ and 100) due to limited computational abilities at that time.

To quantify the simultaneous control of topography, aquifers/aquitards, and groundwater recharge/discharge on geologically controlled flowing wells, Zhang and others (2022) extended Tóth's unit basin model by incorporating an aquitard between two aquifers and assigning a hybrid recharge/discharge boundary at the top of the basin (Figure 29). Specifically, a prescribed head is assigned at the river and a prescribed recharge rate is assigned along the top of the basin from the valley to the divide; when the water table exceeds the elevation of the land surface as defined by Equation (2), the top boundary condition changes to a drain boundary and seepage occurs.

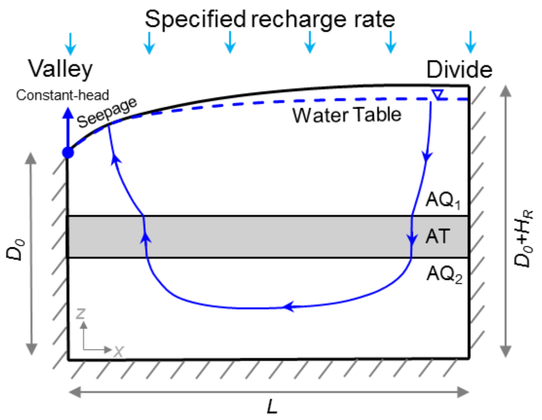


Figure 29 - Conceptual model of a basin with three stratigraphic layers and a river valley. The basin has a constant head boundary at the river, a recharge boundary condition along the top coupled with a head-dependent flux boundary condition that allows outflow when the aquifer head is above the land surface elevation, and no-flow boundary conditions at the other three boundaries. The basin is homogeneous when the aquitard AT has the same hydraulic conductivity as the overlying and underlying aquifers (AQ_1 and AQ_2). The elevation of the topography follows Equation (2).

$$z = D_0 + H_R \cos\left[\frac{\pi}{2}\left(1 - \frac{x}{L}\right)\right] \tag{2}$$

where:

D_0 = ground surface elevation at the river valley (L)

H_R = amplitude of the regional undulation of the topography (L)

L = distance from the valley to the basin divide (L)

In this way, the water table equals the land surface in topographic lows near the river valley and is a subdued replica of the land surface in other parts of the basin. Both aquifers have a hydraulic conductivity K of 1 m/d (3.28 ft/d). In the base case, the hydraulic conductivity of the aquitard K' equals that of the aquifers. Five scenarios of aquifer-aquitard conductivity contrast—with conductivity ratio K/K' equaling 10, 10^2, 10^3, 10^4, and 10^6—are considered to examine the role of aquitards in the occurrence of flowing wells.

Figure 30 shows the basinal flow field from the divide to the river valley for six hydraulic conductivity ratios. By comparing whether the hydraulic head at a point is higher or lower than the elevation of water table, the basin can be divided into recharge and discharge limbs, which are separated by the pink dotted lines. The recharge limb has a component of downward flow, whereas the discharge limb has a component of upward flow. By comparing whether the hydraulic head at a point is higher than the elevation of the overlying land surface, the zone with flowing wells can be obtained. In each case, the zone with flowing wells is located inside the discharge limb.

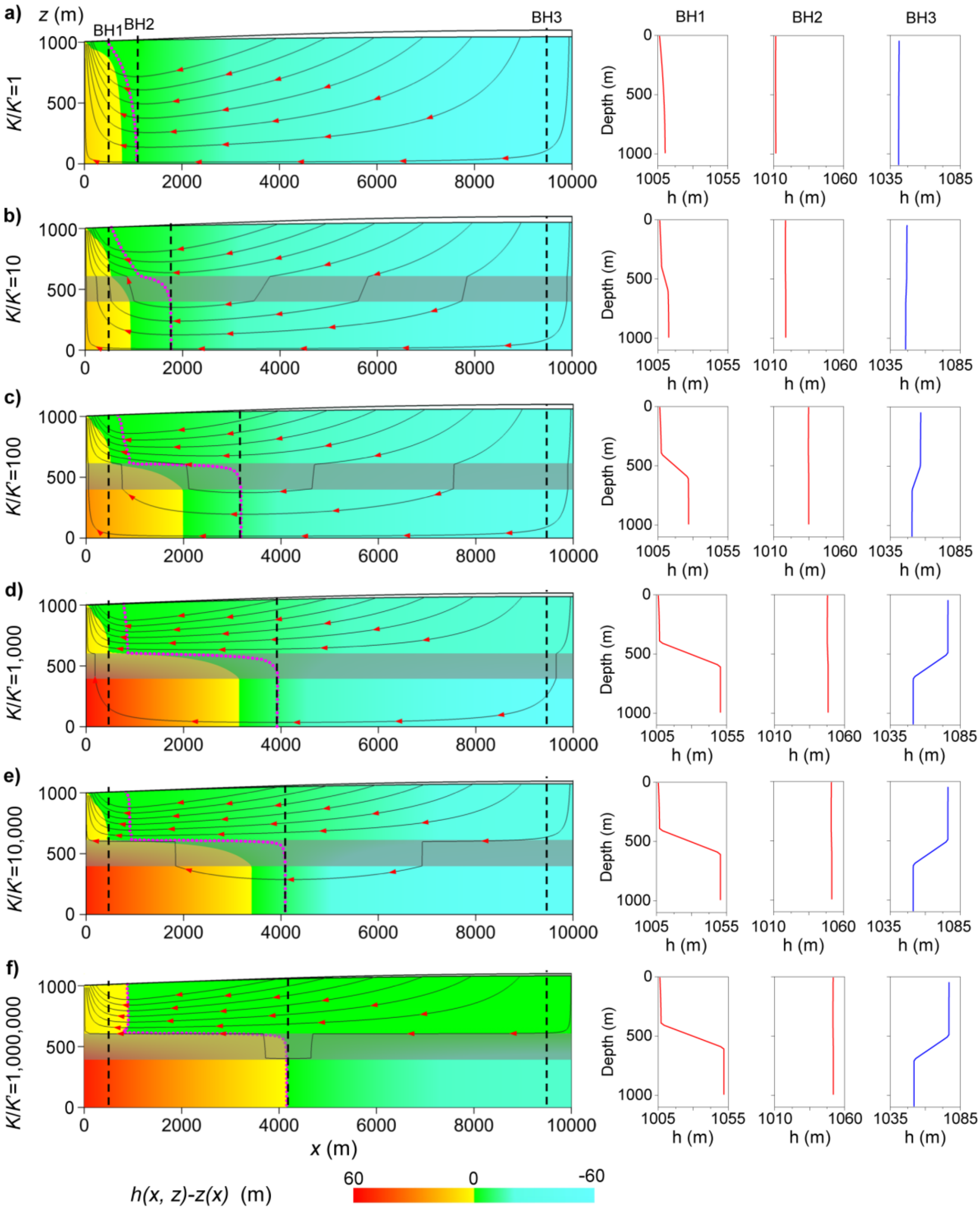


Figure 30 - The distributions of flow paths and zones where head exceeds land surface in basin cross sections along with vertical profiles of hydraulic head in three hypothetical boreholes under six aquifer-aquitard hydraulic conductivity contrasts (*K*/*K*').

- The color coding indicates the magnitude of difference between head in the system and the land surface elevation at the same x location. Wells screened in zones with color in the yellow to red range will be flowing wells, with those in red zones having larger discharge rates.
- The vertical, black, dashed lines in each cross section represent the location of the three hypothetical boreholes.
- The pink dotted line separates the basin into recharge and discharge limbs (i.e., the vertical component of flow is downward in the recharge limb and is upward in the discharge limb).
- The location of BH1 and BH3 are the same in each case, while BH2 is located where the flow field at the bottom of the basin changes from the recharge to the discharge limb.

Cases (a) to (d) are modified from Zhang and others (2022), whereas cases (e) and (f) are modeled for this book. The vertical profiles of head have not been previously published.

In either homogeneous or heterogeneous basins, the zone with flowing wells is located within the discharge limb. This implies that groundwater discharge toward sinks—which is a component of the basinal groundwater flow driven by topography—plays a critical role in geologically controlled flowing wells. An aquitard with lower permeability enlarges the size of the discharge limb as well as the zone with flowing wells, implying that flowing wells are more likely to occur in basins with continuous aquitards. This finding explains why it was long thought that an overlying aquitard is a necessary condition for flowing wells. However, at a very low aquitard hydraulic conductivity, groundwater in the semi-confined aquifer has an extremely low renewability, thus the discharge of the flowing well is not sustainable.

Figure 30 also shows the profiles of hydraulic head in three hypothetical boreholes (BH1 in the discharge limb, BH2 penetrates the boundary of recharge and discharge limbs in the lower aquifer, and BH3 in the recharge limb) for each scenario of K/K'. In the homogeneous case, hydraulic head in BH3 gradually decreases with depth in the recharge limb, and in BH1 gradually increases with depth in the discharge limb.

Because borehole BH2 in each case penetrates the recharge limb of the upper aquifer, there is a trend of decreasing hydraulic head with depth in the upper aquifer. As conductivity ratio K/K' increases (K' decreases), the head difference between the top and the bottom of the aquitard increases, and the depth-increasing trends in the discharge limb and depth-decreasing trends in the recharge limb become less significant. When conductivity ratio K/K' reaches 100, the hydraulic head in the lower aquifer hardly changes with depth. This is consistent with the widespread assumption that groundwater flow in confined aquifers is essentially horizontal.

Based on the works of Hubbert (1940) and Tóth (1962), the pattern of groundwater flow from recharge to discharge areas of a simple basin was well understood. Moreover, Freeze and Witherspoon (1967) showed that various forms of heterogeneity do not change the general flow pattern from recharge to discharge areas. With this background, Heath (1983) produced a plot to show the basinal flow pattern, the role of aquitards, and the time scale of groundwater circulation in a five-layer basin in a USGS water supply paper (Figure 31). This figure has been widely reproduced or slightly modified to show the pattern of groundwater flow from recharge to discharge areas, including the review paper by Tóth (1995) and the classic USGS water supply paper by Winter and others (1998). Because the two influential papers (Tóth, 1995; Winter and others, 1998) did not cite Heath (1983), many people did not know its origin.

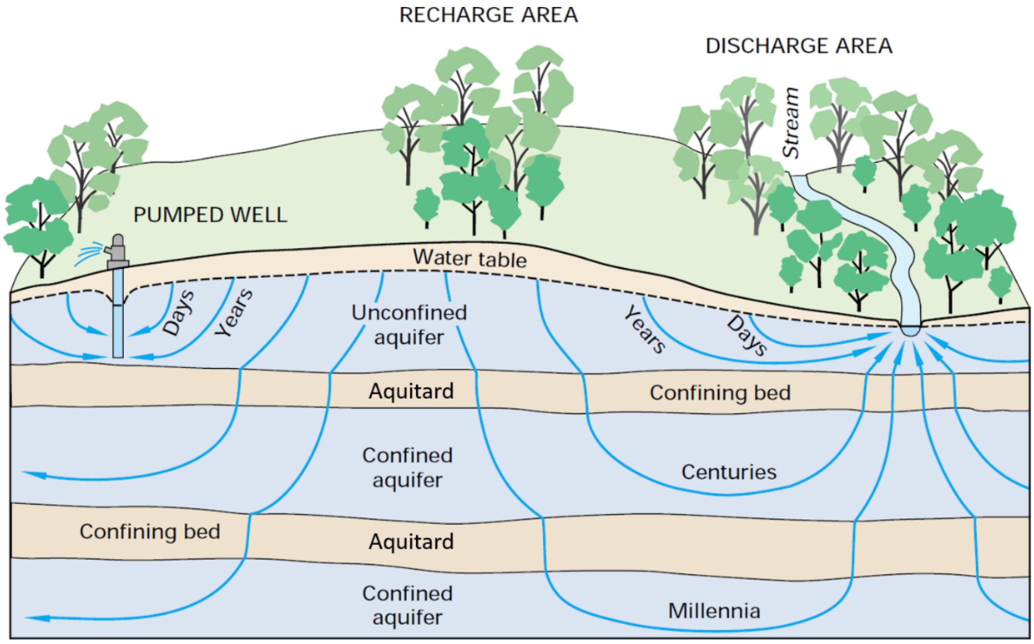


Figure 31 - The pattern and time scales of groundwater flow from recharge to discharge areas in a five-layer basin described by Heath (1983; modified from Winter et al., 1998).

Following the basin configuration shown by Heath (1983), we set the length of the half-basin to be 10 km (6.2 miles), the depth of the basin to be 500 m (1,650 ft) at the valley, and the thicknesses of the two aquitards to be 10 m (32.81 ft). The hydraulic conductivity of the aquifer (K) is set to be 1 m/d (3.28 ft/d). Figure 32 shows the time scales of the four flowlines under three scenarios of K/K′. The increasing time scales from shallow to deep flow paths are consistent with those shown in Heath (1983). As the hydraulic conductivity of the aquitard decreases, groundwater requires more time to flow through the aquitards, thus the longer time scale for each flow path from recharge to discharge area.

