Transient Ground Water Hydraulics

by

Robert E. Glover

Water Resources Publications, LLC

R.E. Glover

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Water Resources Publications, LLC WRP Since 1971 Colorado

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Preface

This book has been written to present developments, formulas and methods which will be useful to engineers for making quantitative evaluations of ground water flow. Because, in irrigated country at any rate, transients will be the rule, the main emphasis will be on transient phenomena. The Dupuit-Forchheimer idealization will be the basis for most of these developments. This idealization, which is described in the text, leads to partial differential equations which are nonlinear in form. To be freed from the difficulties imposed by this nonlinearity "the basic differential equations will generally be linearized by neglecting the changes of saturated thickness which accompany transient flow conditions. This procedure will produce formulas that are, to some degree, approximations. However, if these are used skillfully with aquifer properties obtained from field tests and with awareness of their limitations they will yield results as good as can generally be obtained when application has to be made to aquifers where irregularities and nonuniformity are the rule rather than the exception. When this simplification can be used, and this will include the majority of field cases, the computations are freed of the burdensome details which appear when more elaborate treatments are made. It can be pointed out, in addition, that if the criticisms leveled at these useful approximations were to be taken seriously we should find ourselves obligated to discard the great bulk of engineering formulas used so successfully over the past 200 years since a close scrutiny of their bases will reveal short-comings as bad or worse than those outlined above.

Sometimes conditions will require that a closer evaluation be obtained than can be realized by the formulas derived from the linearized equations. For such cases second approximations or formulas derived by special methods will be supplied.

Even though the task of developing working formulas will be adhered to, parallel or alternative procedures' will be described when it appears that useful results will be obtained.

While the treatment of steady state cases will, in general, be confined to those which represent the terminal state to which a transient state being considered converges, some attention will be given to historical developments and to the steady state flow patterns which were obtained by the early workers. It would be impossible to include all of the developments obtained by previous workers in a text of the size of this one but, where material must be omitted, sufficient references will be included to indicate to the user of this text where this other material can be found.

The chapter is made the basis for organization of the material presented herein. A thumb index is provided so that the chapters may be readily located. Figure, formula and table numbers are identified by a chapter number followed by a sequence number. In this fashion Figure 3-2 is the second figure in Chapter 3.

The tables 1 to 15 inclusive have been machine computed and the reproduction has been made directly from the machine readout sheets to insure accuracy. Checks have been made, where needed, to show that the machine program was working as intended.

Data are seldom available which would permit a determination of a probable error and the number of places which would be appropriate for expressing the final result of a computation of ground water movement. In the material presented, the number of places retained has often been chosen to make the development more easily followed or to clarify comparisons. In the final results figures have been retained on the basis of judgment.

Editor's Note to the Third Printing

Transient Ground Water Hydraulics makes quantitative evaluations of ground water flow in regards to irrigation. This book has been cited and referenced in numerous papers, journals, and books continuously for over 30 years. It is still an important addition to any library that is concerned with quantitative ground water flow. WRP is pleased to re-release this book as part of our *Classic Resource Edition*.

Branka McLaughlin
Publisher/Editor

TABLE OF CONTENTS

Chapter -Index **Page**

Chapter 1 Development of ground water use 1

Chapter 2 Basic differential equations 7

Chapter 3 Well drawing water at the constant rate Q from an aquifer with a free water table 18

Chapter 4 The flowing artesian well 37

Chapter 5 Well with a semi-permeable bed overlying the aquifer 57

Chapter 6 Bank storage 73

Chapter 7 Line source 85

Chapter 8 Parallel drains 89

Chapter 9 Stream depletion due to a well 141

Chapter 10 The use of images and treatment of intermittent operation 146

Chapter 11 Effects of ground water drift 165

Chapter 12 Analogs and Digital Computers 177

Notation and definition of terms 199

Index to Tables 211

Selected References 402

Chapter 1

Development of ground water use

The knowledge that water could be obtained from the earth and that the water so obtained was generally cool, clear and sweet seems to be very ancient. Wells are mentioned frequently in Genesis indicating that the possibility of obtaining water from the ground was well understood perhaps 3500 years ago. Before modern times, wells appear to have been commonly used to supply domestic and stock water since these important uses could be supplied by muscle power which was then the only means available for raising water from them. In early day America there were many dug wells from which water was lifted by means of a bucket and windlass arrangement. These were followed by hand operated pumps. Windmills were later developed which could operate these pumps by harnessing the force of the winds. Many of these were installed not only at farmsteads but in remote locations to provide water for stock. All of these developments had for their purpose the providing of water for the important domestic and stock water uses. Development of gasoline engines gave a more reliable source of power for farm pumps and eliminated the recurring arduous task of pumping water by hand to fill stock tanks during windless periods. The old "Hit and Miss" farm engine became a common sight on farms and continued in use for many years.

Development of more effective well drilling methods, the use of casings, the extension of power lines through rural areas and the development of improved types of pumps opened up the possibility of pumping for irrigation and other heavy uses such as for municipal water supplies.

These developments went almost unnoticed for some time and then problems began to arise. In the river valleys of the arid west where the practice of diverting stream flows for irrigation had long been established it came to be realized that pumps could deplete the flow of the stream to the detriment of the surface diverters operations.

Where replenishment of the ground water reservoir comes from infiltration from precipitation, falling water tables were generally observed as pumping for irrigation developed. Whereas something like one acre foot per year would supply the domestic and stock water needs for a farmstead and the infiltration supply could sustain this demand forever, an economically viable irrigation well would need to lift this much, or more, in a day and the possibility of depletion of the resource came to be recognized. If pump irrigation develops in an area and replenishment is from precipitation a sinking of the water table is to be expected. If the comsumptive use is held within the limits imposed by the supply then the changing water table levels represent only a readjustment toward a new stable configuration but if the supply is overencumbered they have a much more serious portent.

Where municipal supplies come from ground water, the increasing demands caused by population increases can threaten a failure of the supply and cause problems of the most serious kind.

To add to the difficulties outlined above a quantitative assessment of ground water flows cannot be made by the simple procedures which suffice for surface water. An operations study for an irrigation system utilizing surface supplies will, for example, make few mathematical demands beyond the ability to add and subtract. To deal with ground water flows in a similar quantitative way will require much more specialized mathematical techniques. Here the hydraulic and geometric properties of the aquifer must be accounted for and partial differential equations of the second order may need to be solved. Fortunately, the results of these operations can generally be reduced to graphical or tabular form so that the user does not need to deal with the basic mathematical difficulties. It is the purpose of this text to provide such graphs and tables so that the quantitative assessment of ground water flows can be facilitated. It is hoped that these developments will contribute to solutions of some of the serious problems which growing uses of ground water have brought with them.

Occurrence of ground water

Rain and, in some areas, melting snow and ice has supplied over geologic times, percolating waters to build up a body of underground water which exists almost everywhere. Even desert areas receive occasional rains which will contribute deep percolating water to a ground water body. In the more well watered areas of the globe the ground water mound between stream valleys is never completely drained away but, instead, sustains the base flow of the streams so that they continue to flow even though the rains may cease for considerable periods of time.

For those interested in recovering ground water for useful purposes the principal question is not generally the presence of ground water but the presence of sufficient porosity in the formation to sustain a useful flow. For the small quantities of water needed to supply domestic and stock water uses there are vast areas where a well of this capacity can be obtained almost anywhere.

Anything like an exhaustive treatment of productive water bearing formations would be beyond the scope of this work but it will be useful to identify some of the important aquifers of the United States so that some knowledge is acquired concerning the types of formations which may contain water in recoverable amounts (McGuinness).

Coastal sands often yield important supplies of fresh water in spite of being in contact with sea water. The outward flowing fresh water, often replenished from an ample rainfall, holds the saline ocean waters in check (Glover, 1959). The Ghyben-Herzberg rule applies in such cases. This rule holds that for every foot the water table is above sea level there will be 40 feet of fresh water underneath. Extensive aquifers of this type occur along the Atlantic and Gulf coasts.

The Biscayne limestone extends inland from the east coast of Florida. Although it is strong enough to yield drill cores it is nevertheless extremely permeable and is an important source for water supply in the Miami area. Other limestones also yield water in important amounts.

The Dakota sandstone is an extensive and important water bearing formation composed, generally, of a fine sand loosely cemented by iron carbonate. This formation provides an example of a sandstone aquifer.

The Snake Plain aquifer is composed of a series of lava flows. The lavas are extensively cracked presumably due to cooling shrinkage. Permeabilities are surprisingly high. It is an important ground water source in southern and eastern Idaho. The water carrying capabilities of this aquifer are dramatically demonstrated in the "Thousand Springs" area. Here the ground water issues from a cliff in such volumes as to create waterfalls.

The Rathdrum Prairie area in Washington is underlain with an aquifer of exceptional permeability composed of glacial drift.

Many productive aquifers are to be found in alluvial deposits. Perhaps the most extensive of these is the Ogallala formation which extends across parts of Nebraska, Wyoming, Colorado, Kansas, New Mexico and Texas. It is an important source of irrigation water. Alluvial sediments are also widely distributed in river valleys.

Similar to the alluvial deposits are deposits of wind blown sands. Some of these, as the Navajo sandstone in the Utah, Arizona area have become cemented but in other cases they are represented by the sand dune areas in some of the western states. These sand dune areas possess exceptional infiltration capacities. In some areas infiltration rates are so great that no water erosion forms are to be found since even the heaviest rains produce no runoff. Some of the Nebraska rivers originating in the

Sand Hill area are notable for the uniformity of their flows. This is explainable on the basis that almost none of their flow comes from surface runoff while the bulk of it is base flow coming from ground water.

Crystalline rocks are generally unproductive not because water is absent but largely because permeabilities are so low that no useful flow to a well can be maintained. If the rock is fractured, however, as in a fault zone, then productive wells may sometimes be obtained even in formations of this type. Wells yielding around one gallon per minute can often be obtained in rock if a deep hole is drilled and the rock is fractured by pressures.

Chapter 2

Basic differential equations

The differential equations applicable to the flow of ground water are expressions, in mathematical form, of the budgetary requirement that water volumes must be conserved.

For an element of volume dx dy dz below the water table, for example, there can be no accumulation of water volume and it is then necessary to express the requirement that the net flow of water into the element must be and remain zero.

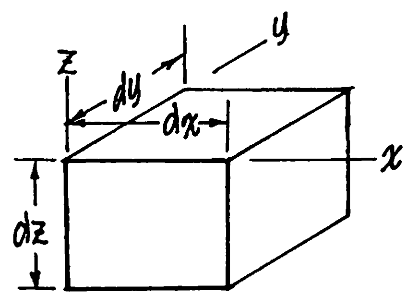


Fig. 2-1 Volume element.

Flows are computed on the basis of Darcy's law which can be expressed in the form

$$q = K\,i\,A$$

where q represents the flow through the area A in an aquifer of permeability K under the influence of the gradient i. If h_1 represents the departure of the pressure from a hydrostatic state the flow through the area $dy\,dz$ in the direction of x will be $-K\,dy\,dz\,\frac{\partial h_1}{\partial x}$ and a similar expression will apply to the flows in the y and z directions. The minus sign is needed because the flow is considered positive in the direction of positive x and if $\frac{\partial h_1}{\partial x}$ is positive the pressure gradient is such as to cause flow in the direction of negative x. The accumulation of flow into the element due to flows in the x direction is $\frac{\partial}{\partial x}(-K\,dy\,dz\,\frac{\partial h_1}{\partial x})\,dx$ or $-K\,dx\,dy\,dz\,\frac{\partial^2 h_1}{\partial x^2}$. The condition that there be no accumulation of flow in the element then takes the form

$$- K\,dx\,dy\,dz\,\frac{\partial^2 h_1}{\partial x^2} - K\,dx\,dy\,dz\,\frac{\partial^2 h_1}{\partial y^2} - K\,dx\,dy\,dz\,\frac{\partial^2 h_1}{\partial z^2} = 0$$

or, simply,

$$\frac{\partial^2 h_1}{\partial x^2} + \frac{\partial^2 h_1}{\partial y^2} + \frac{\partial^2 h_1}{\partial z^2} = 0 \qquad (2\text{-}1)$$

This is known as the Laplace formulation, not because Laplace considered the movement of ground water but because the differential equation obtained is the one which bears his name. Many steady-state solutions have been obtained for it but its use for transient cases has been limited due to the difficulty of dealing with a moving boundary. The formulation is basically sound, however, for both transient and steady state ground water flows. If the moving boundary difficulties could be overcome it would afford a means for overcoming the difficulties caused by nonlinearities which afflict some of the other formulations. Some progress has been made in this direction and more may be expected.

A mathematically advantageous formulation was proposed by Dupuit (1863). He considered an element penetrating the full depth of the aquifer, as shown in the figure below, for flow in the direction of x only

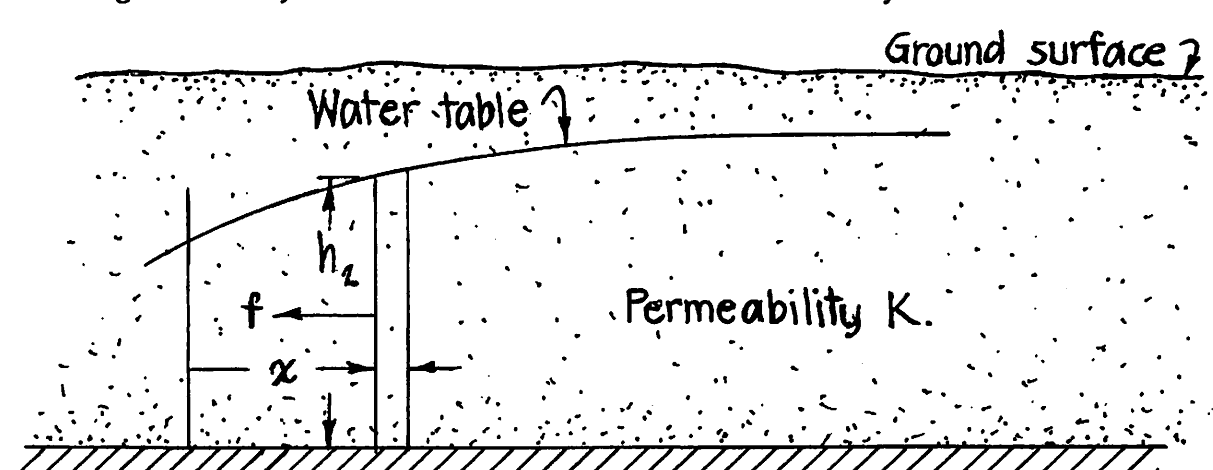


Fig. 2-2 Cross section.

Here the symbol h_2 is used to represent the saturated depth above a level impermeable lower boundary. The flow f through the depth h_2 for a unit width is, if the surface gradient can be considered effective throughout the entire saturated depth, $f = K h_2 \frac{\partial h_2}{\partial x}$. The accumulation of flow in the element during the time dt is $\frac{\partial f}{\partial x} dt$ or $K \frac{\partial}{\partial x} (h_2 \frac{\partial h_2}{\partial x})\, dx\, dt$. The condition that the accumulation of flow in the element must be consistent with the rate of rise of the water table is

$$K \frac{\partial}{\partial x} (h_2 \frac{\partial h_2}{\partial x})\, dx\, dt = V \frac{\partial h_2}{\partial t}\, dx\, dt$$

where V represents the effective voids ratio appropriate for the aquifer. After simplification the relation becomes:

$$K \frac{\partial}{\partial x} (h_2 \frac{\partial h_2}{\partial x}) = V \frac{\partial h_2}{\partial t}$$

It will be noted at once that this is a differential equation of nonlinear form. If a steady state exists the right hand member will be zero and the formulation will then assume the form:

$$\frac{\partial}{\partial x} (h_2 \frac{\partial h_2}{\partial x}) = 0$$

Even though this is nonlinear, solutions are readily obtained. The early workers in the ground water field generally limited their researches to steady state cases.*

Linearization by neglect of small terms

If the transient case be again considered the known intractable nature of nonlinear differential equations will make it desirable to find some way

*Boussinesq, 1904 treated the case of ground water flow to streams on a transient basis.

of linearizing them. The simplest way of doing this is to consider small departures h from an initial saturated depth, as shown in the figure below.

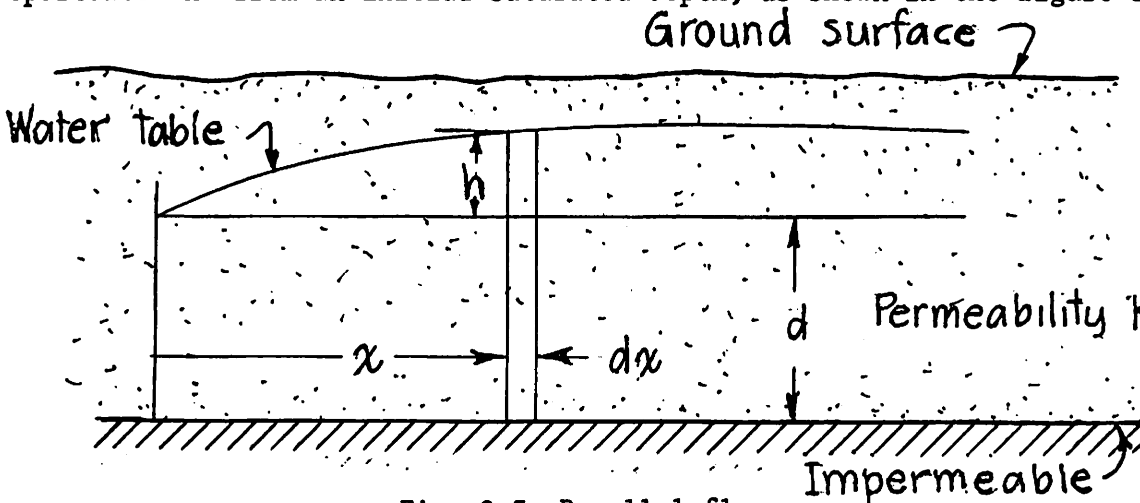


Fig. 2-3 Parallel flow.

By following the reasoning previously used the differential equation for the transient case will take the form:

$$K \frac{\partial}{\partial x} (d+h) \frac{\partial h}{\partial x} = V \frac{\partial h}{\partial t}$$

Linearization can now be obtained, at the expense of approximation, by neglecting h in the term (d + h). This means that the use of any solutions obtained must be restricted to cases where, in fact, h is small compared to d. Experience indicates that this will be a permissible simplification in the great majority of field applications. If the simplification is acceptable the differential equation can be put into the form:

$$\alpha \frac{\partial^2 h}{\partial x^2} = \frac{\partial h}{\partial t} \qquad (2\text{-}2)$$

Where $\alpha = \frac{Kd}{V}$ is the aquifer constant. It is a factor which must be determined by test. Methods of evaluation will be described later in the text. When this simplification is used an important tactical advantage is secured. The differential equation becomes of the type which has been so extensively studied in relation to the conduction of heat in solids. Not only is the differential equation of the same form but the boundary conditions are also analogous. This means that the wealth of material found in this older discipline becomes immediately available for application in the new field. It will be found that this parallelism extends to the important radially symmetrical cases also. It will appear later that the formulas obtained with this linearization are of simpler form and better adapted to engineering applications than those obtained by any other approach. The many advantages offered by this linearization will support a decision to adopt it as the source of the basic system of formulas presented in this text. In the rare cases, where their limitations hinder their use, other more appropriate solutions will be presented to take their place. When this happens complexities may be expected to appear. In some cases these will take the form of new parameters which must be evaluated and applied. In all cases water quantities will be accounted for without approximation.

The linearization of Werner

In the differential equation

$$K \frac{\partial}{\partial x}\left(h_2 \frac{\partial h_2}{\partial x}\right) = V \frac{\partial h_2}{\partial t}$$

The substitution of variable

$$h_2^2 = u$$

is introduced (Werner 1957).

The differential equation is reduced by this means to the form:

$$\frac{K\sqrt{u}}{V} \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$$

Linearization is now accomplished by replacing $\sqrt{u} = h_2$ by some carefully chosen constant representative value H.

If we now set

$$\alpha_w = \frac{KH}{V}$$

The differential equation becomes:

$$\alpha_w \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} \qquad (2\text{-}3)$$

<u>Radial symmetry</u> - Laplace type of formulation

Operation of a well can produce a flow of ground water with radial symmetry as shown in the figure below

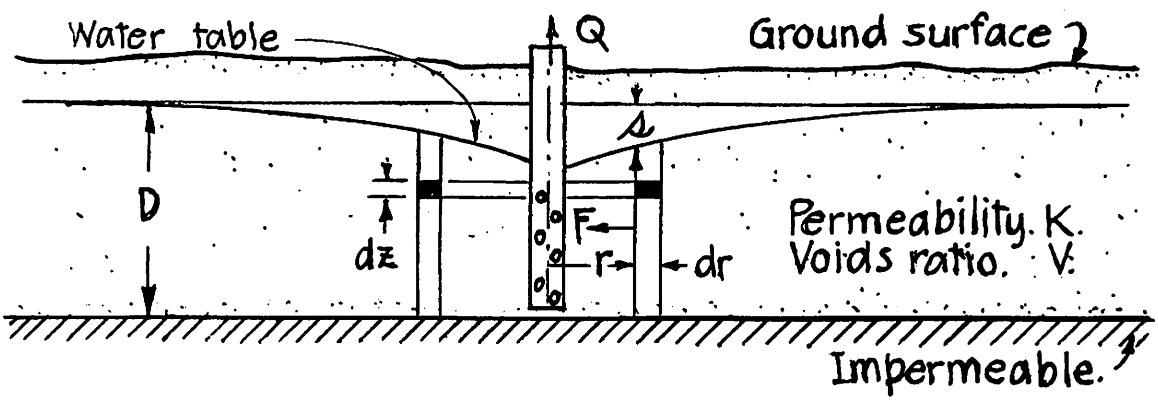


Fig. 2-4 Radial flow.

The flow of ground water into a submerged ring shaped element of volume of the aquifer of cross section dr dz and radius r will be

$$\frac{\partial}{\partial z}(2\pi rKdr\frac{\partial h}{\partial z})dz + \frac{\partial}{\partial r}(a\pi rKdz\frac{\partial h}{\partial r})dr = 0$$

In this expression h represents the departure of the pressure from hydrostatic. The continuity condition which prohibits the accumulation of water in the element dictates the requirement that the sum of these two flows must be zero. After the indicated differentiations are performed and the expression is simplified by dividing out common terms it takes the form:

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} + \frac{\partial^2 h}{\partial z^2} = 0 \qquad (2\text{-}4)$$

This is the Laplace type of formulation where radial symmetry is present.

Radial symmetry - Dupuit-Forchheimer formulation

The flow F through the cylindrical surface of radius r and height (D - s) will be, under the Dupuit-Forchheimer idealization:

$$F = -2\pi r K(D-s) \frac{\partial s}{\partial r}$$

The accumulation of flow in the annulus of dr thickness will be:

$$\frac{\partial F}{\partial r} dr = -2\pi K \frac{\partial}{\partial r}[r(D-s) \frac{\partial s}{\partial r}]dr$$

If, to avoid nonlinearity, s is neglected as being small compared to D then approximately:

$$\frac{\partial F}{\partial r} dr = -2\pi KD \frac{\partial}{\partial r} (r \frac{\partial s}{\partial r}) dr$$

The condition of continuity is then expressible in the form:

$$\frac{\partial F}{\partial r} dr = -2\pi r V dr \frac{\partial s}{\partial t}$$

or after substitution and rearrangement with:

$$\alpha = \frac{KD}{V}$$

it becomes

$$\alpha \frac{\partial}{\partial r} (r \frac{\partial s}{\partial r}) = r \frac{\partial s}{\partial t}$$

or

$$\alpha(\frac{\partial^2 s}{\partial r^2} + \frac{1}{r} \frac{\partial s}{\partial r}) = \frac{\partial s}{\partial t} \qquad (2\text{-}5)$$

This is the linearized differential equation obtained, for radial symmetry conditions, when s is neglected as being small compared to D.

A similar procedure is described in Chapter XIII of the text by Polubarinova-Kochina.

Comparisons of results obtained by these alternative methods will be found later in the text.

Chapter 2.

Problems

(2-1) For steady state conditions $(\partial s/\partial t) = 0$ and formula 2-5 takes the form:

$$\frac{d^2s}{dr^2} + \frac{1}{r}\frac{ds}{dr} = 0 \qquad \text{or} \qquad \frac{d}{dr}\left(r\,\frac{ds}{dr}\right) = 0$$

derive from this a formula for the draw down s, as a function of r, due to pumping of a well, of radius a, at the rate Q when the water table is maintained at the level D when $r = b$. Consider s as negligibly small as compared to D.

Answer:

$$s = \frac{Q}{2\pi KD}\log_e\left(\frac{b}{r}\right)$$

(2-2) Compare this result with that obtained by Dupuit (Rouse and Ince, 1963) and comment on differences or similarities.

(2-3) In the relation $F = -2\pi Kr(D-s)\frac{ds}{dr}$,

presented in the account of the Dupuit-Forchheimer formulation set $F = Q$ and $h_2 = (D-s)$ to obtain

$$Q = 2\pi Krh\frac{dh_2}{dr}$$

Treat this as a nonlinear differential equation and derive a formula for the variation of saturated depth h_2 with r when the water table is maintained at the depth $h_2 = D$ at $r = b$.

Answer:

$$D^2 - {h_2}^2 = \frac{Q}{\pi K}\log_e\left(\frac{b}{r}\right)$$

(2-4) Compare this result with the second form obtained by Dupuit (Rouse and Ince, 1963) and comment on differences or similarities.

(2-5) By using an idealization of the Dupuit-Forchheimer type, as shown in figure 2-3, derive an expression for the steady state form of the water table where a uniform infiltration rate $\dot{i}$ is applied to a strip drained by parallel drains spaced a distance L apart. Note that the continuity condition will require that, if h is considered negligibly small compared to d.

$$Kd\,\frac{dh}{dx} = \dot{i}\left(\frac{L}{2} - x\right)$$

Answer:

$$h = \frac{\dot{i}L^2}{8Kd} - \frac{\dot{i}}{2Kd}\left(\frac{L}{2} - x\right)^2 \qquad \text{At } x = \frac{L}{2} \qquad h_m = \frac{\dot{i}L^2}{8Kd}$$

(2-6) With the conditions shown on figure 2-2 and with $f = i\left(\frac{L}{2} - x\right)$ express the continuity condition and solve the resulting nonlinear differential equation to obtain the variation of h_2 with respect to x, if $h_2 = d$ when $x = 0$ and when $x = L$.

Answer:

$$\frac{h_2^{\,2}}{2} - \frac{d^2}{2} = \frac{iL}{2K}\left(x - \frac{x^2}{L}\right)$$

At $x = \frac{L}{2}$

$$(h_2 - d)_c = d\left[\sqrt{1 + \frac{iL^2}{4Kd^2}} - 1\right]$$

Approximately, at $x = L/2$:

$$(h-d)_c \cong \frac{iL^2}{8Kd} \qquad \text{if } \frac{iL^2}{4Kd^2} \text{ is small compared to unity.}$$

(2-7) Compare the value obtained from (2-5) for $(h+d)$ with the value obtained from (2-6) for h_2 at the point $x = L/2$, which is the point midway between drains. (Suggestion: Develop the radical into a series by use of the binomial theorem on the basis that $(iL^2/4Kd^2)$ is small compared to unity).

Answer:

To a first approximation the results of (2-5) and (2-6) agree.

Chapter 3

Well drawing water at the constant rate Q from an aquifer with a free water table

To treat this case it will be sufficient to solve the linearized differential equation:

$$\alpha \left(\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r}\right) = \frac{\partial s}{\partial t}$$

Subject to the initial and boundary conditions

$$s = 0 \quad \text{when} \quad t = 0 \quad \text{for} \quad r > 0$$

$$-\,2\pi r KD \frac{\partial s}{\partial r} \to Q \quad \text{as} \quad r \to 0$$

The solution, so obtained, will serve as an effective approximation so long as s is small compared to D .

The Boltzman variable, used in the form:

$$z = \frac{r}{\sqrt{4\alpha t}}$$

may be applied to obtain the desired solution. With:

$$\frac{\partial s}{\partial r} = \frac{\partial s}{\partial z}\frac{1}{\sqrt{4\alpha t}} \qquad \frac{\partial s}{\partial t} = -\,\frac{\partial s}{\partial z}\frac{2\alpha r}{(4\alpha t)^{3/2}}$$

$$\frac{\partial^2 s}{\partial r^2} = \frac{\partial^2 s}{\partial z^2}\frac{1}{4\alpha t}$$

Substitution and simplification will produce the ordinary differential equation

$$\frac{d^2 s}{dz^2} + \left(\frac{1}{z} + 2z\right)\frac{ds}{dz} = 0 \,.$$

Let $p = \frac{ds}{dz}$. Then the equation may be reduced to one of the first order.

$$\frac{dp}{dz} + \left(\frac{1}{z} + 2z\right) p = 0 \,.$$

An integrating factor is:

$$e^{\int\left(\frac{1}{z} + 2z\right)dz} = e^{\log_e z + z^2} = z e^{z^2}$$

Then a solution is:

$$pze^{z^2} = C$$

And

$$\frac{ds}{dz} = \frac{Ce^{-z^2}}{z}$$

By integration, if $s = 0$ when $t = 0$

$$s = C\int_{\left(\frac{r}{\sqrt{4\alpha t}}\right)}^{\infty} \frac{e^{-z^2}dz}{z}$$

The flow toward the well is:

$$F = -2\pi KDr\frac{\partial s}{\partial r} = + 2\pi KDCe^{-\frac{r^2}{4\alpha t}}$$

If $F \to Q$ as $r \to 0$ $\quad C \to \left(\frac{Q}{2\pi KD}\right)$

Then, finally, the solution sought is; with the variable of integration changed to u to correspond to the usage of Table 1,

$$s = \frac{Q}{2\pi KD}\int_{\left(r/\sqrt{4\alpha t}\right)}^{\infty} \frac{e^{-u^2}du}{u} \qquad (3\text{-}1)$$

An example to illustrate the use of this formula can be made to serve two purposes. A computation of drawdowns produced by pumping a well can first be made. Then, by considering the computed drawdowns to be observed values, the process of determining aquifer properties from pump test data can be presented. An advantage is conferred by going about the presentation in this way because we will already know what the outcome should be and we will thus be in position to evaluate the effectiveness of the method. The following conditions will be assumed to prevail:

Flow of well Q = 750 gallons per minute

Permeability K = 0.00384 ft/sec D - 66.6 feet

Transmissibility KD = 0.2557 ft^2/sec V = 0.171

Aquifer constant $\alpha = \frac{KD}{V}$ = 1.50 ft^2/sec

In consistent units of feet and seconds:

$Q = 1.6710\ ft^3/sec \qquad (Q/2\pi KD) = 1.0401$

Computations of the drawdowns s for various distances and times are made as shown below. Values of $s/(Q/2\pi KD)$ are obtained from Table 1.

	r (feet)				
Time	10	50	100	500	1000
1 day 86400 sec					
$r/\sqrt{4\alpha t}$	0.01389	0.06945	0.1389	0.6945	1.3890
$s/(Q/2\pi KD)$	3.988	2.381	1.695	0.2909	0.02697
s (feet)	4.148	2.476	1.763	0.302	0.028
1 week 604800 sec					
$r/\sqrt{4\alpha t}$	0.005250	0.02625	0.05250	0.2625	0.5250
$s/Q/2\pi KD)$	4.9609	3.3518	2.6597	1.0828	0.4846
s (feet)	5.160	3.486	2.766	1.126	0.504
1 month 2628000 sec					
$r/\sqrt{4\alpha t}$	0.002518	0.01259	0.02518	0.1259	.2518
$s/(Q/2\pi KD)$	5.6957	4.0871	3.3935	1.7916	1.1217
s (feet)	5.924	4.251	3.530	1.863	1.167
4 months 10512000 sec					
$r/\sqrt{4\alpha t}$	0.001259	0.006296	0.01259	0.06296	0.1259
$s/(Q/2\pi KD)$	6.3888	4.7793	4.0871	2.4786	1.7916
s (feet)	6.645	4.971	4.251	2.578	1.863

Attention may now be turned to the problem of determining aquifer properties from field test data. The computed drawdowns from the above tabulation may now be construed as data obtained from observation wells from which aquifer data are to be derived. The first step is to prepare a plot such as is shown on figure 3-1. To be suitable, the grid must be logarithmic both ways. Such a chart is plotted from data obtained from Table 1.

The test data are plotted on a similar sheet with the observed drawdowns s as ordinates and the quantity $(r/\sqrt{t})$ as abscissa. The fact that use of the Boltzman variable succeeds in converting the partial differential equation 2-5 to an ordinary differential equation will insure that all of the plotted points will fall on a single line irrespective of

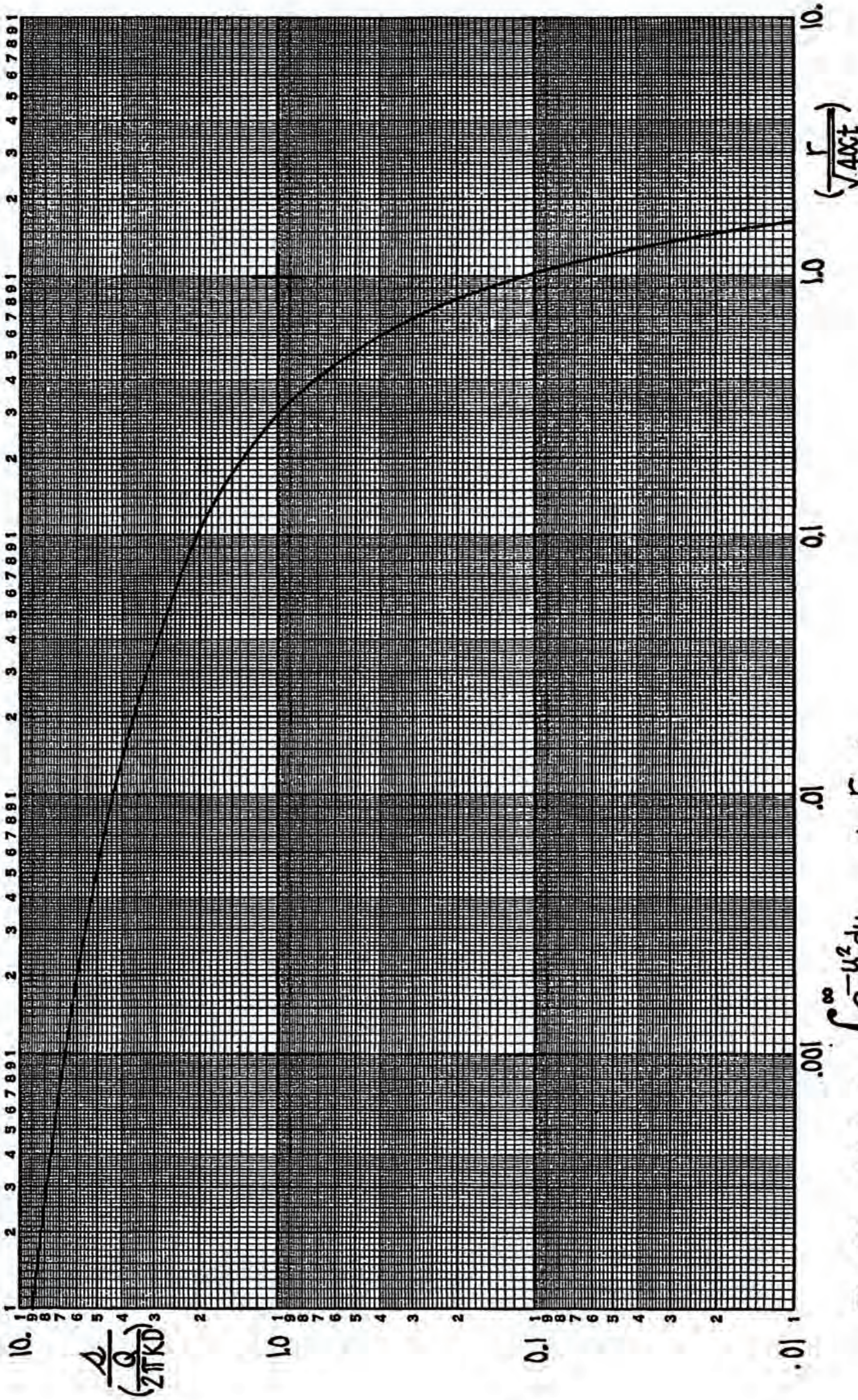


Fig. 3.1 Plot of: $\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}du}{u}$ vs $\left(\frac{r}{\sqrt{4\alpha t}}\right)$

radius and time so long as these two variables occur in the form $(r/\sqrt{t})$. Such a plot is shown in figure 3-2.

The sheet of figure 3-1 is now placed over the plot of figure 3-2 and the two are adjusted, while keeping the axes parallel, until the observed points of figure 3-2 coincide everywhere with the curve of figure 3-1. The position of the cross, at the point 1.0 - 1.0 of figure 3-1 is now marked on figure 3-2.

It may have been noted that, while the parameters of figure 3-1 are pure numbers, the parameters of figure 3-2 are not. Since both scales are logarithmic a shift will represent a multiplication and the missing units can appear in the factor represented by the shift. The following relations are now available:

$$\frac{s}{(Q/2\pi KD)} = 1.00 \qquad s = 1.02$$

$$\frac{r}{\sqrt{4\alpha t}} = 1.00 \qquad \frac{r}{\sqrt{t}} = 2.40 \qquad \frac{Q}{2\pi KD} = 1.04$$

From the first of these:

$$\frac{1.02}{\left(\frac{Q}{2\pi KD}\right)} = 0.981 \qquad KD = \frac{Q(0.981)}{(1.02)(2\pi)} = \frac{(1.671)(0.981)}{6.409} = 0.2588\ ft^2/sec.$$

From the second pair of values, by substitution:

$$\frac{2.40}{\sqrt{4\alpha}} = 0.980 \qquad \alpha = 1.499\ ft^2/sec$$

The values obtained, 0.2558 and 1.499 should be, as is known, 0.256 and 1.50, respectively. The errors are less than one percent. The effective voids ratio is:

$$V = \frac{KD}{\alpha} = \frac{0.2558}{1.499} = 0.171$$

This figure corresponds to $V = 0.171$ as used in the original computation.

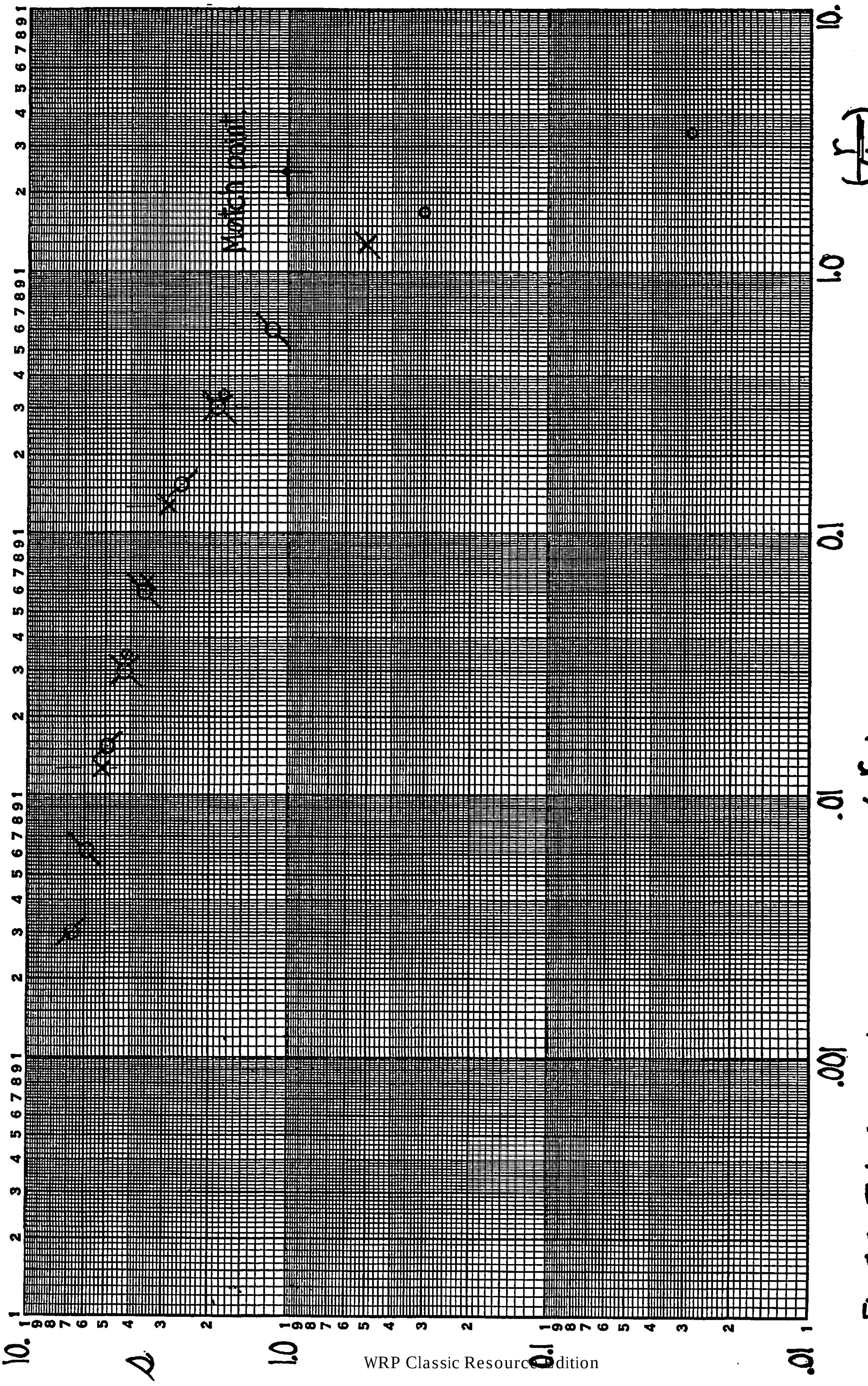


Fig. 3.2 Plot of drawdowns s vs: $(r/\sqrt{t})$.

Some justification for using the point 1.0 - 1.0 of figure 3-1 as a match point needs to be made. The logic of the procedure would be more readily understood if we used a selected one of the observed points of figure 3-2 as the basis of the computation. If the validity of such a procedure be granted, then it may be seen that so long as the observed points of figure 3-2 are adjusted to agree with the curve of figure 3-1, the match point can be shifted to the convenient position 1.0 - 1.0 because the shift represents multiplication by the same factor on both charts. In subsequent work this factor will divide out.

Some scatter is to be expected in a set of points representing field observations because of aquifer irregularities. The presence of impermeable boundaries or of streams where the water level is maintained will also cause irregularities. The "Theory of Aquifer Tests" as treated in U.S.G.S. Water Supply Paper 1536-E of 1962, will supply additional details.

The case where the drawdown s is not everywhere small compared to the initial saturated depth D has been treated by Glover and Bittinger. They obtain a second approximation, based upon Dupuit-Forchheimer concepts, by accounting, approximately, for the restriction of the area available for flow of ground water as a result of drawdown. The relationship obtained is:

$$\frac{y}{\left(\frac{Q}{2\pi KD}\right)} = \frac{1}{\sigma}\left(1-\sqrt{1-2\sigma\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}du}{u}}\right) \tag{3-2}$$

Where:

$$\sigma = \frac{Q}{2\pi KD^2} \tag{3-3}$$

Table 2 has been prepared to facilitate the use of this relationship. An example of its application will be given later in the section on the effect of pumping over an area.

A development of much interest is that of N. S. Boulton who attacks the problem, of estimating drawdowns due to pumping a well, by utilizing the Laplace type of formulation as a basis for deriving formulas.

Water table maintained at $r = b$.

Constant pumping rate Q .

For the ultimate steady state, the flow everywhere must be Q , then,

$$-2\pi r KD \frac{ds}{dr} = Q$$

or

$$\frac{ds}{dr} = \frac{-Q}{2\pi KD}\frac{1}{r}$$

By integration, if $s = 0$ when $r = b$,

$$s = \frac{Q}{2\pi KD} \log_e\left(\frac{b}{r}\right)$$

To meet the initial condition that $s = 0$ for $t = 0$ for $0 < r < b$ use a Fourier-Bessel series of the type

$$s_1 = \sum_{n=1}^{n=\infty} A_n J_o(\beta_n r) e^{-\frac{(\beta_n b)^2 \alpha t}{b^2}}$$

Which is a solution of equation 2-5.

The A_n values are to be obtained from the relation

$$\int_0^b \frac{Q}{2\pi KD} \log_e\left(\frac{b}{r}\right) J_o(\beta_n r)\, r\, dr = A_n \int_0^b J_o^2(\beta_n r)\, r\, dr .$$

or

$$A_n = \frac{\int_0^b \frac{Q}{2\pi KD} \log_e\left(\frac{b}{r}\right) J_o(\beta_n r)\, r\, dr}{\int_0^b J_o^2(\beta_n r)\, r\, dr}$$

Let

$$w = \beta_n r \qquad dr = \frac{dw}{\beta_n} \qquad r = \frac{w}{\beta_n}$$

Then:

$$\int_0^b J_o^2(\beta_n r)\; r\, dr = \frac{1}{\beta_n^2} \int_0^{\beta_n b} J_o^2(w)\; w\, dw = \frac{\beta_n^2 b^2}{\beta_n^2 2} \left[J_1^2(w) + J_o^2(w)\right]_0^{\beta_n b}$$

$$= \frac{b^2}{2} \left[J_1^2(\beta_n b) + J_o^2(\beta_n b)\right]$$

$$\int_0^b \frac{Q}{2\pi KD} \log_e\left(\frac{b}{r}\right) J_o(\beta_n r)\; r\, dr = \frac{Q}{2\pi KD} \int_0^b \log_e\left(\frac{b}{r}\right) J_o(\beta_n r)\; r\, dr\;.$$

By parts, let:

$$u_1 = \log_e\left(\frac{b}{r}\right) \qquad dv_1 = r\, J_o(\beta_n r)\; dr$$

$$du_1 = -\frac{dr}{r} \qquad v_1 = \frac{r\, J_1(\beta_n r)}{\beta_n}$$

Then the last of these integrals is to be evaluated in the following way. Note that the first term vanishes at both limits.

$$\frac{Q}{2\pi KD} \int_0^b \log_e\left(\frac{b}{r}\right) J_o(\beta_n r)\; r\, dr = \frac{Q}{2\pi KD} \frac{rJ_1(\beta_n r)}{\beta_n} \log_e\left(\frac{b}{r}\right)$$

$$\left. + \frac{Q}{2\pi KD} \int \frac{rJ_1(\beta_n r)\; dr}{\beta_n r} \right]_o^b$$

$$= \frac{Q}{2\pi KD} \left. \frac{-J_o(\beta_n r)}{\beta_n^2} \right]_0^b = \frac{Q}{2\pi KD} \frac{1}{\beta_n^2} \left[1 - J_o(\beta_n b)\right]$$

And

$$A_n = \frac{Q}{2\pi KD} \frac{2\left[1 - J_o(\beta_n b)\right]}{(\beta_n b)^2 \left[J_o^2(\beta_n b) + J_1^2(\beta_n b)\right]}$$

Since the boundary conditions will require that $J_o(\beta_n b) = 0$ this expression reduces to:

$$A_n = \frac{Q}{2\pi KD} \frac{2}{(\beta_n b)^2 J_1^2(\beta_n b)} \qquad (3\text{-}4)$$

Then the final form of the solution is:

$$s = \frac{Q}{2\pi KD}\left[\log_e\left(\frac{b}{r}\right) - \sum_{n=1}^{n=\infty} \frac{2J_o(\beta_n r)e^{-(\beta_n b)^2\left(\frac{\alpha t}{b^2}\right)}}{(\beta_n b)^2 J_1^{\,2}(\beta_n b)}\right] \qquad (3\text{-}5)$$

This function may be evaluated by use of Table 6.

Impermeable outer boundary present

The developments described at the beginning of this chapter apply where the outer boundary of the aquifer, or reservoir if the flow is of oil, is so remote that the disturbance created by the well has not had time to reach the outer boundary. All real aquifers are of finite extent, however, and if the removal of water or oil continues long enough the effect of the outer boundary must become apparent. If the actual case can be idealized as being circular, of outer radius b , with a well at the center in the form of a line sink flowing at the rate Q , then the required solution of the differential equation;

$$\alpha_1\left(\frac{\partial^2 y}{\partial r^2} + \frac{1}{r}\frac{\partial y}{\partial r}\right) = \frac{\partial y}{\partial t}$$

is;

$$\frac{y}{\left(\frac{Q}{2\pi KD}\right)} = \left[\log_e\left(\frac{b}{r}\right) + \frac{r^2}{2b^2} - \frac{3}{4} + \frac{2\alpha_1 t}{b^2} - \sum_{n=1}^{n=\infty} \frac{2J_o(\beta_n r)e^{-(\beta_n b)^2\left(\frac{\alpha_1 t}{b^2}\right)}}{(\beta_n b)^2 J_o{}^2(\beta_n b)}\right] \qquad (3\text{-}6)$$

Table 7 contains values of $\frac{y}{\left(\frac{Q}{2\pi KD}\right)}$ for values of $\left(\frac{\alpha_1 t}{b^2}\right)$. The postulates used here are the same as for the flowing artesian well development. It is assumed that the fluid occupies interstices in a porous bed sandwiched between impermeable members and that a pressure is originally present. Under the original conditions the assumption is that a part of the weight of the overburden is carried by this fluid pressure and that when this pressure is relieved by operation of a well the load that it originally carried is shifted to the grain structure which is compressed as a result. The quantity V_1 then represents the quantity of fluid squeezed out of the interstices of a column of the permeable bed having a unit horizontal area and a height D when the pressure on the column is increased by the pressure equivalent to that exerted by a one foot depth of water.

Example

A case of oil flow will be considered. Bottom hole oil pressures will be expressed in equivalent feet of water in conformity with the usages adopted herein. Laboratory determinations of permeability will be expressed in terms of the permeability for water and the permeabilities for other fluids obtained by applying a ratio of viscosities. Then the permeability for oil will be

$$K_o = \frac{K_w \mu_w}{\mu_o}$$

In this manner the units required will be reduced to two; a length, which will be expressed in feet and a time unit which will be expressed in days.

In the first part of the example pressure drawdowns will be computed for a set of assumed well test conditions and in the second part the method of estimating the reservoir properties from the test data will be shown.

As a basis for the computations assume that $a = 7.5$ feet , $b = 750$ feet , $D = 11.0$ feet , $\mu_o = 1.25$ centipoises. Bottom hole pressure with the well shut in is 1500. lb/in^2. From laboratory tests on a sample of the oil sand it has been determined that the porosity ϕ , representing the ratio of the volume of oil in the sample to the gross volume of the sample, is 0.12 and that the compressibility C_t , expressed as the reduction in volume under a pressure of one pound per square inch, to the initial volume is 18.5×10^{-6}. The permeability K_w for water at a temperature such that the viscosity μ_w is 1.0 centipoise is found to be 0.150 ft/day. Then,

$$K_o = \frac{(0.150)(1.00)}{1.250} = 0.120\left(\frac{ft}{day}\right) . \quad K_o D = (0.120)(11.0) = 1.320\left(\frac{ft^2}{day}\right)$$

Bottom hole pressure is (1500)(2.307) = 3460. feet of water.

$$C_w = 18.5(10)^{-6}/2.307 = 8.0 \times 10^{-6} \text{ per foot of water.}$$

$$V_1 = (0.12)(8.0)(10)^{-6}(11.0) = 10.560(10)^{-6} \qquad \text{(Dimensionless)}$$

$$\alpha_1 = \frac{K_oD}{V_1} = \frac{(1.320)(10)^6}{10.560} = 125000.(\frac{ft^2}{day})$$

$$b^2 = 750^2 = 562500.\ ft^2$$

Flow rate; 400 barrels per day or 2246 cubic feet per day. A barrel, as used here, is 42 gallons or 5.615 cubic feet.

$$\frac{Q}{2\pi K_oD} = \frac{2246.}{(6.283)(1.320)} = 270.812 \text{ feet.}$$

The effective radius is 7.5 feet. Then $(\frac{a}{b}) = 0.010$.

Computation of the pressure drawdown at the well is made in the manner shown below. Note that:

$$\frac{\alpha_1 t}{a^2} = \frac{\alpha_1 t}{b^2}\frac{b^2}{a^2} . \qquad (\frac{\alpha_1}{b^2}) = \frac{125000}{562500} = 0.222222$$

Table 1 Computation of pressure reduction at a = 7.5 ft.

Time (minutes)	Time (Days)	$(\frac{\alpha_1 t}{b^2})$	$\underline{y}$* $(\frac{Q}{2\pi K_oD})$	$(\frac{4\alpha_1 t}{b^2})$	$(\frac{a}{4\alpha_1 t})$	$\int_{(\frac{a}{\sqrt{4\alpha_1 t}})}^{\infty} \frac{e^{-u^2}}{u} du$ **	y (feet)
0	0	0		0	∞	0	0
10	.00694	.001542		.006168	.1273	1.780	482.0
20	.01389	.003087		.012348	.0902	2.120	574.1
30	.02083	.004629		.018516	.0735	2.324	629.4
60	.04167	.009260		.037040	.0520	2.669	722.8
90	.06250	.013889	2.842	.055556	.0424	2.871	777.5
120	.08333	.018518	3.002				813.0
150	.10417	.023149	3.116				843.8
180	.12500	.027778	3.210				869.3
240	.16667	.037037	3.358				909.4
360	.25000	.055556	3.561				964.6
720	.50000	.111111	3.910				1058.9
1080	.75000	.166667	4.115				1143.4
1440	1.00000	.222222	4.266				1155.3
1800	1.25000	.277778	4.395				1190.2
2160	1.50000	.333333	4.515				1222.7
2520	1.75000	.388889	4.628				1253.3
2880	2.00000	.444444	4.742				1284.2

*From Table 7.

**From Table 1.

A reference to Table 7 will show that the disturbance caused by production does not reach the outer boundary at $(r/b) = 1$ until $(\alpha_1 t/b^2) = 0.02$. For values of $(\alpha_1 t/b^2)$ smaller than this the appropriate values of $y/(Q/2\pi K_o D)$ can be obtained from Table 1, as shown above. Note that in this case

$$\frac{y}{\left(\frac{Q}{2\pi K_o D}\right)} = \int_{\frac{r_w}{\sqrt{4\alpha_1 t}}}^{\infty} \frac{e^{-u^2} du}{u}$$

Determination of reservoir properties from drawdown test data.

As a first step, prepare a master chart from Tables 7 and 1 showing curves for $y/(Q/2\pi K_o D)$ versus $(\alpha_1 t/b^2)$ for a series of ratios of (r/b) as shown on figure 3-3. Plot the drawdowns y from the above table versus t , treating them as test values, as shown on figure 3-4. Then match the observed y versus t curve against one of the curves of the master chart. It will be found to fit with the curve for $(r/b) = 0.01$. A selected data point and the corresponding match point on the master chart are indicated by crosses. Corresponding to the selected data point $y = 1058.9$ is the point

$$\frac{y}{\left(\frac{Q}{2\pi K_o D}\right)} = 4.0 \ .$$

Since y and Q are both known quantities this can be solved for $K_o D$ then

$$K_o D = \frac{4.0Q}{2\pi y} = \frac{(4.0)(2246)}{(6.283)(1058.9)} = 1.350 \ (ft^2/day)$$

This should be 1.320 ft^2/day as this is the value we began with.

$$\alpha_1 = \frac{K_o D}{V_1} = \frac{(1.350)(10)^6}{10.560} = 127840. \ (ft^2/day)$$

We know this should be 125000 ft^2/day.

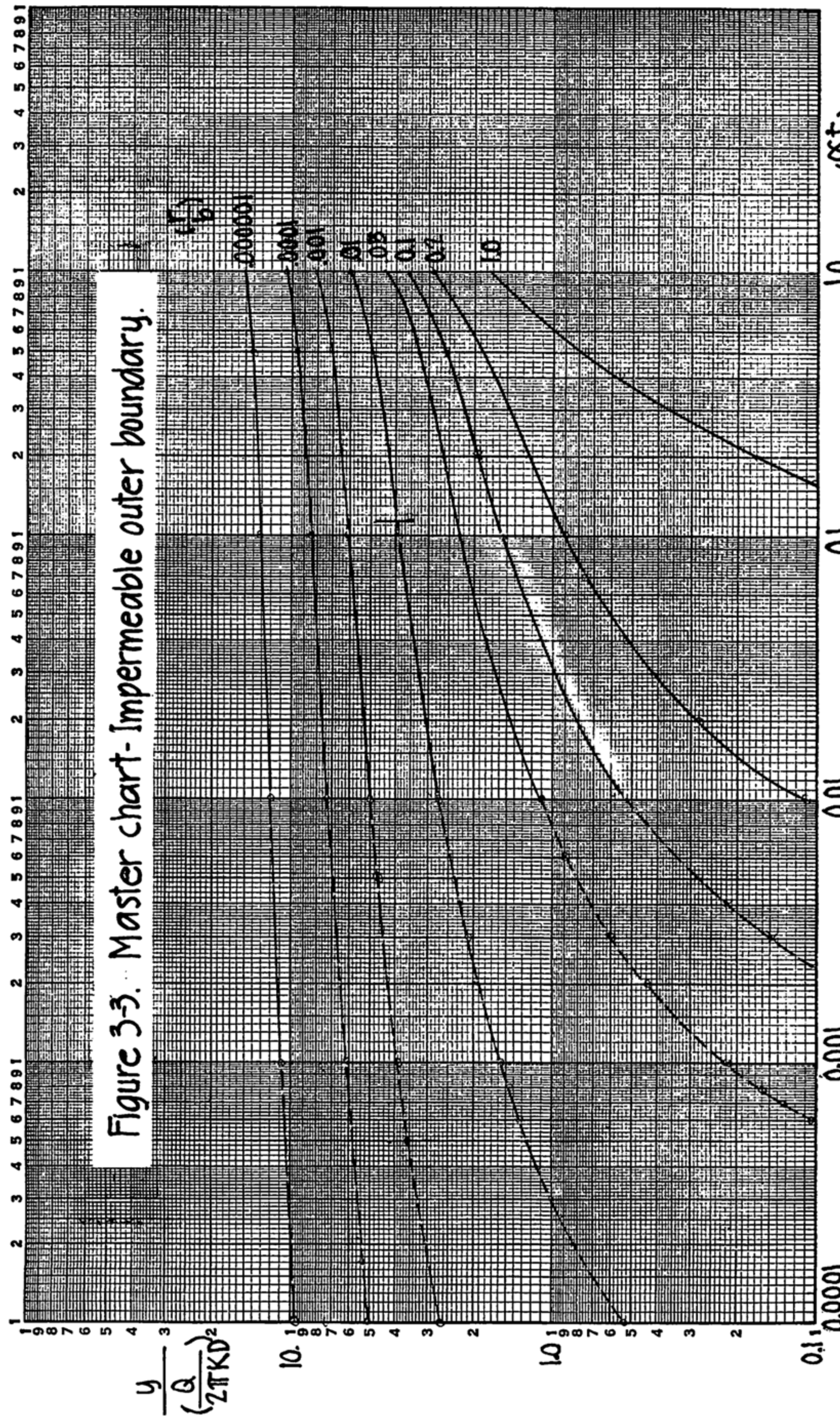


Figure 3-3. Master chart- Impermeable outer boundary.

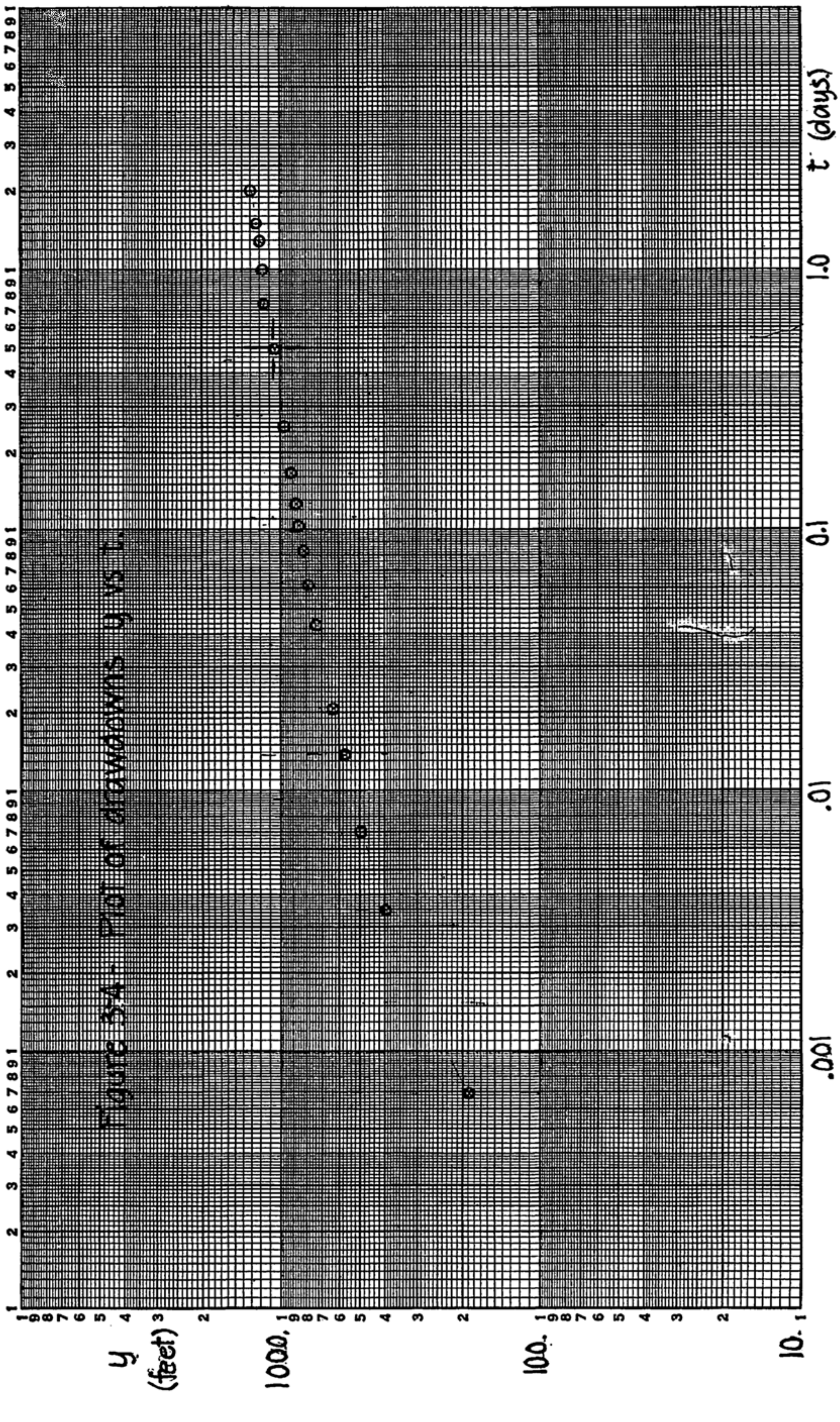


Figure 3-4. Plot of drawdowns y vs t.

From the corresponding abscissas;

$t = 0.500$ days and $(\alpha_1 t/b^2) = 0.113$. Then,

$$b^2 = \frac{\alpha_1 t}{0.113} = \frac{(127840)(0.500)}{0.113} = 565663 \; .$$

$$b = 752. \text{ feet}$$

We know this should be 750 feet. Since the (r/b) ratio was found to be .01 the effective well radius is

$$a = (752)(.01) = 7.52 \text{ feet}$$

Then by this process of curve matching we have recovered the reservoir properties.

Based upon the recovered properties we can infer that the whole volume of oil in the reservoir is

$$\pi b^2 D\phi = \pi(565663)(11.0)(0.12) = 2346053 \text{ ft}^3$$

or 418000 barrels

Chapter 3

Problems

(3-1) Farmers A and B live on adjoining farms and both have wells used to supply irrigation water. Their wells are 2500 feet apart and fully penetrate a saturated thickness of 60 feet. The aquifer constant is $\alpha = 0.75\ ft^2/sec$. How much lowering of the water table will A's well cause at B's well if he lifts 1.2 ft^3/sec for a period of four months? The permeability is .0025 ft/sec and the effective voids ratio is 0.2.

Answer: 0.783 feet.

(3-2) If A's well is gravel packed out to a radius of 1.25 feet what will be drawdown at the outside of the gravel pack after four months of pumping under the conditions described in (1)?

Answer: 11.46 feet.

(3-3) For purposes of checking an analog design a supplemental panel is set up with analog components to represent a circular area of five mile radius. The outer components are connected to the ground wire to represent a maintained water table elevation there. An input lead is connected to the node point at the center of the panel to represent a well pumped at a constant rate of 10.0 ft^3/sec. The analog is designed to represent the conditions $D = 200$ ft, $K = 0.001$ ft/sec, $V = 0.20$. By using Table 6 compute the drawdowns which should be shown by the analog test panel at the end of one year of continuous pumping. Node points are at 0.5 mile intervals.

Answer:

Radius (Miles)	Drawdown (feet)
0	----
0.5	9.421
1.0	4.528
1.5	2.233
2.0	1.063
2.5	0.478

Answer: (3-3) continued

Radius (Miles)	Drawdown (feet)
3.0	0.201
3.5	0.078
4.0	0.028
4.5	0.009
5.0	0

(3-4) The Fox Hills formation in the Denver area is a sandstone member confined between the Pierre Shale below and the Laramie formation above. It outcrops in a roughly elliptical zone which is about 110 miles across in the North-South direction and 70 miles across in the East-West direction.

The Fox Hills sandstone within the outcrop has the form of a synclinal fold and at Denver the top of this member is about 1500 feet below ground level. The bed is filled with water under pressure and in an earlier era flowing artesian wells were obtained from it. If it be assumed that the total flow of the early wells was 10,000 gallons per minute and that the pressure was originally 200 feet of water at Denver estimate how long would be required for the wells to cease to flow if the wells were distributed over a circle nine miles in diameter. Assume that the aquifer properties are $KD = 0.002\ ft^2/sec$, $V = 0.0004$, $\alpha = 5.0\ ft^2/sec$ and that the recharge rate is small compared to the well flow.

Answer: 0.910 years.

Suggestion: If Table 7 does not have the needed value, but does show that there would have been no drawdown at the outer boundary for greater times, then the case can be considered as an infinitely extended aquifer and a solution can be obtained by use of Table 1.

(3-5) Convert the data of Problem (3-1) to units of meters and seconds and solve for a drawdown in meters. Convert this drawdown back to feet and compare with the result obtained for Problem (3-1).

Answer: 0.239 meters or 0.784 feet.

Chapter 4.

The flowing artesian well

The processes of uplift, erosion and deposition have been important factors in producing the sedimentary formations which are prominent features of the earth as we know it today. Operation of the geologic forces which produced these changes was often erratic and it has happened that a permeable bed, such as a sandstone, has been sandwiched in betweem impermeable strata such as shale. If subsequent changes warp these strata into a synclinal fold with the permeable bed exposed at the surface along an outcrop then the conditions shown in figure 4-1 can become a reality. Recharge then occurs

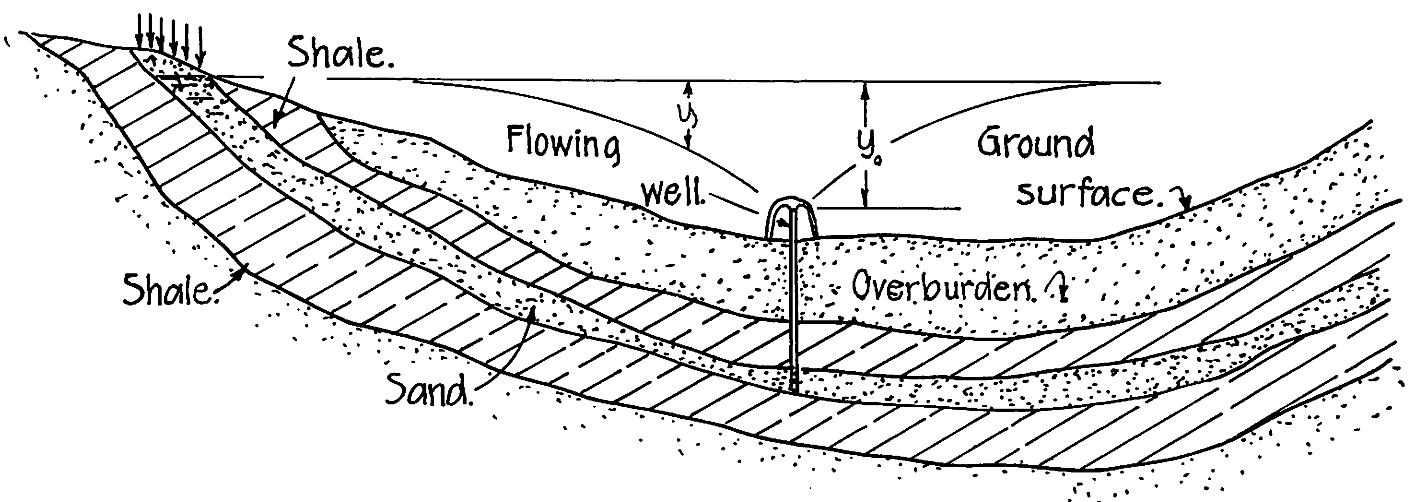


Fig. 4-1 Section showing an artesian stratum.

as a result of precipitation along the outcrop and the permeable member contains water under pressure. If a well is now sunk to the permeable bed, as shown, a flowing well can be obtained.

The combination of conditions needed to produce such a flowing well are not of common occurrence but erratic geologic processes have sometimes produced them. The Fox Hills formation below the Denver area is an example. Here the formation occupies an elongated saucer shaped depression confined between the Laramie formation above and the Pierre Shale below. Outcrops are to be found in the Greeley area to the north, in the plains area to the east and in the Colorado Springs area to the south. To the west of Denver the Fox Hills formation outcrops along the eastern face of the foothills. In earlier times flowing wells could be obtained but the replenishment rate is slow and now wells tapping this formation commonly must be pumped if water is to be obtained. Another example is to be found in the Artesia area of New Mexico.

Because recharge rates are typically low it is easy to overencumber the supply and some areas which once produced flowing wells must now be pumped if water is to be obtained. So long, however, as artesian wells are obtainable it will be important to have an analytical treatment by which the changes of pressure they produce and the rates of flow they can maintain can be estimated.

A treatment can be based upon the idealization that the hydraulic properties of a water bearing stratum can be specified by a permeability K, a thickness D and a yield factor V_1. This latter factor will be much smaller than would be found for a free water table case because under artesian conditions there is no unwatering of the aquifer. The water which flows from the well comes from an expansion of the water due to relief of pressure and from compression of the granular material of the aquifer. It may be noted in passing that the compression possible in a granular material, by application of pressure, is much greater than could be produced in an equal volume of solid material of the same kind. Before the pressures are disturbed, much of the weight of the overburden is sustained by them. When the water pressures are relieved this load is transferred to the grain structure and compression follows.

If we let y represent the pressure reduction in the aquifer, expressed in equivalent feet of water, then the flow toward the well at the radius r

will be:

$$F = -2\pi rKD \frac{\partial y}{\partial r} .$$

The continuity condition is:

$$\frac{\partial F}{\partial r} drdt = -2\pi rdrV_1 \frac{\partial y}{\partial t} dt.$$

or

$$\frac{\partial F}{\partial r} = - 2\pi rV_1 \frac{\partial y}{\partial t}$$

Then, by substitution and rearrangement

$$-2\pi KD(r \frac{\partial^2 y}{\partial r^2} + \frac{\partial y}{\partial r}) = -2\pi rV_1 \frac{\partial y}{\partial t}$$

If

$$\alpha_1 = \frac{KD}{V_1}$$

The relation can be put into the form:

$$\alpha_1 (\frac{\partial^2 y}{\partial r^2} + \frac{1}{r} \frac{\partial y}{\partial r}) = \frac{\partial y}{\partial t}$$

This relation is of the same form as was obtained for the case of the well drawing water from an aquifer with a free water table but, in the present case, no concession needs to be made to avoid nonlinearities and the treatment is exact.

In the previous case the well flow was taken to be constant and the drawdown at the well varied with time. In the present case the pressure drawdown at the well is a constant amount y_o and the discharge of the well varies with time. A similar idealization is often appropriate where a flowing well is producing oil from an extensive oil sand. The differential equation 2-5 is then to be solved subject to the conditions

$y = y_o$ at $r = a$ for $t > 0$

$y = 0$ when $t = 0$ for $r > a$

The required solution has been obtained by a number of investigators. Among them are Nicholson, Smith, Goldstein and Carslaw and Jaeger. A summary of these investigations as well as references to original sources may be found in Carslaw and Jaeger, 1947. The solution so obtained is in the form of an infinite integral. Evaluation is difficult. An alternative approach to a solution for the case where the outer boundary is infinitely remote is to work in terms of a finite outer radius b. This solution will behave as an infinitely remote outer boundary case until the disturbance produced by flow from the well of radius a reaches the outer boundary b. By using a sequence of increasingly remote outer boundaries it is possible to compute with a limited number of terms of the series solution and to extend the outer boundary to as remote a location as may be desired.

The required solution is:

$$y = y_o \left[1 - \sum_{n=1}^{n=\infty} A_n U_o(\beta_n r) e^{-\frac{k^2(\beta_n b)^2}{4} \left(\frac{4\alpha t}{a^2}\right)} \right] \tag{4-1}$$

This solution meets the condition that

$$\frac{\partial y}{\partial r} = 0 \qquad \text{when} \quad r = b.$$

The A_n values are to be computed from the expression:

$$A_n = \frac{\frac{2k}{(\beta_n b)} U_o'(\beta_n a)}{[U_o(\beta_n b)]^2 - k^2[U_o'(\beta_n a)]^2} \tag{4-2}$$

Where

$$U_o(\beta_n r) = J_o(\beta_n a)Y_o(\beta_n r) - Y_o(\beta_n a)J_o(\beta_n r) \tag{4-3}$$

and

$$k = (\frac{a}{b}) \tag{4-4}$$

The $(\beta_n b)$ values are to be obtained as roots of the equation:

$$U_o'(\beta_n b) = 0 \tag{4-5}$$

In these expressions:

$$U_o'(\beta_n r) = \frac{dU_o(\beta_n r)}{d(\beta_n r)}$$

$$J_o'(\beta_n r) = \frac{dJ_o(\beta_n r)}{d(\beta_n r)} = - J_1(\beta_n r)$$

$$Y_o(\beta_n r) = \frac{dY_o(\beta_n r)}{d(\beta_n r)} = - Y_1(\beta_n r)$$

The flow toward the well at the radius r is:

$$F = 2\pi rKD \frac{\partial y}{\partial r} = 2\pi KDy_o \sum_{n=1}^{n=\infty} A_n(\beta_n r)U_o'(\beta_n r) e^{-\frac{k^2(\beta_n b)^2}{4} (\frac{4\alpha_1 t}{a^2})} \tag{4-6}$$

If the flow at the radius a is

$$Q = 2\pi KDy_o G\left(\frac{\sqrt{4\alpha_1 t}}{a}\right) \tag{4-7}$$

Then:

$$G(\frac{\sqrt{4\alpha_1 t}}{a}) = \sum_{n=1}^{n=\infty} A_n(\beta_n a)U_o'(\beta_n a) e^{-\frac{k^2(\beta_n b)^2}{4} (\frac{4\alpha_1 t}{a^2})} \tag{4-8}$$

Values of (y/y_o) can be obtained from Table 3 and of $G(\sqrt{4\alpha t}/a)$ from Table 4.

Values outside the tables

It is quite possible that a need might arise to deal with values which are beyond the scope of the tables. Approximate procedures can be outlined to take care of such cases. For computation of pressure drawdowns the following expression will be useful:

$$y \cong y_o \frac{\displaystyle\int_{\left(\frac{r}{\sqrt{4\alpha_1 t}}\right)}^{\infty} \frac{e^{-u^2}\,du}{u}}{\displaystyle\int_{\left(\frac{a}{\sqrt{4\alpha_1 t}}\right)}^{\infty} \frac{e^{-u^2}\,du}{u}} \qquad \text{if } \left(\frac{\sqrt{4\alpha_1 t}}{a}\right) > 1000. \quad \text{or } \left(\frac{a}{\sqrt{4\alpha_1 t}}\right) < .001 \qquad (4\text{-}9)$$

This expression is based upon the concept that the flow from an artesian well becomes almost steady after the well has been flowing for some time. The formula for drawdowns produced by a well of constant flow is then used and the pressure drawdown y at the radius r is estimated by applying the ratio of the drawdown at the radius r to the drawdown at the radius a to the known drawdown y_o at the radius a.

A comparison of approximate and tabular values is shown in the following table:

Table 4-1 Comparison of approximate and tabular values of (y/y_o) for an artesian well.

$(\frac{r}{a})$	$(\frac{r}{\sqrt{4\alpha_1 t}})$	Tabular values of (y/y_o)	$\int_{(\frac{r}{\sqrt{4\alpha_1 t}})}^{\infty} \frac{e^{-u^2}du}{u}$	Approx. values of (y/y_o)	$(\frac{\sqrt{4\alpha_1 t}}{a})$
1.00	0.001	1.0000	6.6192	1.0000	1000
10.00	0.010	0.6557	4.3166	0.6521	1000
100.00	0.10	0.3118	2.0190	0.3050	1000
1000.00	1.00	0.0188	0.1097	0.0165	1000

Note that $\left(\frac{r}{\sqrt{4\alpha_1 t}}\right) = \left(\frac{a}{\sqrt{4\alpha_1 t}}\right)\left(\frac{r}{a}\right)$ (For this table) $\left(\frac{a}{\sqrt{4\alpha_1 t}}\right) = .001$

Values in the third and fifth columns should be compared. It should be expected that the approximation would improve as times grow longer.

Since it can be shown that, on the basis used for development of formula 4-9,

$$y \cong y_o \, G\left(\frac{\sqrt{4\alpha_1 t}}{a}\right) \int_{\left(\frac{r}{\sqrt{4\alpha_1 t}}\right)}^{\infty} \frac{e^{-u^2} du}{u} \qquad \text{if } \left(\frac{\sqrt{4\alpha_1 t}}{a}\right) > 1000. \qquad (4\text{-}10)$$

a comparison of formulas 4-9 and 4-10 would indicate that an approximation of the following type could be obtained:

$$G\left(\frac{\sqrt{4\alpha_1 t}}{a}\right) \cong \frac{1}{\int_{\left(\frac{a}{\sqrt{4\alpha_1 t}}\right)}^{\infty} \frac{e^{-u^2} du}{u}} \qquad \text{if } \left(\frac{\sqrt{4\alpha_1 t}}{a}\right) > 1000. \qquad (4\text{-}11)$$

A comparison is shown below. In this table only the last two values would be admissable under the restriction imposed on formula 4-11.

Table 4-2 Comparison of approximate and tabular values for $G\left(\frac{\sqrt{4\alpha_1 t}}{a}\right)$

$\left(\frac{\sqrt{4\alpha_1 t}}{a}\right)$	$G\left(\frac{\sqrt{4\alpha_1 t}}{a}\right)$ (tabular)	$\left(\frac{a}{\sqrt{4\alpha_1 t}}\right)$	$\int_{\left(\frac{a}{\sqrt{4\alpha_1 t}}\right)}^{\infty} \frac{e^{-u^2} du}{u}$	$\left(1/\int_{\left(\frac{a}{\sqrt{4\alpha_1 t}}\right)}^{\infty} \frac{e^{-u^2} du}{u}\right)$
100	.22585	0.010000	4.3166	0.2317
200	.19593	0.005000	5.0097	0.1996
300	.18177	0.003333	5.4151	0.1846
400	.17288	0.002500	5.7029	0.1753
500	.16655	0.002000	5.9260	0.1687
600	.16171	0.001667	6.1086	0.1637
700	.15783	0.001428	6.2630	0.1596
800	.15461	0.001250	6.3960	0.1563
900	.15188	0.001111	6.5137	0.1535
1000	.14952	0.001000	6.6192	0.1510
10000	.11146	0.000100	8.9217	0.1121

Values in the second and fifth columns should be compared. At the limit of tabular values the error is about one-half percent. The approximation should improve for values of $(\sqrt{4\alpha_1 t}/a)$ greater than 1000.

Example

Data furnished through the courtesy of Mr. Stan Lohman, of the U.S. Geological Survey, will first be used to make a determination of aquifer properties. The aquifer properties so obtained will then be used to make an estimate of the drawdowns produced by flows at the test well on pressures at an adjacent well. A description of these well tests will be found in the paper by Jacob and Lohman.

A plot of a portion of the $G(\frac{\sqrt{4\alpha_1 t}}{a})$ function is first made on logarithmic cross section paper as shown on figure 4-2. The test data are reduced as shown in table 4-3, and a plot of Q vs $(\sqrt{t}/a)$ is then made on logarithmic paper, as shown on figure 4-3. The observed data are then matched to the type curve data by superimposing the plots and shifting them while keeping the axes parallel. When a satisfactory match is obtained the point 0.1 - 1000 of the type curve is marked on the test data plot. These points are marked on the charts.

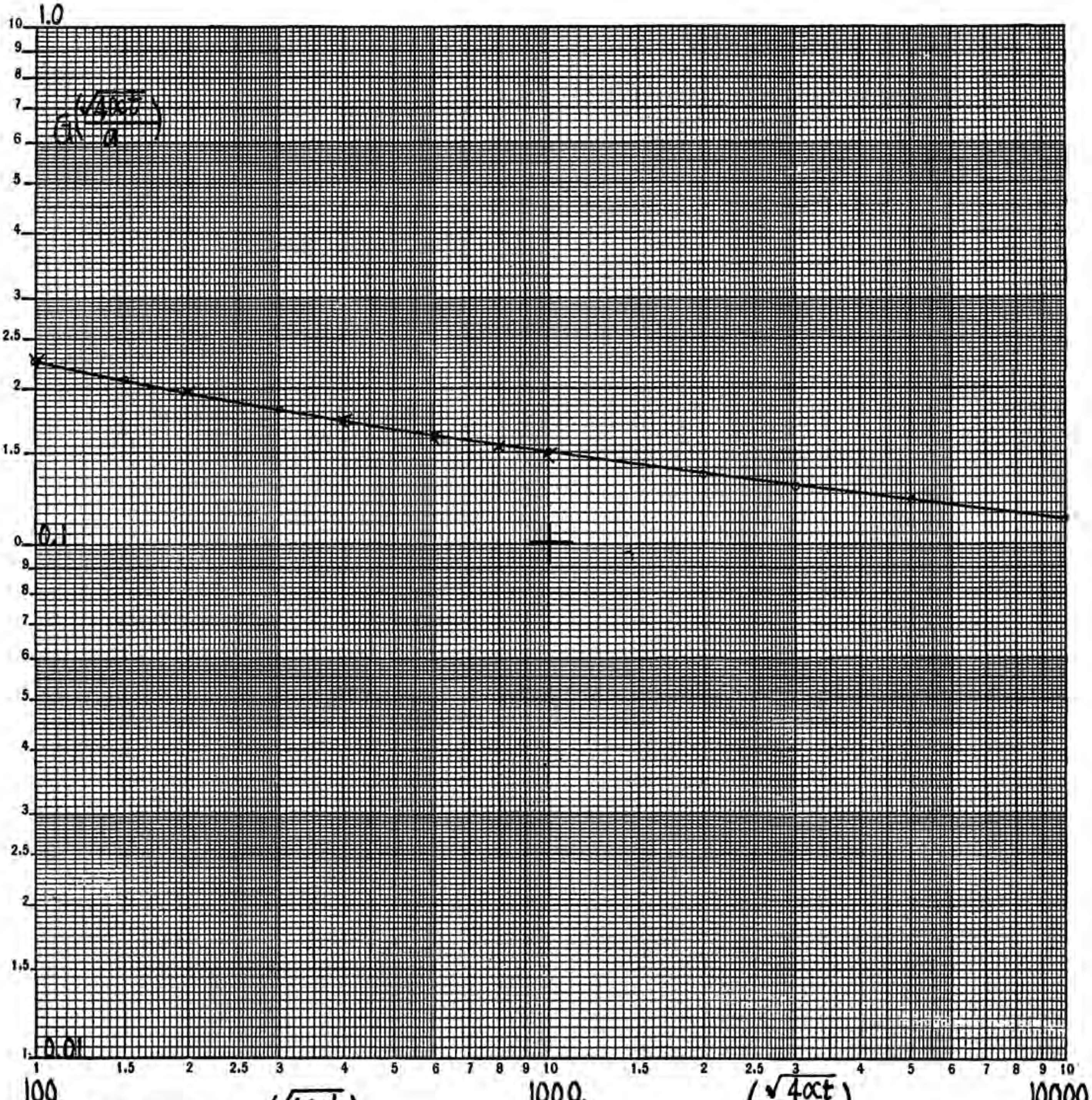


Fig 4-2 Plot of $G\left(\frac{\sqrt{4\alpha t}}{a}\right)$.

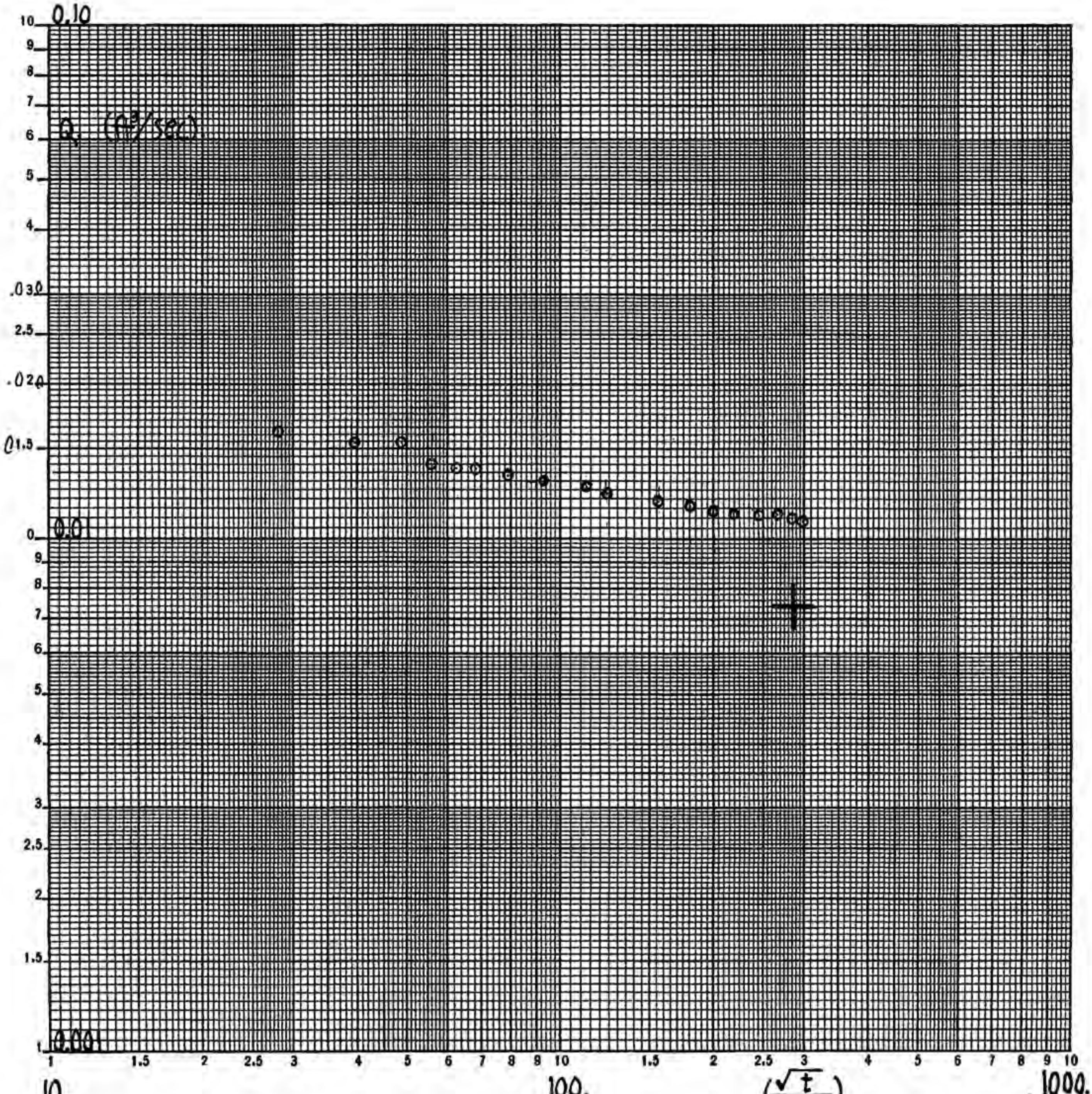


Fig 4.3. Observations.