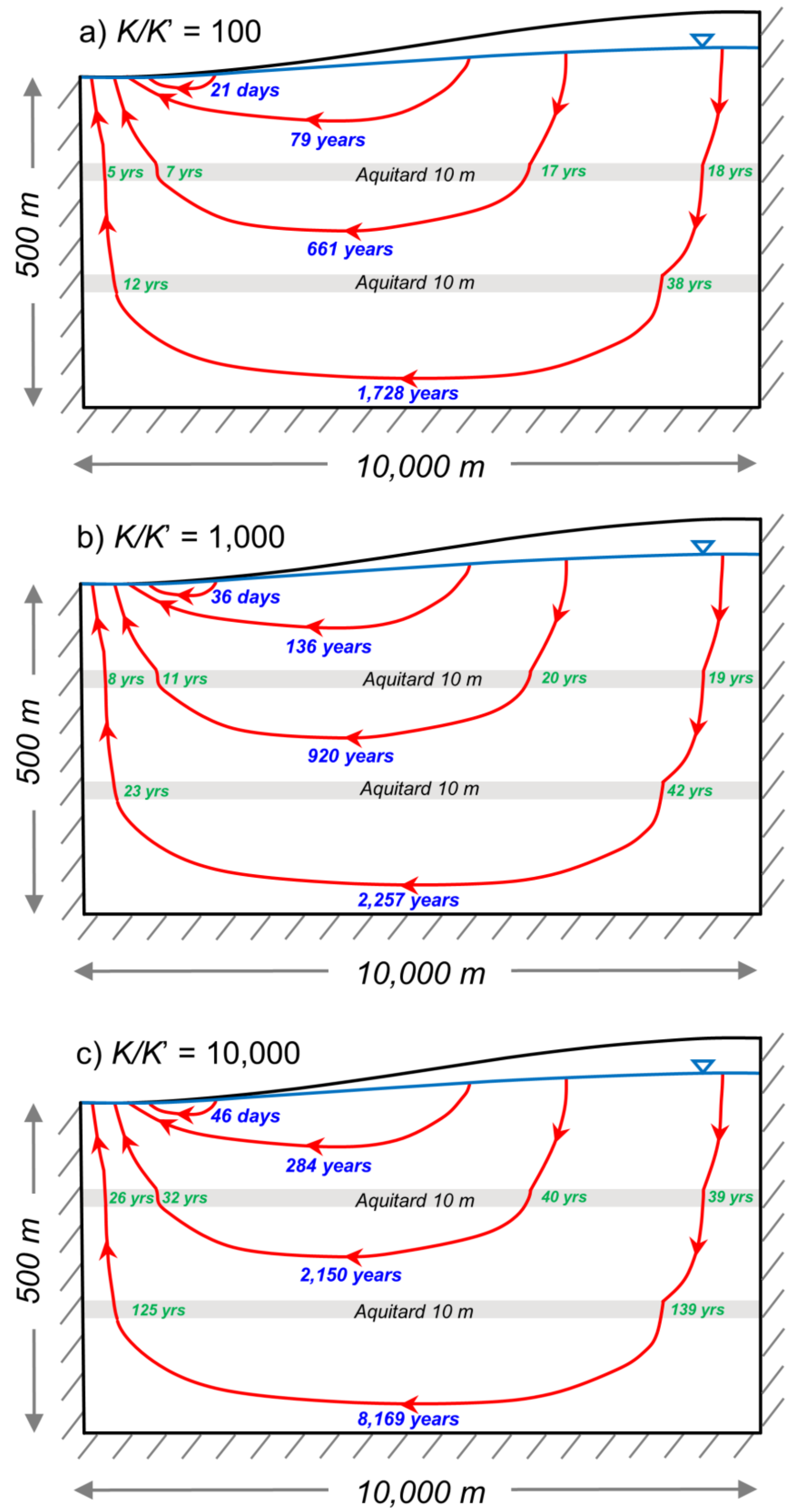


Figure 32 - The time scales of groundwater circulation from recharge to discharge areas along different flow paths (numbers in blue) under three different, aquifer-aquitard hydraulic conductivity contrasts (K/K'). The green numbers indicate the time required for the flow line to reach the other side of the aquitard [conversions: 10 m = 32.81 ft; 500 m = 1,650 ft; 10,000 m = 32,800 ft].

6 Hydraulics of Flowing Wells in a Confined Aquifer with a Flat Initial Potentiometric Surface

6.1 Steady-State Discharge Rate of Flowing Wells in Confined Aquifers

Flowing wells were important sources of water supply in France since the early nineteenth century; this led to discharge rates being measured at different elevations of discharge orifices (Figure 33a) in many flowing wells in the 1840s. Such experiments can be regarded as constant-head tests at multiple drawdowns in a single well. As shown in Figure 33b, the discharge rates increased linearly as the elevation of the discharge orifice decreased in both September and November 1847.

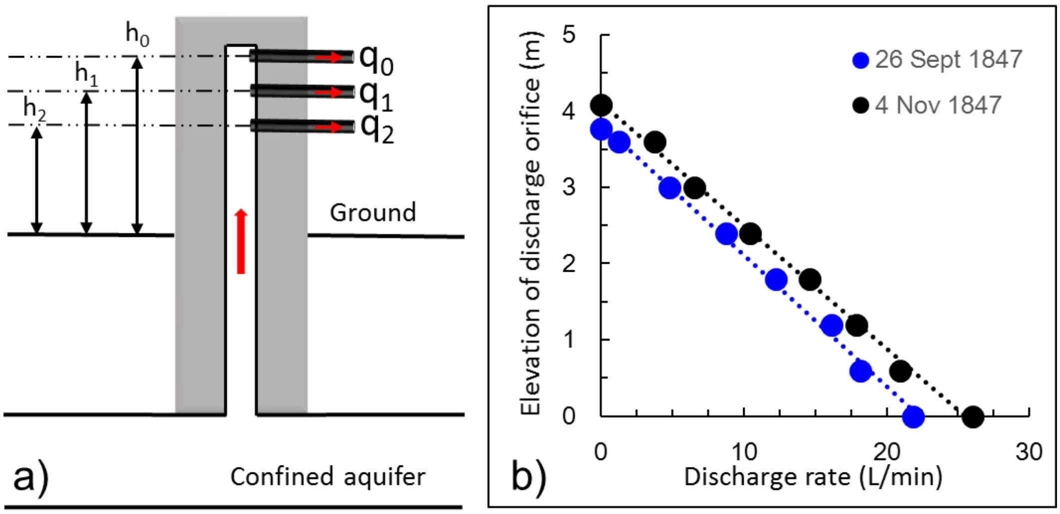


Figure 33 - a) A schematic of a device with three discharge orifices for measuring discharge rate (only one discharge orifice is open during measurement). h_0, h_1, and h_2 are elevations of the three discharge orifices, while q_0, q_1 and q_2 are discharge rates at each orifice; b) Changes in discharge rate with elevation of discharge orifice of a flowing well above the ground surface (data from Darcy, 1856).

According to the Chézy equation that describes head loss at high flow velocities in pipes (Section 3.1), head loss in the well pipe should be proportional to the square of flow rate. Based on the linear trend between head loss and flow rate shown in Figure 33b, it was inferred that head loss is dominated by groundwater flow in the aquifer with low velocities. To confirm the linear trend between head loss and flow rate observed in flowing wells, Darcy designed his famous sand column experiments. The results of the sand column experiments were reported in Appendix D of Darcy (1856). The publication of Darcy's law—which shows a linear relationship between flow rate and hydraulic gradient in porous media—represents the beginning of groundwater hydrology as a quantitative science (Freeze & Back, 1983).

Probably because only Appendix D of Darcy (1856) was translated, Freeze (1994) wrote, *"I'm sure that Darcy went to his grave without the realization that he had unlocked one of nature's basic laws"* (p. 30). In fact, Darcy was familiar with groundwater circulation from topographic highs to lows (Section 4.1), and he was also familiar with head loss during water movement in pipes (Brown, 2002; Ritzi & Bobeck, 2008). Therefore, Darcy knew that he had unlocked one of nature's basic laws.

In 1850, Dupuit succeeded Darcy as Chief Director for Water and Pavements in Paris and became familiar with Darcy's work. The field data shown in Figure 33b triggered Dupuit to quantify the coefficient relating well discharge rate to head loss. Dupuit (1863) realized that groundwater flow would radially converge to the flowing well, and head loss in the confined aquifer would form a cone of depression around the flowing well (Figure 34a).

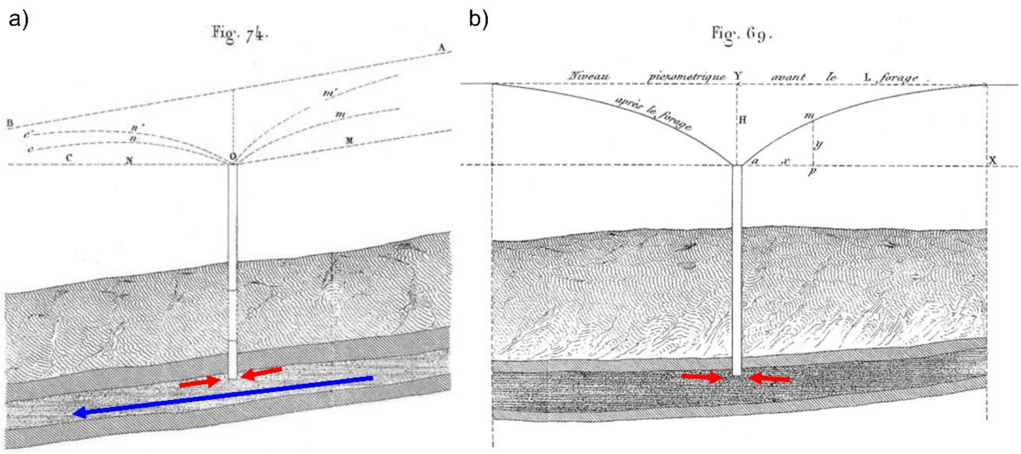


Figure 34 - Diagrams from Dupuit (1863) showing radial flow toward a flowing well penetrating a confined aquifer, which caused head loss with a cone of depression in the potentiometric level (*niveau piezometrique*; transl.: new piezometric). The regional hydraulic gradient is considered in a) and neglected in b). The red arrows in (a) and (b) are added to show the direction of radial flow. The blue arrow in (a) is added to show the direction of regional flow.

By assuming a flat initial potentiometric surface (Figure 34b) based on Darcy's law, the relationship between discharge rate and hydraulic gradient can be written as shown by Equation (3).

$$Q_s = K(2\pi r B)\frac{dH}{dr} = 2\pi T r\frac{dH}{dr} \quad (3)$$

where:

Q_s = discharge rate (L^3T^{-1})

K = hydraulic conductivity (LT^{-1})

B = thickness (L)

T = transmissivity (L^2T^{-1})

r = radius within the cone of depression (corresponds to x in Figure 34 b) (L)

H = corresponding head with reference to the elevation of the discharge orifice (L)

At that time, there were no observation wells near the flowing well. By assuming the hydraulic head is fixed at the outer boundary of the cone of depression, at a distance R away from the flowing well, Dupuit (1863) obtained Equation (4) by integrating Equation (3) from the boundary of the well r_w to the outer boundary of the cone of depression R.

$$Q_s = \frac{2\pi T(H_0 - H_w)}{\ln(R/r_w)} = \frac{2\pi T s_w}{\ln(R/r_w)} \tag{4}$$

where:

H_0 and H_w = hydraulic head at the radius of influence and at the discharge orifice (L)

s_w = drawdown at the discharge orifice (L)

H_0 can be obtained by measuring the initial hydraulic head of the flowing well when the well has been closed for a long time, and H_w equals the elevation of the discharge orifice.

Equation (4) is often referred to as the *Dupuit discharge formula*. Dupuit (1863) pointed out that Equation (4) explains the linear relationship between Q_s and H_w measured in many flowing wells as reported in Darcy (1856) and illustrated in Figure 33. Equation (4) can be transformed to obtain the specific capacity, which is a measure of well productivity. The expression of specific capacity can be written as shown by Equation (5).

$$\frac{Q_s}{s_w} = \frac{2\pi T}{\ln(R/r_w)} \tag{5}$$

Equations (4) and (5) are applicable to steady flow to both pumping and flowing wells in confined aquifers and can be used to estimate transmissivity or hydraulic conductivity of the confined aquifer. However, a major limitation of this method is the difficulty of determining R. This limitation can be overcome by using two monitoring wells within the cone of depression.

By integrating Equation (3) between two monitoring wells with different distances away from the pumping well, Thiem (1906) obtained an equation to determine hydraulic conductivity known as the *Thiem equilibrium method*. However, because the drawdown increases with time, the Thiem equilibrium method failed to obtain a fixed hydraulic conductivity, which—at the turn of 1930s—led to the emergence of transient well hydraulics. The publication of the Theis equation (Theis, 1935) for predicting transient drawdown induced by constant-rate pumping and estimating hydraulic parameters was a revolutionary development in groundwater hydrology. The details of the Thiem and Theis equations are beyond the scope of this book.

6.2 Transient Discharge Rate of Flowing Wells in Infinite Confined Aquifers

Meinzer (1928) pointed out that when a flowing well has been shut off for a period, upon reopening, the discharge rate decreases with time. However, the transient behavior of discharge rate in flowing wells was not quantitatively examined until the 1950s. Based on Smith's (1937) solution for the response of temperature to introduction of a heat sink kept at a constant temperature, Jacob and Lohman (1952) derived an analytical solution of the response of head (drawdown) to constant-drawdown well tests and then obtained the solution of transient discharge rate to flowing wells.