Table 4-3 Artesian well test.

Time of observation	Rate of flow G.P.M.	Time since flow began minutes	Rate of flow $(\frac{ft^3}{sec})$	Time since flow began (seconds)	$(\frac{\sqrt{t}}{a})$
10:29 A.M.	-	0	--	0	0
10:30	7.28	1.	.01622	60	28.07
10:31	6.94	2.	.01546	120	39.69
10:32	6.88	3.	.01533	180	48.61
10:33	6.28	4.	.01400	240	56.13
10:34	6.22	5.	.01386	300	62.76
10:35	6.22	6.	.01386	360	68.75
10:37	5.95	8.	.01326	480	79.38
10:40	5.85	11.	.01303	660	93.08
10.45	5.66	16.	.01261	960	112.26
10:50	5.50	21.	.01225	1260	128.61
10:55	5.34	26.	.01190	1560	143.10
11:00	5.34	31.	.01190	1860	156.26
11:10½	5.22	41.5	.01163	2490	180.80
11:20	5.14	51.	.01145	3060	200.42
11:30	5.11	61.	.01138	3660	219.20
11:45	5.05	76.	.01125	4560	244.66
12:00N	5.00	91	.01114	5460	267.72
12:12 P.M.	4.92	103	.01096	6180	284.83
12:22	4.88	113	.01087	6780	298.34

Shutoff pressure, y_o = 92.33 feet of water.

Well radius 0.276 feet.

Penetration 60 feet.

Thickness of aquifer, D = 100 feet (Estimated from thicknesses at the location of nearby fully penetrating wells).

From chart matching, on the basis that the well is fully penetrating.

$(\frac{Q}{2\pi KDy_o}) = 0.100$ $Q = 0.007400$ ft^3/sec. KD = 0.00012755 ft^2/sec.,

$(\frac{\sqrt{4\alpha_1 t}}{a}) = 1000.$ $(\frac{\sqrt{t}}{a}) = 285.$ $\alpha = 3.076$ ft^2/sec $= (\frac{KD}{V_1})$

Then

K = 0.0000012755 ft/sec.

KD = (0.00012755)(86400) = 11.04 feet^2 per day.

Lohman's value is 11.7 feet^2 per day.

<u>Total production volume</u>

If the pressure reduction is:

$$y = y_o\left[1 - \sum_{n=1}^{n=\infty} A_n U_o(\beta_n r) e^{-\frac{k^2(\beta_n b)^2}{4}(\frac{4\alpha_1 t}{a^2})}\right]$$

The whole production volume out to time t is:

$$P = V_1 \int_a^b 2\pi r y dr = 2\pi V_1 \int_a^b r y dr$$

For the first term, with $k = (\frac{a}{b})$

$$2\pi V_1 y_o \int_a^b r dr = 2\pi y_o V_1 (\frac{b^2 - a^2}{2}) = 8\ y_o KDt(\frac{a^2}{4\alpha_1 t})(\frac{b^2-a^2}{2a^2})$$

Since

$$\int_a^b rU_o(\beta_n r)dr = -\frac{1}{\beta_n^2}[(\beta_n b)U_o'(\beta_n b) - (\beta_n a)U_o'(\beta_n a)] \quad \text{and}$$

because, in this case, $U_o'(\beta_n b) = 0$ this integral reduces to $\frac{(\beta_n a)U_o'(\beta_n a)}{\beta_n^2}$

Then for the second term:

$$2\pi y_o V_1 \int_a^b A_n r U_o(\beta_n r) e^{-\frac{k^2(\beta_n b)^2}{4}\left(\frac{4\alpha_1 t}{a^2}\right)} dr = 8\pi K D y_o t\left(\frac{a^2}{4\alpha t}\right) \sum_{n=1}^{n=\infty} \frac{A_n U_o'(\beta_n a)}{(\beta_n a)} e^{-\frac{(\beta_n a)^2}{4}\left(\frac{4\alpha_1 t}{a^2}\right)}$$

Since $k = \left(\frac{a}{b}\right)$ and $V_1 = \left(\frac{KD}{\alpha}\right)$

Finally, the whole production volume is:

$$P = 8\pi K D y_o t\left(\frac{a^2}{4\alpha_1 t}\right)\left[\frac{(b^2-a^2)}{2a^2} - \sum_{n=1}^{n=\infty} \frac{A_n U_o'(\beta_n a) e^{-\frac{(\beta_n a)^2}{4}\left(\frac{4\alpha_1 t}{a^2}\right)}}{(\beta_n a)}\right]$$

or if

$$H\left(\frac{a^2}{4\alpha_1 t}\right) = \left(\frac{a^2}{4\alpha_1 t}\right)\left[\frac{(b^2-a^2)}{2a^2} - \sum_{n\ 1}^{n\ \infty} \frac{A_n U_o'(\beta_n a) e^{-\frac{(\beta_n a)^2}{4}\left(\frac{4\alpha_1 t}{a^2}\right)}}{(\beta_n a)}\right]$$

$$P = 8\pi K D y_o t \cdot H. \qquad (4\text{-}12)$$

Values of the function H may be found in Table 5. These values have been computed from the relations shown in equations 4-12 above. Values are given for the range of $\frac{\sqrt{4\alpha_1 t}}{a}$ from 3 to 10,000. When an application requires values of H beyond the range of this table the approximate formulas described in the following paragraphs may be used. The graph of figure 4-4 has been prepared using these approximate formulas to extend the range to

$$\left(\frac{\sqrt{4\alpha_1 t}}{a}\right) = 1{,}000{,}000.$$

Extension of the H function table

Beyond the argument $(\sqrt{4\alpha_1 t}/a) = 1000$, the approximate expression

$$G\left(\frac{\sqrt{4\alpha_1 t}}{a}\right) \cong \frac{1}{\displaystyle\int_{\left(\frac{a}{\sqrt{4\alpha_1 t}}\right)}^{\infty} \frac{e^{-u^2}\,du}{u}}$$

has been shown to hold with an error of one percent or less. In this range the quantity $(a/\sqrt{4\alpha_1 t})$ will have a value of 0.001 or less. A series development is given in USBR Monograph 31 of the form:

$$\int_x^{\infty} \frac{e^{-u^2}\,du}{u} = -0.288608 - \log_e x + \frac{x^2}{2} - \frac{x^4}{2!4} + \frac{x^6}{3!6} - \ldots\ldots$$

In the region: $\left(\frac{a}{\sqrt{4\alpha_1 t}}\right) < 0.001$ the approximation

$$\int_x^{\infty} \frac{e^{-u^2}\,du}{u} \cong -0.288608 - \log_e x. \qquad \text{(Approximate for } x < 0.001\text{)}$$

would represent the integral with an accuracy of less than one unit in the sixth decimal place. Then

$$G\left(\frac{\sqrt{4\alpha_1 t}}{a}\right) \cong \frac{1}{-0.288608 - \log_e\left(\frac{a}{\sqrt{4\alpha_1 t}}\right)} \quad . \quad \text{(Approximate for } \left(\frac{\sqrt{4\alpha_1 t}}{a}\right) > 1000 \text{ .}$$

If the constant term is treated as a logarithm $-0.288608 = \log_e 0.749306$, then the approximate expression for G can be put in the form

$$G\left(\frac{\sqrt{4\alpha_1 t}}{a}\right) \cong \frac{1}{\log_e m\left(\frac{\sqrt{4\alpha_1 t}}{a}\right)} \quad \text{or} \quad G\left(\frac{\sqrt{4\alpha_1 t}}{a}\right) \cong \frac{2}{\log_e m^2\left(\frac{4\alpha_1 t}{a^2}\right)}$$

Where $m = 0.749307$ and $m^2 = 0.561461$. $(1/m^2) = 1.78107$. The total production can then be expressed as:

$$P = 2\pi KDy_o \int_0^t G\left(\frac{\sqrt{4\alpha_1 t}}{a}\right) dt$$

or:

$$P \cong 4\pi KDy_o \left[\int_0^t \frac{dt}{\log_e m^2 \left(\frac{4\alpha_1 t}{a^2}\right)} + C_1 \right]$$

Let

$$x = m^2\left(\frac{4\alpha_1 t}{a^2}\right) \qquad dx = \frac{m^2 4\alpha_1 dt}{a^2} \qquad dt = \frac{a^2 dx}{4\alpha_1 m^2}$$

Then, by substitution:

$$P \cong \frac{4\pi KDy_o a^2}{m^2 \quad 4\alpha_1} \left[\int_0^x \frac{dx}{\log_e x} + C_2 \right] \qquad \text{if} \quad \left(\frac{\sqrt{4\alpha_1 t}}{a}\right) > 1000.$$

and:

$$P \cong \frac{4\pi KDy_o t}{m^2} \left(\frac{a^2}{4\alpha_1 t}\right) \left[\int_0^x \frac{dx}{\log_e x} + C_2 \right]$$

$$\cong 7.124290 \; \pi KDy_o t \left(\frac{a^2}{4\alpha_1 t}\right) \left[\int_0^x \frac{dx}{\log_e x} + C_2 \right]$$

Where C_1 and C_2 represent constants of integration.

This can be put in the form:

$$P \cong 8\pi KDy_o t \quad 0.890536 \left(\frac{a^2}{4\alpha_1 t}\right) \left[\int_0^x \frac{dx}{\log_e x} + C_2 \right] \qquad (4\text{-}13)$$

so that

$$H \cong 0.890536 \left(\frac{a^2}{4\alpha_1 t}\right)\left[\int_0^x \frac{dx}{\log_e x} + C_2\right] \qquad (4\text{-}14)$$

The integral which appears here is known as the logarithmic integral. Its value in terms of the exponential integral is given below in the example.

Example

To show the method used for evaluating the logarithmic integral.

Suppose $\left(\frac{\sqrt{4\alpha t}}{a^2}\right) = 1000.$ $\qquad \frac{4\alpha t}{a^2} = 1000000.$

$x = m^2\left(\frac{4\alpha t}{a^2}\right) = 561460.$ $\quad \log_e 561460. = 13.238293$

$$\int_0^x \frac{dx}{\log_e x} = Ei(\log_e x) = 46270.$$

Where:

$$Ei(z) = \int_{-\infty}^{z} \frac{e^{+u} du}{u} .$$

is a tabulated function (Jahnke and Emde, 1945--Dwight, 1958, Department of Commerce, 1966).

An evaluation of the constant C_2 now needs to be made. When $(\sqrt{4\alpha_1 t}/a) = 1000$ the tabular value of H is 0.040671 and the value obtained from the approximation is 0.041205. Based upon these values $C_2 = -0.000534$. When $(\sqrt{4\alpha_1 t}/a) = 10000$ the tabular value is $H = 0.029616$ and the corresponding value obtained from the approximate expression based upon the logarithmic integral is $H = 0.029794$. On this basis $C_2 = -0.000178$. A comparison shows that the approximation value is coming closer to the tabular value as $(\sqrt{4\alpha_1 t}/a)$ increases and that there is only about 0.6 percent difference at the end of the table where $(\sqrt{4\alpha_1 t/a}) = 10\ 000$. Then for values of $(\sqrt{4\alpha_1 t/a})$ beyond the table it would be reasonable to take $C_2 = 0$ and use the relation:

$$H = 0.890536\left(\frac{a^2}{4\alpha_1 t}\right)\int_0^x \frac{dx}{\log_e x} \tag{4-15}$$

It should be realized that the production function H is based upon developments which imply that the reservoir is of infinite extent. Ultimately the disturbances caused by production of the well will reach the outer boundary of any actual finite reservoir and the production will then fall below that estimated by use of the H function.

<u>Example of application</u>

Estimate the total production of the Artesian test well of Table 4-3 if it flowed unchecked for one year

$KD = 0.00012756\ ft^2/sec$ $\alpha = 3.078\ ft^2/sec$ $t = 31536000\ sec$ $a = 0.276\ ft$

$$\left(\frac{\sqrt{4\alpha t}}{a}\right) = \frac{19698.}{0.276} = 71370.$$

This is beyond the H table range. From the graph of figure 4-4

$H = .0240$ $8\pi KD = .003206$ $8\pi KDy_o\ t = (003206)(92.33)(31536000) = 9335000.$

$P = 9335000(.0240) = 224000.ft^3$ or 5.14 acre feet.

As a rough check, the well would have produced

$$\frac{(5.00)(31536000)}{448.8} = 351300.\ ft^3 \text{ or } 8.06 \text{ acre feet.}$$

if the flow observed at noon of the day of the test had been sustained.

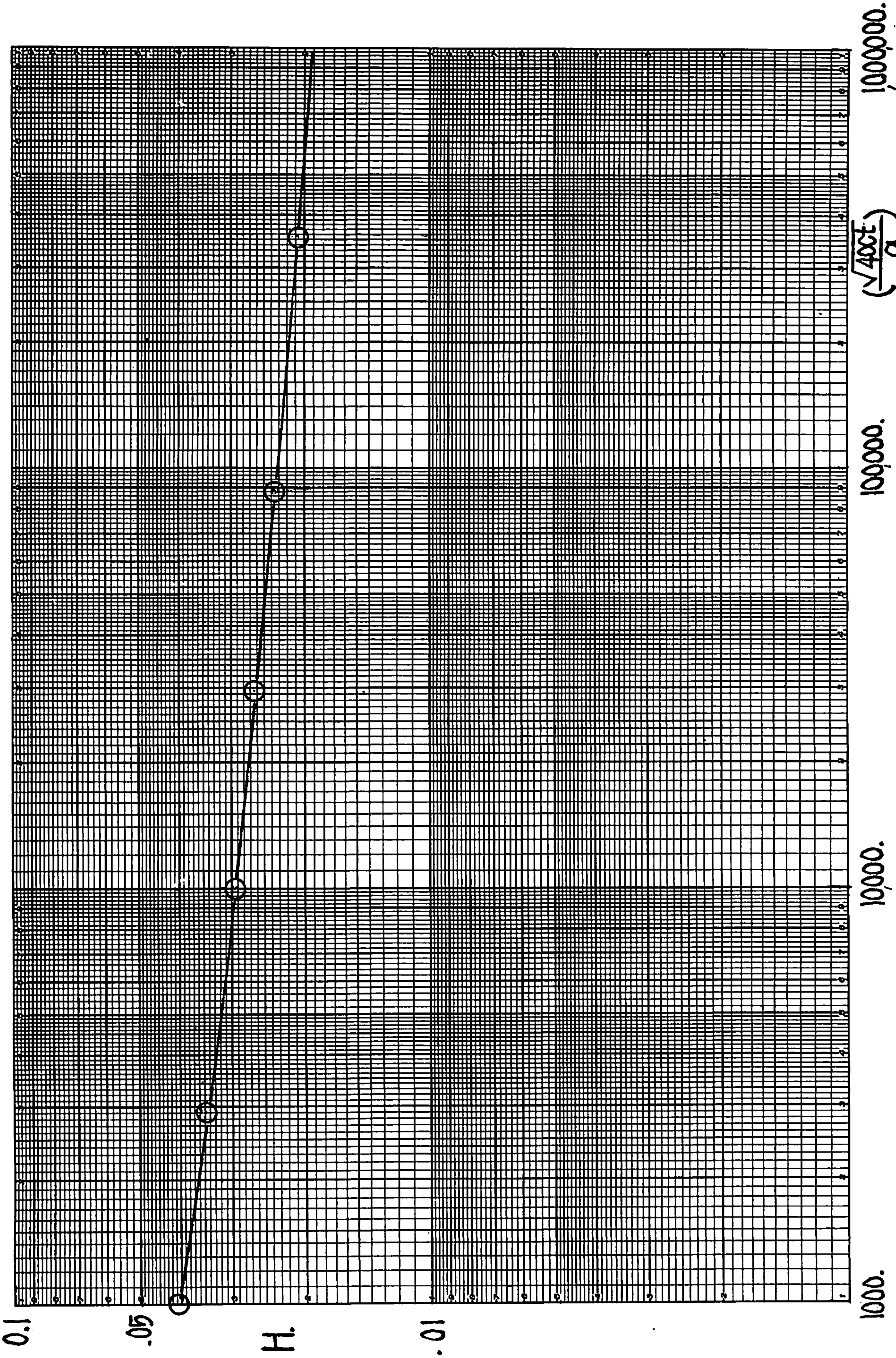


Fig. 4—4 Values of H for large values of $\left(\frac{\sqrt{4\alpha t}}{a}\right)$.

Chapter 4.

Problems.

(4-1) For the Fox Hills aquifer as described for Problem (3-4) compute the pressure drawdown at (r/a) = 1000 to (r/a) = 10000 at the end of the first day of operation if y_o = 200 feet and the diameter of the well casing is six inches.

Answer:

r (feet)	$\frac{r}{a}$	$\frac{y}{y_o}$	y (feet)	r	$\frac{r}{a}$	$\frac{y}{y_o}$	y (feet)
0.25	1	1.0000	200.0	1500	6000	.009006	1.80
250	1000	0.1725	34.5	1750	7000	.004547	.91
500	2000	0.09444	18.9	2000	8000	.002178	.44
750	3000	0.05422	10.8	2250	9000	.000985	.20
1000	4000	0.03074	6.15	2500	10000	.000420	.08
1250	5000	0.01702	3.40				

(4-2) Estimate the flow of the well at the end of each of the first twelve hours of operation.

Answer:

Time (hours)	$(\frac{\sqrt{4\alpha_1 t}}{a})$	$G(\frac{\sqrt{4\alpha_1 t}}{a})$	Q (ft^3/sec)	Time (hours)	$(\frac{\sqrt{4\alpha_1 t}}{a})$	$G(\frac{\sqrt{4\alpha_1 t}}{a})$	Q (ft^3/sec)
0	----	-----	--------	7	2840	0.1295	0.325
1	1073	0.1480	0.372	8	3036	0.1284	0.323
2	1518	0.1408	0.354	9	3220	0.1274	0.320
3	1859	0.1370	0.344	10	3394	0.1266	0.318
4	2149	0.1343	0.338	11	3560	0.1258	0.316
5	2497	0.1317	0.331	12	3718	0.1252	0.315
6	2629	0.1308	0.329				

(4-3) Estimate the total flow of the well for the first 12 hours of operation.

Answer:

$$P = 14560 \text{ ft}^3$$

(4-4) Check the result obtained for (4-3) by use of formula 4-14.

Answer:

$$P = 14330 \text{ ft}^3$$

(4-5) Compare this total flow with what would have been produced if the well had flowed for the full twelve hours at the rate it was flowing at the end of the first hour.

Answer:

16070 ft^3

Chapter 5

Well with a semi-permeable bed overlying the aquifer

A cross section through the well is shown in figure 5-1.

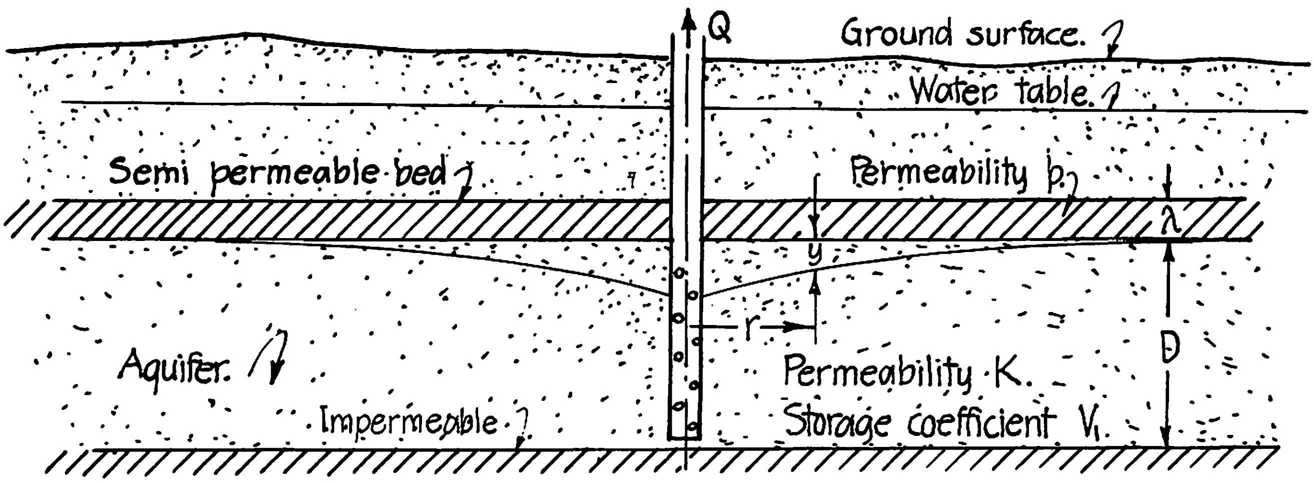


Fig. 5-1 Well drawing water from an aquifer overlain by a semi-permeable bed.

To treat this case it will be needful to modify equation 2-5 of Chapter 2 to account for the seepage of water through the semi-permeable bed. It is assumed that before pumping begins there is a hydrostatic pressure distribution which is continuous from the water table downward through the semi-permeable bed and the aquifer to the impermeable barrier. When pumping begins, withdrawal of water from the aquifer reduces the pressure at the bottom of the semi-permeable bed and causes a downward seepage of water through it. The pressure reduction at the bottom of the semi-permeable bed is represented by y . The downward flow, per unit of horizontal area is then (py/λ) since the downward gradient is (y/λ) . It is supposed that the water table is maintained at its original level above the semi-permeable bed and that movement through the semi-permeable bed is in the vertical direction only. Unwatering of the aquifer does not occur. The storage coefficient V_1 is of the artesian type since the yield of water from the aquifer is produced by expansion of the water and compression of the aquifer by the pressure reduction y . In regard to the compression of the aquifer it may be noted that the weight of the overburden is originally supported, in part, by the hydrostatic pressure and the remainder by contact pressures between grains. When the pressure below the semi-permeable bed is reduced the load originally carried by it is transferred to the aquifer grain structure to cause it to be compressed. This compression causes the aquifer to yield water. The storage coefficient V_1 is given the subscript 1 to indicate that the yield of water is produced by pressure change and not by drainage.

The flow through a cylindrical shell of radius r and height D is, by Darcy's law $F = -2\pi rKD \frac{\partial y}{\partial r}$. The minus sign is appropriate where F is considered positive if toward the well and y is taken to be positive for a pressure reduction. The continuity condition is then:

$$- \frac{\partial}{\partial r}(2\pi rKD \frac{\partial y}{\partial r})\, dr\, dt + 2\pi r \frac{py}{\lambda}\, dr\, dt = -2\pi rV_1 \frac{\partial y}{\partial t}\, dr\, dt\ .$$

After simplification and rearrangement this becomes, with $\alpha_1 = (KD/V_1)$

$$\alpha_1 (\frac{\partial^2 y}{\partial r^2} + \frac{1}{r}\frac{\partial y}{\partial r}) = \frac{\partial y}{\partial t} + \frac{py}{V_1 \lambda} \qquad (5\text{-}1)$$

A solution of this differential equation which meets the conditions

$y = 0$ when $t = 0$ for $r > 0$

$$-2\pi rKD \frac{\partial y}{\partial r} \rightarrow Q \text{ as } r \rightarrow 0 \text{ for } t > 0$$

is:

$$y = \frac{Q}{2\pi KD} \int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2 - \frac{m^2}{u^2}}\, du}{u} \qquad (5\text{-}2)^*$$

*This problem has been treated by Hantush and Jacob in terms of the integral $\int_{\frac{r^2 S}{4Tt}}^{\infty} \frac{e^{-y - \frac{r^2}{4B^2 y}}\, dy}{y}$. See Trans. Amer. Geophysical Union, Vol. 36, 1955.

where

$$m = \frac{r}{2}\sqrt{\frac{p}{\lambda KD}}$$

Values of the integral which appear in equation (5-2) may be found in table 14.

Values outside the table

Since the table covers a finite range of values something needs to be said about the procedure to be used when a point falls outside the tabular range.

In the integrand of formula 5-2 the factor e^{-u^2} is unity when $u = 0$ and decreases as u increases reaching the value 0.0001234 when $u = 3.0$ the factor $e^{-\frac{m^2}{u^2}}$ is zero when u is zero and rises toward unity as u increases. The factor $1/u$ reaches an infinite value when $u = 0$ and decreases toward zero as u increases. The integrand, however, always approaches zero as u approaches zero if $m > 0$. The integral is, therefore, always finite so long as $m > 0$. The integral from zero to infinity can be evaluated and has the value

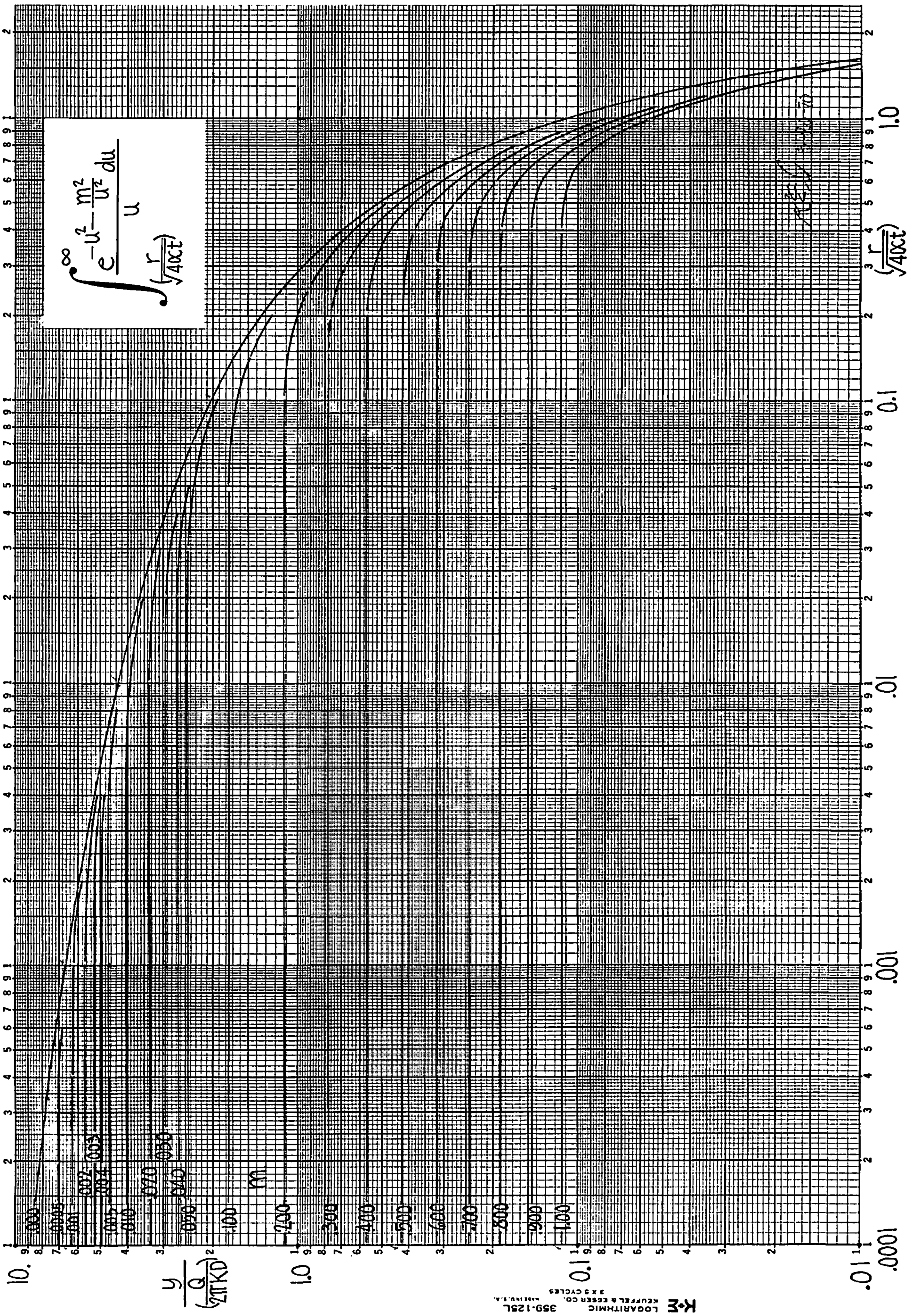
$$\int_0^\infty \frac{e^{-u^2-\frac{m^2}{u^2}}\,du}{u} = K_o(2m). \tag{5-3}$$

This case has an ultimate steady state which is realized when the pressure reduction below the semi-permeable bed becomes great enough to produce a seepage through it sufficient to supply the flow of the well. When the steady state is reached $(\partial y/\partial t)$ becomes zero and the differential equation 5-1 takes the form

$$\frac{d^2y}{dr^2} + \frac{1}{r}\frac{dy}{dr} - \frac{py}{\alpha_1\lambda V_1} = 0. \tag{5-4}$$

Note that

$$(p/\alpha_1\lambda V_1) = (p/\lambda KD)$$



$$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2-\frac{m^2}{u^2}}}{u}\,du$$
$\frac{y}{\left(\frac{Q}{2\pi KD}\right)}$
$\frac{r}{\sqrt{4\alpha t}}$
m
10.
1.0
0.1
.01
.0001
.001
.01
0.1
1.0
.001
.002
.003
.004
.005
.010
.020
.030
.040
.050
.100
.200
.300
.400
.500
.600
.700
.800
.900
1.00
K+E LOGARITHMIC 359-125L
KEUFFEL & ESSER CO.
3 X 5 CYCLES

Fig 5-2

The substitutions:

$$\mu = \frac{y}{\left(\frac{Q}{2\pi KD}\right)} \qquad \text{and} \qquad \rho = r\sqrt{\frac{p}{\lambda KD}}$$

will reduce the above differential equation to the form:

$$\frac{d^2u}{d\rho^2} + \frac{1}{\rho}\frac{du}{d\rho} - \mu = 0 \ . \qquad (5\text{-}5)$$

A solution, subject to the requirement that $\mu \to 0$ as $\rho \to \infty$ is:

$$\mu = K_o(\rho)$$

or

$$\mu = K_o(2m)$$

or

$$y = \left(\frac{Q}{2\pi KD}\right) K_o(2m) \ . \qquad (5\text{-}6)$$

This is the ultimate steady state. The quantity K_o is the modified Bessel's function of the second kind. It has been extensively tabulated. Approximate values can be read from the chart of figure 5-2. This result is in complete agreement with the value given for the integral in formula 5-2. A reference to figure 5-2 will show that, for a given value of r, the plots become horizontal when time increases sufficiently. This horizontal portion indicates that the ultimate steady state has been attained. The value of u corresponding to the horizontal portion of the plots will be found to be in agreement with that given by formula 5-2. It will hold this value even though the value of $(r/\sqrt{4\alpha_1 t})$ is less than the smallest value shown on figure 5-2.

There will be some values in the region where $(r/\sqrt{4\alpha_1 t}) < .001$ and $m < .005$, however, which cannot be treated in this way. In this region the factor e^{-u^2} is between 0.999999 and unity. To a close approximation, therefore, it will be possible to replace the integral of formula 5-2 with

$$\mu = K_o(2m) - \int_0^a \frac{e^{-\frac{m^2}{u^2}}\,du}{u} \qquad (5\text{-}7)$$

Where a represents the limit $\left(\frac{r}{\sqrt{4\alpha_1 t}}\right)$. (Valid if $\left(\frac{r}{\sqrt{4\alpha_1 t}}\right) < 0.001$).

The variable substitution $(m/u) = v$ will put this into the form

$$\mu = K_o(2m) - \int_{\left(\frac{m}{a}\right)}^{\infty} \frac{e^{-v^2}\,dv}{v} \qquad (5\text{-}8)$$

(Valid if $m < .005$. $\left(\frac{r}{\sqrt{4\alpha_1 t}}\right) < .001$).

As an example suppose $m = 0.001$, $a = 0.01$, $\frac{m}{a} = 0.1$, $K_o(2m) = 6.3305$

$$\int_{0.1}^{\infty} \frac{e^{-v^2}\,dv}{v} = 2.01896 \qquad \text{(From Table 1)}$$

Then

$$\frac{y}{\left(\frac{Q}{2\pi KD}\right)} = \mu = 6.33055 - 2.01896 = 4.31159$$

The corresponding tabular value is 4.31158.

A similar approximation can be made where the (m/u) values are small enough to make $e^{-\frac{m^2}{u^2}} \cong 1$. Here the approximation is:

$$\mu = \int_a^{\infty} \frac{e^{-u^2}\,du}{u}$$

Evaluation can be made with the help of Table 1. The open circles on figure 5-2 show approximations obtained in this way.

Example

The following example will illustrate the use of these formulas. An aquifer of 125 feet thickness having a permeability K of 0.0004 ft/sec and a storage coefficient of $V_1 = 0.0009$ is overlain by a bed of glacial till having a thickness of 32 feet and a permeability p, for vertical flow, of $35(10)^{-9}$ ft/sec. A well penetrating the full thickness of the aquifer is pumped at the rate of 0.25 ft^3/sec. It is desired to compute the pressure drawdown, as a function of the radius, at the end of 24 hours of pumping. It is also desired to estimate the final steady state drawdowns and to estimate the time required to establish this steady state.

Solution

With:

$K = 0.00040$ ft/sec

$D = 125$ ft $\quad KD = 0.050\ ft^2/sec$

$V_1 = 0.0009$ (dimensionless) $\quad \alpha_1 = 55.556\ ft^2/sec$

$p = (35.0)(10)^{-9}$ ft/sec

$\lambda = 32.$ ft

$Q = 0.250\ ft^3/sec \quad t = 86400$ sec (24 hours)

$$\left(\frac{Q}{2\pi KD}\right) = \frac{0.250}{(6.2832)(0.050)} = 0.795$$

$$\sqrt{\frac{p}{\lambda KD}} = \sqrt{\frac{35(10)^{-9}}{(32)(.050)}} = \sqrt{(2.19)(10)^{-8}} = 0.0001480$$

The following radii will be used:

Radius (feet)	m	$\left(\frac{r}{\sqrt{4\alpha_1 t}}\right)$	$\int_{\left(\frac{r}{\sqrt{4\alpha_1 t}}\right)}^{\infty} \frac{e^{-u^2-\frac{m^2}{u^2}}du}{u}$	y
10	0.00074	0.002282	5.864	4.66
50	0.00370	0.011410	4.157	3.30
100	0.00740	0.022820	3.452	2.77
500	0.03700	0.114103	1.860	1.48
1000	0.07400	0.228206	1.171	0.93
5000	0.37000	1.141031	0.0623	0.05
10000	0.74000	2.282062	0.0004	0.00

$$\sqrt{4\alpha_1 t} = \sqrt{(4)(5\ 5.555)(86400)} = 4382$$

Computations were made by use of Table 14 and double interpolation was used to obtain the y values. The detail of the computations for the first two radii are shown below. For $m = 0$ the table cannot be used the these values were obtained by the methods described previously.

For $m = 0.00074 \qquad \frac{r}{\sqrt{4\alpha_1 t}} = 0.002282$

m	0.002	0.002282	0.003	
0	5.9260		5.5205	$\mu = 5.864$
.00074	5.957	5.864	5.629	$y = (5.864)(0.795) = 4.661$
.001	6.0049		5.7932	

For $m = 0.00370 \qquad \frac{r}{\sqrt{4\alpha_1 t}} = 0.01141$

m	0.010	0.01141	0.020	
.003	4.2726		3.6125	$\mu = 4.157$
.00370	4.249	4.157	3.605	$y = (4.157)(0.795) = 3.304$
.004	4.2398		3.6039	

The ultimate steady state will be substantially attained when $\eta = 4$ and η is given by the relation:

$$\eta = t\left(\frac{p}{\lambda V_1}\right)^{*}$$

then

$$t = \frac{4\lambda V_1}{p} = \frac{(4)(32)(.0009)}{35(10)^{-9}} = 3291400.\ \text{sec or 38 days.}$$

---

*An independent development is presented in Bureau of Reclamation Monograph 31. This formula is obtained from this source.

The ultimate steady state drawdowns may be computed by use of formula 5-6.

$$y = \left(\frac{Q}{2\pi KD}\right) K_o(2m)$$

The computation can be made as follows:

Radius (feet)	m	$K_o(2m)$	y
10	0.00074	6.632028	5.272
50	0.00370	5.022289	3.992
100	0.00740	4.329351	3.441
500	0.0370	2.724716	2.166
1000	0.0740	2.043074	1.624
5000	0.370	0.620173	0.492
10000	0.740	0.219434	0.174

As a second example we will estimate the pressure drawdown at the radii 10, 100, and 1000 feet for a succession of times. We will use readings from the graph of figure 5-2 and make the computations with a slide rule. The data for the example are as follows:

$p = 125.(10)^{-9}$ ft/sec $K = 0.000625$ ft/sec $D = 40$ ft

$\lambda = 20$ ft $KD = 0.025$ ft^2/sec $V_1 = 0.005$

$\alpha_1 = \frac{KD}{V_1} = 50$ ft^2/sec $Q = 0.240$ ft^3/sec

$$\frac{p}{\lambda KD} = \frac{125.(10)^{-9}}{(20)(.025)} = 250.(10)^{-9}$$

$$m = \frac{r}{2}\sqrt{\frac{p}{\lambda KD}} \qquad \sqrt{\frac{p}{\lambda KD}} = (0.5000)(10)^{-3} \qquad \left(\frac{Q}{2\pi KD}\right) = 1.528$$

The quantities $(r/\sqrt{t})$ are added for purposes to be described later.

Table 5-1.

For r = 10 feet $m = \frac{10}{2}(0.5000)(10)^{-3} = 0.00250$

Time	Time (seconds)	$\left(\frac{r}{\sqrt{4\alpha_1 t}}\right)$	$\frac{y^*}{\left(\frac{Q}{2\pi KD}\right)}$	y (feet)	$\left(\frac{r}{\sqrt{t}}\right)$
1 min	60	0.09128	2.10	3.21	1.290
10 min	600	0.02887	3.28	5.01	0.408
1 hr	3600	0.01178	4.18	6.38	0.167
6 hr	21600	0.00482	5.02	7.66	0.068
12 hr	43200	0.00340	5.20	7.94	0.048
24 hr	86400	0.00240	5.40	8.25	0.034
5 days	432000	0.00108	5.46	8.33	0.015
10 days	864000	0.00076	5.50	8.40	0.011

For r = 100 feet $m = 0.0250$

Time	Time (seconds)	$\left(\frac{r}{\sqrt{4\alpha_1 t}}\right)$	$\frac{y^*}{\left(\frac{Q}{2\pi KD}\right)}$	y (feet)	$\left(\frac{r}{\sqrt{t}}\right)$
1 min	60	0.9128	0.150	0.229	12.90
10 min	600	0.2889	1.00	1.528	4.08
1 hr	3600	0.1178	1.87	2.86	1.67
6 hr	21600	0.0482	2.70	4.13	0.680
12 hr	43200	0.0340	2.90	4.44	0.481
24 hr	86400	0.0240	3.10	4.74	0.340
5 days	432000	0.0108	3.30	5.04	0.152
10 days	864000	0.0076	3.30	5.04	0.108

For r = 1000 feet $m = 0.250$

Time	Time (seconds)	$\left(\frac{r}{\sqrt{4\alpha_1 t}}\right)$	$\frac{y^*}{\left(\frac{Q}{2\pi KD}\right)}$	y (feet)	$\left(\frac{r}{\sqrt{t}}\right)$
1 min	60	9.128	--	--	--
10 min	600	2.887	--	--	--
1 hr	3600	1.178	0.060	0.092	16.67
6 hr	21600	0.482	0.470	0.718	6.680
12 hr	43200	0.340	0.710	1.085	4.82
24 hr	86400	0.240	0.830	1.270	3.40
5 days	432000	0.108	0.920	1.408	1.52
10 days	864000	0.076	0.920	1.408	1.08

*Read from the chart of Fig. 5-2.

Determination of aquifer properties

Aquifer properties can be determined from test data by a curve matching procedure such as has been previously described. For this purpose the data of Table 5-1 are plotted on a chart with logarithmic scales for both ordinate and abscissa. Such a chart is shown on figure 5-3. Observed pressure drawdowns y from test data are then plotted against corresponding values of $(r/\sqrt{t})$ on an identical grid. This second chart is then placed over the master chart of figure 5-2 and adjusted, while keeping the axes parallel, until a fit is obtained. The position of the index of the master chart is then marked on the test data chart. The aquifer properties can then be obtained by the process described in the following example.

It will be advantageous to use the computed drawdowns of the second example as test data because we will then know that results should be obtained and thereby gain some insight as to the effectiveness of the method. Plots of y versus $(r/\sqrt{t})$ are shown on figure 5-3. When this is superposed on the master chart and adjusted to a satisfactory fit the position of the master chart index is indicated by a cross.

For the master chart index

$$\frac{y}{\left(\frac{Q}{2\pi KD}\right)} = 1.00 \qquad \left(\frac{r}{\sqrt{4\alpha_1 t}}\right) = 1.00$$

For the test data chart index

$$y = 1.58 \qquad \left(\frac{r}{\sqrt{t}}\right) = 14.0$$

From the first of these

$$KD = \frac{(1.00)(Q)}{2\ \pi\ y} = \frac{(1.00)(0.24)}{(6.2832)(1.58)} = 0.0242 \text{ (Should be 0.025)}$$

$$K = (.0242)/(40) = 0.000605 \quad \text{(Should be 0.000625)}$$

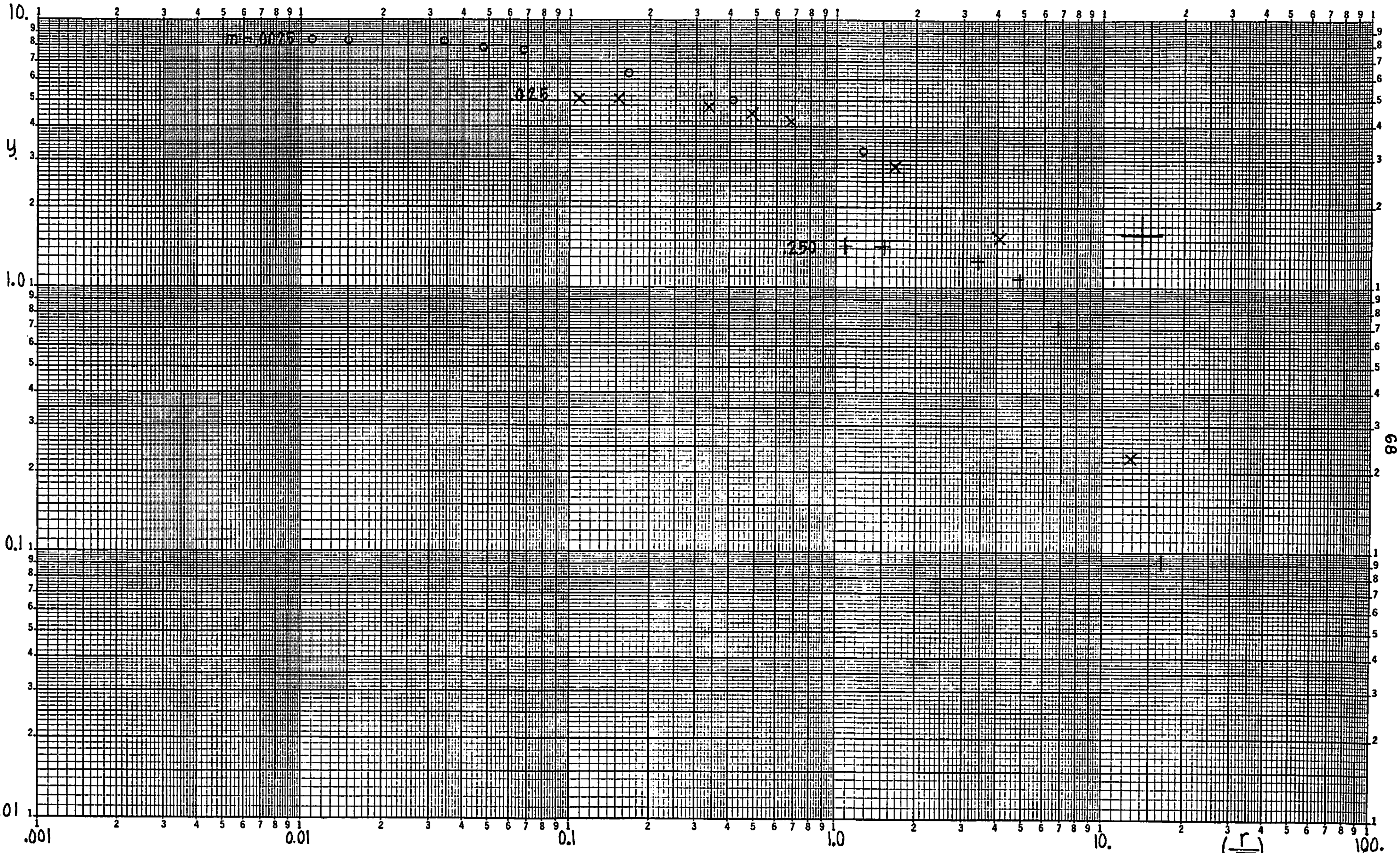


Fig 5-3 Plot of data of table 5-1.

From the second set of values

$$\sqrt{4\alpha_1} = \left(\frac{r}{\sqrt{t}}\right) . \quad \text{Then,} \quad \sqrt{4\alpha_1} = 14.0$$

$$\alpha_1 = \frac{14.0^2}{4} = 49.0 \quad \text{(Should be 50)}$$

For $r = 100$ ft $\qquad m = 0.0250$

$$m = \frac{r}{2}\sqrt{\frac{p}{\lambda KD}} \quad \text{or} \quad \frac{p}{\lambda} = \frac{4m^2 KD}{r^2}$$

then

$$\frac{p}{\lambda} = \frac{(4)(.000625)(.0243)}{(10000)} = 6.075(10)^{-9}$$

$$p = (6.075)(10)^{-9}(20) = 121.5(10)^{-9} \quad \text{(Should be } 125.(10)^{-9}\text{)} .$$

Then all of the aquifer properties have been recovered to an approximation which is close enough for engineering purposes.

Comments

Because there is generally no way to maintain the water table level in the upper bed, drawdowns must occur there to supply the leakage through the semi-permeable bed and it is a fair question, therefore, as to how long the solution described can maintain its validity. In actual applications the solution has worked very well. The explanation seems to be that the pressure drawdowns spread rapidly due to the artesian type values of V_1 and the flow of the well is then drawn from a large area and the drawdown of the water table is very slow. Good correlation of test results is the rule and it will be found that when a semi-permeable bed overlies the aquifer this development will produce correlations while formulas ignoring the presence of the semi-permeable member will not produce correlation.

Applications can be made to situations where the upper member has a low permeability as compared to the lower member and the water table is in the upper member. In such cases λ can represent the saturated thickness of that part of the upper member which is below the water table.

Chapter 5

Problems

(5-1) A permeable water bearing sand having a thickness of 40 feet is overlain by a glacial till having a saturated thickness of 20 feet. A well is sunk to the bottom of the water bearing sand and gravel packed to an effective diameter of two feet. The aquifer properties are:

$K = 0.002$ ft/sec $KD = 0.080$ ft^2/sec $V_1 = 0.0005$

$p = 180 \times 10^{-9}$ ft/sec

If the well is pumped at the rate of 0.250 ft^3/sec what will be the pressure drawdown y below the semi-permeable till bed, after 24 hours of pumping, at the radii $r = 1.0$ ft, 10 ft, 100 ft and 1000 ft?

Answer:

radius (feet)	y (feet)
1.0	3.98
10.0	2.81
100.0	1.72
1000.0	0.61

(Suggestion: for $m = 0$ revert to Table 1)

(5-2) What will be the ultimate steady state drawdown?

Answer: From formula 5-6

radius (feet)	y (feet)
1.0	4.04
10.0	2.89
100.0	1.75
1000.0	0.63

(5-3) Approximately how long will it take to establish the ultimate steady state?

Answer: About 5.7 days, based upon the point from Table 14 identified by $m = 0.200 \frac{r}{\sqrt{4\alpha t}} = 0.06$. This relation is solved for t using $r = 1000$ ft . This value must be accepted as an estimate because the point when stability is reached is difficult to recognize.

Chapter 6

Bank storage

This chapter will be devoted to consideration of those cases where flow is in one direction only. The first case to be treated will be that of a reservoir, with permeable banks, which has remained filled for a long time and is then drawn down as shown in the figure below. It is desired to determine the configuration of the water table in the banks for any time subsequent to the drawdown and to estimate the rate of return flow from them and the total amount of the return flow. To illustrate the various possible approaches this case will be attacked by several methods and a final comparison of results will be made. As a first approach a solution of the differential equation 2-2

$$\alpha \frac{\partial^2 h_1}{\partial x^2} = \frac{\partial h_1}{\partial t}$$

Subject to the requirements

$h_1 = H_o$ for $x > o$ when $t = o$

$h_1 = 0$ for $x = 0$ for $t > 0$

can be sought. The required solution is:

$$h_1 = H_o \frac{2}{\sqrt{\pi}} \int_0^{\left(\frac{x}{\sqrt{4\alpha t}}\right)} e^{-u^2} du \qquad (6\text{-}1)$$

The flow of ground water at $x = 0$ is:

$$F = + KD \left(\frac{\partial h_1}{\partial x}\right)_{x=0}$$

But since, as will be shown later,

$$\frac{\partial h_1}{\partial x} = H_o \frac{2}{\sqrt{\pi}} \frac{e^{-\left(\frac{x^2}{4\alpha t}\right)}}{\sqrt{4\alpha t}}$$

And

$$\left(\frac{\partial h_1}{\partial x}\right)_{x=0} = H_o \frac{2}{\sqrt{\pi}} \frac{1}{\sqrt{4\alpha t}}$$

Then

$$F = \frac{2\ H_o\ KD}{\sqrt{4\pi\alpha t}} \qquad (6\text{-}2)$$

The total amount of return flow is obtained by integrating this expression with respect to t from 0 to t. The result is:

$$q_o = H_o\ V \sqrt{\frac{4\alpha t}{\pi}} \qquad (6\text{-}3)$$

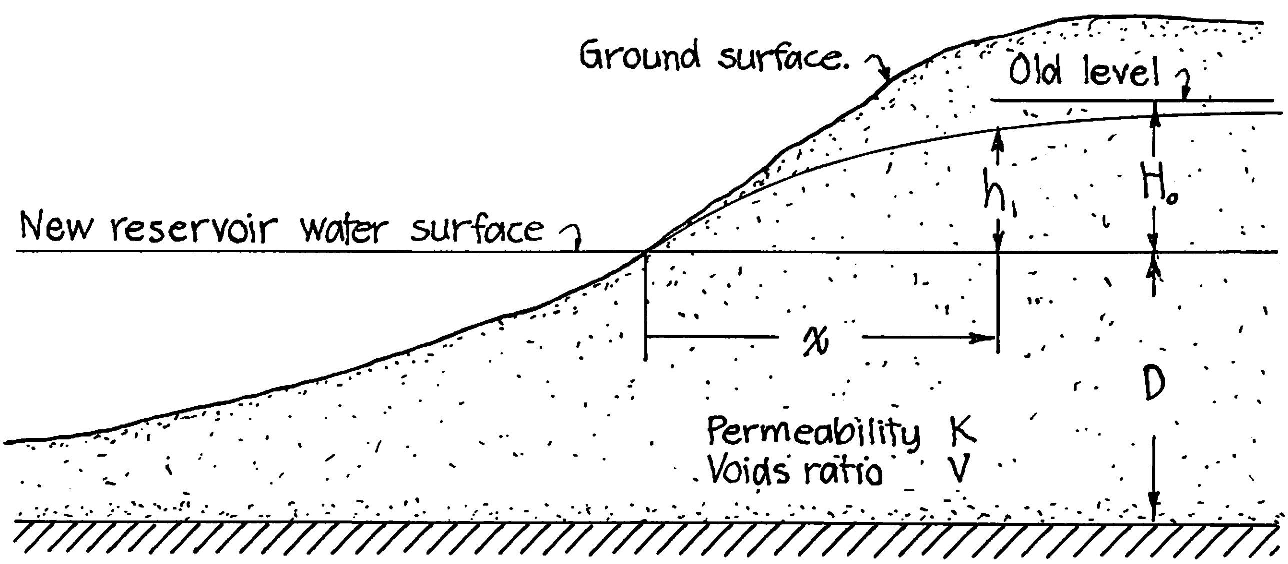


Fig. 6-1 Bank storage.

Differentiation of the integral

In order to check the validity of the expression for h_1 and to compute the derivatives needed for flow determination it is necessary to differentiate the integral which appears in the right hand member of the solution. To find the derivative let the integral be represented by the area under the curve shown below

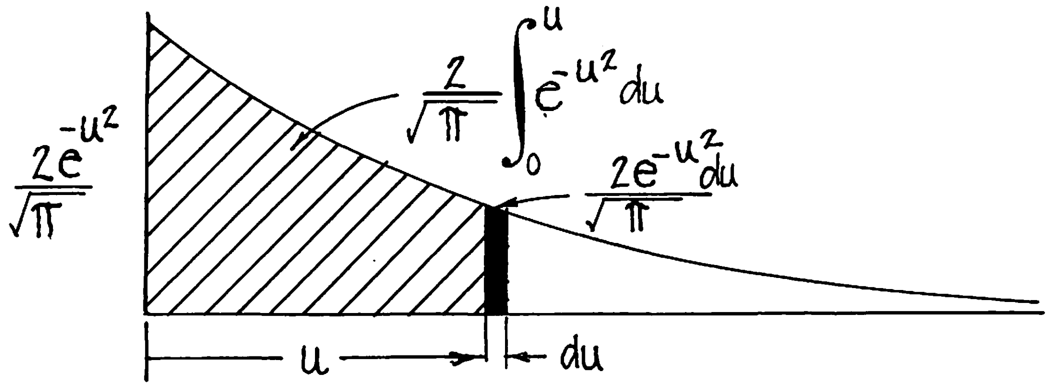


Fig. 6-2 Integral and derivative.

If

$$I = \frac{2}{\sqrt{\pi}} \int_0^u e^{-u^2} du$$

is represented by the singly shaded area then the increment of I as u increases from u to $u + du$ is

$$dI = \frac{2\ e^{-u^2}}{\sqrt{\pi}}\ du$$

or,

$$\frac{dI}{du} = \frac{2\ e^{-u^2}}{\sqrt{\pi}} \qquad (6\text{-}4)$$

The upper limit of the integral is:

$$u_1 = \left(\frac{x}{\sqrt{4\alpha t}}\right)$$

To get $\frac{\partial h_1}{\partial x}$ we note that:

$$\frac{\partial h_1}{\partial x} = \frac{\partial h_1}{\partial u}\frac{\partial u}{\partial x}$$

Then

$$\frac{\partial h_1}{\partial x} = H_o \frac{\partial I}{\partial u}\frac{\partial u}{\partial x} = H_o \frac{2}{\sqrt{\pi}} \frac{e^{-(\frac{x^2}{4\alpha t})}}{\sqrt{4\alpha t}} \qquad (6\text{-}5)$$

This is the derivative used for estimating the flow.

Check of the validity of the solution

By differentiation with respect to x

$$\frac{\partial^2 h_1}{\partial x^2} = -H_o \frac{2}{\sqrt{\pi}} \frac{e^{-(\frac{x^2}{4\alpha t})}}{\sqrt{4\alpha t}} \frac{2x}{4\alpha t} = \frac{-H_o\, 4e^{-(\frac{x^2}{4\alpha t})}}{\sqrt{\pi}\,(4\alpha t)^{\frac{3}{2}}}$$

In the above work the partial derivative of u with respect to x has been evaluated as:

$$\frac{\partial u_1}{\partial x} = \frac{1}{\sqrt{4\alpha t}}$$

In a similar manner

$$\frac{\partial h_1}{\partial t} = H_o \frac{\partial I}{\partial u}\frac{\partial u}{\partial t} = H_o \frac{2\, e^{-(\frac{x^2}{4\alpha t})}}{\sqrt{\pi}} \frac{\partial u}{\partial t} = -H_o \frac{2}{\sqrt{\pi}} \frac{e^{-(\frac{x^2}{4\alpha t})}\, 4\alpha x}{2(4\alpha t)^{\frac{3}{2}}} = -H_o \frac{4\alpha}{\sqrt{\pi}} \frac{e^{-(\frac{x^2}{4\alpha t})}}{(4\alpha t)^{\frac{3}{2}}}$$

Where

$$\frac{\partial u}{\partial t} = \frac{-x4\alpha}{2(4\alpha t)^{3/2}}$$

Then $\alpha \frac{\partial^2 h_1}{\partial x^2} = \frac{\partial h_1}{\partial t}$ and the differential equation is satisfied.

For the first boundary condition

$$h_1 = H_o \quad \text{for} \quad x > 0 \quad \text{when} \quad t = 0$$

$$h_1 \to H_o \frac{2}{\sqrt{\pi}} \int_0^\infty e^{-u^2} du = H_o \quad \text{when} \quad x \to \infty.$$

Since

$$\frac{2}{\sqrt{\pi}} \int_0^{\infty} e^{-u^2} du = 1.00$$

For the second condition

$$h_1 = 0 \quad \text{for} \quad x = 0 \quad \text{for} \quad t > 0$$

We have

$$h_1 = H_o \frac{2}{\sqrt{\pi}} \int_0^0 e^{-u^2} du = 0$$

Then the expression satisfies both the differential equation, and the boundary and initial conditions. The solution we have is therefore unique (Cohen, 1933). This means that although we might possibly find another solution differing outwardly in form from this one it must, nevertheless, yield the same numerical values as this one for a specified value of the parameter $(\frac{x}{\sqrt{4\alpha t}})$.

Application of Werner's idealization

A solution of Werner's differential equation 2-3 satisfying the condition:

$$h_2 = H_o \quad \text{for} \quad x > 0 \quad \text{when} \quad t = 0$$

$$h_2 = 0 \quad \text{for} \quad x = 0 \quad \text{for} \quad t > 0$$

is

$$u = H_o^2 \frac{2}{\sqrt{\pi}} \int_0^{(\frac{x}{\sqrt{4\alpha t}})} e^{-u^2} du$$

Then since $u = h_2^2$

$$h_2 = H_o \sqrt{\frac{2}{\sqrt{\pi}} \int_0^{(\frac{x}{\sqrt{4\alpha t}})} e^{-u^2} du} \qquad (6\text{-}6)$$

This solution is of limited application because the condition $h_2 = 0$ for $x = 0$ is one of complete drawdown at $x = 0$. Treatment of lesser drawdowns runs into the difficulty of meeting the boundary condition:

$$h_2 = \text{constant for } x = 0 \text{ for } t > 0.$$

For the complete drawdown case, however, the solution obtained is a close approximation if $\alpha_w = \frac{K H_o}{2V}$. From this idealization we have obtained a treatment of the extreme case where the water table is drawn down to the barrier at the origin. This case can be considered to be out of reach for the previous development since the drawdown is not small when compared to the original saturated depth.

It will be of interest to compute the rate of ground water flow at the origin.

The flow at x at the time t is:

$$f = K\, h_2 \left(\frac{\partial h_2}{\partial x}\right)_{x = 0}$$

But

$$\frac{\partial h_2}{\partial x} = \frac{H_o \frac{2}{\sqrt{\pi}} \frac{e^{-\left(\frac{x^2}{4\alpha t}\right)}}{\sqrt{4\alpha t}}}{2\sqrt{\frac{2}{\sqrt{\pi}} \int_0^{\left(\frac{x}{\sqrt{4\alpha t}}\right)} e^{-u^2} du}}$$

Then

$$f = \frac{KH_o^{\,2} \frac{2}{\sqrt{\pi}} \frac{e^{-\left(\frac{x^2}{4\alpha t}\right)}}{\sqrt{4\alpha t}} \sqrt{\frac{2}{\sqrt{\pi}} \int_0^{\left(\frac{x}{\sqrt{4\alpha t}}\right)} e^{-u^2} du}}{2\sqrt{\frac{2}{\sqrt{\pi}} \int_0^{\left(\frac{x}{\sqrt{4\alpha t}}\right)} e^{-u^2} du}}$$

or

$$f = \frac{K\, H_o^2\, e^{-\frac{x^2}{4\alpha t}}}{\sqrt{\pi}\ \ \sqrt{4\alpha t}} \tag{6-7}$$

As x approaches zero the flow approaches

$$F_2 = \frac{K\, H_o^2}{\sqrt{4\pi\alpha_w t}} \tag{6-8}$$

A comparison with the previous result will show that this flow is one-half what would have been computed on the basis of the first development but the variation with respect to time remains the same.

Developments of Haushild and Kruse (1962)

These authors employed two methods for extending the range of the drawdown conditions which could be treated. A second approximation development was made by the method of Picard (Agnew, 1942) but a more satisfactory treatment was obtained by utilizing physical concepts such as the following: The flows obtained from the first approximation 6-1 must be a good approximation to the true flows because it is only near the origin that drawdowns become large enough to make the validity of the first approximation questionable. Even in this area it can be expected that gross discrepancies will be absent because the solution must show the right drawdown at the origin where the drawdown is a maximum. Then it is to be expected that a much improved approximation could be obtained if the water table profile were computed on the basis of the first approximation flows but with the true saturated thickness accounted for. These concepts lead to the following formulation

$$K\ (d + h_3)\ \frac{\partial h_3}{\partial x} = KD\ \frac{\partial h_1}{\partial x}$$

Where d represents the saturated depth at the new water surface level. In this expression the right hand member represents the flow, as obtained from the first approximation. The left hand member represents the flow as computed from the Dupuit-Forchheimer idealization in its nonlinear form.

By integration:

$$\frac{(d + h_3)^2}{2} = D\, h_1 + C_1$$

If

$$h_3 = 0 \quad \text{when} \quad x = 0$$

$$C_1 = \frac{d^2}{2}$$

Then

$$(d + h_3)^2 = d^2 + 2\, Dh_1$$

or, after rearrangement:

$$h_3 = \sqrt{2\, D\, h_1 + d^2} - d. \qquad (6\text{-}9)$$

In their work they use:

$$D_1 = (d + \frac{H_o}{2}) \qquad (6\text{-}10)$$

The development of Moody

In a discussion of Paper 3317 by Haushild and Kruse, ASCE Transactions, Vol. 127, Part I, 1962, Mr. W. T. Moody developed a solution of the nonlinear partial differential equation of ground water flow from bank storage when the Dupuit-Forchheimer assumptions hold. He followed the general method described by J. Crank, in "The Mathematics of Diffusion," Oxford, 1957, pages 149-152. An iterative procedure was used to yield an essentially exact solution of the differential equation:

$$\alpha \frac{\partial^2 h}{\partial x^2} + \frac{\alpha}{D}\left(\frac{\partial h}{\partial x}\right)^2 = \frac{\partial h}{\partial t} - \frac{\alpha h}{D}\frac{\partial^2 h}{\partial x^2}$$

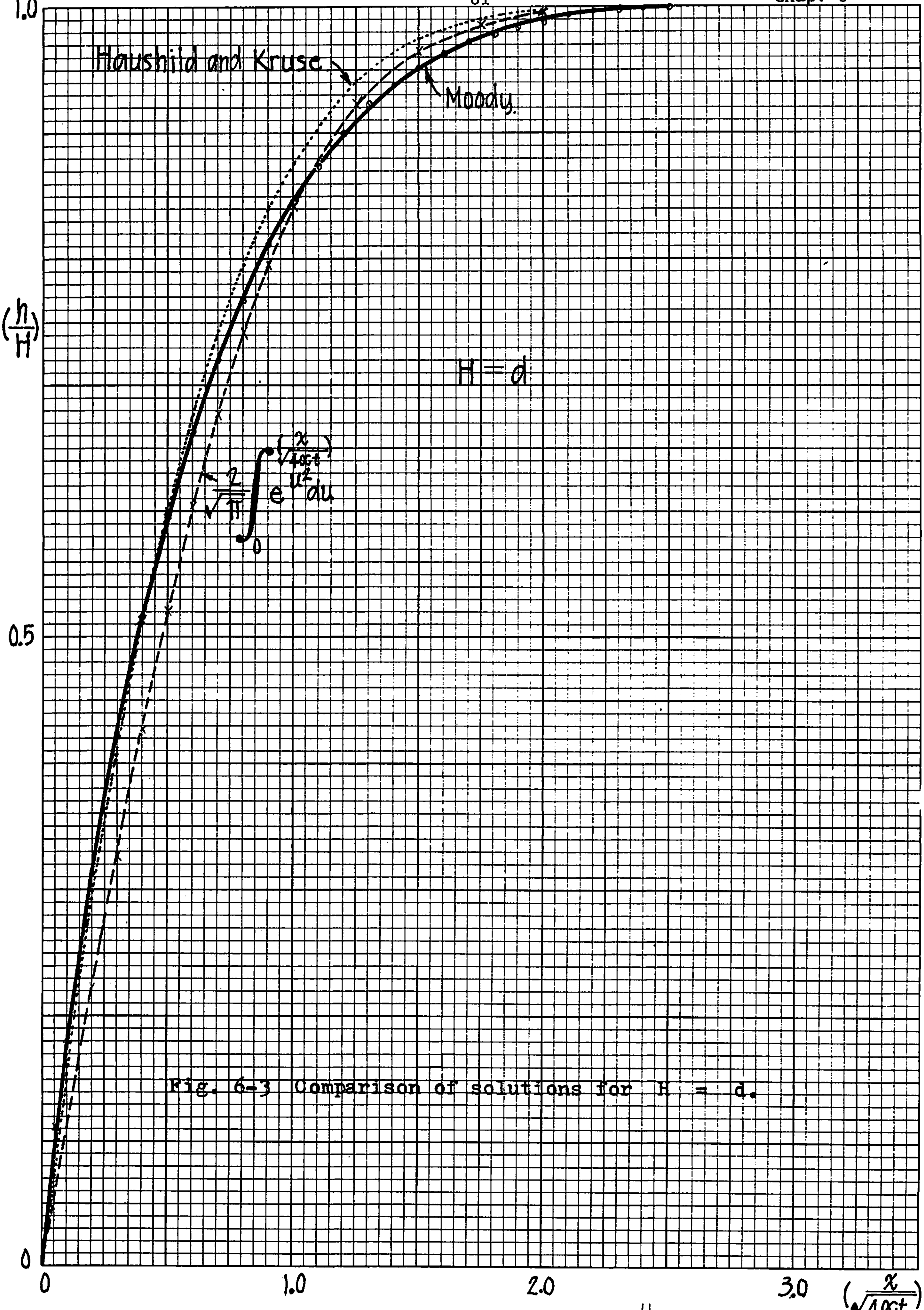


Fig. 6-3 Comparison of solutions for H = d.

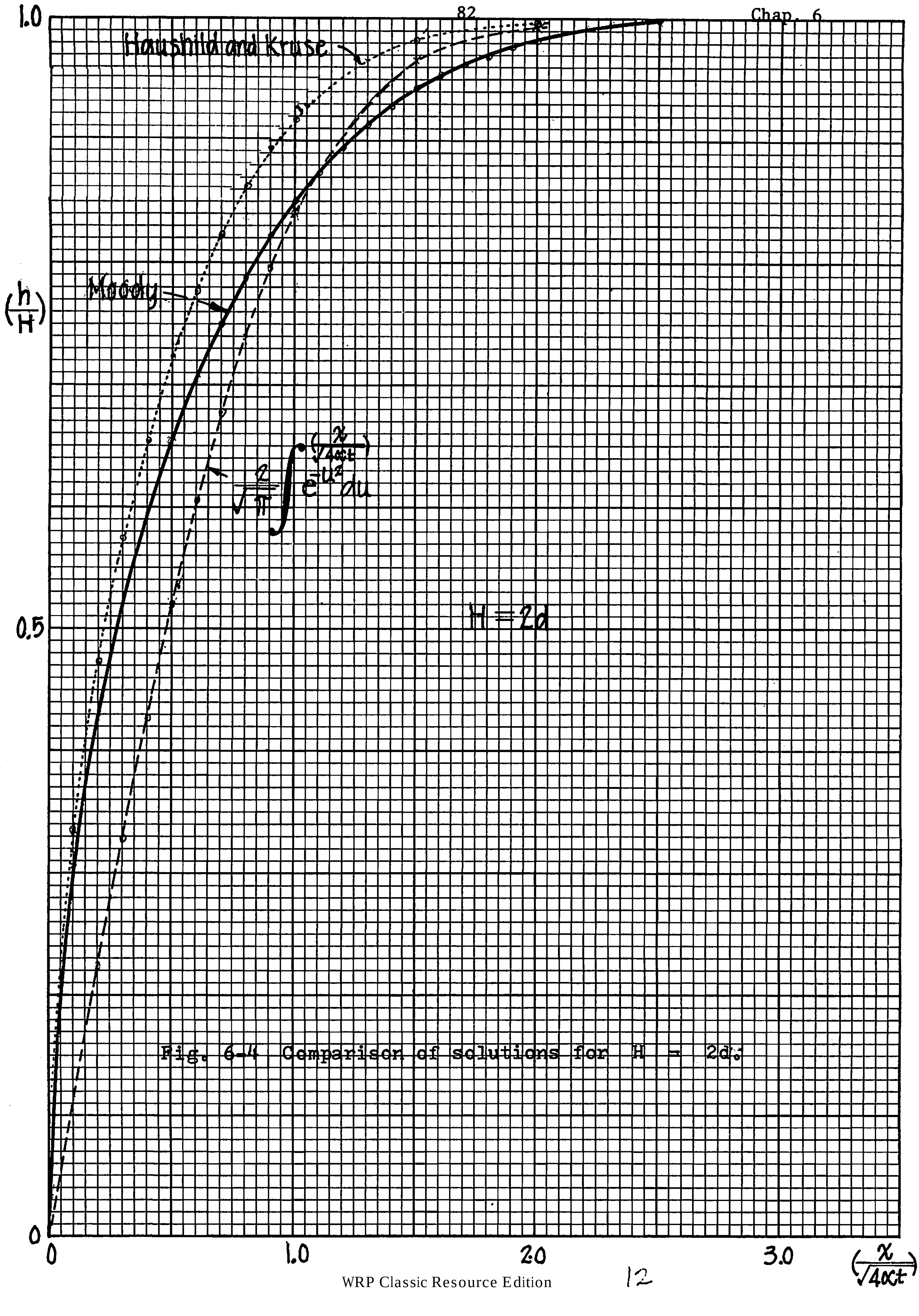


Fig. 6-4 Comparison of solutions for H = 2d.

This is Haushild and Kruse's equation (6). The solution thus developed is included on figures 6-3 and 6-4 for comparison purposes.

Comparisons

When drawdown is complete, d = 0, and it will be found that the Werner and Haushild and Kruse solutions become identical. Furthermore with the Haushild and Kruse choice of $D_1 = d + \frac{H_o}{2}$, when d = 0, reduces to $D_1 = \frac{H_o}{2}$. then the flow computed by formula 6-2 becomes identical with that obtained from the Werner idealization. No choice has yet been made for the value of D to use in the solution obtained from the Werner linearization but it would be reasonable to make the same choice as Haushild and Kruse. If this is done these developments are brought into complete accord. Haushild and Kruse checked their formula for complete drawdown against laboratory test data and found a very good agreement. Analytical developments and laboratory test data are then brought into harmony.

Comparisons with Moody's results are shown on figures 6-3 and 6-4. It is surprising how well the first approximation solution holds up even when applied to drawdowns which could well be considered excessive as judged from the approximations introduced to linearize the differential equation from which it was derived.

Values of the function

$$\frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du$$

can be found in Table 8.

Chapter 6

Problems

(6-1) A reservoir built in a sandy area has an average length of 20,000 feet, has been filled for some time, and is then drawn down 10 feet. What will be the rate of return flow from 40,000 feet of bank, at the end of the first month following drawdown, if the aquifer properties are?

$$KD = 3{,}000{,}000 \text{ ft}^2/\text{year} \qquad V = 0.15$$

$$\alpha = 20{,}000{,}000 \text{ ft}^2/\text{year}$$

Answer: 16.6 ft^3/sec.

(6-2) What will be the total return when one month has elapsed since drawdown?

Answer: 2006 acre feet.

(6-3) If the water level in the reservoir stands, on the average, 40 feet higher than it did before the reservoir was built what will be the leakage rate five years after construction?

Answer: 8.59 ft^3/sec.

(6-4) What would be the accumulated leakage loss at this time?

Answer: 62165 acre feet.

(6-5) If a severe drought caused the reservoir to be emptied, after this ground water storage had been accumulated, would a part then return to the reservoir to supplement the flow obtainable from surface storage?

Answer: Yes.

Chapter 7

Line source

Canal leakage penetrating to the water table can raise a ground water ridge from which ground water can flow both ways as shown in the figure below

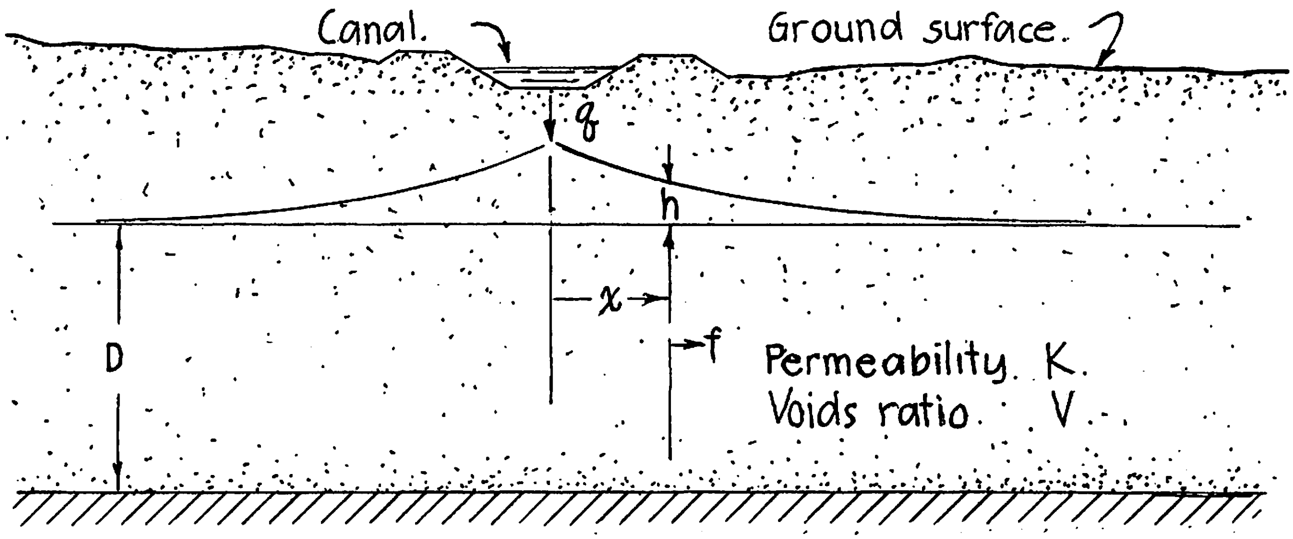


Fig. 7-1 Line source

If it is assumed that conditions are sufficiently uniform to cause a parallel flow the basic differential equation to be satisfied for a first approximation treatment is:

$$\alpha \frac{\partial^2 h}{\partial x^2} = \frac{\partial h}{\partial t}$$

This is to be solved subject to the boundary conditions

$$q = 2f \quad \text{at} \quad x = 0 \quad \text{for} \quad t > 0$$

$$h = 0 \quad \text{when} \quad t = 0 \quad \text{for} \quad x > 0$$

A solution satisfying the differential equation and the boundary conditions is:

$$h = \frac{q\sqrt{4\pi\alpha t}}{2\pi KD}\left(\frac{x}{\sqrt{4\alpha t}}\right)\int_{\left(\frac{x}{\sqrt{4\alpha t}}\right)}^{\infty}\frac{e^{-u^2}du}{u^2} \qquad (7\text{-}1)$$

Values of this integral may be obtained from Table 9. At x = 0 this becomes:

$$(h)_o = \frac{q\sqrt{4\pi\alpha t}}{2\pi KD} \qquad (7\text{-}2)$$

It will be of interest to compute the flow f passing between planes a unit distance apart. This flow is, to a first approximation:

$$f = -KD\frac{\partial h}{\partial x} = -\frac{q\sqrt{\pi}}{2\pi}\left[-\frac{\sqrt{4\alpha t}}{x}e^{-\frac{x^2}{4\alpha t}} + \int_{\left(\frac{x}{\sqrt{4\alpha t}}\right)}^{\infty}\frac{e^{-u^2}du}{u^2}\right]$$

To obtain this result the procedure for differentiating an integral has been followed. This was described previously. In the present case, however, the variation is in the lower limit which introduces a negative sign. An evaluation of the integral is needed. To obtain this integrate by parts with

$$u_1 = e^{-u^2} \qquad dv_1 = \frac{du}{u^2}$$

$$du_1 = -2ue^{-u^2}du \qquad v_1 = -\frac{1}{u}$$

Then since

$$\int u_1 dv_1 = u_1 v_1 - \int v_1 du_1$$

$$\int_{\left(\frac{x}{\sqrt{4\alpha t}}\right)}^{\infty} \frac{e^{-u^2}du}{u^2} = -\frac{e^{-u^2}}{u} - \int \frac{2ue^{-u^2}du}{u}\Bigg]_{\left(\frac{x}{\sqrt{4\alpha t}}\right)}^{\infty}$$

$$= \frac{\sqrt{4\alpha t}\ e^{-\left(\frac{x^2}{4\alpha t}\right)}}{x} - \sqrt{\pi} + \sqrt{\pi}\ \frac{2}{\sqrt{\pi}} \int_0^{\left(\frac{x}{\sqrt{4\alpha t}}\right)} e^{-u^2}du$$

In obtaining this result use has been made of the relation

$$\frac{2}{\sqrt{\pi}} \int_0^{\infty} e^{-u^2}du = 1.0$$

Then by substitution

$$f = -\frac{q\sqrt{\pi}}{2\pi}\left[-\frac{\sqrt{4\alpha t}\ e^{-\left(\frac{x^2}{4\alpha t}\right)}}{x} + \frac{\sqrt{4\alpha t}\ e^{-\left(\frac{x^2}{4\alpha t}\right)}}{x} - \sqrt{\pi} + \sqrt{\pi}\ \frac{2}{\sqrt{\pi}} \int_0^{\left(\frac{x}{\sqrt{4\alpha t}}\right)} e^{-u^2}du\right]$$

or

$$f = \frac{q}{2}\left[1 - \frac{2}{\sqrt{\pi}} \int_0^{\left(\frac{x}{\sqrt{4\alpha t}}\right)} e^{-u^2}du\right] \qquad (7\text{-}3)$$

Values of the integral shown here may be obtained from Table 8. When $x \to 0$ $f \to \frac{q}{2}$ as it should. It will appear later that a formula of this general type will hold for a point source, a line source of finite length or a line source of infinite length.

Chapter 7

Problems

(7-1) A canal constructed in alluvial sediments in a river valley leaks at the rate of one cubic foot per second per mile of length. If water is run in the canal during an irrigating season lasting six months how high a mound will the leakage create under the canal?

Aquifer properties are: K = 0.0040 ft/sec D = 60 ft.

KD = 0.240 ft^2/sec V = 0.160 α = 1.50 ft^2/sec

Answer: 2.16 feet.

(7-2) What rise of the water table is to be expected, at this time, at a distance of one quarter of a mile from the canal?

Answer: 1.68 feet.

(7-3) What will be the ground water flow, per mile of canal, at the quarter mile distance at this time?

Answer: 0.424 cubic feet per second per mile.

Chapter 8

Parallel drains

In areas where natural drainage is inadequate, irrigation will cause the water table to rise progressively until the land becomes water logged. To improve the drainage, parallel drains can be installed. These may take the form of drainage canals or of tile drains laid in a trench and back filled. The latter arrangement has the advantage that the installation of drains does not take any land out of production.

First approximation solution

A solution of the differential equation 2-2

$$\alpha \frac{\partial^2 h}{\partial x^2} = \frac{\partial h}{\partial t}$$

Subject to the conditions

$h = 0$ when $x = 0$ for $t > 0$

$h = 0$ when $x = L$ for $t > 0$

$h = H$ when $t = 0$ for $0 < x < L$

is

$$h = \frac{4H}{\pi} \sum_{n=1,3,5\ldots}^{n=\infty} \frac{e^{-n^2\pi^2\left(\frac{\alpha t}{L^2}\right)}}{n} \sin\left(\frac{n\pi x}{L}\right) \tag{8-1}$$

A cross section normal to the drains is shown in figure 8-1. When $x = \frac{L}{2}$ this expression takes the form

$$h_c = \frac{4H}{\pi} \sum_{n=1,3,5\ldots}^{n=\infty} \frac{e^{-n^2\pi^2\left(\frac{\alpha t}{L^2}\right)}}{n} \sin\left(\frac{n\pi}{2}\right) \tag{8-2}$$

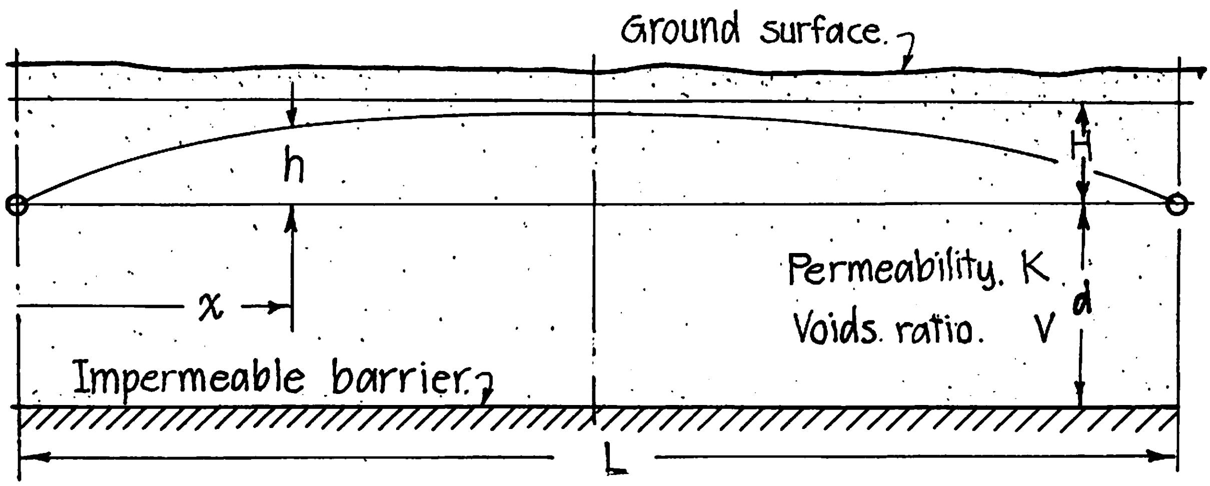


Fig. 8-1 Cross section normal to the line of drains.

Values of this function are listed in Table 10. These values find use in the technically important task of selecting a drain spacing to fit a specified set of field conditions. This is because the most difficult point to drain is midway between the drains. If this point can be drained then every other point will be drained also.

The flow to a drain from one side is

$$Kd \left(\frac{\partial h}{\partial x}\right)_{x=0} = \frac{4KdH}{L} \sum_{n=1,3,5...}^{n=\infty} e^{-n^2\pi^2\left(\frac{\alpha t}{L^2}\right)} \tag{8-3}$$

It may be noted in passing that this function has a singularity at $t = 0$. This value must be disregarded as the infinite gradient obtained from the above formula conflicts with the requirement that, for validity, the gradient must be small compared to unity. It will be shown later that there is a local resistance due to the convergence of the flow approaching a tile drain which limits the flow rate to a finite value.

Another quantity of importance is the fractional part of the drainable volume remaining at the time t. This is obtained from the relation

$$p = \frac{1}{HL}\int_0^L h\,dx = \frac{8}{\pi^2}\sum_{n=1,3,5\ldots}^{n=\infty} \frac{e^{-n^2\pi^2(\frac{\alpha t}{L^2})}}{n^2} \tag{8-4}$$

Values of p are obtainable from Table 11.