For a fully-penetrating flowing well tapping a homogeneous and infinite confined aquifer (Figure 35), when the flowing well is opened, the head at the flowing well drops to the discharge orifice instantaneously, with the drawdown s_w maintained constant at the discharge orifice. As the drawdown s propagates away from the well, water stored in the confined aquifer is gradually consumed. The decreasing head in the confined aquifer leads to a time-decreasing discharge rate Q_w. For convenience of mathematical treatment, the initial potentiometric surface is assumed to be flat, i.e., the regional hydraulic gradient as shown in Figure 34a is not considered.

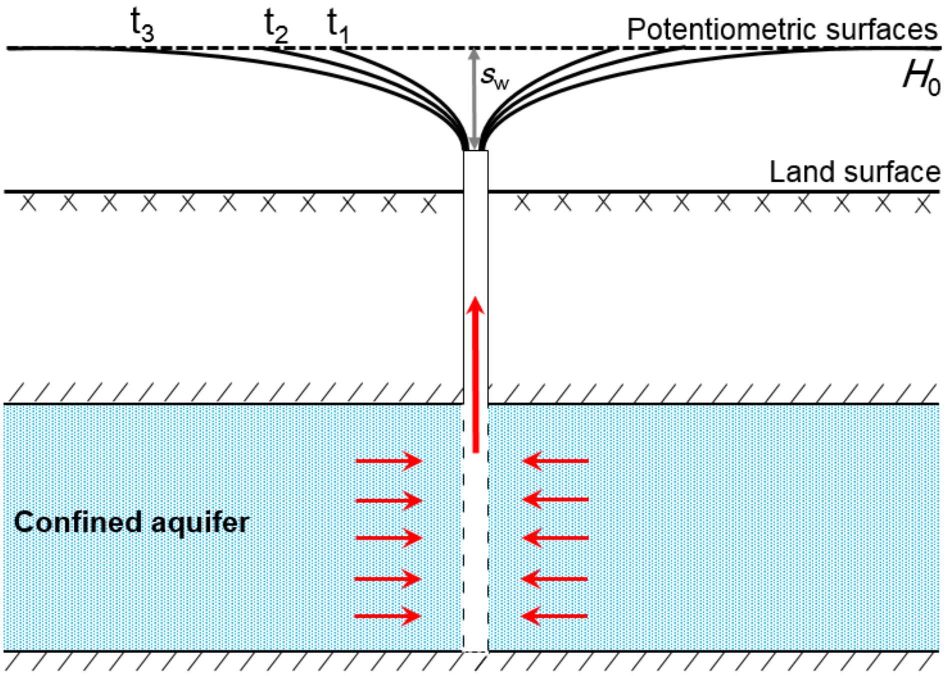


Figure 35 - A schematic diagram showing the response of drawdown to a flowing well that penetrates a confined aquifer. The dashed line at the top represents the initial potentiometric surface before opening the well outlet. The three sets of solid lines represent the evolution of the drawdown. H_0 is the potentiometric head before drilling or opening the flowing well. s_w is the drawdown at the well after drilling or opening the flowing well. The three solid lines refer to the potentiometric surfaces at times t_1, t_2 and t_3, respectively. The red horizontal and red vertical lines indicate the flow direction in the confined aquifer and inside the well screen, respectively.

Following Jacob and Lohman (1952), the mathematical model and the solution for discharge rate are shown in Box 2. The dimensionless discharge rate Q_D, defined as $\frac{Q_w}{2\pi T s_w}$, is a function of dimensionless time t_D. Figure 36 shows the dependence of Q_D on t_D.

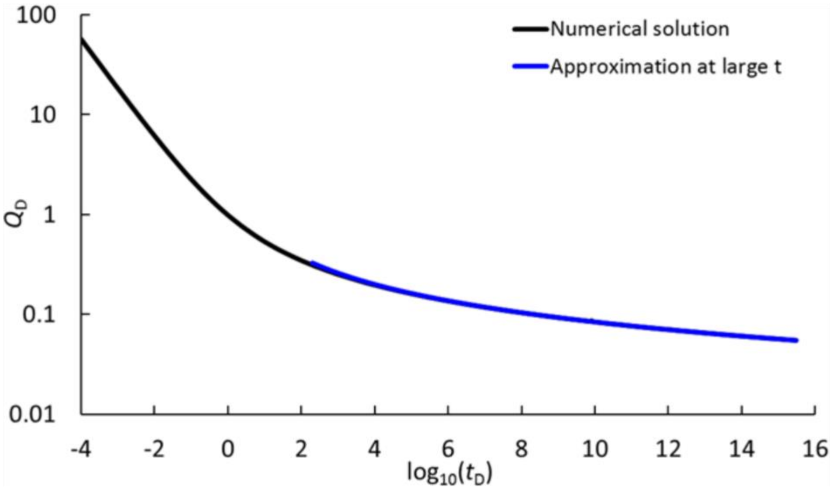


Figure 36 - The curve of dimensionless discharge rate Q_D versus the dimensionless time t_D.

When $t_D = 200$ (corresponding to $\log_{10}(t_D)=2.3$), the difference between Q_D and $2/\ln(2.25t_D)$ is only around 0.05, and the difference between the two terms decreases with t_D. Therefore, when $t_D > 200$, the discharge rate at the flowing well can be expressed as shown by Equation (6).

$$Q_w = \frac{4\pi T s_w}{ln\frac{2.25Tt}{r_w{}^2 S}} \tag{6}$$

The approximate discharge rate shown in Equation (6) can be transformed as shown by Equation (7).

$$\frac{1}{Q_w} = \frac{2.3}{4\pi T s_w}\log_{10}\frac{2.25Tt}{r_w{}^2 S} = \frac{0.183}{T s_w}\log_{10}\frac{2.25T}{r_w{}^2 S} + \frac{0.183}{T s_w}\log_{10} t \tag{7}$$

Equation (7) indicates that $1/Q_w$ has a linear relationship with $\log_{10} t$. Therefore, a plot of $1/Q_w$ versus $\log_{10} t$ can be used to estimate T and S of a confined aquifer.

Storage depletion is the only source of water derived in flowing wells tapping confined aquifers that do not have a regional gradient. The curve of Q_D versus t_D in confined aquifers indicates that it is impossible to reach a steady-state discharge rate.

7 Hydraulics of Flowing Wells in Semi-Confined Aquifers with a Flat Potentiometric Surface

7.1 Transient Discharge Rate

Several years after investigating constant-rate pumping in semi-confined (or leaky) aquifers (Hantush & Jacob, 1955), Hantush (1959) derived a solution of transient discharge rate to a fully-penetrating flowing well in an infinite semi-confined aquifer as illustrated in Figure 37 and described mathematically in Box 3.

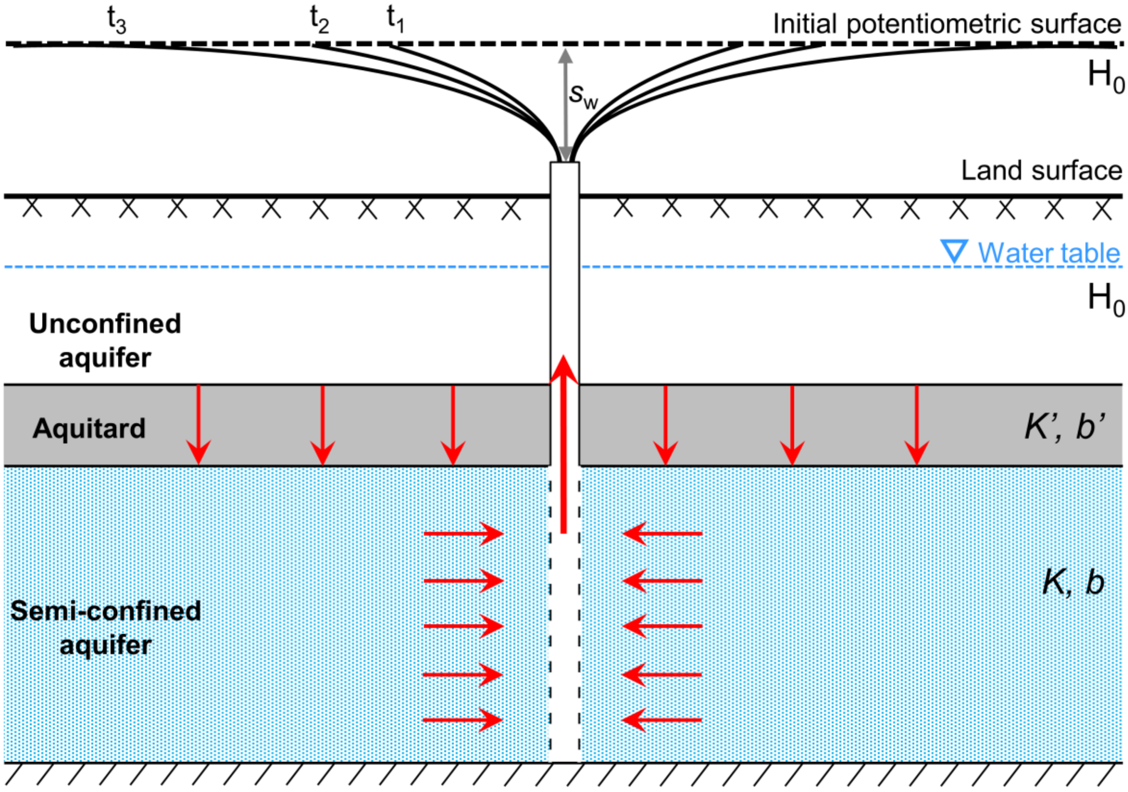


Figure 37 - A schematic showing the response of drawdown to a flowing well that penetrates a semi-confined aquifer. The dashed line at the top represents the initial potentiometric surface before opening the well outlet. A limitation of the model is the assumption that the water table height of the overlying unconfined aquifer equals the initial potentiometric surface of the semi-confined aquifer and sufficiently high K that the water table does not change in response to the discharge. H_0 is the potentiometric head before drilling or opening the flowing well, and s_w is the drawdown at the well after drilling or opening the flowing well. The three solid lines are potentiometric surfaces at times t_1, t_2, and t_3, respectively. K and K' are hydraulic conductivities of the semi-confined aquifer and the aquitard, respectively. b and b' are the thickness of the semi-confined aquifer and the aquitard, respectively. The red horizontal and red vertical lines indicate the flow direction in the confined aquifer and inside the well screen, respectively.

The semi-confined aquifer has a thickness of b and a hydraulic conductivity of K, while the overlying aquitard has a thickness of b' and a hydraulic conductivity of K'. The initial potentiometric surface of the semi-confined aquifer is assumed to be flat and equals H_0. The hydraulic head of the overlying unconfined aquifer is assumed to be maintained at the initial head equaling H_0. If the head in the leaky aquifer is denoted as H, the downward leakage rate through the aquitard v_z can be expressed as shown by Equation (8).

$$v_z = \frac{K'(H_0 - H)}{b'} = \frac{K's}{b'} \tag{8}$$

A term $\left(-{}^{s}/_{B^2}\right)$ is considered in Equation Box 3-1 of Box 3 to account for leakage through the aquitard, where B is called the leakage factor and is a characteristic length of the semi-confined aquifer that reflects its resistance to leakage. B^2 equals $\frac{Kbb'}{K'}$. A large B value implies that the aquitard is almost impermeable.

As shown in Box 3, the dimensionless discharge rate, Q_D, defined as $\frac{Q_w}{2\pi T s_w}$, is a function of dimensionless time t_D and r_w/B. Figure 38 shows the dependence of Q_D on r_w/B and t_D.

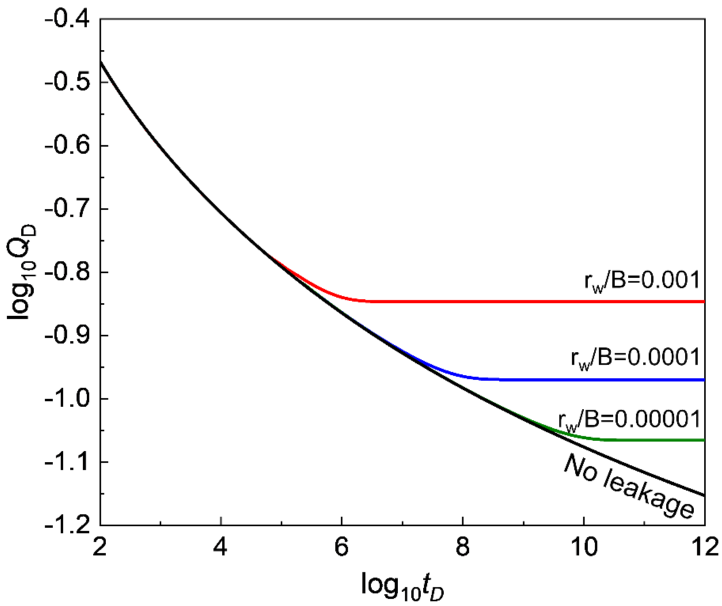


Figure 38 - Relationship between $Q(t_D, r_w/B)$ and t_D for a range of r_w/B. The flattening of the drawdown at the later time indicates steady-state has been reached because of leakage. Note that the well radius, r_w, is fixed but the leakage factor, B, is not fixed. As r_w/B increases, steady-state occurs sooner due to the decreasing B.

7.2 Steady-state Discharge Rate

When t_D approaches infinity, the equation of discharge rate reduces as shown by Equation (9).

$$Q_s = 2\pi T s_w \frac{r_w}{B}\frac{K_1(r_w/B)}{K_0(r_w/B)} \tag{9}$$

For small values of x, $xK_1(x) = 1$ and $K_1(x) = \ln(1.123/x)$. Because r_w/B is usually smaller than 0.01, Equation (9) can be expressed as Equation (10).

$$Q_s = \frac{2\pi T s_w}{K_0(r_w/B)} = \frac{2\pi T s_w}{\ln\left(1.123\frac{B}{r_w}\right)} \tag{10}$$

Equation (10) indicates that the specific capacity Q_s/s_w of a flowing well is controlled by T, r_w, and B. This equation can be used to interpret the discharge rate versus elevation of discharge orifices as shown in Figure 33b. A major advantage of Equation (10) over Equation (5) is that the control of inter-aquifer leakage on discharge rate of the flowing well is considered. In the absence of a regional hydraulic gradient, the downward leakage from the overlying unconfined aquifer is the only source of water to maintain the steady-state cone of depression and the steady-state discharge rate of a flowing well.

Note that in a semi-confined aquifer, the expression of Q_s/s_w for the steady-state discharge rate induced by a constant-drawdown aquifer test is the same as that for steady-state drawdown induced by a constant-rate pumping test. The equation of steady-state drawdown induced by constant-rate pumping was derived by De Glee (1930), and can also be obtained by approximating to a very large t_D in the expression of transient drawdown induced by constant-rate pumping (Jacob, 1946).

8 Hydraulics of Flowing Wells in Basins with a Background Basinal Flow Field

8.1 Transient Discharge Rate and Water Sources in a Semi-Confined Aquifer of a Layered Basin

As was shown in Figure 30, the occurrence of flowing wells in semi-confined aquifers is closely related to upward groundwater flow in the discharge area. To numerically obtain the transient discharge rate of a flowing well tapping the semi-confined aquifer, the two-dimensional cross section shown in Figure 29 is extruded along the y-axis to obtain a three-dimensional basin (Figure 39). The three-dimensional synthetic basin has a specified head boundary condition along the river valley in the top model layer, a hybrid recharge/discharge boundary condition in other parts of the top layer (Section 5.3), and zero-flux boundary conditions at the four lateral boundaries and at the bottom.

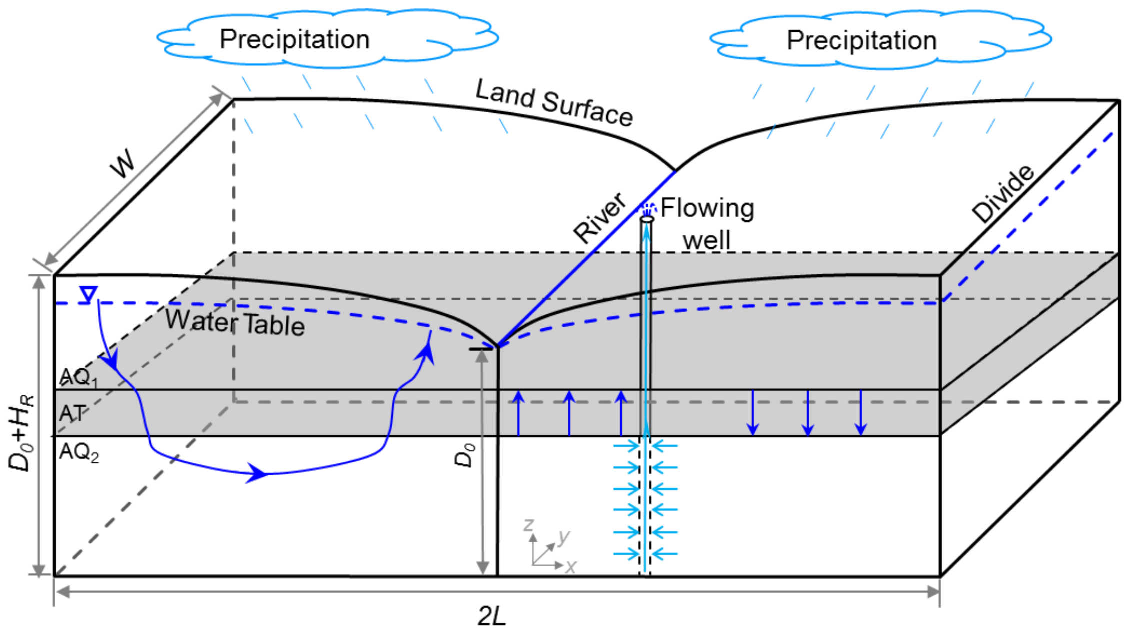


Figure 39 - The conceptual model of a three-dimensional layered river valley basin with a flowing well in the right half of the basin (modified from Zhang et al., 2022). The three-dimensional basin is obtained by first including the mirror image of the cross section shown in Figure 29 on the other side of the river to form a complete basin and then extending it along the y-axis. The basin is homogeneous when the aquitard AT has the same hydraulic conductivity as the overlying and underlying aquifers (AQ1 and AQ2).

The length of the synthetic basin ($2L$) and the width of the synthetic basin (W) are both set to 20 km (12.43 miles). To obtain the transient behavior of a flowing well, the flowing well is located at x = 500 m (0.31 miles) and y = 10,000 m (6.2 miles)—that is, 500 m (1,640 ft) away from the river valley, 9,500 m (5.9 miles) away from the no-flow boundary at the divide, and 10,000 m (6.2 miles) away from the no-flow boundary at the other two lateral boundaries. Figure 40 shows the dimensionless discharge rate Q_D of flowing wells tapping the semi-confined aquifer versus dimensionless time t_D under three different conductivity ratios (i.e., K/K' of 1, 10, and 100).

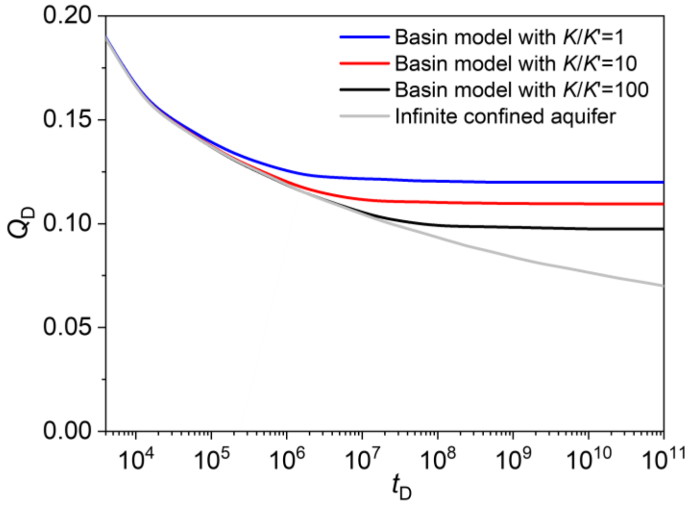


Figure 40 - Dimensionless discharge rate (Q_D) of the flowing well tapping the semi-confined aquifer versus dimensionless time (t_D) in the three-layer basin (modified from Zhang et al., 2022). The gray line represents that of a flowing well tapping an infinite confined aquifer. Steady state flow is greater when more water can penetrate the aquitard (i.e., when K/K' is smaller).

Figure 40 also shows the curve of Q_D versus t_D for a flowing well tapping an infinite confined aquifer, which is approximated by a basin with a length and a width of 200 km (≅124 miles) in the numerical model. For flowing wells tapping the semi-confined aquifer, the curves of Q_D versus t_D coincide with the curve for the infinite confined aquifer in the early stage but deviate from it at later times. As the conductivity ratio K/K' decreases (K' increases), a stronger inter-aquifer leakage leads to an earlier deviation from the curve for the infinite confined aquifer.

The curves of Q_D versus t_D obtained by the basin model with a regional gradient shown in Figure 40 are similar to those in semi-confined aquifers without a regional gradient shown in Figure 38. Like sources of water in other production wells, the total discharge rate from the flowing well can be partitioned into two parts: storage consumption and capture from neighboring aquifers (Figure 41). Detailed descriptions of storage consumption and capture can be found in Konikow and Bredehoeft (2020) and are not repeated here.

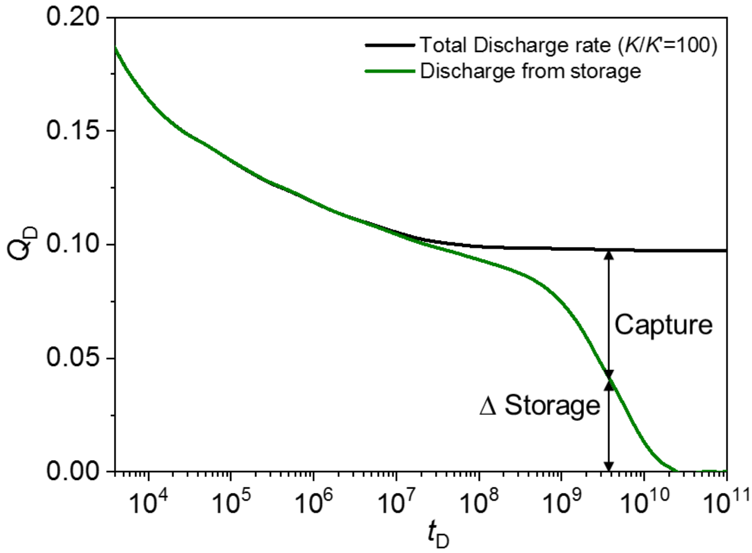


Figure 41 - The partitioning of dimensionless discharge rate Q_D into storage consumption in the bounded semi-confined aquifer and capture from the overlying aquifer (modified from Zhang et al., 2022). At the later stage, no storage is consumed, and all discharge is from capture. Capture is the sum of recharge, which has a positive sign, and discharge, which has a negative sign. Before pumping begins recharge equals discharge and the absolute value of discharge decreases as water flows to the well instead of to the river.

In the basin model with a regional gradient, before installing a well, there is upward inter-aquifer leakage through the aquitard in the discharge limb, which constitutes discharge of the semi-confined aquifer, and downward inter-aquifer leakage through the aquitard in the recharge limb, which constitutes recharge of the semi-confined aquifer (Figure 42a). After installing a flowing well, although there is drawdown in the semi-confined aquifer, the head near the flowing well is still higher than the land surface and the head of the upper aquifer. The lowered head near the flowing well in the discharge limb leads to a smaller upward leakage through the aquitard, which constitutes decreased discharge of the semi-confined aquifer. The lowered head in the recharge limb of the semi-confined aquifer also leads to increased downward inter-aquifer leakage through the aquitard, which constitutes increased recharge to the semi-confined aquifer (Figure 42b). Therefore, capture comprises two parts: decreased discharge and increased recharge. This is different from what we pointed out in Section 7.2 that in the absence of a regional gradient, the downward leakage from the overlying unconfined aquifer is the only source of capture.

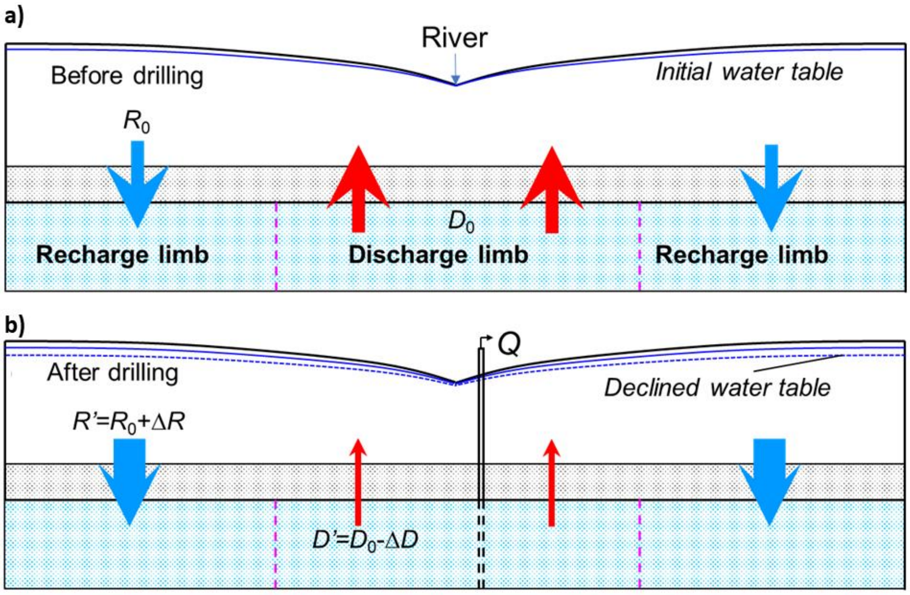


Figure 42 - A schematic representation of inter-aquifer leakage in the basin before a) and after b) installing a flowing well near the river (modified from Zhang et al., 2022). The larger downward blue arrows in (b) represent increased recharge, and the smaller upward red arrows in (b) represent decreased discharge. The flow field before installing a flowing well can be found in Figure 30.

To sum up, by considering the basinal background flow field, it is found that storage depletion, increased downward inter-aquifer leakage in the recharge limb, and decreased upward inter-aquifer leakage in the discharge limb all contribute to groundwater discharge in the flowing well. Zhang and others (2022) found that decreased discharge (upward leakage) in the discharge limb plays a much greater role than increased recharge (downward leakage) in the recharge limb. This recognition of the sources of water in flowing wells has implications for sampling groundwater from flowing wells to represent water that has been subjected to long times of water-rock interaction.