Application of Werner's method

The first approximation solution is also a solution of Werner's differential equation. The boundary and initial conditions are also appropriate but since h_2 is measured from the barrier the case represented is one where the drain is on the barrier. Then

$$\frac{h_2}{H} = \sqrt{\frac{4}{\pi}\sum_{n=1,3,5\ldots}^{n=\infty} \frac{e^{-n^2\pi^2(\frac{\alpha t}{L^2})}}{n} \sin\left(\frac{n\pi x}{L}\right)} \tag{8-5}$$

The development of Boussinesq

A transient state drainage treatment has been contributed by J. Boussinesq (Boussinesq 1904). He used concepts very similar to those employed by Dupuit in that he assumed the surface gradient to apply throughout the saturated depth. In our notation the condition of continuity would take the form

$$\frac{\partial}{\partial x}\left(Kh_2 \frac{\partial h_2}{\partial x}\right) = V \frac{\partial h_2}{\partial t} \tag{8-6}$$

The development applies where the drains are on the barrier at a distance L apart. The distance x is measured from one of the drains toward the other. The drainable depth h_2 has the value H at $x = \frac{L}{2}$ when the time $t = 0$. Let

$$U = \frac{h_2}{H}$$

$$\xi = \frac{x}{L}$$

$$\eta = \frac{KH}{VL^2} t$$

Then the differential equation takes the form

$$\frac{\partial}{\partial \xi} (U \frac{\partial U}{\partial \xi}) = \frac{\partial U}{\partial \eta} \qquad (8\text{-}7)$$

A possible type of solution is

$$U = WY$$

Where W is a function of ξ only and Y is a function of η only. Substitution of this product into the differential equation permits a separation of the variables and yields two ordinary differential equations one in W and the other in Y. In the case of Y the relation is

$$\frac{1}{Y^2} \frac{dY}{d\eta} = -C$$

And a solution satisfying the conditions $Y = 1$ when $\eta = 0$ is

$$Y = \frac{1}{C\eta + 1}$$

The differential equation for W is of the nonlinear form

$$\frac{d^2W}{d\xi^2} + \frac{1}{W} (\frac{dW}{d\xi})^2 = -C$$

This can be reduced to a first order differential equation by the substitutions

$$\frac{dW}{d\xi} = p \qquad \frac{d^2W}{d\xi^2} = p \frac{dp}{dW}$$

After substitution the above differential equation becomes

$$p \frac{dp}{dW} + \frac{p^2}{W} = -C$$

Where it may be noted that W has now become the independent variable. A further substitution

$$v = p^2 \qquad \frac{dv}{dW} = 2p \frac{dp}{dW}$$

reduces it to the linear form

$$\frac{dv}{dW} + \frac{2v}{W} = -2C$$

A solution is

$$vW^2 = -\frac{2CW^3}{3} + C_2$$

or

$$p^2 = -\frac{2CW}{3} + \frac{C_2}{W^2}$$

If $p = 0$ when $W = 1$ $C_2 = \frac{2C}{3}$ and

$$\frac{W}{\sqrt{1-W^3}} \frac{dW}{d\xi} = \sqrt{\frac{2C}{3}}$$

By integration subject to the condition that $W = 0$ when $\xi = 0$.

$$\int_0^W \frac{WdW}{\sqrt{1-W^3}} = \sqrt{\frac{2C}{3}}\,\xi \qquad (8\text{-}8)$$

When $\xi = \frac{1}{2}$ $W = 1$ then

$$\int_0^1 \frac{WdW}{\sqrt{1-W^3}} = \sqrt{\frac{C}{6}}$$

The integral of this relation can be evaluated with the aid of the Beta and Gamma functions (Osgood 1933, p. 485). The evaluation is

$$\int_0^1 \frac{WdW}{\sqrt{1-W^3}} = \frac{\Gamma(\frac{1}{2})\ \Gamma(\frac{2}{3})}{3\Gamma(\frac{7}{6})} = 0.86237$$

then

$$C = (6)\ (0.86237)^2 = 4.46209 \qquad \sqrt{\frac{2C}{3}} = 1.72474$$

The following table is reproduced through the courtesy of Mr. W. T. Moody.

ξ	W
0.0	0.0
0.005	0.412
0.10	0.575
0.15	0.692
0.20	0.782
0.25	0.853
0.30	0.908
0.35	0.949
0.40	0.978
0.45	0.994
0.50	1.000

Finally

$$\frac{h}{H} = \frac{W}{4.46\left(\frac{KHt}{VL^2}\right) + 1} \qquad (8\text{-}9)$$

This development is of value because it gives indications as to what drainage performance is to be expected when the drains are near the barrier. The first approximation solution gives little guidance in such cases. It may be noted that the initial condition of a uniform drainable depth is not met by the Boussinesq development. It is interesting to note also that the pattern of decrease is not of a descending exponential type but here takes an algebraic form.

The Method of Brooks

A second approximation solution which remains valid when the drainable depth is not negligibly small compared to the saturated depth below the drains was obtained by Brooks 1963 by application of the Pioncare, Lighthill, Kuo method. Good results were obtained where drainable depth were as great as the saturated depth below the drains (H/d = 1.0). He also developed a second approximation of the type described by Haushild and Kruse, 1962. This formula can be expressed in our notation as

$$h_1 = -D_a + \sqrt{D_a^2 + 2D_a h_0 + \left(\frac{H}{2}\right)^2} \qquad (8\text{-}10)$$

Where $D_a = (d + \frac{H}{2})$ and h_0 comes from the first approximation. It is obtained by computing the drainable depths on a nonlinear basis based upon the flows obtained from the first approximation. The original paper may need to be consulted. His origin is placed midway of the original drainable depth.

The Method of Dumm, Tapp and Moody

This procedure was developed at the U.S. Bureau of Reclamation* to provide an orderly approach to the problem of determining drain spacings. It was recognized that there would be applications where the drainable depth would not be small when compared to the barrier depth. It was also understood that where the drainable depth is not small, in the above sense, the basic differential equation would be nonlinear in form which would mean that the principle of superposition would not apply since the sum of two solutions is then not a solution. As a consequence of this concept the superposition of uniform increments of drainable depth originating in uniform irrigations was abandoned in favor of a drainage pattern representing observed configurations after a number of irrigations had been made. Computations of drainage progress were begun anew with each irrigation. The increment of added drainable depth being added to the depth obtained from the preceding calculation. Computations are made for the point midway between drains. The pattern chosen to represent the data obtained from field observations is

$$h = 8H \left(\frac{x}{L} - \frac{3x^2}{L^2} + \frac{4x^3}{L^3} - \frac{2x^4}{L^4}\right) \tag{8-11}$$

A first approximation solution having this initial configuration is given as

$$h_1 = \frac{192H}{\pi^5} \sum_{m=0}^{m=\infty} \frac{[(2m+1)^2\ \pi^2 - 8]\ e^{-(2m+1)^2\ \pi^2 \left(\frac{\alpha t}{L^2}\right)}}{(2m+1)^5} \sin \frac{(2m+1)\pi x}{L} \tag{8-12}$$

At the point midway between drains this takes the form

*See also USBR Eng. Monograph 31, 1966.

$$\left(\frac{h_1}{H}\right)_{\frac{L}{2}} = \frac{192}{\pi^3} \sum_{n=1,3,5...}^{n=\infty} \frac{(-1)^{\frac{n-1}{2}} \left(n^2 - \frac{8}{\pi^2}\right) e^{-n^2\pi^2\left(\frac{\alpha t}{L^2}\right)}}{n^5} \qquad (8\text{-}13)$$

The initial drainable depth at the point midway between drains will here be represented by H_m . The quantity D_m is expressible in the notation of this volume as $D_m = (d + H_m)$ and the aquifer constant is, correspondingly, $\alpha_m = \frac{KD_m}{V}$. The quantity q_m represents the rate of discharge to unit length of drains from the space between two drains, or, if a single drain is considered, it represents the flow to unit length of drain, from both sides. The quantity W_m represents the total flow to a unit length of drain from both sides. The quantity m is given by the relation $m = \left(\frac{H_c}{d + H_c}\right)$. The quantity h_m represents the drainable depth midway between drains at the time t .

For the case where the drains are on the barrier Boussinesq's solution is used.

The intermediate case was treated by comparison with field tests. It was found that the first approximation formula could be used providing an aquifer constant of the form:

$$\alpha = \frac{K\,D_a}{V} \qquad \text{was used, where} \qquad D_a = \left(d + \frac{H}{2}\right) .$$

<u>Moody's development</u>

A development by Moody 1966 does away with the need to make such a choice. He used a computer to solve the nonlinear differential equation for a series of drain positions ranging from a location near the water table to a location on the barrier. He produced a table giving the maximum water table height, the discharge to the drains and the volume of water removed. This is in dimensionless form and covers the entire range of possible drain positions between the water table and the barrier. The table is reproduced here as Table 8-1 through the courtesy of Mr. W. T. Moody and of the U. S. Bureau of Reclamation.*

The procedures described have been presented to the Profession in a series of papers and much comment has been received both favorable and unfavorable. The authors have made changes to meet the unfavorable comments and have correlated their methods with field data from Australia, Canada and the United States. The method has also been correlated with the results of laboratory studies. There is also accumulating the experience with field installations for which drain spacings have been selected by application of the method. The experience with such installations is understood to have been satisfactory. So far as this writer is aware, this method is the most carefully worked out, has received the most searching scrutiny, and has been more extensively tested against laboratory and field data than any method proposed for determining the spacing of drains which will, on the one hand, provide satisfactory drainage and on the other hand avoid the excessive expenditures incurred when the drains are spaced closer than necessary. Some valuable by-products were obtained in the period of development, which has now covered 12 years and a study of the original papers is therefore recommended to those who become seriously involved in the task of selecting drain spacings. One of these is the need to account for the local resistance to flow near the drain. This problem will be dealt with later in the text.

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*From unpublished USBR data. For a Graphical Presentation see ASCE Paper No. 4835, by William T. Moody, on "Nonlinear Differential Equation of Drain Spacing," Journal of the Irrigation and Drainage Division, June 1966, pp. 1-9, inclusive.

Table 8-1

Values of Dimensionless Parameters for Water Surface Height (h_m/H_m) Flow Rate (q_mL/KD_mH_m) and Volume Drained (W_m/VLH_m) for Given Parameters of Time ($\alpha_m t/L^2$). For notation see page 98.

	m = 1.0			m = 0.9			m = 0.8			m = 0.7			m = 0.6		
$\frac{\alpha_m t}{L^2}$	$\frac{h_m}{H_m}$	$\frac{q_mL}{KD_mH_m}$	$\frac{W_m}{VLH_m}$	$\frac{h_m}{H_m}$	$\frac{q_mL}{KD_mH_m}$	$\frac{W_m}{VLH_m}$	$\frac{h_m}{H_m}$	$\frac{q_mL}{KD_mH_m}$	$\frac{W_m}{VLH_m}$	$\frac{h_m}{H_m}$	$\frac{q_mL}{KD_mH_m}$	$\frac{W_m}{VLH_m}$	$\frac{h_m}{H_m}$	$\frac{q_mL}{KD_mH_m}$	$\frac{W_m}{VLH_m}$
0.	1.	1.9746	0.	1.	3.2249	0.	1.	4.4752	0.	1.	5.7254	0.	1.	6.9757	0.
0.0002	1.0000	2.0845	0.0004	1.0000	3.3219	0.0007	1.0000	4,5506	0.0009	1.0000	5.7710	0.0012	1.0000	6.9831	0.0014
0.0004	0.9999	2.1915	0.0008	0.9999	3.4138	0.0013	0.9999	4.6199	0.0018	0.9999	5.8103	0.0023	0.9999	6.9852	0.0028
0.0006	0.9999	2.2928	0.0013	0.9999	3.4980	0.0020	0.9999	4.6809	0.0028	0.9999	5.8419	0.0035	0.9999	6.9816	0.0042
0.0008	0.9998	2.3898	0.0018	0.9998	3.5763	0.0027	0.9998	4.7355	0.0037	0.9998	5.8679	0.0047	0.9998	6.9745	0.0056
0.0010	0.9997	2.4815	0.0023	0.9997	3.6482	0.0035	0.9997	4.7835	0.0047	0.9997	5.8882	0.0058	0.9997	6.9634	0.0070
0.0012	0.9996	2.5687	0.0028	0.9996	3.7144	0.0042	0.9996	4.8259	0.0056	0.9996	5.9040	0.0070	0.9996	6.9496	0.0084
0.0014	0.9995	2.6508	0.0033	0.9995	3.7752	0.0050	0.9995	4.8632	0.0066	0.9995	5.9155	0.0082	0.9995	6.9331	0.0098
0.0016	0.9994	2.7282	0.0038	0.9994	3.8310	0.0057	0.9994	4.8959	0.0076	0.9994	5.9234	0.0094	0.9994	6.9146	0.0112
0.0018	0.9993	2.8011	0.0044	0.9992	3.8820	0.0065	0.9992	4.9242	0.0085	0.9992	5.9280	0.0106	0.9992	6.8942	0.0125
0.0020	0.9991	2.8695	0.0049	0.9991	3.9286	0.0073	0.9991	4.9487	0.0095	0.9991	5.9297	0.0118	0.9991	6.8724	0.0139
0.0040	0.9968	3.3459	0.0112	0.9967	4.2107	0.0155	0.9967	5.0459	0.0196	0.9967	5.8462	0.0236	0.9967	6.6097	0.0274
0.0060	0.9932	3.5618	0.0182	0.9931	4.2880	0.0240	0.9931	4.9969	0.0296	0.9930	5.6788	0.0351	0.9930	6.3289	0.0403
0.0080	0.9885	3.6382	0.0254	0.9884	4.2695	0.0325	0.9883	4.8910	0.0395	0.9882	5.4908	0.0463	0.9880	6.0630	0.0527
0.0100	0.9831	3.6422	0.0327	0.9829	4.2061	0.0410	0.9826	4.7644	0.0492	0.9824	5.3042	0.0570	0.9821	5.8192	0.0646
0.0120	0.9770	3.6084	0.0399	0.9766	4.1224	0.0494	0.9762	4.6328	0.0586	0.9758	5.1267	0.0675	0.9753	5.5979	0.0760
0.0140	0.9706	3.5551	0.0471	0.9699	4.0306	0.0575	0.9693	4.5034	0.0677	0.9685	4.9609	0.0776	0.9678	5.3971	0.0870
0.0160	0.9638	3.4921	0.0541	0.9628	3.9368	0.0655	0.9619	4.3791	0.0766	0.9608	4.8070	0.0873	0.9597	5.2146	0.0976
0.0180	0.9567	3.4248	0.0610	0.9555	3.8441	0.0733	0.9542	4.2612	0.0852	0.9528	4.6644	0.0968	0.9512	5.0481	0.1079
0.0200	0.9496	3.3563	0.0678	0.9480	3.7542	0.0809	0.9462	4.1499	0.0936	0.9444	4.5321	0.1060	0.9424	4.8954	0.1178
0.0400	0.8779	2.7630	0.1287	0.8711	3.0466	0.1483	0.8637	3.3255	0.1677	0.8557	3.5911	0.1863	0.8472	3.8409	0.2041
0.0600	0.8133	2.3404	0.1795	0.8006	2.5708	0.2042	0.7868	2.7936	0.2285	0.7721	3.0019	0.2519	0.7566	3.1944	0.2741
0.0800	0.7569	2.0182	0.2229	0.7387	2.2134	0.2519	0.7191	2.3981	0.2803	0.6984	2.5663	0.3074	0.6766	2.7178	0.3330

Table 8-1--Continued

	m = 1.0			m = 0.9			m = 0.8			m = 0.7			m = 0.6		
$\frac{\alpha_m t}{L^2}$	$\frac{h_m}{H_m}$	$\frac{q_m L}{KD_m H_m}$	$\frac{W_m}{VLH_m}$	$\frac{h_m}{H_m}$	$\frac{q_m L}{KD_m H_m}$	$\frac{W_m}{VLH_m}$	$\frac{h_m}{H_m}$	$\frac{q_m L}{KD_m H_m}$	$\frac{W_m}{VLH_m}$	$\frac{h_m}{H_m}$	$\frac{q_m L}{KD_m H_m}$	$\frac{W_m}{VLH_m}$	$\frac{h_m}{H_m}$	$\frac{q_m L}{KD_m H_m}$	$\frac{W_m}{VLH_m}$
0.1000	0.7076	1.7613	0.2607	0.6845	1.9296	0.2933	0.6598	2.0848	0.3250	0.6338	2.2214	0.3551	0.6069	2.3401	0.3834
0.1200	0.6643	1.5514	0.2937	0.6369	1.6982	0.3295	0.6077	1.8291	0.3641	0.5773	1.9398	0.3967	0.5460	2.0315	0.4270
0.1400	0.6260	1.3772	0.3230	0.5947	1.5061	0.3614	0.5617	1.6168	0.3985	0.5274	1.7059	0.4330	0.4926	1.7750	0.4650
0.1600	0.5918	1.2309	0.3490	0.5572	1.3446	0.3899	0.5207	1.4384	0.4290	0.4833	1.5093	0.4651	0.4455	1.5595	0.4983
0.1800	0.5612	1.1067	0.3723	0.5235	1.2076	0.4154	0.4841	1.2869	0.4562	0.4439	1.3423	0.4936	0.4038	1.3767	0.5276
0.2000	0.5336	1.0004	0.3934	0.4932	1.0903	0.4383	0.4511	1.1571	0.4806	0.4087	1.1994	0.5190	0.3668	1.2205	0.5536
0.4000	0.3576	0.4494	0.5275	0.3006	0.4811	0.5835	0.2450	0.4839	0.6314	0.1941	0.4637	0.6705	0.1501	0.4274	0.7019
0.6000	0.2689	0.2541	0.5951	0.2046	0.2650	0.6550	0.1467	0.2481	0.7011	0.1000	0.2146	0.7345	0.0654	0.1738	0.7578
0.8000	0.2155	0.1632	0.6358	0.1477	0.1644	0.6969	0.0923	0.1413	0.7388	0.0534	0.1081	0.7654	0.0292	0.0752	0.7813
1.0000	0.1798	0.1136	0.6630	0.1106	0.1099	0.7238	0.0596	0.0855	0.7610	0.0290	0.0569	0.7813	0.0132	0.0335	0.7916
1.2000	0.1542	0.0836	0.6825	0.0848	0.0771	0.7422	0.0391	0.0537	0.7746	0.0159	0.0306	0.7898	0.0060	0.0151	0.7962
1.4000	0.1350	0.0641	0.6971	0.0661	0.0561	0.7554	0.0259	0.0345	0.7833	0.0088	0.0167	0.7944	0.0027	0.0068	0.7983
1.6000	0.1200	0.0507	0.7085	0.0522	0.0419	0.7651	0.0173	0.0226	0.7889	0.0048	0.0092	0.7969	0.0012	0.0031	0.7992
1.8000	0.1081	0.0411	0.7176	0.0415	0.0319	0.7725	0.0116	0.0149	0.7926	0.0027	0.0051	0.7983	0.0006	0.0014	0.7996
2.0000	0.0983	0.0340	0.7251	0.0333	0.0246	0.7781	0.0078	0.0099	0.7950	0.0015	0.0028	0.7991	0.0003	0.0006	0.7998

Table 8-1--Continued

$\frac{\alpha_m t}{L^2}$	m = 0.5 $\frac{h_m}{H_m}$	m = 0.5 $\frac{q_m L}{KD_m H_m}$	m = 0.5 $\frac{W_m}{VLH_m}$	m = 0.4 $\frac{h_m}{H_m}$	m = 0.4 $\frac{q_m L}{KD_m H_m}$	m = 0.4 $\frac{W_m}{VLH_m}$	m = 0.3 $\frac{h_m}{H_m}$	m = 0.3 $\frac{qL}{KDH}$	m = 0.3 $\frac{W_m}{VLH_m}$	m = 0.2 $\frac{h_m}{H_m}$	m = 0.2 $\frac{q_m L}{KD_m H_m}$	m = 0.2 $\frac{W_m}{VLH_m}$	m = 0.1 $\frac{h_m}{H_m}$	m = 0.1 $\frac{q_m L}{KD_m H_m}$	m = 0.1 $\frac{W_m}{VLH_m}$
0.	1.	8.2260	0.	1.	9.4762	0.	1.	10.7265	0.	1.	11.9768	0.	1.	13.2271	0.
0.0002	1.0000	8.1873	0.0017	1.0000	9.3837	0.0019	1.0000	10.5724	0.0021	1.0000	11.7538	0.0024	1.0000	12.9280	0.0026
0.0004	0.9999	8.1452	0.0033	0.9999	9.2905	0.0038	0.9999	10.4217	0.0042	0.9999	11.5389	0.0047	0.9999	12.6428	0.0052
0.0006	0.9999	8.1006	0.0049	0.9999	9.1994	0.0056	0.9999	10.2786	0.0063	0.9999	11.3389	0.0070	0.9999	12.3808	0.0077
0.0008	0.9998	8.0559	0.0065	0.9998	9.1132	0.0074	0.9998	10.1470	0.0084	0.9998	11.1584	0.0092	0.9998	12.1482	0.0101
0.0010	0.9997	8.0100	0.0081	0.9997	9.0289	0.0093	0.9997	10.0214	0.0104	0.9997	10.9887	0.0115	0.9997	11.9319	0.0125
0.0012	0.9996	7.9640	0.0097	0.9996	8.9484	0.0111	0.9996	9.9043	0.0124	0.9996	10.8331	0.0136	0.9996	11.7363	0.0149
0.0014	0.9995	7.9174	0.0113	0.9995	8.8698	0.0128	0.9995	9.7920	0.0143	0.9995	10.6858	0.0158	0.9995	11.5527	0.0172
0.0016	0.9994	7.8710	0.0129	0.9994	8.7943	0.0146	0.9994	9.6863	0.0163	0.9994	10.5489	0.0179	0.9994	11.3842	0.0195
0.0018	0.9992	7.8245	0.0145	0.9992	8.7206	0.0164	0.9992	9.5845	0.0182	0.9992	10.4185	0.0200	0.9992	11.2247	0.0218
0.0020	0.9991	7.7784	0.0160	0.9991	8.6494	0.0181	0.9991	9.4879	0.0201	0.9991	10.2961	0.0221	0.9991	11.0764	0.0240
0.0040	0.9967	7.3369	0.0311	0.9967	8.0298	0.0347	0.9967	8.6914	0.0382	0.9967	9.3247	0.0416	0.9966	9.9327	0.0449
0.0060	0.9929	6.9468	0.0454	0.9929	7.5339	0.0503	0.9928	8.0932	0.0550	0.9928	8.6277	0.0596	0.9927	9.1404	0.0640
0.0080	0.9879	6.6062	0.0590	0.9878	7.1220	0.0649	0.9876	7.6129	0.0707	0.9875	8.0819	0.0763	0.9873	8.5315	0.0816
0.0100	0.9818	6.3079	0.0719	0.9815	6.7716	0.0788	0.9812	7.2129	0.0855	0.9809	7.6345	0.0920	0.9805	8.0385	0.0982
0.0120	0.9748	6.0447	0.0842	0.9742	6.4686	0.0921	0.9737	6.8718	0.0996	0.9731	7.2570	0.1068	0.9725	7.6264	0.1138
0.0140	0.9670	5.8106	0.0961	0.9661	6.2027	0.1047	0.9652	6.5758	0.1130	0.9643	6.9321	0.1210	0.9633	7.2738	0.1287
0.0160	0.9586	5.6008	0.1075	0.9573	5.9669	0.1169	0.9560	6.3152	0.1259	0.9546	6.6480	0.1346	0.9530	6.9671	0.1430
0.0180	0.9496	5.4113	0.1185	0.9479	5.7558	0.1286	0.9460	6.0834	0.1383	0.9441	6.3964	0.1476	0.9420	6.6966	0.1566
0.0200	0.9403	5.2392	0.1291	0.9380	5.5651	0.1399	0.9356	5.8752	0.1503	0.9330	6.1713	0.1602	0.9302	6.4553	0.1698
0.0400	0.8382	4.0756	0.2210	0.8285	4.2969	0.2371	0.8184	4.5066	0.2525	0.8077	4.7061	0.2672	0.7964	4.8964	0.2814
0.0600	0.7402	3.3725	0.2956	0.7231	3.5380	0.3150	0.7053	3.6920	0.3340	0.6869	3.8357	0.3521	0.6679	3.9695	0.3695
0.0800	0.6540	2.8542	0.3571	0.6307	2.9771	0.3799	0.6068	3.0875	0.4015	0.5825	3.1860	0.4221	0.5579	3.2730	0.4417

Table 8-1--Continued

	m = 0.5			m = 0.4			m = 0.3			m = 0.2			m = 0.1		
$\frac{\alpha_m t}{L^2}$	$\frac{h_m}{H_m}$	$\frac{q_m L}{KD_m H_m}$	$\frac{W_m}{VLH_m}$	$\frac{h_m}{H_m}$	$\frac{q_m L}{KD_m H_m}$	$\frac{W_m}{VLH_m}$	$\frac{h_m}{H_m}$	$\frac{q_m L}{KD_m H_m}$	$\frac{W_m}{VLH_m}$	$\frac{h_m}{H_m}$	$\frac{q_m L}{KD_m H_m}$	$\frac{W_m}{VLH_m}$	$\frac{h_m}{H_m}$	$\frac{q_m L}{KD_m H_m}$	$\frac{W_m}{VLH_m}$
0.1000	0.5792	2.4427	0.4099	0.5510	2.5306	0.4348	0.5225	2.6046	0.4583	0.4940	2.6652	0.4804	0.4658	2.7128	0.5014
0.1200	0.5142	2.1061	0.4553	0.4823	2.1649	0.4817	0.4506	2.2087	0.5063	0.4194	2.2381	0.5293	0.3890	2.2537	0.5509
0.1400	0.4576	1.8263	0.4945	0.4230	1.8611	0.5218	0.3891	1.8803	0.5471	0.3564	1.8846	0.5704	0.3251	1.8750	0.5920
0.1600	0.4081	1.5914	0.5287	0.3716	1.6065	0.5564	0.3365	1.6058	0.5819	0.3031	1.5904	0.6051	0.2717	1.5617	0.6263
0.1800	0.3646	1.3926	0.5584	0.3269	1.3915	0.5864	0.2912	1.3750	0.6116	0.2579	1.3446	0.6344	0.2272	1.3020	0.6549
0.2000	0.3263	1.2231	0.5846	0.2880	1.2090	0.6123	0.2523	1.1801	0.6371	0.2196	1.1385	0.6592	0.1900	1.0862	0.6787
0.4000	0.1136	0.3805	0.7266	0.0844	0.3285	0.7459	0.0618	0.2758	0.7605	0.0448	0.2261	0.7715	0.0321	0.1814	0.7796
0.6000	0.0414	0.1331	0.7735	0.0256	0.0971	0.7837	0.0155	0.0682	0.7901	0.0092	0.0464	0.7941	0.0054	0.0307	0.7965
0.8000	0.0153	0.0485	0.7902	0.0078	0.0295	0.7950	0.0039	0.0171	0.7975	0.0019	0.0096	0.7988	0.0009	0.0052	0.7994
1.0000	0.0057	0.0180	0.7964	0.0024	0.0090	0.7985	0.0010	0.0043	0.7994	0.0004	0.0020	0.7997	0.0002	0.0009	0.7999

Moody's development is based upon an initial configuration given by the relation $h = H_m[1 - (\frac{2x_m}{L})^4]$. His origin of coordinates is at midspan. The following notation applies to his development.

$D_m = (d + H_m)$.

H_m represents the initial drainage depth, at midspan, at time zero.

h_m the drainable depth, at midspan, at the time t.

$m = \frac{H_m}{d + H_m} = \frac{H_m}{D_m}$. $\quad H_m = m\,D_m$

q_m = the rate of discharge from the area between two drains, per unit length along the drains.

W_m = the total quantity of water removed from the area between two drains, per unit length along the drains, up to the time t.

x_m = distance from midspan measured horizontally toward a drain.

$\alpha_m = \frac{KD_m}{V}$

Table 8-1--Continued

m = 0

$\frac{\alpha_m t}{L^2}$	$\frac{h_m}{H_m}$	$\frac{q_m L}{K D_m H_m}$	$\frac{W_m}{V L H_m}$
0.	1.	16.	0.
0.001	0.9998	12.9402	0.0139
0.002	0.9992	11.8719	0.0263
0.003	0.9983	11.1239	0.0377
0.004	0.9969	10.5388	0.0486
0.005	0.9952	10.0560	0.0589
0.006	0.9931	9.6447	0.0687
0.007	0.9906	9.2866	0.0782
0.008	0.9877	8.9698	0.0873
0.009	0.9845	8.6862	0.0961
0.010	0.9808	8.4298	0.1047
0.020	0.9279	6.7266	0.1795
0.030	0.8579	5.7562	0.2415
0.040	0.7844	5.0790	0.2955
0.050	0.7137	4.5459	0.3436
0.060	0.6478	4.0957	0.3867
0.070	0.5874	3.7014	0.4257
0.080	0.5324	3.3496	0.4609
0.090	0.4825	3.0332	0.4928
0.100	0.4372	2.7475	0.5217
0.200	0.1629	1.0238	0.6963
0.300	0.0607	0.3816	0.7613
0.400	0.0226	0.1422	0.7856
0.500	0.0084	0.0530	0.7946
0.600	0.0031	0.0198	0.7980
0.700	0.0012	0.0074	0.7993
0.800	0.0004	0.0027	0.7997
0.900	0.0002	0.0010	0.7999
1.000	0.0001	0.0004	0.8000

Parallel drains from the Laplace standpoint*

Consider the expression

$$p = (d-y) + \sum_{n=1}^{n=m} A_n \cosh\left(\frac{n\pi y}{L}\right) \sin\left(\frac{n\pi x}{L}\right) \quad . \tag{8-14}$$

where p represents a pressure. It is measured in feet of water and represents a departure from the pressures appropriate to a static state. This is a solution of the Laplace differential equation.

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = 0$$

which meets the requirement that $\frac{\partial p}{\partial y} = 0$ when $y = 0$. The coordinate y is measured upward from the barrier. The quantities A_n are to be chosen to meet the initial conditions that

$$p = H \quad \text{when} \quad y = d \quad \text{for} \quad 0 < x < L$$

The term (d-y) represents the hydrostatic pressure which would be present if the water table were at the level of the water surface maintained in the drains. The quantity p represents a pressure in feet of water and the terms under the summation sign represent the additional pressures present when the water table is above the level of the drains. The pressure p will be zero when

$$0 = (d-y) + \sum_{1}^{m} A_n \cosh\left(\frac{n\pi y}{L}\right) \sin\left(\frac{n\pi x}{L}\right) \quad .$$

*This presentation follows closely that of the paper on "Parallel Drains from the Laplace Standpoint," by Robert E. Glover, which appeared in the Journal of the American Water Resources Association, Vol. 8, No. 1, February 1972, pp. 50-54 inclusive. The development is presented here through the courtesy of the A.W.R.A.

If the quantity n has an upper limit m, which implies a finite number of terms in the series, and $\frac{n\pi y}{L}$ is everywhere small compared to unity; then

$$\cosh\left(\frac{n\pi y}{L}\right) \doteq 1$$

$$y_0 = d + \sum_{1}^{m} A_n \sin\left(\frac{n\pi x}{L}\right) .$$

This expression represents a water table profile. If a uniform increment of depth H reaches the water table, due to deep percolation from a uniform application of irrigation water, then the pressure imposed at drain level by the water table profile can be represented initially by the expression

$$p_0 = \frac{4H}{\pi} \sum_{n=1,3,5\ldots}^{n=\infty} \frac{1}{n} \sin\left(\frac{n\pi x}{L}\right) . \tag{8-15}$$

If the series is terminated at the mth term then the expression can represent approximately a uniform increment of depth H. The relationships considered up to this point do not involve the element of time. This factor can now be introduced.

After the increment is applied, water will flow to the drains and the water table will begin to sink. The flows to each drain accounted for by the individual terms of the series will be:

$$K \int_0^d \left(\frac{\partial p}{\partial x}\right)_0 dy = A_n K \frac{n\pi}{L} \int_0^d \cosh\left(\frac{n\pi y}{L}\right) dy = A_n K \sinh\left(\frac{n\pi y}{L}\right)\Big]_0^d$$

$$= A_n K \sinh\left(\frac{n\pi d}{L}\right)$$

The volume above the line $y = d$ is, approximately, for each term

$$S_n \cong \int_0^L (y-d)\ dx = A_n \cosh\left(\frac{n\pi d}{L}\right) \int_0^L \sin\left(\frac{n\pi x}{L}\right) dx \cong 2\ A_n \frac{L}{n\pi} \cosh\left(\frac{n\pi d}{L}\right) .$$

The continuity condition for each term is, since there is flow out at $x = 0$ and at $x = L$

$$V \frac{\partial S_n}{\partial t} = 2\ K \int_0^d \left(\frac{\partial p}{\partial x}\right)_0 dy$$

In this expression t represents time. By substitution:

$$\frac{dA_n}{dt} \frac{2LV}{n\pi} \cosh\left(\frac{n\pi d}{L}\right) = 2\ A_n\ K \sinh\left(\frac{n\pi d}{L}\right)$$

or if

$$\beta = \frac{Kn\pi}{LV} \tanh\left(\frac{n\pi d}{L}\right)$$

$$\frac{dA_n}{dt} + \beta\ A_n = 0$$

This is a differential equation whose solution is:

$$A_n = B_n\ e^{-\beta t} .$$

Where the B_n quantities are new constants. Then all of the requirements described previously will be met to a close approximation if

$$p \cong \frac{4H}{\pi} \sum_{n=1,3,5\ldots}^{n=m} \frac{e^{-\beta t}}{n} \sin\left(\frac{n\pi x}{L}\right) \frac{\cosh\left(\frac{n\pi y}{L}\right)}{\cosh\left(\frac{n\pi d}{L}\right)} + (d-y) \qquad (8\text{-}16)$$

This expression remains an exact solution of equation 1.

When

$$\frac{n\pi d}{L} << 1 \qquad \tanh\left(\frac{n\pi d}{L}\right) \cong \frac{n\pi d}{L}$$

and with

$$\beta \cong \frac{\alpha n^2 \pi^2}{L^2} \qquad \alpha = \frac{Kd}{V}$$

Then, approximately,

$$h \cong \frac{4H}{\pi} \sum_{n=1,3,5\ldots}^{n=m} \frac{e^{-n^2\pi^2 \left(\frac{\alpha t}{L^2}\right)}}{n} \sin\left(\frac{n\pi x}{L}\right) + (d-y) \ .$$

The formula derived from the Dupuit-Forchheimer idealization is then recovered but with the important exception that here

$$\alpha = \frac{Kd}{V} \qquad (8\text{-}17)$$

If the solution is limited to three terms and $\frac{\pi y}{L} < 0.1$ everywhere then the Dupuit-Forchheimer and Laplace solutions become essentially identical in form and an approximation to the initial condition is obtained which is close enough for practical purposes. Since the higher ordered terms vanish rapidly it is permissible to use Table 10 for computation of the remaining drainable depth at $x = L/2$ except that here the aquifer constant must be of the form $\alpha = Kd/V$.

Examples

Use of the first approximation solution and the Laplace type solution will be illustrated by use of an example adapted from Dumm's 1964 paper. The given data are:

Depth from ground surface to barrier	30 ft
Depth from ground surface to drain	8 ft

Drainable depth produced by an irrigation	0.46 ft
Permeability	K = 10 ft/day
Effective voids ratio	0.18
Drain spacing	L = 1450 ft

We will compute the remaining drainable depth at the point midway between drains at the end of successive seven day periods. Values for three months, six months, and one year are added. The differences between the values obtained by the first approximation and the Laplace type solution are due only to the difference in the α values. These differences are small. The drainable depth at the center does not respond immediately to the action of the drains. The part remaining, however, responds very quickly due to drainage taking place in the immediate neighborhood of the drain.

Computation by first approximation

$$K = 10 \text{ ft/day} \quad D_a = (d + \frac{H}{2}) = (22 + \frac{0.46}{2}) = 22.23 \text{ ft}$$

$$V = 0.18 \quad \alpha = \frac{KD_a}{V} = \frac{(10)\ (22.23)}{0.18} = 1235 \frac{\text{ft}^2}{\text{day}} \quad L = 1450 \text{ ft}$$

$$\frac{\alpha}{L^2} = \frac{1235}{1450^2} = 0.0005874 \frac{1}{\text{day}} \qquad H = 0.46 \text{ ft}$$

Time Days	$\frac{\alpha t}{L^2}$	$\frac{h_c}{H}$	h_c	p	Remarks
0	0	1.000	0.460	1.000	Values of $\frac{h_c}{H}$ can be
7	0.004112	1.000	0.460	0.855	read from Table 10.
14	0.008224	1.000	0.460	0.795	Values of p can be
21	0.012335	0.997	0.459	0.749	read from Table 11.
28	0.016447	0.988	0.454	0.711	
35	0.020559	0.973	0.448	0.676	
42	0.024671	0.951	0.473	0.646	
49	0.028782	0.926	0.426	0.617	
56	0.032894	0.897	0.413	0.591	
63	0.037006	0.868	0.399	0.566	

Time Days	$\frac{\alpha t}{L^2}$	$\frac{h_c}{H}$	h_c	p	Remarks
70	0.041118	0.838	0.385	0.543	
77	0.045229	0.807	0.371	0.520	
84	0.049341	0.777	0.357	0.479	
91	0.053453	0.748	0.344	0.499	Three months
182	0.106906	0.443	0.204	0.282	Six months
365	0.214400	0.153	0.070	0.098	One year

Computation by the Laplace type solution

$\alpha = \frac{Kd}{V} = \frac{(10)\ (22)}{0.18} = 1222\ (ft^2/day)\ (\alpha/L^2) = 0.0005813$

Time Days	$\frac{\alpha t}{L^2}$	$\frac{h_c}{H}$	h_c	p	Remarks
0	0	1.000	0.460	1.000	
7	0.004069	1.000	0.460	0.845	
14	0.008138	1.000	0.460	0.796	
21	0.012207	0.997	0.459	0.751	
28	0.016276	0.989	0.455	0.712	
35	0.020346	0.974	0.448	0.678	
42	0.024417	0.953	0.438	0.647	
49	0.028484	0.928	0.427	0.619	
56	0.032553	0.900	0.414	0.593	
63	0.036622	0.871	0.401	0.568	
70	0.040691	0.841	0.387	0.545	
77	0.044760	0.811	0.373	0.523	
84	0.048829	0.781	0.359	0.502	
91	0.052898	0.752	0.346	0.482	Three months
182	0.105797	0.448	0.206	0.285	Six months
365	0.212174	0.157	0.072	0.100	One year

Note: $\frac{\pi y_m}{L} = \frac{(3.1416)\ (22.46)}{1450} = 0.0487$

These two examples yield closely similar results because the drainable depth is small compared to the saturated depth below the drains. It will be profitable to now consider a somewhat extreme case where the drainable depth is nearly equal to the saturated depth below the drains and to again compare the results of computations made by several methods.

As an example of a case where the drains are located about midway between the water table and the barrier, data from a field installation supplied by Mr. Ray Winger of the Bureau of Reclamation will be used. The data are:

Depth of barrier below ground surface	16 ft
Drain depth	9 ft
Permeability	1.4 (ft/day)
Effective voids ratio	0.093
Maximum allowable water table height 3 ft below ground surface or 6 ft above the drains	
Drain spacing	510 ft

Computation of the drainable depth midway between drains by the method of Moody.

$D_m = (16 - 3) = 13$ ft. $\alpha_m = \frac{(1.4)(13)}{0.093} = 195.7(\text{ft}^2/\text{day})$.

$m = \frac{6}{13} = 0.462$.

Time Days	$(\frac{\alpha_m t}{L^2})$	$(\frac{h_m}{H_m})$	h_m
0	0	1.000	6.00
20	.0150	.960	5.76
40	.0301	.882	5.29
60	.0451	.809	4.85
80	.0602	.736	4.42
100	.0752	.666	4.00
120	.0903	.606	3.64
140	.1053	.556	3.34
160	.1204	.506	3.04
180	.1354	.463	2.78
270	.2031	.328	1.97
365	.2746	.227	1.36

Time	$(\frac{\alpha_m t}{L^2})$	$(\frac{h_m}{H_m})$	h_m
42	0.7584	0.015	0.09
49	0.8848	0.008	0.05

One day is 24 hours. $\frac{\alpha_m t_1}{L^2} = \frac{(195.7)(24)}{510^2} = \frac{4696.8}{260100} = 0.018057$

Computation by first approximation method.

$D_a = (d + \frac{H_o}{2}) = (7 + \frac{6}{2}) = 10$ ft $\quad \alpha = \frac{KD_a}{V} = \frac{(1.4)(10)}{0.093} = 150.5$ (ft^2/day).

$$\frac{\alpha t_1}{L^2} = \frac{(150.54)(24)}{510^2} = 0.01389$$

Time Days	$(\frac{\alpha t}{L^2})$	$(\frac{h_c}{H_o})$ *	h_c
0	0	1.0000	6.00
20	.0116	.9979	5.99
40	.0231	.9599	5.76
60	.0347	.8844	5.31
80	.0463	.7992	4.80
100	.0579	.7164	4.30
120	.0694	.6409	3.85
140	.0816	.5687	3.41
160	.0926	.5103	3.06
180	.1042	.4551	2.73
270	.1562	.2724	1.63
365	.2112	.1583	0.95

*Read from Table 10.

Computation by the method of Brooks.

Time Days	h_c	$(h_c - \frac{H_o}{2})$	h_1	$(h_1 + \frac{H_o}{2})$
0	6.00	3.00	3.00	6.00
20	5.99	2.99	2.99	5.99
40	5.76	2.76	2.81	5.81
60	5.31	2.31	2.46	5.46
80	4.80	1.80	2.04	5.04
100	4.30	1.30	1.62	4.62
120	3.85	0.85	1.22	4.22
140	3.41	0.41	0.83	3.83
160	3.06	0.06	0.50	3.50
180	2.73	-0.27	0.18	3.18
270	1.63	-1.37	-0.96	2.04
365	0.95	-2.05	-1.75	1.25

Notes: The numbers in the column headed h_c are those of the first approximation. The numbers in the column headed $(h_c - \frac{H_o}{2})$ are those of the first approximation referred to an origin $(H_o/2)$ above the level of the drains. The column headed h_1 is Brooks second approximation, computed by use of formula 8-10. The figures in the last column are those of the previous column referred back to drain level. They compare with the first approximation figures in the column headed h_c.

Reference: Brooks, R.H., 1963, ASCE Paper 3420.

Laplace type solution.

$d = 7.0$ ft $\qquad \alpha_L = \frac{Kd}{V} = \frac{(1.4)(7.0)}{0.093} = 105.4\ (ft^2/day)$

$L = 510$ ft $\qquad H_o = 6.00$ ft

Time Days	$(\frac{\alpha_L t}{L^2})$	$(\frac{h_c}{H_o})$	h_c
0	0	1.000	6.00
20	.0081	1.000	6.00
40	.0162	.989	5.93
60	.1243	.953	5.72
80	.0324	.901	5.40
100	.0405	.842	5.05
120	.0486	.782	4.69
140	.0567	.725	4.35
160	.0648	.670	4.02
180	.0729	.619	3.72
270	.1094	.432	2.60
365	.1479	.296	1.77

Note: The values in the column headed (h_c/H_o) were obtained from Table 10.