8.2 Vertical Profiles of Discharge Rate in Flowing Wells with Long Screens

When the hydraulic conductivity of the aquitard is two or more orders of magnitude lower than that of the aquifer, the semi-confined aquifer is dominated by horizontal flow and the aquitard is dominated by vertical flow (Figure 30). In previous analytical studies of groundwater flow in semi-confined aquifers as shown in Figure 35, it is usually assumed that vertical groundwater flow can be neglected in the semi-confined aquifer. In fact, as shown in Figure 30, in either a homogeneous or a heterogeneous basin, there are components of upward groundwater flow in both recharge and discharge limbs; that is, groundwater flow is not totally horizontal in a semi-confined aquifer even if the hydraulic conductivity of the aquitard is low.

In the discharge limb of homogeneous basins, as a result of the depth-increasing hydraulic head, long-screened wells—including flowing wells and non-flowing wells—can experience both inflow and outflow along the screen length (Zhang et al., 2018). In the deep part of the discharge limb, because hydraulic head in the aquifer is higher than the mean head in the well, groundwater inflows from the aquifer to the well, whereas in the shallow part of discharge limb, because hydraulic head in the aquifer is lower than the mean head in the well, groundwater outflows from the well to the aquifer. The co-occurrence of inflow and outflow in a long-screen well creates a circuit for vertical flow within the well, which was termed intraborehole flow or wellbore flow in the literature (Reilly et al., 1989; Zinn & Konikow, 2007).

To be consistent with our discussion in Section 5.3 and Section 8.1, we use the conceptual model shown in Figure 39 to illustrate vertical profiles of radial discharge rate (Darcy velocity) of flowing wells. Figure 43 shows the vertical distribution of dimensionless radial discharge rate $q/q_{\max}$ for flowing wells tapping the semi-confined aquifer in the discharge area (x = 400 m, 1,312 ft) of the layered basin under four different hydraulic conductivity ratios. The maximum q, q_{max}, occurs at the bottom of the screen. When the aquitard has a hydraulic conductivity two orders of magnitude lower than the aquifer (i.e., K/K' equals 100), the radial inflow velocity is quite uniform (Figure 43a). However, when the aquitard has the same hydraulic conductivity as the aquifer, the radial inflow velocity increases significantly with depth (Figure 43d), which is a direct result of the increasing hydraulic head with depth in the discharge area.

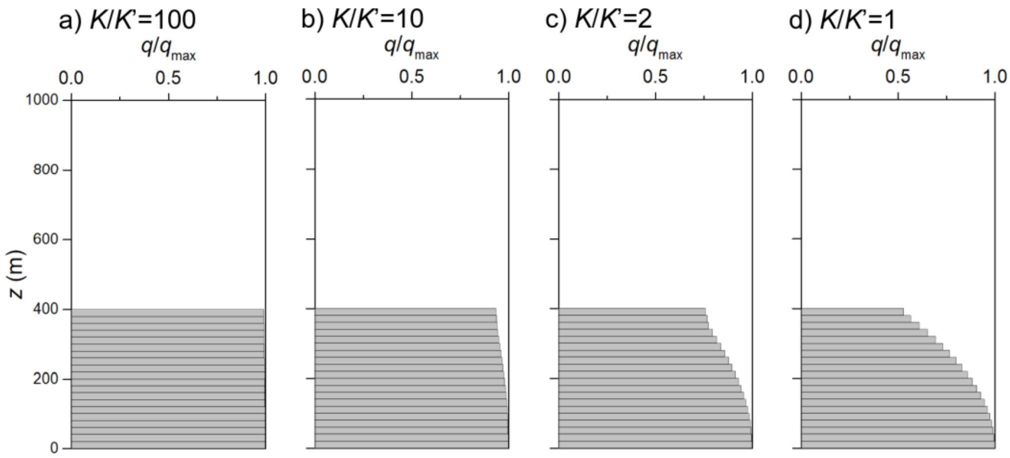


Figure 43 - Vertical variation of dimensionless flow rate in a flowing well tapping the deep part of a basin (located at x =400 m, 1,312 ft) with different *K*/*K*'. As *K*' increases, the trend of increasing flow rate with depth becomes more significant.

In a homogeneous basin, the trend of increasing flow rate with depth is more obvious in longer screens (Figure 44). Moreover, when the screen length is long enough (Figure 44b and c), then at depth, groundwater flows from the aquifer to the well resulting in a cumulative inflow rate of Q_{in}, while in shallow portions of the screen water flows from the well to the aquifer resulting in a cumulative outflow rate of Q_{out}. Inflow occurs where the larger hydraulic head of the aquifer is higher than head in the flowing well, whereas outflow occurs where hydraulic head in the aquifer is the lower than head in the well. If

Q_{in} is larger than Q_{out}, cumulative flow rate at the well outlet is above 0, which explains the cause of water overflow at the well outlet.

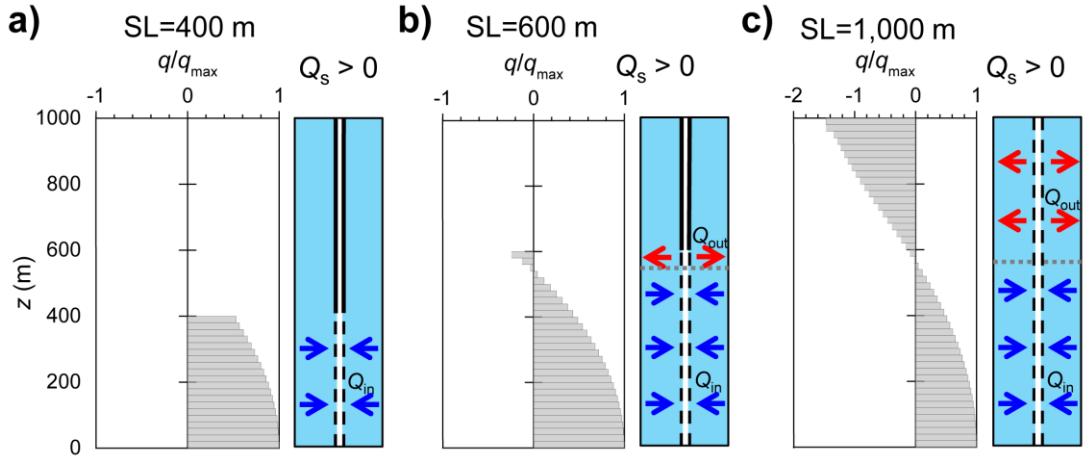


Figure 44 - Vertical variations of dimensionless flow rate in flowing wells with different screen lengths (SL) in a homogeneous basin. a) A flowing well with $Q_{in} > Q_{out} = 0$; b) a flowing well with $Q_{in} > Q_{out} > 0$ and c) a flowing well with $Q_{in} > Q_{out} > 0$. In all the cases shown $Q_s > 0$ because Q_{in} exceeds Q_{out}.

Figure 45 shows the vertical variations of dimensionless radial discharge rate of fully penetrating flowing wells installed at different distances away from the river. When a fully penetrating flowing well is installed at the river valley ($x = 0$), there is no outflow segment because the hydraulic head of the aquifer is higher than the head of flowing well at all depths (Figure 45a). As the distance away from the valley increases, the length of outflow segment and cumulative outflow Q_{out} increases (Figure 45b and c). When $x = 550$ m, the well does not overflow because Q_{in} is balanced by Q_{out}, which corresponds to the scenario with occurrence of intraborehole flow. For the case shown in Figure 45b, the flowing well is a form of intraborehole flow with $Q_{in} > Q_{out}$.

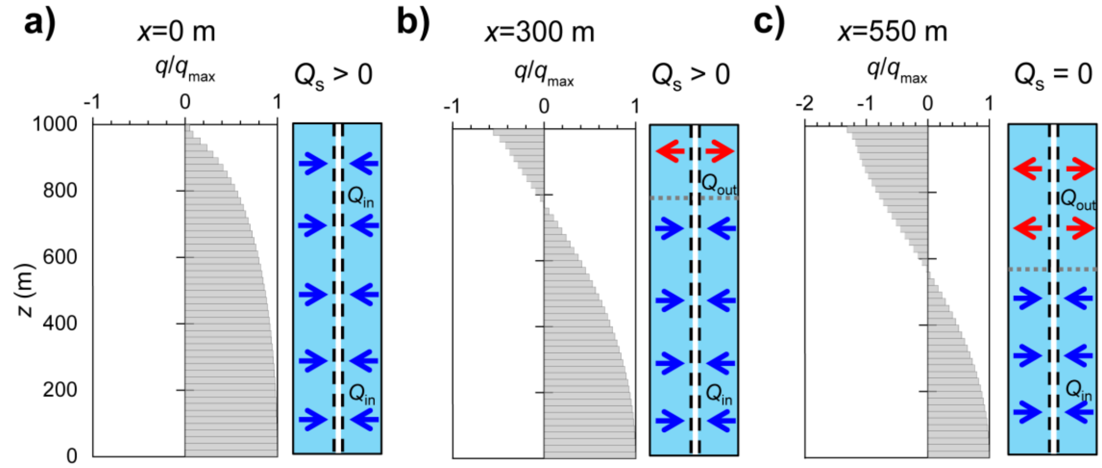


Figure 45 - Vertical variations of dimensionless radial discharge rate of fully penetrating flowing wells installed at different distances away from the valley. The cases (a) and (b) are flowing wells, whereas the case (c) is a non-flowing well.

As shown in Figure 45 - a and Figure 45b, in a flowing well with a long screen, even when there is no aquitard, groundwater in the shallow part of the aquifer does not enter the flowing well. This indicates that the chemistry of water discharging from the well is representative of deep groundwater. A geophysical flow log, if available, would be useful in identifying the source zone.

9 Wrap-up

The journey of how quantitative groundwater science evolved began in France with the work of Darcy and Dupuit, who were the first to apply a hydraulic-engineering perspective to groundwater. Their work was triggered by the interesting phenomenon of overflow in the flowing wells, which were widespread in the first half of the nineteenth century as a result of the development of cable-tool drilling. As a trained hydraulic-engineer, Dupuit plotted the first potentiometric surface in a confined aquifer with consideration of head loss through the aquifer. The focus on groundwater research shifted from France to America where the US Geological Survey, established in 1879, became the leader in groundwater research with a focus on field studies. Here, the framework was also strongly influenced and biased by flowing wells, but the thinking had a geological rather than a hydraulic-engineering perspective. The focus was on groundwater as a major water resource in aquifers and how groundwater manifests in these most permeable strata of sedimentary basins. The bounding strata with lower permeability, which were referred to as confining beds (layers, units), were believed to be critical to occurrence of flowing wells. This conceptual model of geologically controlled flowing wells, which comes from the study of flowing wells in past centuries and is still common on recently published diagrams aimed at introductory groundwater education, is misleading because it bears little resemblance to reality.

Early researchers in the 1800s already had perceptions about where the water in confined aquifers originated and adopted intuitive conceptualizations of the recharge occurring where the confined aquifers outcropped and then moved in a plug flow manner akin to pipe flow. These were the early concepts for what we now refer to as the theory of groundwater flow systems. There was much that was useful in the early geology-based concepts for flow systems but some important aspects were incorrect and remained so until M. K. Hubbert published his physics-based theory of groundwater flow in 1940. His work was greatly expanded by analytical and numerical studies by J. Toth and R. A. Freeze in the 1960s. When computers became readily accessible for groundwater computations, numerical results showed cross-formational flow through aquitards and indicated that aquifers do not need to outcrop to receive recharge. These results also demonstrated that the presence of aquitards could enlarge the zone with flowing wells. This resulted in modern thinking about groundwater flow systems in an aquifer system (aquifer-aquitard system).

The hydraulics of pumping aquifers assuming full confinement, horizontal flow, and steady conditions that originated with Dupuit underwent little advancement until 1935 when C. V. Theis of the US Geological Survey provided the transient solution. This accomplished for aquifer hydraulics what the 1940 work of Hubbert did for the development of groundwater flow system analysis. Theis (1935) and Hubbert (1940) laid the foundations for many advances that began in the 1940s in aquifer hydraulics and in the 1960s for the flow system theory.

The minor advancement of concepts related to aquifer hydraulics and groundwater flow systems between about 1880 and 1935 can be attributed to the fact that knowledge about them was trapped in an early paradigm in which confined aquifers were the singular focus and aquitards were an appendage—both with respect to groundwater flow systems and aquifer-aquitard hydraulics. A paradigm shift was needed—but, as is typical of science, such shifts take time. This book describes the establishment of the early paradigm in which outcropping aquifers and confining beds governed the groundwater flow system to the shift to a new paradigm that became well established by the twenty-first century wherein the flow system is governed by the spatial arrangement of aquifers, aquitards, and topography.

Although the earliest study of well hydraulics by Dupuit (1863) aimed to quantify the discharge rate of a flowing well, later developments of transient well hydraulics in the 1930s through the 1950s aimed to estimate hydraulic properties based on the response of water levels to pumping in infinite confined or semi-confined aquifers with a horizontal initial head. The assumptions of infinite aquifers and horizontal initial head were also applied to studies of transient discharge rates from flowing wells in the 1950s. In fact, the assumption of horizontal initial head does not influence estimated discharge rate and hydraulic properties but leads to an incorrect understanding of the source of groundwater derived in flowing wells. Thus, aquifer hydraulics needs to be combined with groundwater flow system analysis. The study of hydraulics of long-screen flowing wells in the discharge area of a homogeneous basin, which has depth-increasing hydraulic head, revealed the occurrence of inflow from the aquifer to the well in the deep part and outflow from the well to the aquifer in the shallow part of the well. This finding is useful for interpreting hydrochemical tracers in the discharge limb of a regional flow system by sampling from flowing wells.