The results of these computations are shown on figure 8-2. The solid heavy line represents Brooks second approximation which will here be used as a basis for comparison. The light solid line shows the results obtained by use of the first approximation solution. It holds up surprisingly well even though here the drainable depth is almost half of the original saturated depth and nearly equal to the saturated depth below the drains. The dashed curve shows the results obtained from using Moody's computer solution. This solution and the first and second approximation solutions are not strictly comparable because they have different initial conditions. The initial

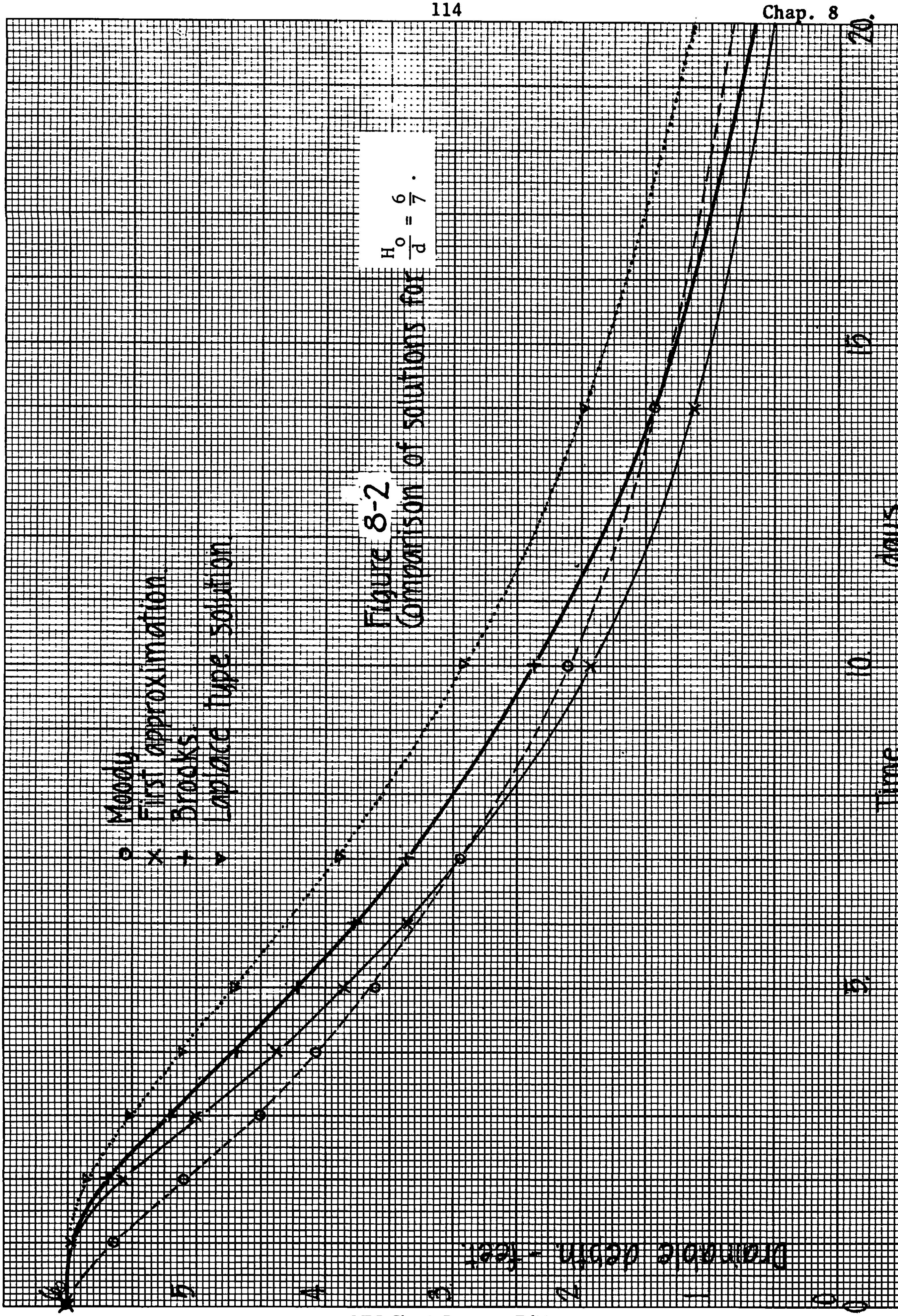


Figure 8-2
Comparison of solutions for $\frac{H_o}{d} = \frac{6}{7}$.

condition of Moody's solution, however, represents closely a configuration which would appear at an early epoch in the drainage of a uniform drainable depth. If Moody's initial configuration is superimposed on such a chart as that of figure 7 of USBR Monograph 31, it will be found to correspond nearly to the profile for $\frac{\alpha t}{L^2} = 0.014$. A second approximation curve obtained by use of Brooks formula indicates that the parameter should be about $\frac{\alpha t}{L^2} = 0.010$ to produce a close fit. Since the value of $\frac{\alpha t_1}{L^2} = 0.015$ for time 20 days is substantially this amount it can be concluded that if Moody's curve is shifted to the right about 20 days on figure 8-2 the effect of the differing initial conditions will be accounted for. If this is done Moody's result and Brook's second approximation will be in close agreement over the first ten days. The solution obtained from the Laplace formulation is similar in shape to the second approximation curve of Brooks but lies above it. The reason for this seems to be that this solution accounts for the head loss needed to produce vertical as well as horizontal flow whereas the other solutions account for the horizontal component of flow only. A particle of water initially at the water table ten feet back from the drain, for example, has to travel six feet vertically and 10 feet horizontally to reach the drain. The solution of the Laplace equation accounts for this but the solutions derived on the Dupuit-Forchheimer basis only account for the horizontal ten feet of distance. The drainage is therefore slowed near the drain and the drainage of water remote from the drain is also slowed because it cannot reach the drain until the water close to the drain is disposed of. This comparison brings out the important effect of flow convergence near the drain. More will be said on this point later.

Selection of drain spacings

The formulas described can be used as a means for computing drain spacings on a cut and try basis. The procedure will be illustrated by use of the first approximation formula. The h_c/H values will be obtained from Table 10. The computation will be based upon figures adapted from Lee D. Dumm's 1964 paper. The allowable rise of the water table at mid-span at the end of the irrigation season is 4 ft. It will be assumed that this height is attained at the end of the previous irrigation season.

Data are:

$K = 10$ ft/day

$D = 22$ ft

$V = 0.18$

$KD = 220$ ft^2/day

$\alpha = \frac{KD}{V} = 1222.23$ ft^2/day

$$\frac{\alpha}{L^2} = \frac{1222.23}{1500^2} = 0.00054321$$

Try a spacing of 1500 feet.

Application	Time Days	Drainable Depth ft	$(\frac{\alpha t}{L^2})$	$(\frac{h_c}{H})$	h_c
Apr 22*	132	0.46	0.0717	0.6267	0.288
June 6	87	0.46	0.0473	0.7920	0.364
July 1	62	0.46	0.0337	0.8887	0.409
July 21	42	0.46	0.0228	0.9616	0.442
Aug 4	28	0.46	0.0152	0.9917	0.456
Aug 18	14	0.46	0.0076	0.9998	0.460
Sept 1	0	0.46	0	1.0000	0.460
	365	4.00	0.1983	0.1799	0.720
				Total	3.599

*Snowmelt

This spacing can be widened. Try a spacing of 1700 ft.

Application	Time Days	Drainable Depth ft	$(\frac{\alpha t}{L^2})$	$(\frac{h_c}{H})$	h_c
Apr 22*	132	0.46	0.0558	0.731	0.336
June 6	87	0.46	0.0368	0.869	0.400
July 1	62	0.46	0.0262	0.942	0.433
July 21	42	0.46	0.0178	0.984	0.453
Aug 4	28	0.46	0.0118	0.998	0.459
Aug 18	14	0.46	0.0059	1.000	0.460
Sept 1	0	0.46	0	1.000	0.460
	365	4.00	0.1544	0.277	1.110
				Total	4.111

*Snowmelt

This spacing is too wide. By interpolation, a spacing of 1657 feet would just meet the requirements of a four-foot rise at the end of the irrigation season.

The method of M. Maasland

The previous treatments of drainage by parallel drains has been based upon the concept of a drainable depth which comes into existence at time zero. This is an idealization of furrow irrigation practices where the irrigation water is applied during a brief interval of time and produces, by deep percolation, a drainable depth H.

The Maasland approach is somewhat different and possesses certain advantages which will be described later. He assimilates the deep percolation from a succession of irrigations to an average infiltration rate i. The consequences of such an idealization may be approached through the use of the formula for drainage of a single uniform drainable depth H (f. 8-1). This is:

$$h = \frac{4H}{\pi} \sum_{n=1,3,5...}^{n=\infty} \frac{e^{-n^2\pi^2 (\frac{\alpha t}{L^2})}}{n} \sin \left(\frac{n\pi x}{L}\right)$$

Suppose the drainable depth dH appears at the time ξ where ξ represents a time variable running between 0 and t . The variable ξ indicates the time of occurrence of an event whose effect is to be computed at the time t .

The drainable depth at the point x at the time t will then be given by

$$h = \frac{i}{V} \int_0^t \frac{4}{\pi} \sum_{n=1,3,5...}^{n=\infty} \frac{e^{-n^2\pi^2 (\frac{\alpha(t-\xi)}{L^2})}}{n} \sin \left(\frac{n\pi x}{L}\right) d\xi$$

Where $\frac{i}{V}$ is the rate of rise of the water table due to the constant infiltration rate i . The infiltration is considered to be entire water.

By integration

$$h = \frac{i4}{V\pi} \frac{L^2}{\alpha\pi^2} \sum_{n=,1,3,5...}^{n=\infty} \frac{e^{-n^2\pi^2 (\frac{\alpha(t-\xi)}{L^2})}}{n^3} \sin \left(\frac{n\pi x}{L}\right) \Bigg]_0^t$$

or

$$h = \frac{i4L^2}{Kd\pi^3} \sum_{n=,1,3,5...}^{n=\infty} \frac{1}{n^3} \sin \left(\frac{n\pi x}{L}\right)$$

$$- \frac{i4L^2}{Kd\pi^3} \sum_{n=1,3,5...}^{n=\infty} \frac{e^{-n^2\pi^2 (\frac{\alpha t}{L^2})}}{n^3} \sin \left(\frac{n\pi x}{L}\right) \qquad (8\text{-}18)$$

At $x = L/2$, the point midway between the drains,

$$\frac{n\pi x}{L} = \frac{n\pi}{2}$$

Also

$$\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \ . \ . \ . \ = \frac{\pi^3}{32}$$

This is one of the Euler numbers. A plot of formula 8-18 is shown on figure 8-3.

The summation of descending exponentials disappears in time. Then there is an ultimate steady state given by the summation which is free of exponentials. This is:

$$h_c = \frac{i4L^2}{Kd\pi^3} \sum_{n=1,3,5...}^{n=\infty} \frac{1}{n^3} \sin\left(\frac{n\pi}{2}\right) = \frac{i4L^2}{Kd\pi^3} \frac{\pi^3}{32} = \frac{iL^2}{8Kd} \qquad (8\text{-}19)$$

An independent development for the steady state will be of interest. The statement that the flow is equal to the supply is

$$Kd \frac{dh}{dx} = i \left(\frac{L}{2} - x\right).$$

By integration, if $h = 0$ when $x = 0$

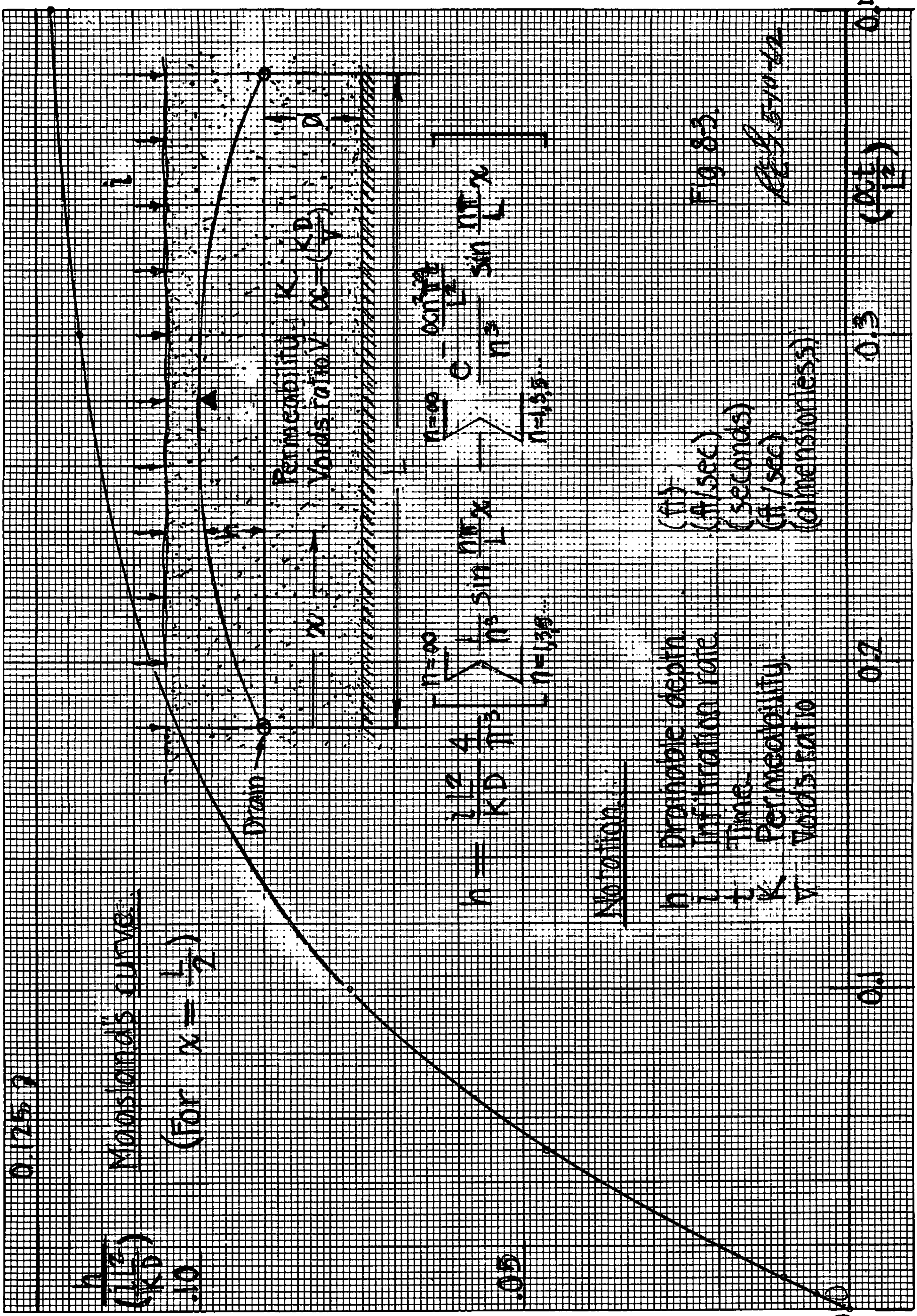
$$h = \frac{i}{2Kd} x(L - x).$$

When $x = \frac{L}{2}$

$$h_c = \frac{iL^2}{8Kd} \qquad (8\text{-}20)$$

as before. It will be found that the term

$$\frac{i4L^2}{V\alpha\pi^3} \sum_{n=1,3,5...}^{n=\infty} \frac{1}{n^3} \sin\left(\frac{n\pi x}{L}\right)$$ is the Fourier series


0.125 ?
Maasland's curve
(For x = L/2)
h / (iL²/KD)
.10
.05
0
0.1
0.2
0.3
0.4
(αt/L²)
Permeability K
Voids ratio V
α = (KD/V)
i
Drain
x
D
L
h = (iL²/KD) (4/π³) [Σ (n=1,3,5 … ∞) (1/n³) sin (nπ/L)x − Σ (n=1,3,5 … ∞) (e^(−αn²π²t/L²) / n³) sin (nπ/L)x]
Notation
h Drainable depth (ft)
i Infiltration rate (ft/sec)
t Time (seconds)
K Permeability (ft/sec)
V Voids ratio (dimensionless)
Fig 8.5

which represents

$$h = \frac{i}{2Kd} x (L - x) .$$

Formulas of the type of equation 8-20 have been used to estimate drain spacings. It will be clear that this procedure implies that the irrigation season is long enough to establish the ultimate steady state. Under ordinary conditions the irrigation season is too short to establish an ultimate steady state and the result is that drain spacings obtained by use of ultimate steady state relations are closer than necessary to provide drainage. To put this in other words, the use of ultimate steady state formulas, based upon the concept of a continuous infiltration rate i, neglects the favorable effects of the winter drain-out period.

This difficulty can be substantially overcome if the effects of a succession of seasonal applications are considered. This is a case of intermittent operation as treated in Chapter 11. With an irrigation pattern as illustrated on figure 8-4 the effects of previous irrigations and cessation of irrigations can be treated in the following way. The height of the water table midway between drains at the end of the last irrigation season is of interest. Here T represents the yearly period and $T/3$ the irrigation period.

Suppose

$$(\alpha T/L^2) = 0.2$$

then the computation is made in the following way.

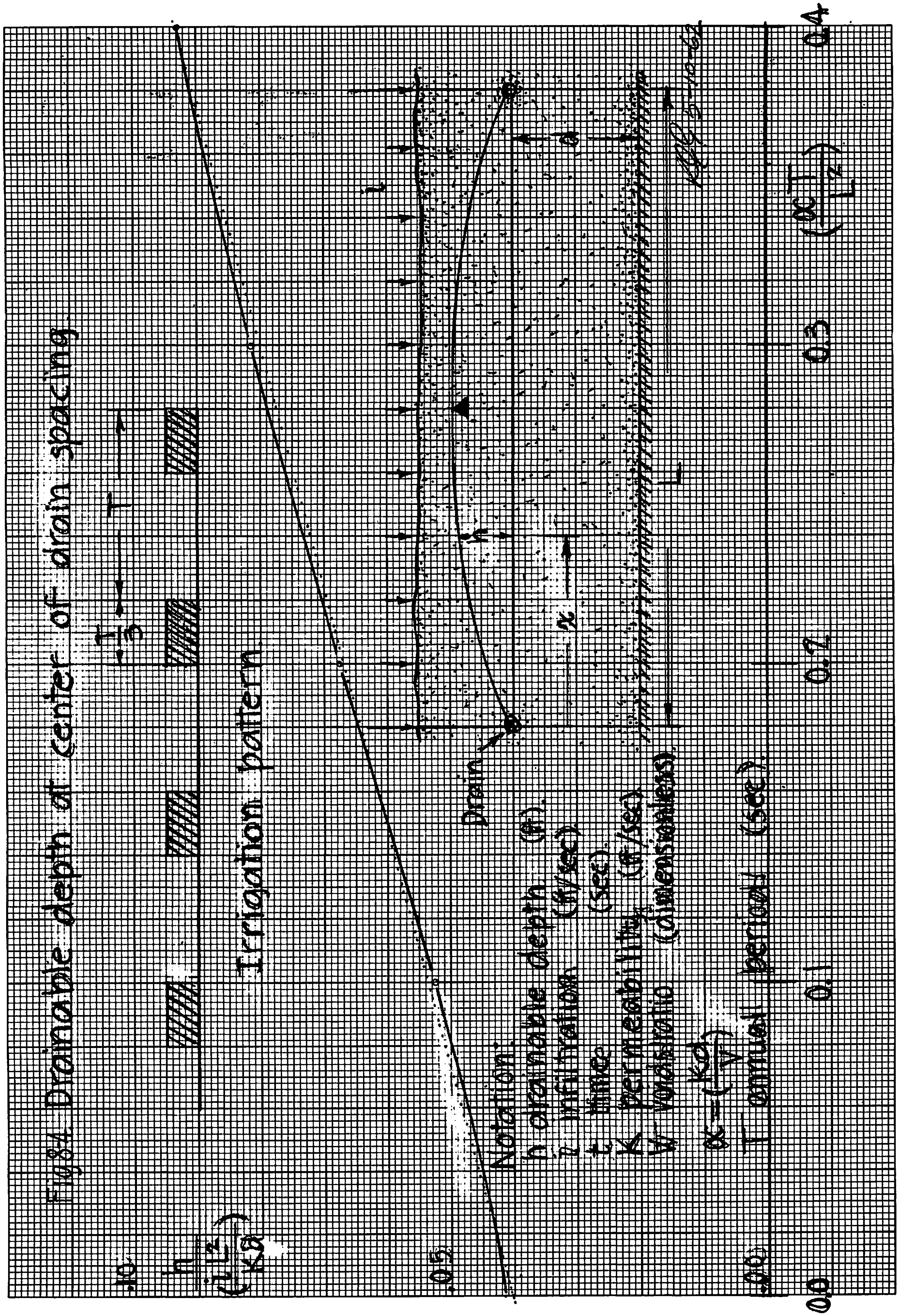


Fig 8-4 Drainable depth at center of drain spacing
Irrigation pattern
T/3
T
Drain
L
x
a
h
Notation:
h drainable depth (ft)
r infiltration (ft/sec)
t time (sec)
K permeability (ft/sec)
V voids ratio (dimensionless)
α = (Ka/V)
T annual period (sec)
h/(rL²/Ka)
(αT/L²)
1.0
0.5
0.0
0.0
0.1
0.2
0.3
0.4

Table 8-2 Computation of $h/(\frac{iL^2}{Kd})$ for intermittent irrigation. $(\alpha T/L^2) = 0.2$. Irrigation period T/3 .

$(\alpha t/L^2)$		$h/(\frac{iL^2}{Kd})$
0.0667		+0.0585
0.2000		-0.1071
0.2667		+0.1137
0.4000		-0.1225
0.4667		+0.1250
	Total	+0.0676

(Compare with figure 8-4.)

The first figure represents the effect at the end of the last irrigation period which is 1/3 of a year in length. Then

$$(\alpha t/L^2) = (0.2/3) = 0.0667$$

The corresponding value for $h/(\frac{iL^2}{Kd})$ is obtained from figure 8-3. The next figure represents a cessation of irrigation at the end of the previous irrigation period. Here $(\alpha t/L^2) = 0.2000$ because the time is one year. The third figure with $(\alpha t/L^2) = (4)\ (0.2)/3 = 0.2667$ accounts for the beginning of the previous irrigation period. The fourth and fifth figures, together, account for the irrigations made two years previous. As the $(\alpha t/L^2)$ values grow larger the two values of the pair approach equality. This explains how convergence can be obtained even though the values are growing larger with time. A series of such computations will permit the construction of a chart such as shown on figure 8-4. With specified values of i , L , K , d , α , T an $(\alpha T/L^2)$ value can be computed and an $h/(\frac{iL^2}{Kd})$ value can be read directly from the chart which includes the effects of irrigations in previous years. An h value can then immediately be computed. A cut and try procedure for estimating drain spacings can then be used which will take account of the effects of irrigations in previous years.

A direct approach to this problem can be made if it is noted that the graph of figure 8-4 is nearly a straight line. The straight line approximation shown has the formula

$$\frac{h_c}{\left(\frac{iL^2}{Kd}\right)} = 0.040 + 0.1325 \left(\frac{\alpha T}{L^2}\right) \qquad \text{For } 0 < \left(\frac{\alpha T}{L^2}\right) < 0.4$$

From which, by rearrangement,

$$\left(\frac{L^2}{\alpha T}\right) = 25 \left(\frac{h_c V}{iT}\right) - 3.3125 \tag{8-21}$$

The problem of determining a drain spacing for the conditions of the problem used to illustrate application of the first approximation solution of Chapter 8 may now be reconsidered. With

$K = 10$ ft/day $\qquad V = 0.18$

$D_a = 22.225$ ft $\qquad K\,D_a = 222.25$ ft/day

$\alpha = 1234.7\ ft^2/day$

Irrigation applications contributing 0.46 ft of drainable depth were made on June 6, July 1, July 21, August 4, August 18 and September 1. A similar contribution from snowmelt was indicated for April 22. In all (7) (0.46) = 3.22 feet of drainable depth were contributed in 132 days. To accommodate this to our chart conditions we can assume that these applications were made in 1/3 year or 122 days. Then

$$i = \frac{(3.22)\ (0.18)}{122} = 0.00475\ \frac{ft}{day}$$

The allowable drainable depth at the center of the span is 4.0 ft and T = 365 days. Then

$$\left(\frac{hV}{iT}\right) = \frac{(4.0)\ (0.18)}{(0.00475)\ (365)} = \frac{0.72}{1.734} = 0.4152$$

$$\left(\frac{L^2}{\alpha T}\right) = (25)\ (0.4152) - 3.3125 = 7.0675$$

$$L^2 = (7.0675)\ (1234.7)\ (365) = 3185100$$

$$L = 1785 \text{ feet}$$

This compares with Dumm's estimate of 1450 feet. The difference is largely due to a difference in assumed intervals between applications. In our case it was about 17 days whereas his last three irrigations were made at 14 day intervals. A corresponding infiltration rate would be

$$i = \frac{(3)\ (0.46)\ (0.18)}{(3)\ (14)} = 0.00591\ \frac{\text{ft}}{\text{day}}$$

and

$$\frac{hV}{iT} = \frac{(4.0)\ (0.18)}{(0.00591)\ (365)} = \frac{0.720}{2.157} = 0.3338$$

$$\frac{L^2}{\alpha T} = (25)\ (0.3338) - 3.3125 = 5.0325$$

$$L^2 = (5.0325)\ (1234.7)\ (365) = 2265000 \text{ ft}^2$$

$$L = 1506 \text{ feet}$$

The ultimate steady state formula would give

$$L = \sqrt{\frac{8\ h_c\ Kd}{i}} = \sqrt{\frac{(8)\ (4.0)\ (222.25)}{0.00591}} = 1097 \text{ feet}$$

This is admittedly too short for the reasons mentioned previously. The trial procedure described in the paragraph on "Selection of drain spacings" is much shortened if the trial value is close. Chart 8-4 and formula 8-21 can provide good trial values.

Flow of water to drains

The flow of drainage water from the width L between drains to the two drains bordering the width is:

$$q_2 = 2Kd \left(\frac{\partial h}{\partial x}\right)_0 = \frac{8iL}{\pi^2} \sum_{n=1,3,5...}^{n=\infty} \frac{1}{n^2} - \frac{8iL}{\pi^2} \sum_{n=1,3,5...}^{n=\infty} \frac{e^{-n^2\pi^2\left(\frac{\alpha t}{L^2}\right)}}{n^2}$$

Since the cosine terms which arise as a result of the indicated differentiations are 1 when $x = 0$. But

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ . \ . \ . \ = \frac{\pi^2}{8}$$

This is one of the Bernoulli numbers. Then

$$\frac{q_2}{iL} = 1 - \frac{8}{\pi^2} \sum_{n=1,3,5...}^{n=\infty} \frac{e^{-n^2\pi^2\left(\frac{\alpha t}{L^2}\right)}}{n^2} \qquad (8\text{-}22)$$

This can be put in the form:

$$\frac{q_2}{iL} = 1 - p \qquad (8\text{-}23)$$

where values of p can be obtained from Table 11.

As an example of the use of this result we may compute the flow of drainage water from the width between drains using the data of an example from Chapter 8. It is worthwhile to note that the flow so obtained will be appropriate for the flow of drainage water to one drain from both sides. With

$\alpha = 1222.23\ ft^2/day$ $\quad \left(\frac{\alpha t}{L^2}\right) = 0.06627$

$L = 1500\ ft$ $\quad i = 0.00591\ ft/day$

$t = 122\ days$

From tables

$$p = 0.4217$$

$$1 - p = 0.5783$$

Then

$$q_2 = iL(1-p) = (0.00591)\ (1500)\ (0.5783) = 5.1266\ ft^2/day$$

This means that each drain must be able to pick up and carry away a little over five cubic feet of drainage water per foot of drain per day. This estimate can be expected to be below that obtained by the methods which account for the initial rush of water to the drains immediately following the application of irrigation water. Drains designed in this way could be expected to run at maximum capacity for a few days following irrigation. Some comparisons will be found in the paragraph on "Local convergence losses."

Local convergence losses

Where tile drains are used, the flow, which has been occupying the entire saturated depth, must converge toward the drain. This means that the flow must pass through restricted areas and it must be expected that increased head losses will be required to move the flow to the drain.

The following development has for its purpose the evaluation of these convergence losses.

Consider the expression (Byerly)

$$p_1 = \frac{p_0}{\pi} \log_e \left[\cosh^2 \left(\frac{\pi x}{d}\right) - \cos^2 \left(\frac{\pi y}{d}\right)\right]$$

where p_1 represents the pressure needed to drive the flow. It is measured in feet of water. It is a solution of the Laplace equation

$$\frac{\partial^2 p_1}{\partial x^2} + \frac{\partial^2 p_1}{\partial y^2} = 0$$

By differentiation

$$\frac{\partial p_1}{\partial x} = \frac{p_0}{\pi}\left[\frac{2\cosh\left(\frac{\pi x}{d}\right)\sinh\left(\frac{\pi x}{d}\right)}{\cosh^2\left(\frac{\pi x}{d}\right) - \cos^2\left(\frac{\pi y}{d}\right)}\right]\frac{\pi}{d}$$

Then

$$\frac{\partial p_1}{\partial x} = 0 \quad \text{if} \quad x = 0 \quad \text{when} \quad y > 0$$

By differentiation with respect to y

$$\frac{\partial p_1}{\partial y} = \frac{p_0}{\pi}\left[\frac{2\cos\left(\frac{\pi y}{d}\right)\sin\left(\frac{\pi y}{d}\right)}{\cosh^2\left(\frac{\pi x}{d}\right) - \cos^2\left(\frac{\pi y}{d}\right)}\right]\frac{\pi}{d}$$

Then

$$\frac{\partial p_1}{\partial y} = 0 \quad \text{when} \quad y = d$$

$$= 0 \quad \text{when} \quad y = 0 \quad \text{if} \quad x > 0$$

The idealization is as shown in figure 8-5.

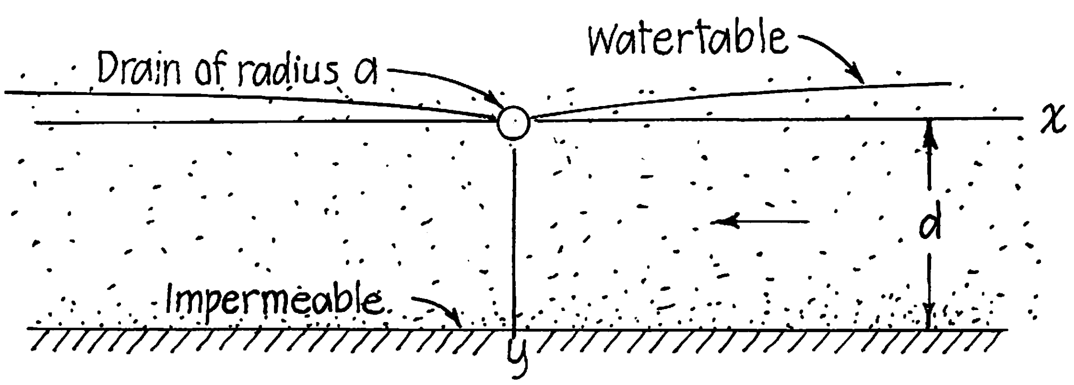


Fig. 8-5 Drain.

The origin is at the center of the drain and a sink is located there. The flow approaches from the right and goes to the sink without crossing the y axis. The idealization neglects the presence of saturated depth above the elevation of the drains. The tacit assumption is also made that the flow enters the drain through a quadrant. The quantity p_1 can be considered as the pressure which drives the flow. The flow approaches the drain along a strip of uniform width d. The pressure difference between the point $y = 0$ and x and the point $y = 0$ and $x = a$ is

$$p_1 - p_3 = \frac{p_0}{\pi} \log_e \frac{[\cosh^2 (\frac{\pi x}{d}) - 1]}{[\cosh^2 (\frac{\pi x}{a}) - 1]}$$

The pressure gain out to x due to the uniform flow in the strip is:

$$p_2 = \frac{2p_0 x}{d}$$

The gradient, when x is large compared to d, can be inferred from the expression for $(\partial p_1/\partial x)$. When $(\pi x/d) >> 1$ then $\cosh (\frac{\pi x}{d})$ and $\sinh (\frac{\pi x}{d})$ become large compared to unity and nearly equal while $\cos^2 (\frac{\pi y}{d})$ can never exceed unity. Then when $(\pi x/d) >> 1$, $\frac{\partial p}{\partial x} \rightarrow 2\, p_0/d$. The above expression for p_2 can be derived from this result. It represents the head loss which would be needed to drive the flow

$$Kd \frac{\partial p_1}{\partial x} = 2Kp_0$$

from the point x to the origin if there were no convergence.

The pressure loss due to convergence is:

$$[p_1 - p_2 - p_3] = \frac{p_0}{\pi} \log_e [\cosh^2 (\frac{\pi x}{d}) - 1] - \frac{p_0}{\pi} \log_e [\cosh^2 (\frac{\pi a}{d}) - 1] - \frac{2x}{d}$$

when $(\frac{\pi x}{d})$ is large compared to unity then $\cosh^2 (\frac{\pi x}{d})$ will be large compared to unity and

$$\cosh^2 (\frac{\pi x}{d}) - 1 \doteq \cosh^2 (\frac{\pi x}{d}) \quad (\text{If } \frac{x}{d} >> 1)$$

under these conditions also

$$\cosh^2 (\frac{\pi x}{d}) \doteq \frac{e^{(\frac{2\pi x}{d})}}{4} \qquad (\text{If } \frac{x}{d} >> 1)$$

then approximately

$$\frac{p_0}{\pi} \log_e [\cosh^2 (\frac{\pi x}{d}) - 1] \doteq \frac{p_0}{\pi} (\frac{2\pi x}{d}) \log_e e - \frac{p_0}{\pi} \log_e 4 = [\frac{2x}{d} - 0.44127]\ p_0$$

If (x/d) is large compared to unity the quantity 0.44127 can be dropped. If $(\pi a/d)$ is small compared to unity

$$\cosh^2 (\frac{\pi a}{d}) \doteq 1 + (\frac{\pi a}{d})^2 + \frac{1}{3} (\frac{\pi a}{d})^4 + \ldots$$

and approximately

$$[\cosh^2 (\frac{\pi a}{d}) - 1] \doteq (\frac{\pi a}{d})^2$$

so that

$$\frac{p_0}{\pi} \log_e [\cosh^2 (\frac{\pi a}{d}) - 1] \doteq \frac{2p_0}{\pi} \log_e (\frac{\pi a}{d}) .$$

Finally

$$[p_1 - p_2 - p_3] \doteq \frac{2p_0 x}{d} - \frac{2p_0}{\pi} \log_e (\frac{\pi a}{d}) - \frac{2p_0 x}{d}$$

Figures 8-6 and 8-7 are reproduced here through the courtesy of the American Concrete Institute.

These figures illustrate the close relationship of the mathematical treatments of the flow of heat in solids and the flow of groundwater. These charts first appeared in a paper on "Insulation for Protection of New Concrete in Winter" by L. H. Tuthill, R. E. Glover, C. H. Spencer and W. B. Bierce in the Journal of the Concrete Institute for November 1951. The charts appear on pages 262 and 264.

The local resistance incident to converging flow to a drain here replaces the resistance to heat flow produced by form insulation.

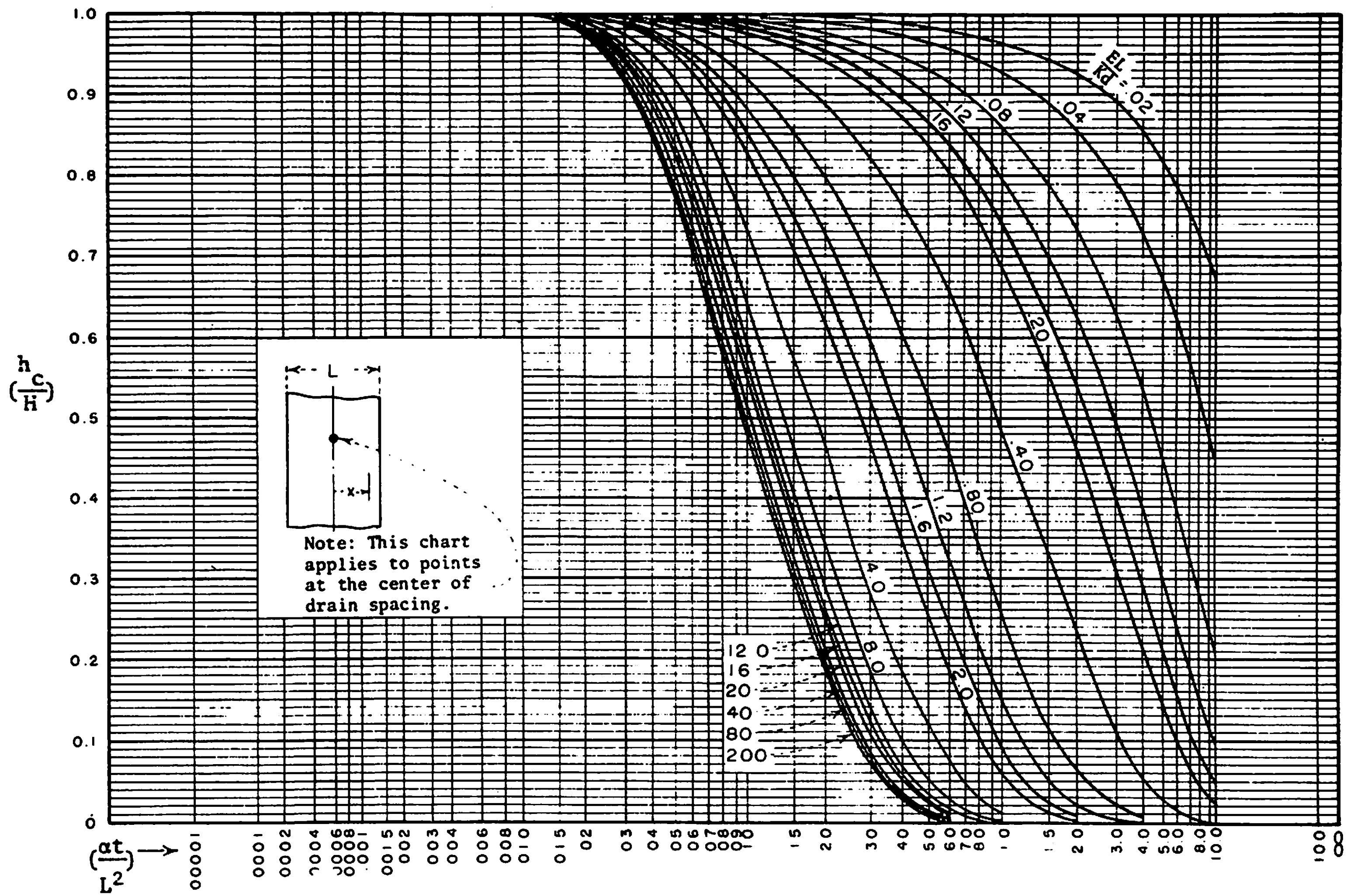


Fig. 8-6 (h_c/H) vs $(\alpha t/L^2)$ taking account of local flow resistance at the drain.

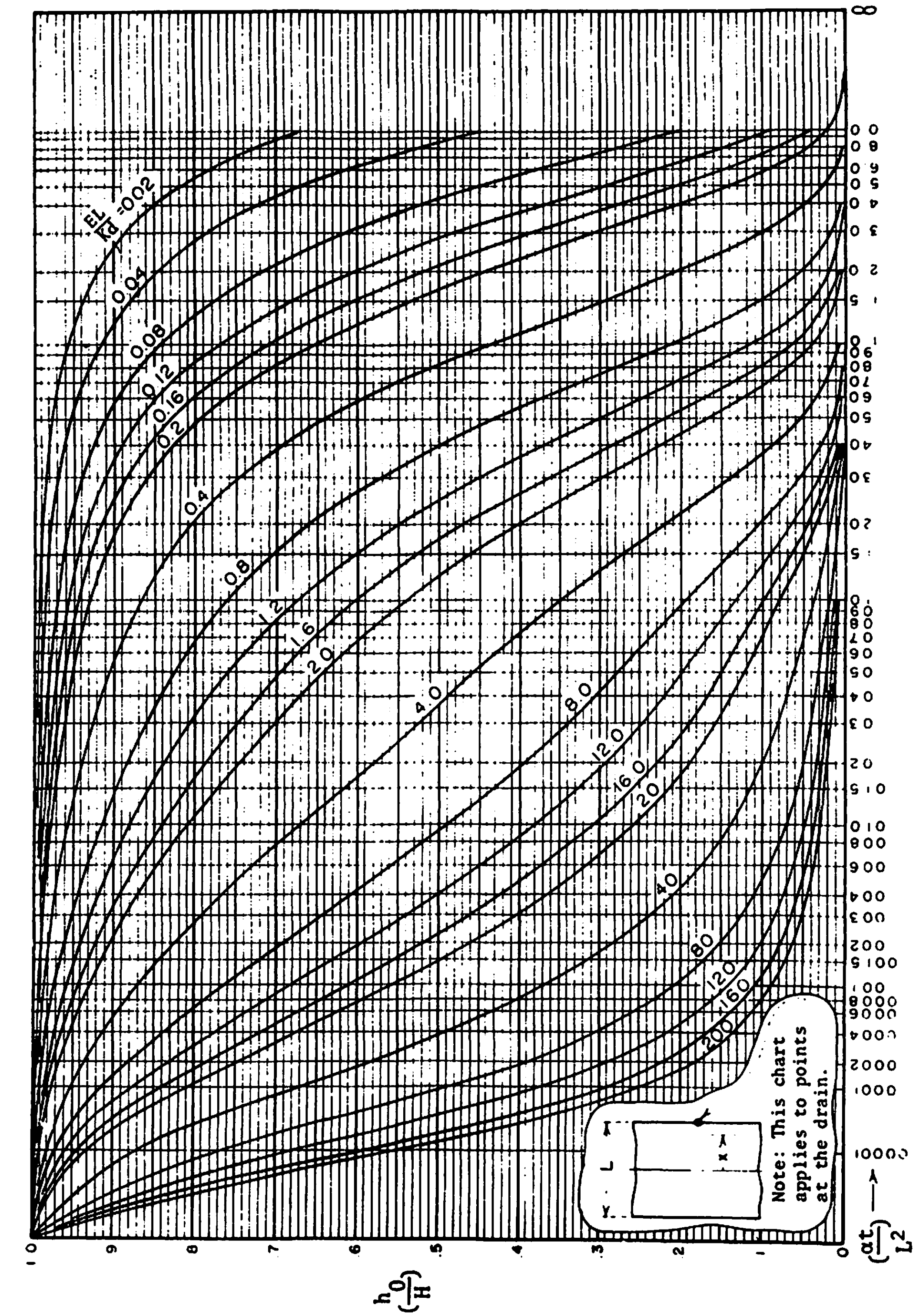


Fig. 8-7 (h_0/H) vs $(\alpha t/L^2)$ taking account of local flow resistance at the drain.

or

$$[p_1 - p_2 - p_3] \doteq \frac{2p_0}{\pi} \log_e \left(\frac{d}{\pi a}\right)$$

We need a ratio expressing the flow rate and the head causing the flow. Set

$$E\, h_0 = q \tag{8-24}$$

Where h_0 represents the head required to overcome the convergence losses and q represents the flow to unit length of drain from one side. Then:

$$E = \frac{q}{h_0} = \frac{2Kp_0}{\frac{2p_0}{\pi} \log_e \left(\frac{d}{\pi a}\right)} = \frac{\pi K}{\log_e \left(\frac{d}{\pi a}\right)} \tag{8-25}$$

As an example of the use of this result the problem whose solution is given in the paragraph on "Selection of drain spacings" will be resolved taking the convergence losses into account. In so doing, the charts prepared for an analogous problem in the flow of heat will be used. With

$K = 10$ ft/day $\qquad \alpha = 1222.23$ ft^2/day

$d = 22$ ft $\qquad Kd = 220$ ft^2/day

$a = 0.5$ ft $\qquad L = 1500$ ft

$\left(\frac{d}{\pi a}\right) = \frac{22}{1.5708} = 14.006 \qquad \left(\frac{d}{\pi a}\right) >> 1$

$\log_e 14.006 = 2.63949$

$$E = \frac{\pi K}{\log_e \left(\frac{d}{\pi a}\right)} = \frac{(3.1416)\ (10)}{2.63949} = 11.902 \text{ (ft/sec)}$$

$$\frac{EL}{KD} = \frac{(13.843)\ (1500)}{220} = 81.152 \text{ (Dimensionless)}$$

At the end of the irrigation season the depth of water at the drains is estimated in the following manner:

Application	Time Days	Drainable Depth (feet)	$(\frac{\alpha t}{L^2})$	$(\frac{h_0}{H})$**	h_0 (feet)
Apr 22*	132	0.46	0.0717	0.025	0.0115
June 6	87	0.46	0.0445	0.030	0.0138
July 1	62	0.46	0.0337	0.035	0.0161
July 21	42	0.46	0.0228	0.045	0.0184
Aug 4	28	0.46	0.0152	0.055	0.0253
Aug 18	14	0.46	0.0076	0.082	0.0377
Sept 1	0	0.46	0	1.000	0.4600
	365	4.00	0.1938	0.010	0.0400
					0.6228

*Snowmelt
**From chart of figure 8-7.

Then if water flows to the drain from both sides

$$2q = 2Eh = 2\ (11.902)\ (0.6228) = 14.825\ \text{ft}^2/\text{day}\ .$$

This value is about three times as high as was obtained for this case by using the Maasland idealization. The reason for the difference is that the value computed above is a peak value whereas the Maasland value is in the nature of an average. It would be good engineering to design the drains to carry the peak flows since, otherwise, the computed drainage performance could not be obtained. It may be noted also that the additional cost of a slightly larger tile would be a small part of the cost of installing the drains.