10 Exercises

Exercise 1 - Analytical Analysis of a Zone with Flowing Wells in a Homogeneous Basin

The occurrence of flowing wells in topographic lows of homogeneous, unconfined aquifers can be described analytically. The undulating topography is characterized by a cosine function; the undulating water table is a subdued replica of the topography. Wang and others (2015) introduced a damping factor α [-] to establish the relationship between the water table configuration $z_{WT}(x, \alpha)$ and the undulating topography $z_T(x)$. The function of water table undulation in a cross section can be written as shown by the following equation.

$$z_{WT}(x,\alpha) = \alpha \cdot z_T(x) = \alpha \cdot \left[H_R - H_R \cos\left(\frac{\pi x}{L}\right)\right]$$

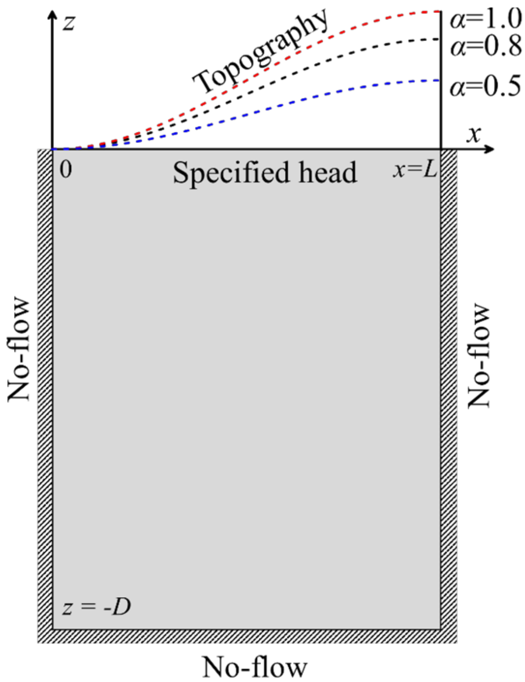
where:

x = the horizontal distance from the valley to the divide (L)

α = damping factor relating water table elevation to undulating topography (dimensionless)

H_R = the amplitude of the topographic undulation (L)

When $\alpha = 1$, the configuration of the water table is an exact copy of the topography; when $\alpha < 1$, the water table has a smaller amplitude than the topography.



The geometry of a unit basin with a length of L, depth of $|D|$.

Following Jiang and others (2011), the hydraulic head in a homogeneous and isotropic unit basin with a depth of D and a length of L can be derived as shown by the following equation.

$$h(x,z) = \alpha H_R - \alpha H_R \cos\left(\frac{\pi}{L}x\right)\frac{\cosh\left(\frac{\pi}{L}(D-z)\right)}{\cosh\left(\frac{\pi}{L}D\right)}$$

The head exceeding land surface can be obtained by the difference between the hydraulic head and the undulating topography, which is written as shown here.

$$(h(x,z) - z_T(x)) = H_R(\alpha - 1) + H_R \cos\left(\frac{\pi}{L}x\right)\left(1 - \alpha\frac{\cosh\left(\frac{\pi}{L}(D-z)\right)}{\cosh\left(\frac{\pi}{L}D\right)}\right)$$

Based on the second equation shown in this exercise (i.e., the expression for hydraulic head $h(x, z)$) not for the difference between head and the land surface, the hydraulic head can be calculated by using software such as Excel or MATLAB. Please try to plot the spatial distribution of head exceeding land surface by fixing D, L, and H_R, and changing α using Surfer or MATLAB (or any other software that has the capability of plotting contours) and compare it to the topographical elevation to determine where head exceeds the surface.

Solution to Exercise 1↴

Return to where text links to Exercise 1↥

11 References

Alley, W., & Alley, R. (2017, September 21). *High and dry: Meeting the challenges of the world's growing dependence on groundwater*. Yale Scholarship Online. https://doi.org/10.12987/yale/9780300220384.001.0001↗.

Anderson, M. P. (2008). *Groundwater: Benchmark papers in hydrology*. IAHS Press. https://iahs.info/Publications-News/?category=6↗.

The Social Economist. (1822). No. 1: Bored springs or artificial fountains obtained by boring the earth. *Monthly Magazine and British Register, 54*.

Arago, M. (1835). On springs, artesian wells, and sprouting fountains. *The Edinburgh New Philosophical Journal, 18*.

Batu, V. (1998). *Aquifer hydraulics: A comprehensive guide to hydrogeologic data analysis*. John Wiley & Sons.

Bear, J. (1972). *Dynamics of fluids in porous media*. American Elsevier.

Bear, J. (1979). *Hydraulics of groundwater*. McGraw-Hill.

Biswas, A. K. (1970). *The history of hydrology*. North-Holland.

Bond, F. T. (1865). The geology of mineral springs. *Population Science Review, 4*.

Botham, C. J. (2021, October 22). The Grenelle artesian well of Paris. *On Verticality*. https://www.onverticality.com/blog/grenelle-artesian-well↗.

Bouwer, H. (1978). *Groundwater hydrology*. McGraw-Hill.

Brassington, R. (2017). *Field hydrogeology* (3rd ed.). John Wiley & Sons.

Bredehoeft, J. D., Neuzil, C. E., & Milly, P. C. D. (1983). *Regional flow in the Dakota aquifer: A study of the role of confining layers* (Water Supply Paper 2237). U.S. Geological Survey. https://pubs.usgs.gov/wsp/2237/report.pdf↗.

Bredehoeft, J. D. (2018). The Toth revolution. *Groundwater, 56*(1), 157–159. https://doi.org/10.1111/gwat.12592↗.

Brown, G. O. (2002). Henry Darcy and the making of a law. *Water Resources Research, 38*(7), 11-1–11-12. https://doi.org/10.1029/2001WR000727↗.

Buckland, W. (1836). *Geology & mineralogy, considered with reference to natural theology* (Vol. 1). Routledge. https://doi.org/10.4324/9780203489468↗.

Carpenter, L. G. (1891). *Artesian wells of Colorado and their relation to irrigation*. The State Agricultural College, Fort Collins, CO.

Chamberlin, T. C. (1885). *The requisite and qualifying conditions of artesian wells* (Fifth Annual Report). US Geological Survey.

Commander, D. P. (2005). Artesian water. In J. H. Lehr & J. Keeley (Eds.), *Water encyclopedia* (pp. 1–5). John Wiley & Sons. https://doi.org/10.1002/047147844X.gw56↗.

Darcy, H. (1856). *Les fontaines publiques de la ville de Dijon* [The public fountains of the village of Dijon]. Dalmont.

Darton, N. H. (1897). *Preliminary report on artesian waters of a portion of the Dakotas* (Seventeenth Annual Report). US Geological Survey.

Darton, N. H. (1905). *Preliminary report on the geology and underground water resources of the central Great Plains* (Professional Paper 32). US Geological Survey. https://doi.org/10.3133/pp32↗.

Darton, N. H. (1909). *Geology and underground waters of South Dakota* (Water Supply Paper 227). US Geological Survey. https://doi.org/10.3133/wsp227↗.

Darwin, E. (1785). An account of an artificial spring of water. *Philosophical Transactions of the Royal Society of London, 75*.

Dassargues, A. (2018). *Hydrogeology: Groundwater science and engineering*. CRC Press. https://doi.org/10.1201/9780429470660↗.

Daubrée, A. (1887). *Les eaux souterraines à l'époque actuelle* [Groundwater today]. Dunod.

Davis, S. N., & De Wiest, R. J. (1966). *Hydrogeology*. John Wiley & Sons.

Davie, T. (2008). *Fundamentals of hydrology*. Routledge. https://doi.org/10.4324/9780203933664↗.

De Glee, G. J. (1930). *Over grondwaterstromingen bij wateronttrekking door middel van putten* [About groundwater flows during water extraction by means of wells]. J. Waltman.

de Marsily, G. (1986). *Quantitative hydrogeology*. Academic Press.

de Vries, J. J. (2007). History of groundwater hydrology. In J. W. Delleur (Ed.), *The handbook of groundwater engineering* (2nd Ed.). CRC Press.

De Wiest, R. J. M. (1965). *Geohydrology*. John Wiley & Sons.

Deming, D., Cranganu, C., & Lee, Y. (2002). Self-sealing in sedimentary basins. *Journal of Geophysical Research Solid Earth, 107*(B12), ETG 2-1–ETG 2-9. https://doi.org/10.1029/2001JB000504↗.

Domenico, P. A. (1972). *Concepts and models in groundwater hydrology*. McGraw-Hill.

Domenico, P. A., & Schwartz, F. W. (1998). *Physical and chemical hydrogeology*. John Wiley & Sons.

Driscoll, F. G. (1986). *Groundwater and wells* (2nd ed.). Johnson Division.

Duffy, C. J. (2017). The terrestrial hydrologic cycle: An historical sense of balance. *WIREs Water, 4*, e1216. https://doi.org/10.1002/wat2.1216↗.

Dupuit, J. (1863). *Études théoriques et pratiques sur le mouvement des eaux dans les canaux découverts et à travers les terrains perméables* [Theoretical and practical studies on the movement of water in open channels and through permeable soils] (2nd ed.). Dunod.

Farey, J. (1823). On artesian wells and boreholes. *Monthly Magazine and British Register*, *56*.

Farvolden, R. N. (1961). *Groundwater resources Pembina area, Alberta*. Research Council of Alberta. https://static.ags.aer.ca/files/document/ESR/ESR_1961_04.pdf↗.

Fetter, C. W. (1980). *Applied hydrogeology* (1st ed.). C. E. Merrill.

Fetter, C. W. (1994). *Applied hydrogeology* (3rd ed.). Pearson Education.

Fetter, C. W. (2001). *Applied hydrogeology* (4th ed.). Pearson Education.

Fetter, C.W. (2004). Hydrogeology: A short history, Part 1. *Groundwater, 42*, 790–792. https://doi.org/10.1111/j.1745-6584.2004.tb02734.x↗.

Fitts, C. R. (2013). *Groundwater science* (2nd ed.). Academic Press. http://doi.org/10.1016/C2009-0-62950-0↗.

Freeze, R. A. (1994). Henry Darcy and the fountains of Dijon. *Groundwater, 32*(1), 25–30. https://doi.org/10.1111/j.1745-6584.1994.tb00606.x↗.

Freeze, R. A., & Back, W. (1983). *Benchmark papers in geology: Physical hydrogeology*. Hutchinson Ross.

Freeze, R. A., & Cherry, J. A. (1979). *Groundwater*. Prentice-Hall. https://gw-project.org/books/groundwater/↗.

Freeze, R. A., & Witherspoon, P. A. (1967). Theoretical analysis of regional groundwater flow: 2. Effect of water-table configuration and subsurface permeability variation. *Water Resources Research, 3*(2), 623–634. https://doi.org/10.1029/WR003i002p00623↗.

Fuller, M. L. (1906). Significance of the term "artesian." In M. L. Fuller (Ed.), *Underground-water papers*. US Government Printing Office.

Garnier, F. (1822). *De l'art du Fontenier Sondeur et des puits artésiens* [The art of Fontenier Sounder and artesian wells]. Imprimerie de Madame Huzard.

Habermehl, M. A. (1980). The Great Artesian Basin, Australia. *BMR Journal of Australian Geology and Geophysics, 5*, 9–38. https://ecat.ga.gov.au/geonetwork/srv/api/records/a05f7892-b9de-7506-e044-00144fdd4fa6↗.

Habermehl, M. A. (2020). The evolving understanding of the Great Artesian Basin (Australia), from discovery to current hydrogeological interpretations. *Hydrogeology Journal, 28*(1), 13–36. https://doi.org/10.1007/s10040-019-02036-6↗.

Hantush, M. S. (1959). Nonsteady flow to flowing wells in leaky aquifers. *Journal of Geophysical Research, 64*(8), 1043–1052. https://doi.org/10.1029/JZ064i008p01043↗.

Hantush, M. S., & Jacob, C. E. (1955). Non-steady radial flow in an infinite leaky aquifer. *Eos, Transactions American Geophysical Union, 36*(1), 95–100. https://doi.org/10.1029/TR036i001p00095↗.

Heath, R. (1983). *Basic ground-water hydrology*. US Geological Survey. Water-Supply Paper 2220. https://pubs.usgs.gov/wsp/2220/report.pdf↗.

Héricart de Thury, L. É. F. (1829). *Considérations géologiques et physiques sur la cause du jaillissement des eaux des puits forés ou fontaines artificielles* [Geological and physical considerations on the cause of the gushing of water from drilled wells or artificial fountains]. Paris.

https://www.bavarikon.de/object/bav:BSB-MDZ-00000BSB10706934#↗.

Héricart de Thury, L. É. F. (1830). Observations on the cause of the spouting of overflowing wells or artesian fountains. *The Edinburgh New Philosophical Journal*, *9*.

Hiscock, K. (2005). *Hydrogeology: Principles and practice*. John Wiley & Sons.

Hölting, B., & Coldewey, W. G. (2019). *Hydrogeology*. Springer.

https://doi.org/10.1007/978-3-662-56375-5↗.

Hou, G., Liang, Y., Xiaosi, S. U., Zhao, Z., Tao, Z., Yin, L., Yang, Y., & Wang, X. (2008). Groundwater systems and resources in the Ordos Basin, China. *Acta Geologica Sinica, 82*(5), 1061–1069. https://doi.org/10.1111/j.1755-6724.2008.tb00664.x↗.