It remains to assess the effect of the convergence losses upon the drainage performance. The following computation will provide drainable depth values at midspan which can be compared with similar values where the convergence loss was neglected.

Application	Time Days	Drainable Depth (feet)	$(\frac{\alpha t}{L^2})$	$(\frac{h_c}{H})$	h_c (feet)
Apr 22*	132	0.46	0.0717	0.625	0.288
June 6	87	0.46	0.0445	0.825	0.380
July 1	62	0.46	0.0337	0.900	0.414
July 21	42	0.46	0.0228	0.960	0.442
Aug 4	28	0.46	0.0152	0.990	0.455
Aug 18	14	0.46	0.0076	1.000	0.460
Sept 1	0	0.46	0	1.000	0.460
	365	4.00	0.1938	0.210	0.840
					3.739

*Snowmelt
**From chart of figure 8-6 with (EL/KD) = 81.2.

A comparison of the results of this computation with a similar one in the paragraph on selection of drain spacings will show that the effect of local convergence on drain performance is not great. As would be expected, the computations show a slower drainage when local convergence losses are accounted for.

Return flows from irrigations

The idealizations described in this chapter can be adapted to the task of estimating the pattern of return flows supplied by deep percolations originating in irrigations. These return flows are often an important part of the water supply in irrigated areas. Figures 8-8 and 8-9 show how the conditions in a river valley can be correlated with the parallel drain idealization.

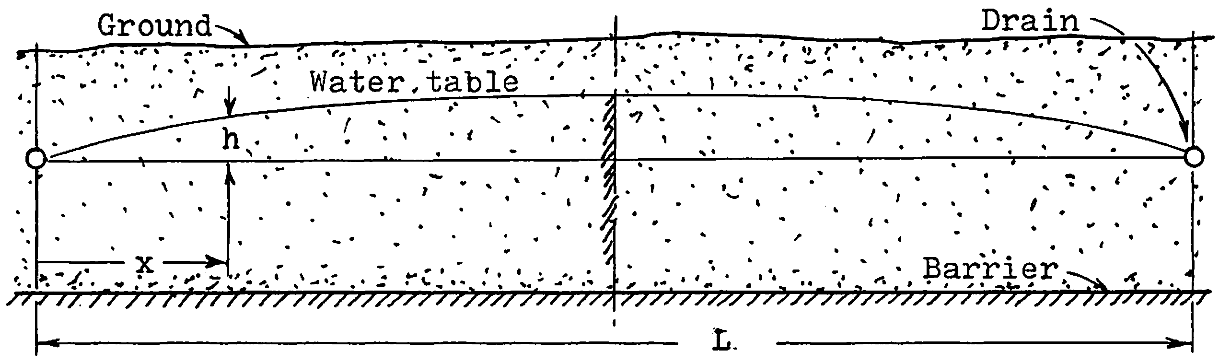


Fig. 8-8 Parallel drain idealization.

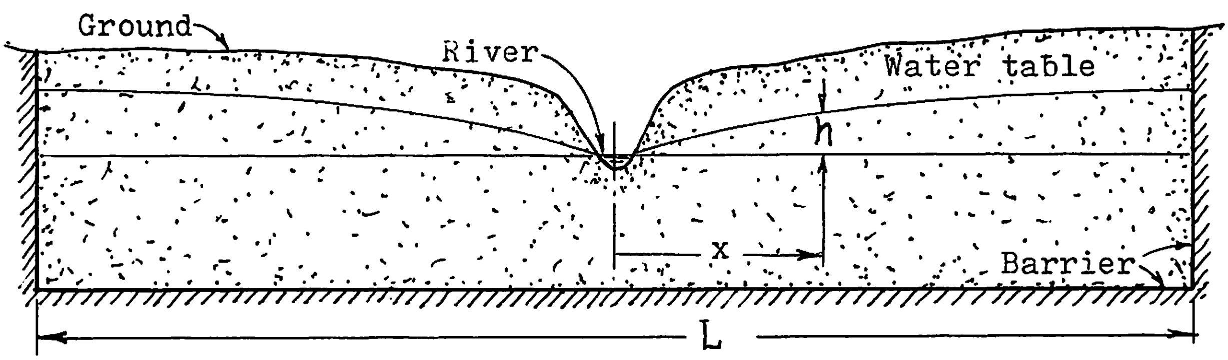


Fig. 8-9 Idealization of a river valley.

Because there is no flow across the line midway between drains, as shown in figure 8-8, the figure may be cut in two there and rearranged to bring the drains into coincidence, as shown in figure 8-9. Here the river replaces the drains. Use will be made of idealization 8-9 later. Mathematically, the idealizations of figures 8-8 and 8-9 are identical so long as L represents both the valley width and the drain spacing.

Chapter 8

Problems

(8-1) Drainage is needed in an irrigated area. Furrow irrigation is practiced and the deep percolation from each irrigation produces a drainable depth of 0.5 ft. Drainable increments originate on the following dates, the first of these being an accumulation of moisture received during the spring.

Date	Interval (Days)	Drainable Depth (feet)	Days to Sept 2
May 1	0	0.5	124
May 27	26	0.5	98
June 17	21	0.5	77
July 8	21	0.5	56
July 22	14	0.5	42
Aug 5	14	0.5	28
Aug 19	14	0.5	14
Sept 2	14	0.5	0
	124	4.0	

The drains are installed at a depth of 8 ft below ground surface and the water table is to be kept 4 ft below ground surface. The barrier is 28 ft below ground level. With a permeability of 15 ft per day estimate the drain spacing by use of formula 8-20 if V = 0.12. K = 15 ft/day KD = 300 ft^2/day α = 2500 ft^2/day.

Answer: L = 1497 ft

(8-2) Estimate the required drain spacing by use of formula 8-21.

Answer: L = 1995 ft

(8-3) Check the suitability of a 2000 ft spacing using the method described in the paragraph on selection of drain spacings. As a basis for the computation use $\alpha = (Kd/V)$ as required by the Laplace type solution.

Answer: The water table height at the end of the irrigating season is 3.98 ft. This is satisfactory.

(8-4) If the drain of problem (8-3) is an 8-inch diameter tile bedded in a 4-inch thickness of gravel estimate, by use of formula 8-25, the approximate convergence loss factor E .

Answer: $E = 20.9$ ft/day.

(8-5) If the convergence loss h_1 of equation 8-24 were applied to drive the flow q through a length ℓ of the aquifer of depth d the equivalence could be explained by the relation

$$E\ h_1 = q = \frac{Kdh_1}{\ell}$$

Where ℓ represents an equivalent length of aquifer then

$$\ell = \frac{Kd}{E}$$

By using this relation estimate how much the drain spacing of problem (8-3) would need to be shortened to compensate for the convergence loss incurred by the use of tile drains instead of open drainage canals.

Answer: $\ell = 14.4$ ft $L = 2000 - (2)\ (14.4) = 1971$ ft.

(8-6) Check this estimate by use of figure 8-6 and the procedure of problem (8-3).

Answer: $\frac{EL}{Kd} = 137$. The water table height at the end of the irrigating season is 3.95 ft. This is satisfactory.

(8-7) By use of figure 8-7 compute the drainable depth at the drain and estimate the flow to the drains at the end of the irrigation season on September 2.

Answer: Drainable depth 0.606 ft. $q_1 = E\ h_1 = 12.7$ ft^2/day or 12.7 cubic ft per day per ft of drain from one side.

(8-8) Check this figure on the basis that the ultimate steady state has been reached and the drains must carry away the infiltration as fast as it is received.

Answer: $q_1 = \frac{iL}{2} = 4.22$ ft^2/day or 4.22 cubic ft per day, per ft of drain, from one side. Transient conditions produce greater demands on drain capacity.

Chapter 9

Stream depletion due to a well

Formula 3-1 of Chapter 3 is appropriate where a well draws water, at the constant rate Q, from an aquifer of uniform properties and of infinite extent. The presence of a flowing stream may impose a condition of no drawdown along its course. If the course of the stream can be idealized as a straight line the condition of no drawdown can be imposed by use of an image well as explained in Chapter 10. In this case the image well is a recharge well of strength -Q and is located at the same distance from the stream as the pumped well and directly across the stream from it. The gradients imposed transverse to the stream by this combination can be computed from equation 3-1 and the flows produced by them can be summed along the whole stream length. In the mathematical sense this will be from $-\infty$ to $+\infty$. The depletion flow will be zero at time zero and will gradually rise toward Q as time increases.

The depletion of the stream by the well, computed in this way is given by the expression

$$\frac{q_1}{Q} = 1 - \frac{2}{\sqrt{\pi}} \int_0^{\left(\frac{x_1}{\sqrt{4\alpha t}}\right)} e^{-u^2} du \qquad (9\text{-}1)$$

For a given case this can be evaluated by use of Table 8. Details of this development are given in the paper by Glover and Balmer 1954.

Example

A well is located one mile from a stream. The aquifer properties are $KD = 0.270$ ft²/sec $V = 0.17$ $\alpha = 1.59$ (ft²/sec). It is desired to estimate what part of the flow of this well will be depleting the stream after the pumping has continued for three months.

With x_1 = 5280 ft = t (3)(2628000) = 7884000 seconds

$$\sqrt{4\alpha t} = \sqrt{(4)(1.59)(7884000)} = 7077 \quad \left(\frac{x_1}{\sqrt{4\alpha t}}\right) = 0.746$$

From Table 8: $\frac{2}{\sqrt{\pi}} \int_0^{0.746} e^{-u^2} du = 0.70858$

then

$$\frac{q_1}{Q} = 1 - 0.70858 = 0.29142$$

and the stream depletion at this time is about 29 percent of the well flow. If the well had maintained a flow of $Q = 1.50$ ft^3/sec the stream depletion, at this time, would be

$$q_1 = (1.50)(0.29142) = 0.437 \text{ (ft}^3\text{/sec)}.$$

The pattern of stream depletion due to a well can be of interest. If equation 3-1 is written in the form:

$$s = \frac{Q}{2\pi KD} \int_{\frac{\sqrt{x^2+z^2}}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2} du}{u}$$

Then the cross stream gradient is

$$\frac{\partial s}{\partial x} = \frac{-Q}{2\pi KD} \frac{x e^{-\left(\frac{x^2+z^2}{4\alpha t}\right)}}{x^2+z^2}$$

The image well will produce a similar gradient. Then the flow from the stream, per unit length of the stream, will be:

$$f = 2KD \frac{\partial s}{\partial x} = \frac{Q}{\pi x_1} \frac{x_1^2 \, e^{-\left(\frac{x_1^2+z^2}{4\alpha t}\right)}}{(x_1^2+z^2)} \qquad (9\text{-}2)$$

The chart of figure 9-1 has been prepared by use of this expression.

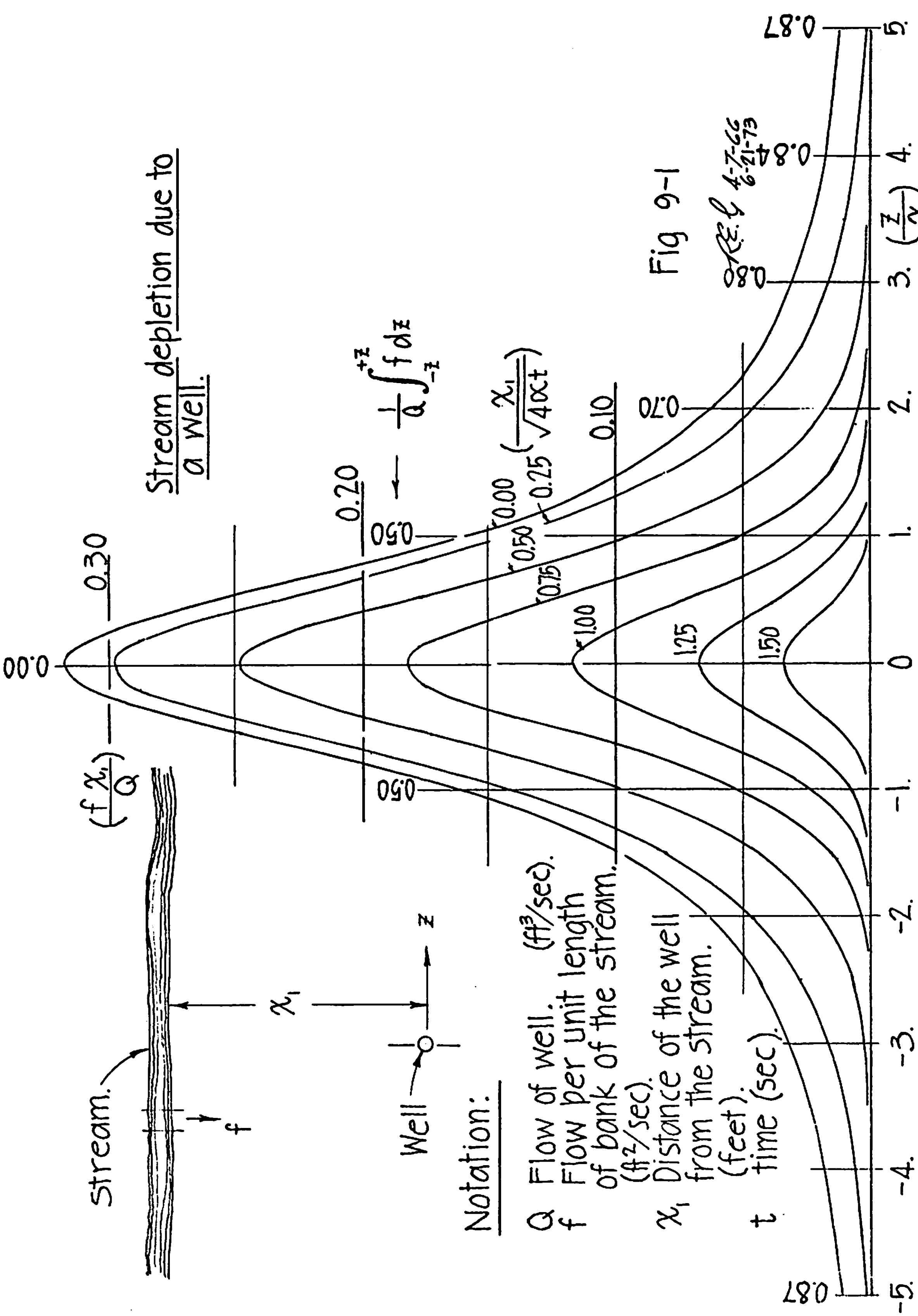

Stream depletion due to a well.
Stream.
Well
Notation:
Q Flow of well. (ft³/sec).
f Flow per unit length of bank of the stream. (ft²/sec).
x₁ Distance of the well from the stream. (feet).
t time (sec).
(f x₁ / Q)
(1/Q ∫ f dz)
(x₁ / √(4αt))
(z / x₁)
0.30
0.20
0.10
0.00
0.25
0.50
0.75
1.00
1.25
1.50
0.70
0.80
0.84
0.87
-5. -4. -3. -2. -1. 0 1. 2. 3. 4. 5.

Fig 9-1

An ultimate steady state is reached when $(x_1/\sqrt{4\alpha t})$ becomes zero. Under these conditions the flow coming from the reach -z to +z is given by

$$\int_{-z}^{+z} f dz = \frac{Q}{\pi x_1} \int_{-z}^{+z} \frac{x_1{}^2 dz}{(x_1{}^2+z^2)} = \frac{2Q}{\pi} \arctan \frac{z}{x_1} \qquad (9\text{-}3)$$

The figures on the chart which read vertically show values of this integral. When the ultimate steady state is reached, for example, one-half of the flow of the well will come from a reach of the river $2x_1$ long centered on the well. A similar reach $10x_1$ long will supply 87 percent of the well flow.

Chapter 9

Problems

(9-1) For the problem (3-1) of Chapter 3 estimate the stream depletion caused by this well if it is one-half mile from the river. Make the computation for the end of each six month interval out to the end of five years.

Answer:

Time (months)	Stream depletion (ft^3/sec)
0	0
6	0.705
12	0.841
18	0.905
24	0.943
30	0.970
36	0.990
42	1.005
48	1.017
54	1.028
60	1.036

(9-2) By using figure 9-1 or formula 9-2 make a plot of f vs z for time six months, based upon the conditions of problem 9-1.

Answer:

(z/x_1)	f (ft^2/sec)
0	0.0001249
1	0.0000539
2	0.0000138
3	0.0000033
4	0.0000007
5	0.0000001

Chapter 10

The use of images and treatment of intermittent operation

These two topics can well be treated together because both are concerned with procedures for extending the usefulness of basic types of solutions. The principle underlying these methods comes from the work of mathematicians. One of the fundamental questions to be settled relates to the conditions to be imposed if the solution of a differential equation is to be unique. Names associated with these investigations are those of Cauchy, Kowalewsky and Darboux (Cohen 1933). For our purposes the results of their investigations can be reduced to the following formulation. The solution of a differential equation is unique if:

(a) The solution satisfies the differential equation.

(b) The solution meets the appropriate boundary conditions.

(c) The solution meets the appropriate initial conditions.

The procedures for using images and for accounting for intermittent operation are best illustrated by use of examples.

Example 1

It is desired to account for the influence of a nearby stream on the drawdown produced by pumping a well. It is supposed that the well draws water from an aquifer which underlies the stream and that the stream is able to maintain the water table level along its course. The basic solution 3-1 is appropriate for an aquifer of infinite extent from which the flow of the well Q is supplied from water initially stored in the aquifer. It may be noted, in passing, that the basic solution does not converge to an ultimate steady state but, instead, the water table level sinks continuously to supply the flow of the well.

The boundary condition imposed by the presence of the stream can be met if the stream location is represented by a straight line and a recharge well is introduced on the far side of the stream and at a distance of x_1 from it. The hypothetical recharge well is to be located directly opposite the pumped well. Its recharge is to match the rate of discharge of the pumped well and the initiation of pumping and recharge are to be simultaneous. With this arrangement the drawdown produced by the pumped well along the course of the stream is everywhere annulled by the rise of the water table produced by the recharge well. It is to be understood that the recharge well has no real existence. It has the status of a mathematical ruse whose sole purpose is to accomplish a meeting of the boundary condition imposed by the stream. The solution, consisting of the algebraic sum of the drawdown of the pumped and recharge wells, is to be considered valid only on the side of the stream occupied by the pumped well.

For the conditions x_1 = 1320 ft (¼ mile)

α = 1.50 ft^2/sec Q = 1.5 ft^3/sec KD = 0.255 V = 0.17

x = 100. ft y = 0 t = 15,768,000 seconds (6 months).

Then, for the pumped well

$$\left(\frac{Q}{2\pi KD}\right) = \frac{1.5}{(6.2832)(0.255)} = 0.9362$$

$$\left(\frac{r_1}{\sqrt{4\alpha t}}\right) = \frac{100}{\sqrt{(4)(1.5)(15768000)}} = 0.01028$$

From tables:

$$\int_{\left(\frac{r_1}{\sqrt{4\alpha t}}\right)}^{\infty} \frac{e^{-u^2}\,du}{u} = 4.28993 \qquad s_1 = (0.9362)(4.28993) = 4.016 \text{ ft}$$

For the image well

$$r_2 = 1320 + 1320 - 100 = 2540 \text{ ft}$$

$$\left(\frac{r_2}{\sqrt{4\alpha t}}\right) = \frac{2540.}{9727.} = 0.26113$$

From Table 1.

$$\int_{\left(\frac{r_2}{\sqrt{4\alpha t}}\right)}^{\infty} \frac{e^{-u^2}du}{u} = 1.08765 \qquad s_2 = (0.9362)(1.08765) = 1.108 \text{ ft}$$

Then the drawdown at this point, with the river present, will be 4.016 - 1.018 = 2.998 feet. If the river were absent the drawdown would be 4.016 feet.

The present situation does lead to an ultimate steady state since the stream provides a supply. The integral which appears in the drawdown formula can be developed into a series of the form:

$$\int_{z}^{\infty} \frac{e^{-u^2}du}{u} = -0.288608 - \log_e z + \frac{z^2}{1!2} - \frac{z^4}{2!4} + \frac{z^6}{3!6} - \ldots$$

For the two wells the drawdown will be

$$s_3 = s_1 - s_2 = \left(\frac{Q}{2\pi KD}\right)\left[-0.288608 - \log_e\left(\frac{r_1}{\sqrt{4\alpha t}}\right) + \left(\frac{r_1^2}{8\alpha t}\right) - \ldots + 0.288608 + \log_e\left(\frac{r_2}{\sqrt{4\alpha t}}\right) - \left(\frac{r_2^2}{8\alpha t}\right) + \ldots\right]$$

As time increases the quantities $\left(\frac{r_1}{\sqrt{4\alpha t}}\right)$ and $\left(\frac{r_2}{\sqrt{4\alpha t}}\right)$ will become small compared to unity and all of the power terms will become negligibly small. When this happens an ultimate steady state is reached in which

$$s_4 = \left(\frac{Q}{2\pi KD}\right)\log_e\left(\frac{r_2}{r_1}\right) \tag{10-1}$$

It will be of interest to compute the ultimate steady state drawdown at the chosen point.

With

$$\left(\frac{r_2}{r_1}\right) = \frac{2540}{100} = 25.40$$

$$\log_e 25.40 = 3.23475$$

$$s_4 = (0.9362)(3.23475) = 3.028 \text{ feet}$$

This is only slightly greater than the transient state drawdown computed for six months of pumping.

If the origin is taken at the point where the line between the wells intersects the idealized stream location and x is counted positive toward the recharge well while y is counted positive downstream then s_4 will be found to be a solution of the Laplace steady state equation:

$$\frac{\partial^2 s_4}{\partial x^2} + \frac{\partial^2 s_4}{\partial y^2} = 0$$

As another example we may consider the case of a well one mile away from the river of the previous example and inquire as to how much the presence of a shale barrier two miles back from the river, on the well side, will alter the stream depletion pattern produced by the well. If the aquifer extended back from the stream to a great distance the stream depletion due to the well would be given by the expression

$$\frac{q_1}{Q} = 1 - \frac{2}{\sqrt{\pi}} \int_0^{\left(\frac{x_1}{\sqrt{4\alpha t}}\right)} e^{-u^2} du$$

A computation of the stream depletion to be expected under these conditions is shown below.

Time (months)	$(\frac{x_1}{\sqrt{4\alpha t}})$	$(\frac{q_1}{Q})$
0	∞	0.0000
1	1.3297	0.0600
2	0.9402	0.1836
3	0.7677	0.2776
6	0.5428	0.4427
9	0.4432	0.5308
12	0.3838	0.5873
18	0.3134	0.6576
24	0.2714	0.7011
36	0.2216	0.7540
48	0.1919	0.7861
60	0.1717	0.8081

For one month, considered to be (1/12) year:

$$(\frac{x_1}{\sqrt{4\alpha t_1}}) = \frac{5280}{\sqrt{(4)(1.5)(2628000)}} = \frac{5280.}{3971.} = 1.3297$$

When the effect of the impermeable shale boundary is to be accounted for two boundary conditions must be met. In the above computation the condition that there would be no change in the water table level at the stream was met by use of a single recharge well image. The formula used recognizes this. The condition that there be no flow across the shale boundary must now be imposed. If we image the pumped well in the shale boundary there will be no flow if the image well is a pumped well. To meet the boundary condition of no change in water table level at the river we image both of the above wells in the line representing the course of the stream. These will be recharge wells. Introduction of the recharge wells will cause a flow at the line representing the shale boundary and to rectify this we image the recharge wells in the shale boundary thereby doing a modicum of damage to the boundary at the stream. To counteract this we image the new wells in the stream boundary. Each change rectifies conditions at one boundary but disturbs them slightly at the other. Each change, however, improves the result. The series obtained is unending but convergent. The figures in the last column show what the depletion would be if the well were pumped for six months and then shut down. The values are obtained by superposition of $(\frac{q_1}{Q})$ values from the previous column. As an example, for time 12 months, $(q_1/Q) = 0.6838 - 0.4641 = 0.2197$. A comparison of the total (q_1/Q) values from the above table with the (q_1/Q) values of the previous table will show that the effect of the barrier begins to be discernible at the end of three months and at the end of 60 months (5 years) there is a very significant difference as will be seen by reference to table 10-1.

The case of a well in the corner between a stream and a tributary can be treated by use of images in the pattern shown below.

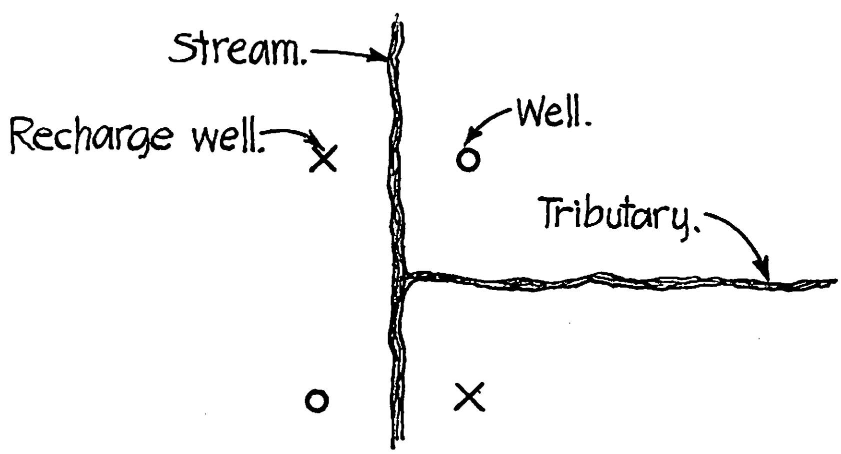


Fig. 10-1 Well in a corner.

The well is first imaged in the stream and then the well and its image are imaged in the line of the tributary. The solution is valid in the corner where the well is located. The circles represent pumped wells and the crosses represent recharge wells.

The problem of meeting the appropriate initial conditions arises when the effects of intermittent operation are to be evaluated. As in the previous treatment, examples provide the best approach to an understanding of the principles involved. Consider, for example, the case of a well which is pumped for six months and is then shut down for six months. It will be convenient to use the well of the preceding example. We must arrange the computation so that the solution satisfies the differential equation and meets the initial and boundary conditions. Suppose we try to meet these requirements by superimposing a recharge, at the pump location, which will nullify the flow of the well and compute the stream depletion thereafter as the algebraic sum of the pumped well and recharge well effects. If we check this proposed procedure against the requirements we find that the first requirement is met since the sum of two solutions of a linear differential equation is also a solution. The sum of the two solutions satisfies precisely the initial conditions at the time of change because the initial conditions are then those conditions which have been established by the pumped well. The boundary condition that the sum of the two solutions should conform to a condition of no well discharge is also met. Then we find that this sum of two solutions does meet the requirements for a unique solution as worked out by the mathematicians. Its application will disclose no shortcomings. A method for handling intermittent operations can then be outlined as follows. Where operations are intermittent each factor, once placed in operation, must be assumed to go on forever. New conditions are to be accounted for by superimposing additional solutions in such a way as to meet the appropriate initial and boundary conditions.

An example of these procedures can be supplied by estimating the stream depletion produced by the well of the previous problem if it is operated continuously for six months and is then shut down. The result is shown in the last column of Table 10-1. These values are obtained by subtracting from each quantity in the "Total" column the value from the same column for a time six months less. For the first six months the values in the two columns are identical because there is no value to subtract. For the end of the seventh month the computation is made by subtracting from the seven month value the one month value, thus:

$$\left(\frac{q_1}{Q}\right) = 0.5098 - 0.0600 = 0.4498$$

The other values are found similarly. It may be noted that the stream depletion continues after the pump is shut down. One year after pumping started and six months after shutdown the stream depletion is still given by the ratio $(q_1/Q) = 0.2197$. The actual depletion at this time is:

$$q_1 = Q\,(0.2197)$$

This is still a little more than one-fifth of the pumping rate.

An important application of this principle is met when making computations of the return flows from irrigations. Operations studies are generally made in terms of acre foot volumes and with time intervals of months because records are commonly kept in these units.

To develop a computation procedure for this use we may start with the Maasland development of Chapter 8. The return flow from both sides of the valley is given by the expression:

$$\frac{q_2}{iL} = 1 - \frac{8}{\pi^2} \sum_{n=1,3,5\ldots}^{n=\infty} \frac{e^{-n^2\pi^2\left(\frac{\alpha t}{L^2}\right)}}{n^2}$$

Table 10-1 Sample computation of the stream depletion due to operation of a well between a river and a barrier

Time months	Pumped well + $(\frac{x_1}{\sqrt{4\alpha t}})$	Pumped well + $(\frac{q_1}{Q})$	Image in Shale bn'dry + $(\frac{x_1}{\sqrt{4\alpha t}})$	Image in Shale bn'dry + $(\frac{q_1}{Q})$	First recharge Well image − $(\frac{x_1}{\sqrt{4\alpha t}})$	First recharge Well image − $(\frac{q_1}{Q})$	Second recharge Well image − $(\frac{x_1}{\sqrt{4\alpha t}})$	Second recharge Well image − $(\frac{q_1}{Q})$	Pumped well image + $(\frac{x_1}{\sqrt{4\alpha t}})$	Pumped well image + $(\frac{q_1}{Q})$	Pumped well image + $(\frac{x_1}{\sqrt{4\alpha t}})$	Pumped well image + $(\frac{q_1}{Q})$	Total $(\frac{q_1}{Q})$	Six months pumping $(\frac{q_1}{Q})$
0	∞	0	∞	0	∞	0	∞	0	∞	0	∞	0	0.0000	0.0000
1	1.3297	0.0600	3.9891	0	6.6485	0	9.3079	0	11.9673	0	14.6267	0	0.0600	0.0600
2	0.9402	0.1863	2.8207	0		0		0		0		0	0.1836	0.1836
3	0.7677	0.2776	2.3031	0.0011		0		0		0		0	0.2787	0.2787
4	0.6648	0.3471	1.9946	0.0048		0		0		0		0	0.3519	0.3519
5	0.5947	0.4003	1.7840	0.0116		0		0		0		0	0.4119	0.4119
6	0.5428	0.4427	1.6286	0.0213	2.7142	0.0001		0		0		0	0.4641	0.4641
7	0.5026	0.4772	1.5078	0.0330	2.5129	0.0004		0		0		0	0.5098	0.4498
8	0.4701	0.5062	1.4104	0.0461	2.3506	0.0009		0		0		0	0.5514	0.3678
9	0.4432	0.5308	1.3297	0.0600	2.2126	0.0017		0		0		0	0.5891	0.3104
10	0.4205	0.5521	1.2615	0.0744	2.1024	0.0030	2.9434	0		0		0	0.6235	0.2716
11	0.4009	0.5708	1.2028	0.0889	2.0046	0.0046	2.8064	0		0		0	0.6551	0.2432
12	0.3839	0.5872	1.1516	0.1034	1.9193	0.0067	2.6870	0.0001		0		0	0.6838	0.2197
15	0.3433	0.6273	1.0300	0.1452	1.7166	0.0152	2.4033	0.0007		0		0	0.7566	0.1675
18	0.3134	0.6576	0.9402	0.1836	1.5671	0.0257	2.1939	0.0019	2.8207	0		0	0.8136	0.1298
21	0.2902	0.6815	0.8705	0.2183	1.4508	0.0402	2.0312	0.0041	2.6115	0.0002		0	0.8557	0.0991
24	0.2714	0.7011	0.8143	0.2495	1.3571	0.0550	1.9000	0.0072	2.4428	0.0006	2.9857	0	0.8890	0.0760
27	0.2559	0.7174	0.7677	0.2776	1.2795	0.0704	1.7913	0.0113	2.3031	0.0011	2.8149	0.0001	0.9145	0.0588
30	0.2428	0.7313	0.7283	0.3030	1.2139	0.0860	1.6994	0.0163	2.1850	0.0020	2.6705	0.0002	0.9342	0.0446
33	0.2315	0.7434	0.6944	0.3261	1.1573	0.1020	1.6203	0.0220	2.0833	0.0032	2.5462	0.0003	0.9490	0.0345
36	0.2216	0.7540	0.6648	0.3471	1.1081	0.1171	1.5513	0.0283	1.9946	0.0048	2.4378	0.0006	0.9611*	0.0269
42	0.2052	0.7717	0.6156	0.3840	1.0259	0.1468	1.4363	0.0422	1.8467	0.0090	2.2570	0.0014	0.9771*	0.0160
48	0.1919	0.7861	0.5758	0.4155	0.9596	0.1748	1.3435	0.0574	1.7274	0.0146	2.1112	0.0028	0.9868*	0.0097
54	0.1810	0.7980	0.5429	0.4426	0.9048	0.2007	1.2667	0.0732	1.6286	0.0213	1.9905	0.0049	0.9929*	0.0061
60	0.1717	0.8082	0.5150	0.4664	0.8583	0.2248	1.2016	0.0893	1.5450	0.0289	1.8883	0.0076	0.9970*	0.0041
120	0.1214	0.8637	0.3614	0.6093	0.6069	0.3907	0.8497	0.2294	1.0925	0.1223	1.3352	0.0590	1.0332	

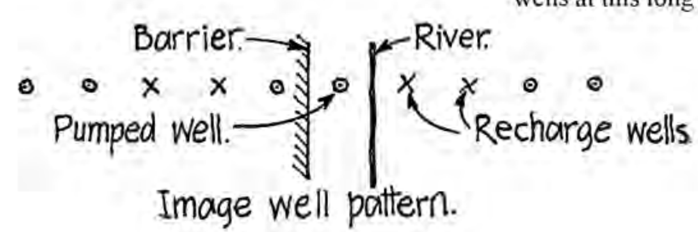
*Additional images would decrease these values slightly.

(1.0332) because we neglected lining wells at this long time

$$\frac{(1)(5280)}{\sqrt{4\alpha t_1}} = 1.3297 \qquad \frac{(7)(5280)}{\sqrt{4\alpha t_1}} = 9.3079$$

$$\frac{(3)(5280)}{\sqrt{4\alpha t_1}} = 3.9891 \qquad \frac{(9)(5280)}{\sqrt{4\alpha t_1}} = 11.9673$$

$$\frac{(5)(5280)}{\sqrt{4\alpha t_1}} = 6.6485 \qquad \frac{(11)(5280)}{\sqrt{4\alpha t_1}} = 14.6267$$



The total return is given by the relation:

$$R = \frac{\int_0^t q_2 dt}{iLt} = 1 + \frac{8}{\pi^4}\left(\frac{L^2}{\alpha t}\right)\sum_{n=1,3,5\ldots}^{n=\infty} \frac{e^{-n^2\pi^2\left(\frac{\alpha t}{L^2}\right)}}{n^4} - \frac{1}{12}\left(\frac{L^2}{\alpha t}\right) \tag{10-2}$$

In obtaining this result use has been made of the identity

$$\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \ldots = \frac{\pi^4}{96}$$

Values of the R function may be found in Table 12.

The following developments will be difficult to follow unless some explanation is made concerning purposes, methods and details of procedures. The purpose will be to develop a set of factors which can be used to estimate return flows from waters reaching the water table in a valley traversed by a stream. It is supposed that the valley floor is composed of permeable sediments occupying a trench eroded in less permeable materials, as shown by figure 8-9, that the valley is of a uniform average width L and that the river follows a course down the middle of the valley. It is further supposed that the permeable sediments of the valley floor are of a uniform character whose transient ground water flow characteristics can be expressed by a single value of the aquifer constant α. It is intended that the factors obtained will have, for a given valley, a permanent usefulness so that they can be put into the memory of a digital computer, for example, for the purpose of estimating return flows needed for operations studies and that these factors will be valid for any amounts reaching the water table in any months preceding the month for which the return flow is to be estimated. Following generally accepted procedures the quantities derived will be expressed in volumes accruing during a month. The factors will be derived for a unit volume $(iLt_1) = 1$ reaching the water table during a one month period t_1.

The factors will be useful for estimating stream depletions due to distributed pumping by considering the amounts removed by the pumps as a negative infiltration. It is supposed that the infiltrations reaching the water table or the depletions due to pumping can be idealized as being uniformly distributed over the width of the valley and, in order to facilitate the computation work, a month will be assumed to be a uniform (1/12) year. It will be recognized that some fraction of the water applied will reach the water table but for convenience of computation the whole amount of water applied may be assumed to reach the water table and the appropriate fraction applied to the final result. This procedure was followed in the preparation of Tables 10-3 and 10-4.

Factors could be obtained for any number of prior months but after some time has passed the total return flow from earlier months will be small. To expedite computations it will, therefore, be found desirable to limit the computation of individual return flows to a selected number of months and to approximate the return flow from all previous months by applying a residue factor to an average monthly increment reaching the water table, or, in the case of pumping, removed from it. In the computation shown in Table 10-3 return flows are computed for 48 months.

The residue is the difference between the sum of the factors and unity. In the example given the sum of factors is 0.98714 and the residue is 1 - 0.98714 = 0.01286. This is the residue factor to be applied to an average monthly increment. In the example, the average monthly increment is arbitrarily obtained from the year 1954 and is 27,350 acre feet per month. The contribution for all months before October 1954 to the return flow for September 1958 is therefore estimated to be (27350)(0.01286) = 352. acre feet. This is added in as shown in Table 10-4.

Where pumping is practiced in areas served by surface diversions a composite return flow fraction can be specified. Where there is considerable variation of valley width the valley length can be divided into reaches, a set of factors derived for each reach, and these combined later to yield a single set of factors as described above. It will be found that where application times run forward, as shown in Table 10-3 the factors obtained from Table 10-2 must be reversed in order to match the appropriate factor to the increment reaching the water table during a previous month. In the computation of Table 10-3, for example, the return flow for September is (60300)(0.14140) = 8526 acre feet. The factor 0.14140 is the factor for month 1 of Table 10-2. The estimate of return flow for August is (82100)(0.11720) = 9623 acre feet. The factor 0.11720 is the factor for month 2 of Table 10-2. Similarly, the return for September coming from the July application should be obtained by applying to it the factor 0.07628 of Table 10-2 for the third month. Having outlined the purposes to be served and the methods to be used the detail of the development of the set of factors can now be approached.

What is needed is a sequence of factors representing the return flow during some selected month due to application of irrigation water at a unit rate in the first month. To get this sequence we proceed in the manner shown in Table 10-2 below. For the sake of brevity only a computation for the first 12 months is shown. A month is considered to be (1/12) year or 2,628,000 seconds. This interval is here represented by the symbol t_1 .

The column "R times months" represents the total return from the accumulation of infiltration at the rate $iLt_1 = 1$ at the stated time. The application is here supposed to be continuous out to the stated time. The "First difference" represents the total return, accumulating out to the stated time, from the application $iLt_1 = 1$ made during the first month only. The "Second difference" represents the return flow accumulating during the stated month from the unit application $iLt_1 = 1$ made during the first month only. The return flow accumulating during some specified month due to an accumulation of infiltration during the first month is found by multiplying the accumulated quantity by the appropriate factor from the "Second difference" column. Since the factors in the "Second difference" column are dimensionless the product will take the units used to express the accumulation. Then if N acre feet of water reached the water table in month 1 the return flow from it accumulating during the month selected from the "Time-months" column of Table 10-2 will be N times the corresponding factor from the "Second difference" column. It is implied that the unit quantity of water reaching the water table during month 1 accumulates at a uniform rate during month 1.

Table 10-2

Partial return in some specified month due to application of water in a previous month

Time months	$(\frac{\alpha t}{L^2})$	R	R times months	First difference	Second difference
0	0	0	0	---	---
1	0.0088375	0.1414034	0.1414034	0.1414034	0.14140
2	0.0176750	0.2000021	0.4000042	0.2586008	0.11720
3	0.0265125	0.2449622	0.7348867	0.3348824	0.07628
4	0.0353500	0.2828602	1.1314409	0.3965542	0.06167
5	0.0441874	0.3162288	1.5811440	0.4497031	0.05315
6	0.0530249	0.3463434	2.0780605	0.4969166	0.04721
7	0.0618624	0.3739350	2.6175448	0.5394843	0.04257
8	0.0706999	0.3994691	3.1957527	0.5782079	0.03872
9	0.0795374	0.4232571	3.8093137	0.6135610	0.03529
10	0.0883749	0.4455212	4.4552115	0.6458978	0.03234
11	0.0972124	0.4664286	5.1307150	0.6755035	0.02960
12	0.1060498	0.4861115	5.8333385	0.7026234	0.02712

Note: For one month with $\alpha = 1.50$ ft^2/sec $L = 21120$ ft.

$$\left(\frac{\alpha t_1}{L^2}\right) = \frac{(1.50)(2628000)}{(21120)^2} = 0.008837487$$

The R values are obtained from Table 12 by linear interpolation. The "R times months" values are obtained by multiplying the R values by the values in the "Time months" column at the left. The difference between successive values in the "R times months" column is shown as the first difference. The "Second difference" values are obtained in the same way from the "First difference" values. The "Second difference" values are given to only five places since linear interpolation and loss of figures by differencing will not justify more places.

An example of a return flow computation is shown in Table 10-3 below. The irrigation water applied is shown for each month for the years 1954 to 1958 inclusive. The "Factors" come from Table 10-2. They will be seen to be in reverse order for the reason previously explained. The factor for month 1 is being applied to the irrigation volume used during September 1958. The "Sum of products" is the sum of the products of the "Factor" and the "Irrigation water applied" in the adjacent column. Since the volumes of water applied are expressed in acre feet and the "Factor" is dimensionless the "Sum of products" represents a volume, of return flow, in acre feet. When the estimated 352 acre feet correction is added to the "Sum of products" it represents the return flow which would have accumulated during the month of September 1958 if all of the irrigation water applied during all previous years had reached the water table.

Table 10-3
Computation of return flow for September 1958

$\alpha = 1.50\ ft^2/sec$ $L = 21120\ ft$

Year	Month	Irrigation water applied A.F.	Factor	Year	Month	Irrigation water applied A.F.	Factor
	Jan.	0			Jan.	300	.01234
		2300				2100	1350
		2100				10300	1468
		33100				7200	1610
		52600				19900	1750
1954		49000		1957		92700	1915
		43700				114900	2084
		45500				106300	2279
		37500				83200	2480
		33700	.00119			21700	2712
		27300	127			2100	2960
	Dec.	1400	141		Dec.	1100	3234
	Jan.	500	148		Jan.	0	3529
		0	170			0	3872
		4900	180			0	4257
		36200	200			1800	4721
		48500	214			49700	5315
1955		32800	239	1958		86200	6167
		41300	254			79500	.07628
		46800	282			82100	.11720
		35800	304		Sept.	60300	.14140
		40400	337			20200	
		5500	362			2300	
	Dec.	0	401		Dec.	0	
	Jan.	0	432	Sum of products		45288	.98714
		0	475				
		4500	514				
		29800	566				
		44000	614				
1956		53700	674				
		42300	730				
		51100	801				
		36100	869				
		33600	.00955				
		1900	.01035				
	Dec.	700	1137				

Notes:

1 - 0.98714 = .01286 (Residue)
Average month (1954) is 27350 A.F.
Estimate of contribution from years 1954 and before: (27350)(.01286) = 352. A.F.

Similar computations for each month provide the data for the following table. The sum of products shown at the bottom of Table 10-3 would be the return flow due to application of irrigation water from October 1954 to September 1958 if all of the applied water reached the water table. Allowances must be made for application prior to October 1954 and for the irrigation water consumed. The following table shows the return flows estimated on the basis that one-third of the applied water is consumed and two-thirds returns to the water table.