Hubbert, M. K. (1940). The theory of ground-water motion. *The Journal of Geology*, *48*(8, part 1). https://doi.org/10.1086/624930↗.

Hudak, P. F. (2004). Groundwater and the hydrologic cycle. In P. F. Hudak, *Principles of hydrogeology* (2nd ed.). CRC Press. https://doi.org/10.1201/9781003040613↗.

Jacob, C. E. (1946). Radial flow in a leaky artesian aquifer. *Eos, Transactions American Geophysical Union, 27*(2), 198–208. https://doi.org/10.1029/TR027i002p00198↗.

Jacob, C. E., & Lohman, S. W. (1952). Nonsteady flow to a well of constant drawdown in an extensive aquifer. *Eos, Transactions American Geophysical Union, 33*(4), 559–569.

https://doi.org/10.1029/TR033i004p00559↗.

Ji, T. T., Jiang, X.-W., Gou, L. F., Jin, Z. D., Zhang, H., Wan, L., Han, G. L., Guo, H. M., & Wang, X. S. (2022). Behaviors of lithium and its isotopes in groundwater with different concentrations of dissolved CO_2. *Geochimica et Cosmochimica Acta, 326*(June), 313–327. https://doi.org/10.1016/j.gca.2022.03.038↗.

Jiang, X.-W., Cherry, J., & Wan, L. (2020). Flowing wells: Terminology, history and role in the evolution of groundwater science. *Hydrology and Earth System Sciences, 24*(12), 6001–6019. https://doi.org/10.5194/hess-24-6001-2020↗.

Jiang, X.-W., Wan, L., Wang, J., Yin, B., Fu, W., & Lin, C. (2014). Field identification of groundwater flow systems and hydraulic traps in drainage basins using a geophysical method. *Geophysical Research Letters, 41*(8), 2812–2819. https://doi.org/10.1002/2014GL059579↗.

Jiang, X.-W., Wan, L., Wang, X. S., Wang, D., Wang, H., Wang, J. Z., Zhang, H., Zhang, Z. Y., & Zhao, K. Y. (2018). A multi-method study of regional groundwater circulation in the Ordos Plateau, NW China. *Hydrogeology Journal, 26*(5), 1657–1668. https://doi.org/10.1007/s10040-018-1731-4↗.

Jiang, X.-W., Wang, X. S., Wan, L., & Ge, S. (2011). An analytical study on stagnant points in nested flow systems in basins with depth-decaying hydraulic conductivity. *Water Resources Research, 47*(1), W01512. https://doi.org/10.1029/2010WR009346↗.

Konikow, L. F., & Bredehoeft, J. D. (2020). *Groundwater resource development: Effects and sustainability*. The Groundwater Project. https://gw-project.org/books/groundwater-resource-development/↗.

Kuhn, T. S. (1962). *The structure of scientific revolutions.* University of Chicago Press.

Johnson, B. J., Allen, D. M., & Wei, M. (2020, November 21–23). *Assessing the hydrogeologic conditions of flowing artesian bedrock wells in the Okanagan Basin, British Columbia* [Conference session]. GeoConvention 2020. https://geoconvention.com/wp-content/uploads/abstracts/2020/58222-assessing-the-hydrogeologic-conditions-of-flowing.pdf↗.

Jones, J. F. (1962). *Reconnaissance groundwater study, Swan Hills and adjacent areas, Alberta*. Research Council of Alberta.

Kasenow, M. (2010). *Applied ground-water hydrology and well hydraulics* (3rd ed.)Water Resources Publications. https://www.wrpllc.com/books/agwhh2.html↗.

Konikow, L. F. (2013). *Groundwater depletion in the United States (1900–2008)* (Scientific Investigations Report 2013–5079). US Geological Survey. https://doi.org/10.3133/sir20135079↗.

Kruseman, G. P., & de Ridder, N. A. (1990). *Analysis and evaluation of pumping test data* (2nd ed.). International Institute for Land Reclamation and Improvement. https://edepot.wur.nl/70972↗.

LaMoreaux, P. E., LaMoreaux, J. W., Soliman, M. M., Memon, B. A., & Assaad, F. A. (2008). *Environmental hydrogeology* (2nd ed.). CRC Press. https://doi.org/10.1201/9781420055023↗.

Lionnais, (1805). *Historie de la ville de Nancy, Description de la fontaine artesienne de Jarville, Nancy* [History of the village of Nancy, Description of the artesian fountain of Jarville, Nancy].

Lohman, S. W. (1972). *Ground-water hydraulics* (Professional Paper 708). US Geological Survey. https://doi.org/10.3133/pp708↗.

Macintosh, A. (1827). *The repertory of patent inventions, and other discoveries and improvements in arts, manufactures, and agriculture*. Bibliotheca Regia Monacensis.

Mádl-Szőnyi, J. (2008). The contribution of József Tóth to modernization of Hungarian hydrogeology. *Central European Geology, 51*(3), 189–201. https://doi.org/10.1556/CEuGeol.51.2008.3.2↗.

Margat, J., Pennequin, D., & Roux, J.-C. (2013). History of French hydrogeology. In N. Howden & J. Mather (Eds.), *History of hydrogeology* (pp. 59–99). Taylor & Francis. https://doi.org/10.1201/b12766↗.

Mariotte, E. (1717). *Oeuvres de Mr. Mariotte*, 2 Vols. (701 pp.). Edited by P. Van der Aa, Leiden.

Mather, J. D. (2013). The progress of hydrogeology in Britain: 1600 to 2000. In N. Howden & J. Mather (Eds.). *History of hydrogeology* (pp. 347–379). Taylor & Francis. https://doi.org/10.1201/b12766↗.

Mays, L. W. (2012). *Ground and surface water hydrology*. John Wiley & Sons.

McKay, B. R., Hainstock, H. N., & Graham, G. (1936). Groundwater resources of the rural municipality of Last Mountain Valley, Number 250, Saskatchewan. *Geological Survey of Canada, 90*.

McWhorter, D. B., & Sunada, D. K. (1977). *Ground-water hydrology and hydraulics*. Water Resources Publications.

Meinzer, O. E. (1923). *Outline of ground-water hydrology, with definitions* (Report 494). US Geological Survey.

Meinzer, O. E. (1928). Compressibility and elasticity of artesian aquifers. *Economic Geology, 23*(3), 263–291. https://doi.org/10.2113/gsecongeo.23.3.263↗.

Meinzer, O. E., & Hard, H. A. (1925). *The artesian water supply of the Dakota sandstone in North Dakota, with special reference to the Edgeley quadrangle* (Water Supply Paper 520-E). US Geological Survey. https://doi.org/10.3133/wsp520E↗.

Meiter, J. A. (2019). *The guide to Wisconsin flowing artesian wells*. Neshkoro.

Meyboom, P. (1962). Patterns of groundwater flow in the Prairie Profile. In *Proceedings of Third Canadian Hydrology Symposium* (pp. 5–33). National Research Council of Canada.

Meyboom, P. (1966). *Groundwater studies in the Assiniboine River drainage basin: Part 1 The evaluation of a flow system in south-central Saskatchewan* (Bulletin 139). Geological Survey of Canada.

Meyer, G., Davis, G., & LaMoreaux, P. E. (1988). Historical perspective. In W. Back, J. S. Rosenshein, & P. R. Seaber (Eds.), *Hydrogeology, The geology of North America* (Vol. 0–2). The Geological Society of America.

Molz, F. J., Boman, G. K., Young, S. C., & Waldrop, W. R. (1994). Borehole flowmeters: Field application and data analysis. *Journal of Hydrology*, 163(3), 347–371. https://doi.org/10.1016/0022-1694(94)90148-1↗

Morris, J. (1849). *A series of large geological diagrams, illustrating the principles of this important and practical science…With explanatory notes upon each diagram*. Reynolds.

Nonner, J. C. (2003). *Introduction to hydrogeology* (2nd ed.). A. A. Balkema.

Norton, W. H. (1897). Artesian wells of Iowa (Annual Report 6, pp. 113–448). Iowa Geological Survey. https://doi.org/10.17077/2160-5270.1330↗.

Ordens, C. M., McIntyre, N., Underschultz, J. R., Ransley, T., Moore, C., & Mallants, D. (2000). Preface: Advances in hydrogeologic understanding of Australia's Great Artesian Basin. *Hydrogeology Journal, 28*, 1–11. https://doi.org/10.1007/s10040-019-02107-8↗.

Perrault, P. (1967). *De l'origine des fontaines* [The origin of fountains]. (A. LaRocque, Transl.). Hafner Publishing. (Original work published 1674).

Pinder, G. F., & Celia, M. A. (2006). *Subsurface hydrology*. Wiley Science. https://www.geokniga.org/bookfiles/geokniga-subsurface-hydrology.pdf↗.

Poeter, E., & Hsieh., P., 2020, Graphical Construction of Groundwater Flow Nets. The Groundwater Project, Guelph, Ontario, Canada.

Poiseuille, J. L. (1841). Recherches expérimentales sur le mouvement des liquides dans les tubes de trés-petits diamétres, comptes rendus [Experimental research on the movement of liquids in tubes of very small diameters, reports]. Académie des Sciences de Paris.

Poland, J. E., Lofgren, B. E., & Riley, F. S. (1972). *Glossary of selected terms useful in studies of the mechanics of aquifer systems and land subsidence due to fluid withdrawal* (Water Supply Paper 2025). US Geological Survey. https://doi.org/10.3133/WSP2025↗.

Price, M. (1996). *Introducing groundwater*. Chapman & Hall, London.

Reilly, T. E., Franke, O. L., & Bennett, G. D. (1989). Bias in groundwater samples caused by wellbore flow. *Journal of Hydraulic Engineering 115*(2), 270–276. https://doi.org/10.1061/(ASCE)0733-9429(1989)115:2(270)↗.

Ritzi, J. R. W., & Bobeck, P. (2008). Comprehensive principles of quantitative hydrogeology established by Darcy (1856) and Dupuit (1857). *Water Resources Research, 44*(10), W10402. https://doi.org/10.1029/2008WR007002↗.

Rushton, K. R. (2003). *Groundwater hydrology: Conceptual and computational models*. John Wiley & Sons.

Schwartz, F. W., & Zhang, H. (2003). *Fundamentals of groundwater*. John Wiley & Sons.

Sen, Z. (2015). *Practical and applied hydrogeology*. Elsevier.

Slichter, C. S. (1902). *The motions of underground waters* (Water Supply Paper 67). US Geological Survey.

Singhal, B., & Gupta, R. P. (2010). *Applied hydrogeology of fractured rocks*. Springer. https://doi.org/10.1007/978-90-481-8799-7↗.

Smith, L. P. (1937). Heat flow in an infinite solid bounded internally by a cylinder. *Journal of Applied Physics*, *8*(6), 441–448. https://doi.org/10.1063/1.1710319↗.

Storrow, C. S. (1835). *A treatise on water-works for conveying and distribution supplies of water*. Hilliard Gray.

Theis, C. V. (1935). The relation between the lowering of the Piezometric surface and the rate and duration of discharge of a well using ground-water storage. *Eos, Transactions American Geophysical Union, 16*(2), 519–524. https://doi.org/10.1029/tr016i002p00519↗.

Thiem, G. (1906). *Hydrologische methoden* [Hydrological methods]. Gebhardt.

Todd, D. K. (1959). *Groundwater hydrology*. John Wiley & Sons.

Todd, D. K., & Mays, L. W. (2004). *Groundwater hydrology* (3rd ed.). John Wiley & Sons.

Tolman, C. F. (1937) *Groundwater*. McGraw-Hill.

Tóth, J. (1962). A theory of groundwater motion in small drainage basins in central Alberta. Canada. *Journal of Geophysical Research*, *67*(11), 4375–4388. https://doi.org/10.1029/JZ067i011p04375↗.

Tóth, J. (1963). A theoretical analysis of groundwater flow in small drainage basins. *Journal of Geophysical Research*, *68*(16), 4795–4812. https://doi.org/10.1029/jz068i016p04795↗.

Tóth, J. (1966). Mapping and interpretation of field phenomena for groundwater reconnaissance in a prairie environment, Alberta, Canada. *International Association of Scientific Hydrology Bulletin, 11*(2), 20–68. https://doi.org/10.1080/02626666609493458↗.

Tóth, J. (1995). Hydraulic continuity in large sedimentary basins. *Hydrogeology Journal, 3*, 4–16. https://doi.org/10.1007/s100400050250↗.

United Nations. (2022). *Groundwater: Making the invisible visible* (UN World Water Development Report 2022). UNESCO. https://www.unwater.org/publications/un-world-water-development-report-2022↗.

van der Gun, J. (2022). *Large aquifer systems around the world*. The Groundwater Project. https://doi.org/10.21083/978-1-77470-020-4↗.

van der Gun, J. (2019, April 26). The global groundwater revolution. *Oxford Research Encyclopedia of Environmental Science* (pp. 1–30). Oxford University Press. https://oxfordre.com/environmentalscience/view/10.1093/acrefore/9780199389414.001.0001/acrefore-9780199389414-e-632↗.

Versluys, J. (1930). The origin of artesian pressure. *Proceedings of the Royal Academy of Science, 33*, 214–222.

Vogel, H. U. (1993). The great well of China. *Scientific American (June)*, 116–121.

Wang, J. Z., Jiang, X.-W., Wan, L., Wörman, A., Wang, H., Wang, X., & Li, H. (2015). An analytical study on artesian flow conditions in unconfined-aquifer drainage basins. *Water Resources Research, 51*(10), 8658–8667. https://doi.org/10.1002/2015WR017104↗.

Water Science School. (2019, October 7). Flowing artesian well in Brunswick, Georgia, USA [Photograph]. https://www.usgs.gov/media/images/flowing-artesian-well-brunswick-georgia-usa↗.

Whitaker, W. (1888). *Chronological list of works referring to underground water, England and Wales*. Appendix, 13th Report of the 57th Meeting of the British Association Committee appointed for the purpose of investigating the circulation of underground waters in the permeable formations of England and Wales and the quantity and character of water supplied to various towns and districts from these formations (pp. 384–414), Manchester, August/September 1887. John Murray.

Williamson, W. H. (2013). The history of hydrogeology in Australia. In N. Howden & J. Mather (Eds.), *History of hydrogeology* (pp. 7–27). Taylor & Francis. https://doi.org/10.1201/b12766↗.

Winter, T. C., Harvey, J. W., Franke, O. L., & Alley, W. M. (1998). *Ground water and surface water: A single resource* (Circular 1139). US Geological Survey. https://pubs.usgs.gov/publication/cir1139↗.

Zhang, Y. P., Jiang, X.-W., Cherry, J., Zhang, Z. Y., Wang, X. S., & Wan, L. (2022). Revisiting hydraulics of flowing artesian wells: A perspective from basinal groundwater hydraulics. *Journal of Hydrology, 609*(June), 127714. https://doi.org/10.1016/j.jhydrol.2022.127714↗.

Zhang, Z. Y., Jiang, X.-W., Wang, X. S., Wan, L., & Wang, J. Z. (2018). A numerical study on the occurrence of flowing wells in the discharge area of basins due to the upward hydraulic gradient induced wellbore flow. *Hydrological Processes, 32*(11), 1682–1694. https://doi.org/10.1002/hyp.11623↗.

Zinn, B. A., & Konikow, L. F. (2007). Effects of intraborehole flow on groundwater age distribution. *Hydrogeology Journal, 15*(June), 633–643. https://doi.org/10.1007/s10040-006-0139-8↗.

12 Boxes

Box 1 - Terms Used for Flowing Wells in 33 Textbooks

Table Box 1-1 Terms used for flowing wells in 33 textbooks by country and author(s).

Book citation	Country	Flowing well	Free flowing well	Overflowing well	Flowing artesian well	Artesian flowing well	Free flowing artesian well	Overflowing artesian well	Artesian well
Alley & Alley (2017)	USA				√				√
Batu (1998)	USA	√							
Bear (1972)	Israel	√							√
Bear (1979)	Israel	√							√
Brassington (2017)	UK								√
Dassargues (2018)	Belgium				√	√			
Davie (2008)	UK								√
de Marsily (1986)	France	√							√
Deming et al. (2002)	USA								√
Domenico & Schwartz (1998)	USA	√							√

Driscoll (1986)	USA				√				√
Fetter (2001)	USA	√			√				
Fitts (2013)	USA		√						√
Freeze & Cherry (1979)	Canada	√			√				
Heath (1983)	USA	√			√				
Hiscock (2005)	UK			√*					
Hölting & Coldewey (2019)	Germany								√
Hudak (2004)	USA	√			√				
Kasenow (2010)	USA	√				√			
Kruseman & Ridder (1990)	The Netherlands		√						√
LaMoreaux et al. (2008)	USA					√			
Lohman (1972)	USA	√			√				
Mays (2012)	USA	√							
McWhorter & Sunada (1977)	USA				√				
Nonner (2003)	The Netherlands						√		

Pinder & Celia (2006)	USA	√							√
Price (1996)	UK			√					√
Rushton (2003)	UK							√ *	√ *
Schwartz & Zhang (2003)	USA	√			√				
Sen (2015)	Turkey				√				
Singhal & Gupta (2010)	India	√							
Todd & Mays (2004)	USA	√							

The ticks with a * represent the word *well* is replaced by *borehole* in the terms.

Return to where text linked to Box 1↑

Box 2 - The Mathematical Model and Solution of a Transient Discharge Rate of Flowing Wells in Infinite Confined Aquifers

Following Jacob and Lohman (1952), the mathematical model and the solution for discharge rate are presented in this box. The mathematical model can be written as shown in Equation Box 2-1.

$$\begin{cases} T\left(\frac{\partial^2 s}{\partial r^2}+\frac{1}{r}\frac{\partial s}{\partial r}\right)=S\frac{\partial s}{\partial t} & r>r_w \\ s(r,0)=0 & r>r_w \\ s(\infty,t)=0 & t>0 \\ s(r_w,t)=s_w & t>0 \end{cases} \quad \text{(Box 2-1)}$$

where:

S = storage coefficient [-] of the confined aquifer, which equals the product of specific storage S_s (L^{-1}) and thickness of the aquifer b (L)

The solution for s is described by Equation Box 2-2.

$$s=s_w A\left(t_D,\frac{r}{r_w}\right) \quad \text{(Box 2-2)}$$

where:

$t_D = \dfrac{Tt}{S{r_w}^2}$

$$A\left(t_D,\frac{r}{r_w}\right) = 1-\frac{2}{\pi}\int_0^\infty \frac{J_0(x)Y_0\left(\frac{r}{r_w}x\right)-Y_0(x)J_0\left(\frac{r}{r_w}x\right)}{{J_0}^2(x)+{Y_0}^2(x)}\exp(-t_D x^2)\frac{dx}{x}$$

J_0 *and* Y_0 = zero-order Bessel functions of the first and second kinds, respectively

The discharge rate at the flowing well is given by Equation Box 2-3a and 2-3b.

$$Q_w=-2\pi T r_w\frac{\partial s(r_w,t)}{\partial r}=2\pi T s_w G(t_D) \quad \text{(Box 2-3a)}$$

$$Q_D=\frac{Q_w}{2\pi T s_w}=G(t_D) \quad \text{(Box 2-3b)}$$

where:

$$G(t_D) = \frac{4t_D}{\pi}\int_0^\infty x e^{-t_D x^2}\left[\frac{\pi}{2}+\arctan\left(\frac{Y_0(x)}{J_0(x)}\right)\right]dx$$

Q_D = dimensionless discharge rate, $\frac{Q_w}{2\pi T s_w}$

The infinite integral for $G(t_D)$ cannot be integrated directly thus needs to be evaluated numerically for the practical range of the parameters involved.

Return to where text linked to Box 2↥

Box 3 - The Mathematical Model of a Transient Discharge Rate of Flowing Wells in Semi-Confined Aquifers with a Flat Potentiometric Surface

Hantush (1959) derived a solution of transient discharge rate to a fully-penetrating flowing well in an infinite semi-confined aquifer as described in this box. The mathematical model can be written as shown in Equation Box 3-1.

$$\begin{cases} \frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} - \frac{s}{B^2} = \frac{S}{T}\frac{\partial s}{\partial t} & r > r_w \\ s(r,0) = 0 & r > r_w \\ s(\infty, t) = 0 & t > 0 \\ s(r_w, t) = s_w & t > 0 \end{cases} \qquad \text{(Box 3-1)}$$

where:

$$B^2 = \frac{Kbb'}{K'}$$

The solution of s is described by Equation Box 3-2.

$$\frac{s}{s_w} = \frac{K_0\left(\frac{r}{B}\right)}{K_0\left(\frac{r_w}{B}\right)} + \frac{2}{\pi}\exp\left(-t_D\frac{r_w^2}{B^2}\right)\int_0^\infty \frac{\exp(-t_D x^2)}{x^2 + \frac{r_w^2}{B^2}} \cdot \frac{J_0\left(\frac{r}{r_w}x\right)Y_0(x) - Y_0\left(\frac{r}{r_w}x\right)J_0(x)}{J_0^2(x) + Y_0^2(x)}xdx \qquad \text{(Box 3-2)}$$

where:

K_0 = zero-order modified Bessel function of the second kind

The discharge rate of the flowing well is given by Equation Box 3-3a and 3-3b.

$$Q_w = -2\pi T r_w \frac{\partial s(r_w, t)}{\partial r} = 2\pi T s_w G(t_D, r_w/B) \qquad \text{(Box 3-3a)}$$

$$Q_D = \frac{Q_w}{2\pi T s_w} = G(t_D, r_w/B) \qquad \text{(Box 3-3b)}$$

where:

$$G\left(t_D, \frac{r_w}{B}\right) = \frac{r_w}{B}\frac{K_1\left(\frac{r_w}{B}\right)}{K_0\left(\frac{r_w}{B}\right)} + \frac{4}{\pi^2}exp\left(-t_D\frac{r_w^2}{B^2}\right) \cdot \int_0^\infty \frac{x\,exp(-t_D x^2)}{J_0^2(x) + Y_0^2(x)} \cdot \frac{dx}{x^2 + \frac{r_w^2}{B^2}}$$

K_1 = first-order modified Bessel function of the second kind

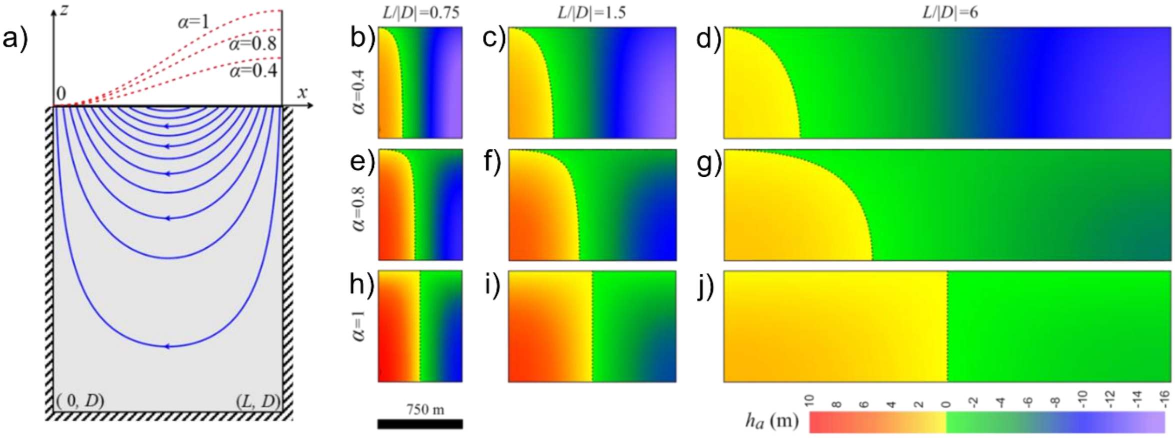
The infinite integral in function $G\left(t_D, \frac{r_w}{B}\right)$ cannot be integrated directly and should be evaluated numerically for the practical range of the parameters involved.

Return to where text linked to Box 3↥

13 Exercise Solutions

Solution Exercise 1

The example given by Wang and others (2015) is shown below.



a) The geometry of a unit basin with a length of L, depth of $|D|$, and three scenarios of water table undulations (corresponding to different α);
b) through j) The distribution of head exceeding land surface (indicated by the dashed red line of α=1 in (a)) in a unit basin under three different water table undulations and basin length/width ratios ($L/|D|$) (modified from Wang et al., 2015).

Return to Exercise 1

Return to where text links to Exercise 1

14 About the Authors

Xiao-Wei Jiang serves as Professor of Hydrogeology at China University of Geosciences (Beijing), and as one of the co-chairs of the Regional Groundwater Flow Commission, International Association of Hydrogeologists. He entered China University of Geosciences (Beijing) as an undergraduate student in 1999 and started his graduate studies in 2003 at the same university. During his PhD study, he began to focus on physics and chemistry of groundwater flow systems. He became a faculty member of China University of Geosciences (Beijing) after obtaining his PhD in 2011, and was promoted to associate professor in 2013 and to full professor in 2016. He taught such courses as hydraulics and groundwater hydraulics for undergraduate students, and the theory of groundwater flow systems for graduate students. The philosophy of informing students about how groundwater knowledge was created led him to explore the history of hydrogeology. The observations of numerous flowing wells in the field and the pursuit of answers to sources of water in flowing wells led to his research on the hydraulics of flowing wells from the perspective of groundwater flow systems. His PhD Dissertation on aquifer systems and groundwater flow systems was awarded one the 100 National Excellent PhD Dissertations of China.

John Cherry, after study in the US and post-doctoral study in France, joined the University of Waterloo in 1971 for field research on the migration and fate of contaminants in groundwater and their remediation. He co-authored *Groundwater* with R. A. Freeze (1979) and co-edited/co-authored several chapters in the book *Dense Chlorinated Solvents in Groundwater* (1996). He is the founding Director of the University Consortium for Field-focused Groundwater Contamination Research. At the G360 Centre for Groundwater Research, University of Guelph, he participates in research on groundwater monitoring technologies and creating safe wells for rural people in remote terrains. He was chair of the Canadian Expert Panel on Environmental Impacts of Shale Gas Development (2012–2014). He is a Foreign Member of the US Academy of Engineering. He received the Lee Kwan Yew Water Prize in 2016 and the Stockholm Water Prize in 2020.

Please consider signing up to the GW-Project mailing list to stay informed about new book releases, events, and ways to participate in the GW-Project. When you sign up to our email list it helps us build a global groundwater community. Sign up↗.

THE GROUNDWATER PROJECT