Table 10-4 Summary of estimated return flows - 1958

Month	Sum of products A.F.	Correction A.F.	Total A.F.	Return flow A.F.
Jan.	27440	352	27792	18528
Feb.	24770	352	25122	16748
Mar.	22541	352	22893	15262
Apr.	20804	352	21156	14104
May	25985	352	26337	17558
June	35257	352	35609	23739
July	40874	352	41226	27484
Aug.	44930	352	45282	30188
Sept.	45288	352	45640	30427
Oct.	40003	352	40355	26903
Nov.	33770	352	34122	22748
Dec.	29279	352	29631	19754
		Total for the year		263443
	Average monthly return flow			21954

A comparison of quantities in Tables 10-3 and 10-4 will show that while the applications ordinarily go to zero in the winter the return flows continue throughout the year at a nearly steady rate. The return flows peak in September at about 30,000 acre feet per month and reach a minimum of about 14,000 A.F./mo in April. Deviations from the average of about 22,000 A.F./mo reach only about 8000 A.F./mo. This is due to the equalizing effect of the ground water reservoir.

Chapter 10

Problems

(10-1) From the data of problem (9-1) estimate the stream depletion at the end of each six months for a seasonal operation in which the pump is on for each six months of the growing season and off for each six months of the nongrowing season.

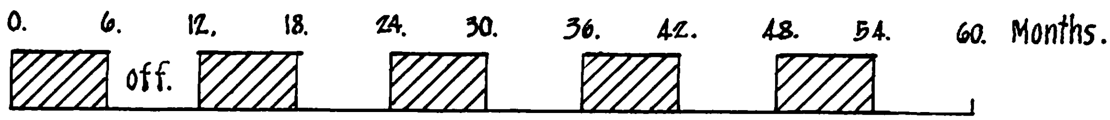


Fig. 10-2 Pumping pattern.

Answer:

Time (months)	Depletion rate (ft^3/sec)	Time (months)	Depletion rate (ft^3/sec)
0	0	36	0.207
6	0.752	42	0.865
12	0.145	48	0.220
18	0.820	54	0.876
24	0.186	60	0.230
30	0.848		

(10-2)

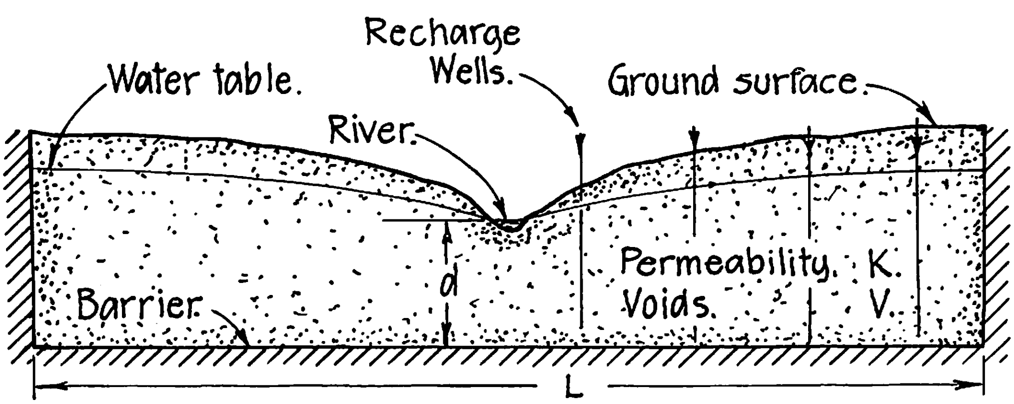


Fig. 10-3 Cross section of a river valley.

The cross section of a drained strip shown on fig. 8-3 is symmetrical about the center, and has a zero gradient there, so the center line is a line of no flow. The flow pattern would be unchanged, therefore, if the section were cut in two and the two parts rearranged to bring the drains into coincidence, as shown above, to give an idealization of conditions where a river has back-filled a pre-existing trench, in an impermeable formation, with permeable alluvial sediments. This situation occurs frequently. Here the river represents the drains which collect return flow supplied by deep percolation from irrigations when the valley floor is farmed.

It should be possible to make an independent approximate check of the validity of formula 8-23 by replacing the uniformly distributed recharge by four recharge wells, as shown on the above figure. Since the two halves are symmetrical the computation can be made for each half separately. Check the point for $(\alpha t/L^2) = 0.1$ by using the method of images.

Answer: The images in the barrier, of the first eight images in the river, must be included.

$(\frac{q_2}{iL}) = 0.696$ From Maasland's method 0.698

(10-3) Suppose the reservoir of problem (6-1) had been in service for five years and then was emptied to supply water during a drought. If the drawdown can be idealized as a drop of 10 feet at the beginning of each of the first four months of a six month irrigation period estimate the rate of ground water return flow at the end of the irrigation period. Assume that the reservoir was kept filled to an average 40 foot depth before the beginning of the irrigation period.

Answer: 24.0 ft^3/sec.

(10-4) If the above reservoir was an off stream reservoir whose bottom was well above the water table would there be any ground water return on drawdown?

Answer: No.

(10-5) For the case of problem (10-3) estimate the total ground water return during the irrigating season.

Answer: 13,860 acre feet.

Chapter 11

Effects of ground water drift

Comments

The reason for considering the effects of ground water movement under the action of a regional gradient is not that the effects are generally important but because administrators have been and can be confronted with objections which, in the absence of definitive answers, can be very perplexing. How, for example, does one answer the objection of a man who has been denied a permit to drill a well because the water supply is overincumbered when he points out that the water table in his area has a gradient of perhaps ten feet to the mile and that the cones of depression which would result from the operation of his well would be continually swept away by the ground water drift caused by the aquifer gradient and that his proposed well could, therefore, not run out of water?

Effect of ground water drift on drawdown

The ground water drift under consideration is that caused by a regional gradient which maintains a steady movement of the ground water below any fixed point on the ground.

The drawdown due to removal of the quantity of water W at $r = 0$ when $t = 0$ is given by the expression

$$s = \frac{W\, e^{-\left(\frac{r^2}{4\alpha t}\right)}}{4\pi KDt} \qquad (11\text{-}1)$$

This is a solution of equation 2-4 which satisfies the condition $s = 0$ when $t = 0$ for $r > 0$ and the above requirement. Such a depression, once created, would move with the water in the aquifer at the drift rate

$$f = \frac{K\gamma}{V} \qquad (11\text{-}2)$$

Where γ represents the regional gradient. The factor V accounts, approximately, for the increased velocity caused by movement through the interstices instead of the gross area. Consider a well located as shown in figure 11-1 below

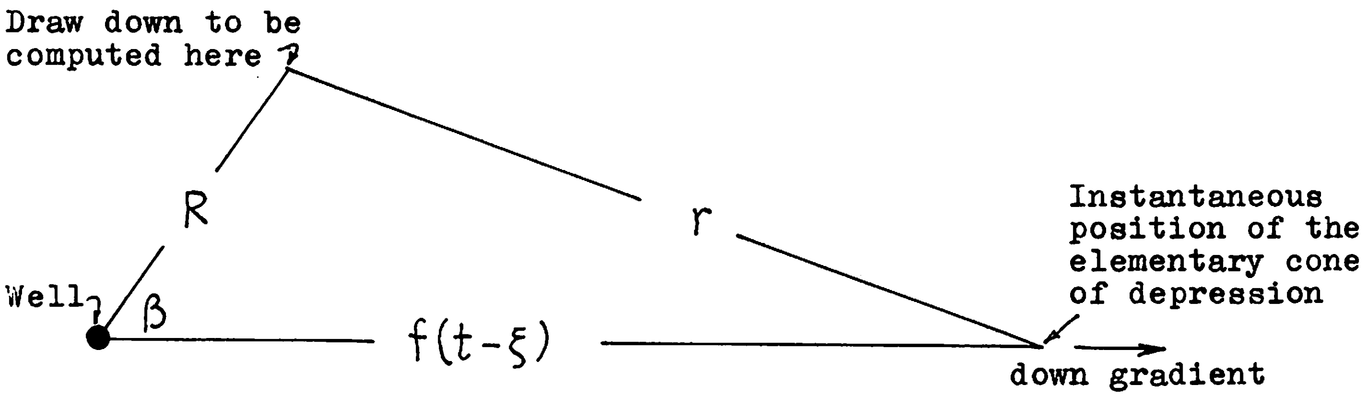


Fig. 11-1 Positions

Let ξ represent a time variable running between 0 and t. Consider ξ to represent the time at which an event occurs and t to be the time at which the effect of the event is to be computed. If, during the time interval $d\xi$ at time ξ, the well removes the quantity of water $dW = Qd\xi$ then the elementary cone of depression caused by this removal will be:

$$ds = \frac{Qd\xi \; e^{-\frac{r^2}{4\alpha(t-\xi)}}}{4\pi KD(t-\xi)}$$

If the point at which the drawdown is to be computed is defined by the radius R and the angle β measured from the direction of drift then, by the cosine law:

$$r^2 = R^2 + f^2(t-\xi)^2 - 2Rf(t-\xi)\cos\beta$$

and

$$\frac{r}{4\alpha(t-\xi)} = \frac{R^2 + f^2(t-\xi)^2 - 2Rf(t-\xi)\cos\beta}{4\alpha(t-\xi)}$$

or

$$\frac{r}{4\alpha(t-\xi)} = \frac{R^2}{4\alpha(t-\xi)} + \frac{f^2(t-\xi)}{4\alpha} - \frac{Rf\cos\beta}{2\alpha}$$

The drawdown at the radius R from the well can then be expressed as

$$s = \int_0^t \frac{Q d\xi\, e^{-\frac{R^2}{4\alpha(t-\xi)} - \frac{f^2(t-\xi)}{4\alpha} + \frac{Rf\cos\beta}{2\alpha}}}{4\pi KD(t-\xi)}$$

or

$$s = \frac{Q\, e^{\frac{Rf\cos\beta}{2\alpha}}}{4\pi KD} \int_0^t \frac{e^{-\frac{R^2}{4\alpha(t-\xi)} - \frac{f^2(t-\xi)}{4\alpha}}\, d\xi}{(t-\xi)}$$

Let

$$u^2 = \frac{f^2(t-\xi)}{4\alpha} . \qquad \frac{1}{(t-\xi)} = \frac{f^2}{4\alpha u^2} .$$

$$2udu = -\frac{f^2 d\xi}{4\alpha} . \qquad d\xi = \frac{-8\alpha u du}{f^2} .$$

By substitution

$$s = \frac{-2Q\, e^{\frac{Rf\cos\beta}{2\alpha}}}{4\pi KD} \int_{\sqrt{\frac{f^2 t}{4\alpha}}}^{0} \frac{e^{-u^2 - \frac{1}{4}\left(\frac{Rf}{2\alpha}\right)^2 \frac{1}{u^2}}\, du}{u}$$

or

$$s = \frac{Q\, e^{\frac{Rf\cos\beta}{2\alpha}}}{2\pi KD} \int_0^{\sqrt{\frac{f^2 t}{4\alpha}}} \frac{e^{-u^2 - \frac{1}{4}\left(\frac{Rf}{2\alpha}\right)^2 \frac{1}{u^2}}\, du}{u} \qquad (11\text{-}3)$$

The integral which appears here can be evaluated by use of Table 14.

It is now possible to evaluate the effect of a regional gradient. The following example will illustrate: Suppose a well is operated for 25 years at the average rate of 1.0 ft^3/sec. The regional gradient in the area is 22 feet per mile or $\gamma = 0.004167$.

Aquifer properties are:

$D = 120$ ft $\quad K = 0.001$ ft/sec $\quad KD = 0.120\ ft^2/sec$

$V = 0.15 \quad \alpha = 0.80\ ft^2/sec$

The drift rate is:

$$f = \frac{K\gamma}{V} = \frac{(0.001)(0.004167)}{0.15} = 27.78(10)^{-6}\text{ft/sec or } 876.0 \text{ ft/yr.}$$

$$\frac{Q}{2\pi KD} = \frac{1.00}{(6.2832)(0.120)} = 1.326 \text{ ft}$$

Twenty five years is 788,400,000 seconds. The drawdown at 1320 ft or 1/4 mile from the well is to be found.

If the water table is level

$$\left(\frac{r}{\sqrt{4\alpha t}}\right) = \frac{1320}{\sqrt{(4)(0.8)(788400000)}} = 0.026$$

$$s = \frac{Q}{2\pi KD}\int_{\left(\frac{r}{\sqrt{4\alpha t}}\right)}^{\infty} \frac{e^{-u^2}du}{u} = (1.326)(3.350) = 4.44 \text{ ft}$$

For the aquifer with drift

$$\left(\frac{Rf}{2\alpha}\right) = \frac{(1320)(27.78)(10)^{-6}}{(2)(0.8)} = 0.02292 \qquad m = \left(\frac{Rf}{4\alpha}\right) = 0.01146$$

$$e^{\left(\frac{Rf}{2\alpha}\right)} = 1.02318 \qquad e^{-\left(\frac{Rf}{2\alpha}\right)} = 0.97734 \quad m^2 = \left(\frac{Rf}{4\alpha}\right)^2 = \frac{1}{4}\left(\frac{Rf}{2\alpha}\right)^2$$

$$\sqrt{\frac{f^2t}{4\alpha}} = \sqrt{\frac{(27.78)^2(10)^{-12}(0.788)(10)^9}{3.2}} = 0.4360$$

From Table 14

$(\frac{Rf}{4\alpha})$	$\int_0^\infty \frac{e^{-u^2-(\frac{Rf}{4\alpha})^2\frac{1}{u^2}}}{u}du$	$\int_{0.430}^\infty \frac{e^{-u^2-(\frac{Rf}{4\alpha})^2\frac{1}{u^2}}}{u}du$	$\int_0^{0.430} \frac{e^{-u^2-(\frac{Rf}{4\alpha})\frac{1}{u^2}}}{u}du$
.010	4.02846	0.64354	3.38492
.020	3.33654	0.64306	2.69348

	$\int_0^\infty \frac{e^{-u^2-(\frac{Rf}{4\alpha})^2\frac{1}{u^2}}}{u}du$	$\int_{0.440}^\infty \frac{e^{-u^2-(\frac{Rf}{4\alpha})^2\frac{1}{u^2}}}{u}du$	$\int_0^{0.440} \frac{e^{-u^2-(\frac{Rf}{4\alpha})^2\frac{1}{u^2}}}{u}du$
.010	4.02846	0.62453	3.40393
.020	3.33654	0.62407	2.71247

	$\int_0^{0.436} \frac{e^{-u^2-(\frac{Rf}{4\alpha})^2\frac{1}{u^2}}}{u}du$	
.010	3.39633	(Linear interpolation was used)
.01146	3.29538	
.020	2.70471	

Then for $\beta = (\pi/2)$ where $\cos\beta = 0$ $e^{-(\frac{Rf}{2\alpha})\cos\beta} = 1.000$

s = (1.326)(3.29538) = 4.370 ft. Computation for the upstream and downstream points is made as follows:

β	$\cos\beta$	$e^{(\frac{Rf}{2\alpha})\cos\beta}$	s
0	1.000	1.02318	4.471
$\pi/2$	0	1.00000	4.370
π	-1.000	0.97734	4.271

It may be noted that the drawdown values computed for the aquifer with and without drift are almost the same. During the entire 25 year period the drift has been only (876.0)(25) = 21900 feet or about four miles. The elementary cones of depression which left the well at the beginning of the pumping period are now about this far from the well but they have long since been filled in. The only active cones are the young ones near the well and it is to be expected, therefore, that the drawdown patterns with and without drift would be closely similar.

Effect of ground water drift on stream depletion caused by pumping

In an irrigated river valley surface diverters convey water in their canals to the outer edge of the valley to irrigate the lands between the canal and the river. Deep percolation losses from the applied water reach the water table and cause a flow of ground water back to the stream. This is the "return flow" which is a common feature of irrigation projects. If irrigation by pumping is also practiced the wells will intercept some of the return flow and reduce the flow of the stream below what it would be if the pumping were not done. The question arises as to whether the ground water drift influences the depletion pattern sufficiently so that it should be taken into account in making estimates of stream depletion.

Evaluation of the effect of the drift can be made as before by summing the effects of the elementary cones of depression which are created by the well and are thereafter carried along with the ground water drift. The drawdowns produced by the removal of the quantity of water W at the radius r at the time t is, as given previously

$$s = \frac{W\,e^{-\left(\frac{r^2}{4\alpha t}\right)}}{4\pi KDt}$$

Let x and y represent rectangular coordinates measured, respectively, toward and along the stream. Then

$$r^2 = x^2 + y^2$$

By differentiation

$$\frac{\partial r}{\partial x} = \frac{x}{r}$$

The cross stream gradient is:

$$\frac{\partial s}{\partial x} = \frac{\partial s}{\partial r}\frac{\partial r}{\partial x} = \frac{-2Wxe^{-(\frac{r^2}{4\alpha t})}}{16\alpha\pi KDt^2}.$$

The flow into the bank, per unit length of stream is, when $x = x_1$

$$- KD\frac{\partial s}{\partial x} = \frac{2Wx_1e^{-(\frac{r^2}{4\alpha t})}}{16\alpha\pi t^2}$$

where x_1 represents the distance from the well to the stream. The total flow is:

$$\int_{-\infty}^{+\infty} - KD\frac{\partial s}{\partial x}\,dy = \frac{2Wx_1e^{-(\frac{x_1^2}{4\alpha t})}}{16\alpha\pi t^2}\int_{-\infty}^{+\infty} e^{-(\frac{y^2}{4\alpha t})}\,dy$$

Let

$$u^2 = (\frac{y^2}{4\alpha t}) \qquad dy = \sqrt{4\alpha t}\,du$$

Then

$$\int_{-\infty}^{+\infty} e^{-(\frac{y^2}{4\alpha t})}\,dy = \sqrt{4\alpha t}\int_{-\infty}^{+\infty} e^{-u^2}\,du = \sqrt{4\alpha t}\,\sqrt{\pi}$$

and the total flow is:

$$\int_{-\infty}^{+\infty} - KD\frac{\partial s}{\partial x}\,dy = \frac{2Wx_1e^{-(\frac{x_1^2}{4\alpha t})}\sqrt{4\alpha t}}{16\alpha t^2 \quad \sqrt{\pi}}$$

If an image is placed on the other side of the stream to satisfy the boundary condition that the drawdown at the stream is zero then the flow is doubled. Then if the stream maintains the water table level along its course the flow into the bank will be,

$$2\int_{-\infty}^{+\infty} - KD \frac{\partial s}{\partial x}\, dy = \frac{4\alpha W x_1 e^{-(\frac{x_1^2}{4\alpha t})}}{\sqrt{\pi}\,(4\alpha t)^{3/2}} .$$

Let ξ represent a time variable running between 0 and t. As before, ξ will represent the time at which an event occurs while t will represent the time at which the effect of the event is to be computed. The lifetime will be $(t - \xi)$. When the rate of drift is represented by f as before, the stream depletion at the time t due to removal of the volume of water $Qd\xi$ at the time ξ is:

$$dq = \frac{4\alpha Q d\xi (x_1 - f(t-\xi)) e^{-\frac{(x_1 - f(t-\xi))^2}{4\alpha(t-\xi)}}}{\sqrt{\pi}\,(4\alpha(t-\xi))^{3/2}}$$

But since

$$\frac{(x_1 - f(t-\xi))^2}{4\alpha(t-\xi)} = \frac{x_1^2}{4\alpha(t-\xi)} - \frac{2x_1 f}{4\alpha} + \frac{f^2(t-\xi)}{4\alpha}$$

The total flow is:

$$q = \frac{4\alpha Q x_1 e^{(\frac{x_1 f}{2\alpha})}}{\sqrt{\pi}(4\alpha)^{3/2}} \int_0^t \frac{e^{-\frac{x_1^2}{4\alpha(t-\xi)} - \frac{f^2(t-\xi)}{4\alpha}}}{(t-\xi)^{3/2}}\, d\xi$$

$$- \frac{4\alpha Q f e^{(\frac{x_1 f}{2\alpha})}}{\sqrt{\pi}(4\alpha)^{3/2}} \int_0^t \frac{e^{-\frac{x_1^2}{4\alpha(t-\xi)} - \frac{f^2(t-\xi)}{4\alpha}}}{(t-\xi)^{1/2}}\, d\xi$$

Let

$$u^2 = \frac{f^2(t-\xi)}{4\alpha} . \qquad d\xi = -\frac{4\alpha}{f^2}\, 2u\, du$$

$$\frac{x_1^2}{4\alpha(t-\xi)} = (\frac{x_1 f}{4\alpha})^2 \frac{1}{u^2} = \frac{1}{4}(\frac{x_1 f}{2\alpha})^2 \frac{1}{u^2}$$

Then, after substitution

$$q = \frac{Q\, e^{(\frac{x_1 f}{2\alpha})}}{\sqrt{\pi}} \left(\frac{x_1 f}{2\alpha}\right) \int_0^{\sqrt{\frac{f^2 t}{4\alpha}}} \frac{e^{-u^2 - \frac{1}{4}(\frac{x_1 f}{2\alpha})^2 \frac{1}{u^2}}\, du}{u^2}$$

$$- \frac{2Q\, e^{(\frac{x_1 f}{2\alpha})}}{\sqrt{\pi}} \int_0^{\sqrt{\frac{f^2 t}{4\alpha}}} e^{-u^2 - \frac{1}{4}(\frac{x_1 f}{2\alpha})^2 \frac{1}{u^2}}\, du \,. \qquad (11\text{-}4)$$

The integrals which appear here can be evaluated with the aid of Tables 15 and 13.

Comparison computation

A pump one mile from a river delivers 1.5 ft^3/sec. The aquifer properties are

$K = 0.00384$ ft/sec $D = 66.6$ ft $V = 0.171$

$\alpha = 1.50$ ft^2/sec $KD = 0.256$ ft^2/sec. The water table gradient toward the river is ten feet per mile or $\gamma = 0.001894$. The stream depletion due to six months of pumping is to be evaluated.

Solution for no water table gradient

$$\frac{q_1}{Q} = 1 - \frac{2}{\sqrt{\pi}} \int_0^{(\frac{x_1}{\sqrt{4\alpha t}})} e^{-u^2}\, du$$

Six months is 15,768,000 seconds or (1/2) year.

$$\sqrt{4\alpha t} = \sqrt{(4)(1.5)(15768000)} = \sqrt{9468000} = 9726.66 \text{ ft}$$

$$\left(\frac{x_1}{\sqrt{4\alpha t}}\right) = \frac{5280}{9726.66} = 0.54284 \,.$$

From Table 8:

$$\frac{2}{\sqrt{\pi}} \int_0^{0.54284} e^{-u^2}\, du = 0.55733$$

Then

$$\frac{q_1}{Q} = 1 - 0.55733 = 0.44267 \quad q_1 = (1.5)(0.44267) = 0.664 \text{ ft}^3/\text{sec.}$$

With a water table gradient of $\gamma = 0.001894$

$$f = \frac{K\gamma}{V} = \frac{(0.00384)(0.001894)}{0.171} = 0.0000425319$$

$$\sqrt{\frac{f^2 t}{4\alpha}} = 0.0689490 \qquad (\frac{x_1 f}{2\alpha}) = 0.0748561$$

$$m = \frac{1}{2}(\frac{x_1 f}{2\alpha}) = 0.0374280 \qquad e^{2m} = 1.07773$$

From Table 15 $\displaystyle\int_0^{0.0689} \frac{e^{-u^2-\frac{m^2}{u^2}} du}{u^2}$

m	0 to ∞	.0689 to ∞	0 to .068
.030	27.82507	11.96689	15.85818
.037428			10.83312
.040	20.45226	11.35910	9.09316

From Table 13 $\displaystyle\int_0^{0.0689} e^{-u^2-\frac{m^2}{u^2}} du$

m	0 to ∞	.0689 to ∞	0 to .0689
.030	0.83460	0.81231	.02229
.037428			.02060
.040	0.81807	0.79806	.02001

Then

$$\frac{q_1}{Q} = (0.60804)(.0748561)(10.83188) - (0.60804)(2)(0.02060)$$

$$= 0.46797$$

$$q = (1.5)(0.46797) = 0.702 \text{ ft}^3/\text{sec} .$$

The stream depletion at this time is increased about 5.7 percent. Again, the effect is a minor one. In this example it was assumed that none of the flow of the well returned to the water table in the area. When well water is used for irrigation a considerable part of the pumped water can return to the water table and, in such cases, only the part consumed depletes the stream.

Chapter 11

Problems

The time spent in interpolation is often a large part of the work of making computations based upon tabular values. When the table is a double entry table the time which must be allotted to interpolation must be increased. It is sometimes possible to modify the problem slightly so that tabular values may be used directly and still obtain results which are suitable for the intended purposes. The following problems are selected to illustrate this point.

(11-1) Remake the computation illustrating the use of formula 11-3 by changing the time and the radius R slightly so that the values needed can be read directly from Table 14.

Answer: m = 0.010 R = 1152 feet $\sqrt{\frac{f^2t}{4\alpha}} = 0.44$

t = 25.455 years

For no water table gradient s = 4.637 feet.

With a water table gradient present

β	s (feet)
0	4.604
$(\pi/2)$	4.513
π	4.424

The conclusion that the drift will not significantly alter the observed drawdowns is confirmed.

(11-2) Make a recomputation of the example of the use of formula 11-4 by slightly changing the values of f and t to cause the values of m and $\sqrt{\frac{f^2t}{4\alpha}}$ to be such as can be read directly from the tables.

Answer:

$m = 0.040$ $f = 45.455 \times 10^{-6}$

$\sqrt{\dfrac{f^2 t}{4\alpha}} = 0.070$ $t = 14,229,000$ seconds

For no gradient $q_1 = 0.629$ ft^3/sec

With the gradient present $q_1 = 0.641$ ft^3/sec

The difference is 1.9 percent. Either this value or the 5.7 percent obtained from the interpolated values indicate that the effect is a minor one.

Chapter 12

Analogs and Digital Computers

The analytical methods which have been described in the previous chapters are pretty well restricted to cases which can be idealized as having regular boundaries and uniform aquifer conditions. When attempts are made to extend these methods to treat cases having irregular boundaries and nonuniform aquifer conditions serious mathematical difficulties are encountered. To avoid these, analog or digital methods may be used.

The electric analog is based upon a similarity of the laws of flow of the electric current and the flow of ground water. A flow of electricity can, then, represent a flow of ground water, a voltage represent a water table elevation and, when transient changes are modeled, a capacitance can represent the ability of the aquifer to store water on a rise of the water table. In a common form of analog the electrical components are mounted on pegboard sheets. These sheets are commonly four feet wide and eight feet long. They are about 3/16 inch thick and have perforations at one-inch centers both ways. The holes scale about 9/32 inch in diameter. Nylon insulating bushings incorporating a "banana plug" type of connector are commercially available. These can be mounted in the holes in the pegboard and secured by means of a locknut. A metal prong is provided to which connections may be soldered. The bushings come in an assortment of colors so that important points on the network can be identified by a colored bushing. Stream courses can be marked by blue bushings and will show up clearly if white bushings are used for the other node points. This type of analog is fundamentally a direct current device.

Before the bushings are installed, a map to the proper scale should be mounted on the pegboard sheet. It helps to have a reversed map mounted on the back of the pegboard. This map should show what would be seen if the map on the front of the panel were placed face down on a light table and viewed from the back. If these two maps are carefully coordinated the map on the back side of the pegboard will be found to be of the greatest help when the electrical components are being mounted. Having these elements on the back will free the front of the pegboard for reference and operation. The panels can be mounted on a frame to hold them at a convenient height. Casters on the frames will make it easy to move them around. When several panels are needed to cover the area of interest they may be hinged together so that the assembly can be folded up and rolled out of the way when not in use. A panel of this type with the maps and electrical components in place ready to go, will cost about a dollar per node point. When capacitors are provided transient ground water movements can be modeled. By making use of this analogy the field problem can be taken into the laboratory and tests can be run which would be prohibitively costly and time consuming if tried in the field. Such an analog is shown on figures 12-1 and 12-2. The network arrangement is shown in figure 12-3.

A method of selecting analog components to meet the needs of a specific problem will be described later but it may be stated here that the analog builder has two basic choices. He can choose to use small resistances and capacitances to create a very high speed analog or he can use large resistances and capacitances to create an analog which will be slowed sufficiently to permit the use of a direct writing oscillograph as a readout device. If the high speed choice is made an oscilloscope is the only device with a high enough writing speed to be useful for readout purposes. To get permanent records will require a camera to photograph the face of the oscilloscope tube. Even so, it will generally be necessary to use an electronic device to impose the

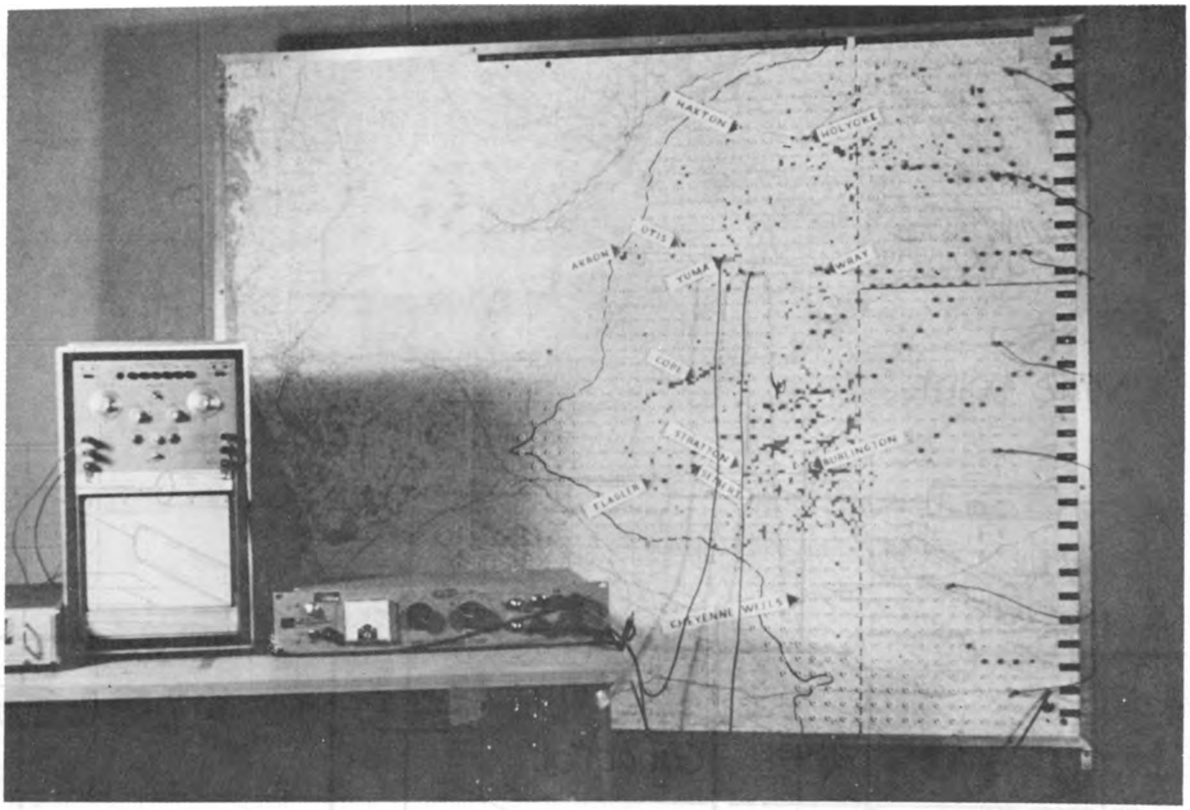


Fig. 12-1. Transient type analog.

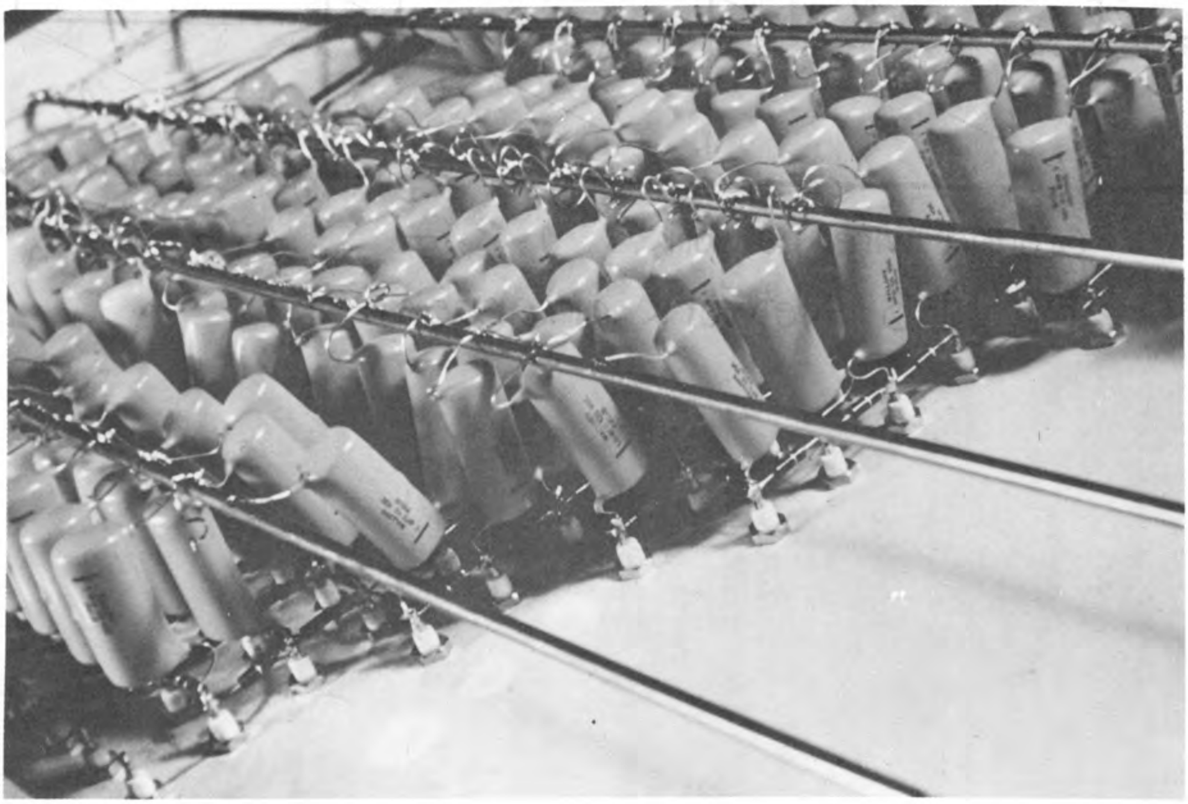


Fig. 12-2. View of back of panel showing capacitors and ground wires.

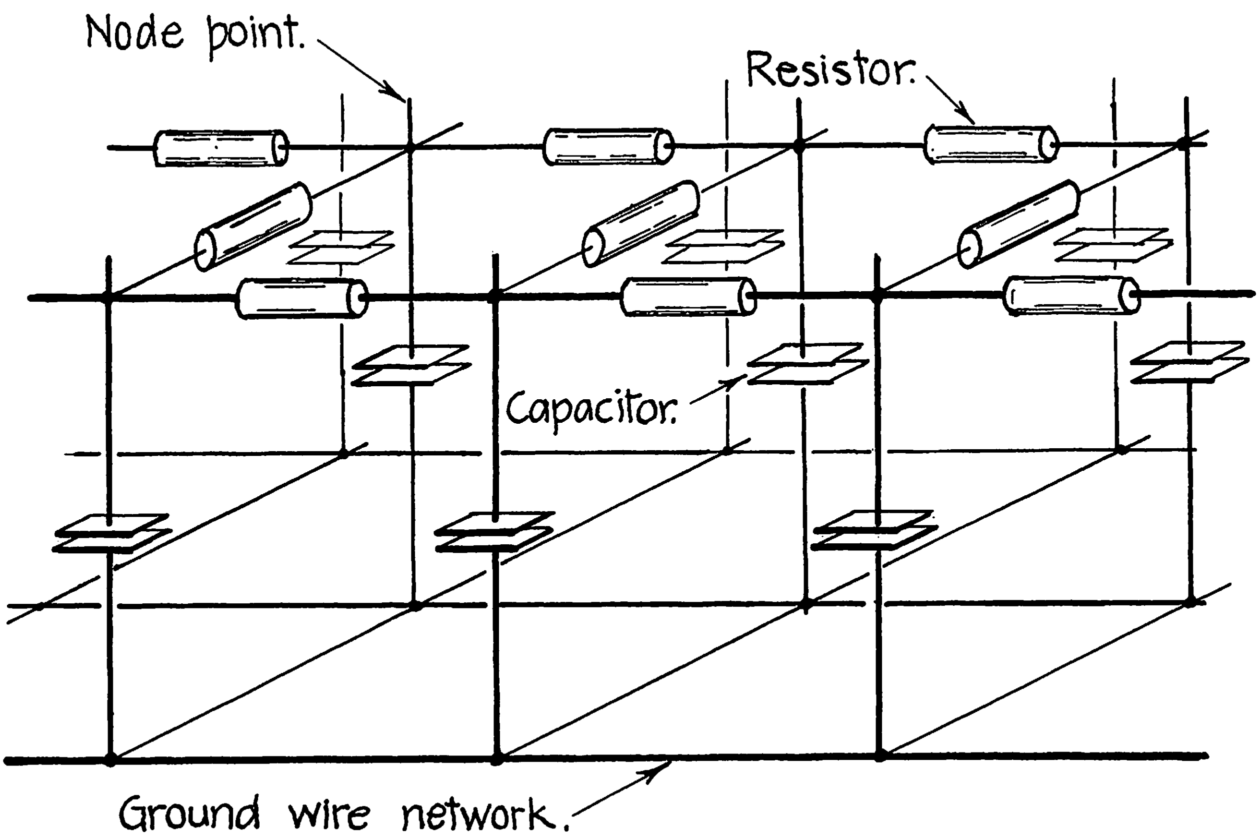


Fig 12-3 Network arrangement.

problem conditions on the analog network repeatedly. If the repetitions are made often enough persistence of vision will cause the successive traces on the oscilloscope tube to appear as a steady pattern. This may be read, or photographed when a permanent record is needed. Cameras of the Polaroid type which give a finished picture in a few seconds are very convenient for this use.

When the slower type of analog is used a finished record is obtained directly from the direct writing oscillograph. A complete record, including the making of notes and identifications can be made in about 20 seconds but this rate of operation with direct writing equipment is made at some cost. With resistances of the order of 1 million ohms and capacitances of the order of 1 microfarad the network resistance, from a selected node point to ground, may be of the order of 10 million ohms. A readout instrument with an input impedence of around 1 million ohms connected between a node point and ground would essentially short circuit the network and cause it to give spurious readings. It will be essential, then, to interpose an electronic servo device which will sense the network voltage and impose it on the oscillograph input without drawing current from the network. Such devices are commercially available or can be constructed. Such a device is generally unnecessary with the high speed type of analog. Inputs into the high speed analog can be controlled by use of resistances which are high compared to the network resistance but with the low speed analog the network resistance is already so high that some other method of control must be found. This takes the form of another electronic servo device which will feed a constant current into a network node point without being influenced by network voltage changes. With the low speed analog an excessive amount of time may be required for network potentials to drain to ground when a sequence of runs is to be made. To overcome this difficulty a multi-contact switch can be installed and connected between selected node points and the ground. With 30 contacts 30 such connections can be made. This switch must be opened, of course, before the next run is made. A time controlled relay with a pilot light which comes on when the switch is activated and goes off when the relay opens the circuit is a useful arrangement. Tests will show how long these drainage contacts need to remain closed. Something like 20 seconds may be needed.

A simpler type of analog which is, however, restricted to steady state conditions can be made with the use of a conducting sheet. Such a sheet may be obtained by rubbing a soft pencil lead on paper but this is difficult to make uniform. A much better material is sold commercially under the name "Teledeltos" paper. It is made for use on recording instruments. The back of the paper has a conducting coating and if this is in contact with a metal surface and a metal stylus is in contact with the face side it will leave a mark on the paper if a suitable voltage is applied between the metal surface and the stylus. The conducting surface on the back side is of interest for our purposes. The resistance measured across a square inch is about 3000 ohms. A conducting ink is commercially available. It is used primarily for making complex electrical circuits. It flows well from a drawing pen. Being based on the use of silver it is more expensive than ordinary inks. Manufacturers of recording instruments and electrical shops are possible sources of these materials. An analog of this type representing the "Northern High Plains" area in Colorado is shown in figure 12-4. Even though this type of analog is appropriate for steady state conditions it can often be used effectively for transient conditions by combining its use with analytical methods. When so used it provides an effective means for establishing boundary conditions. An example of such a use will be given later under the heading: Combination methods.

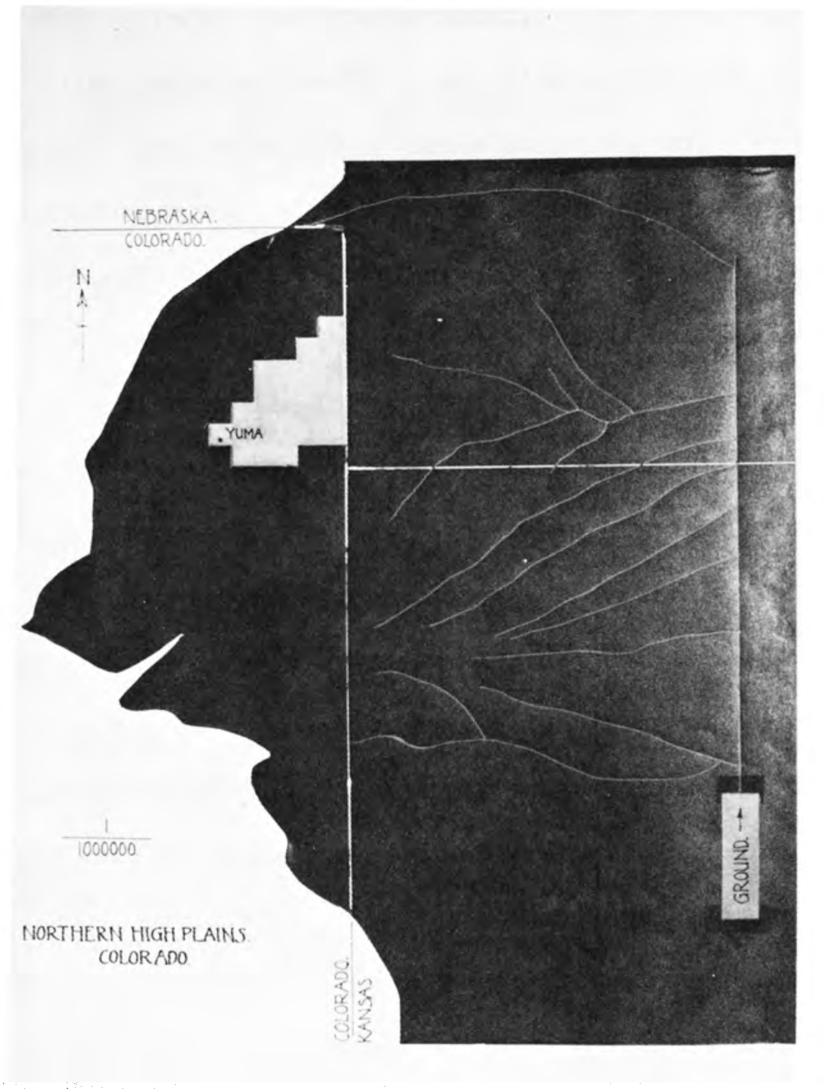


Fig. 12-4. Steady state analog.

When a digital computer is available an arithmetical approach can be used. The computation procedure is basically a simple one but the amount of computation required would be prohibitive if desk computers were to be used. The basic differential equation of ground water flow comprises two fundamental requirements. These are that Darcy's law is obeyed and that water volumes must be accounted for. The second of these is the so-called continuity requirement. One way to go about the computation would be as follows: Suppose the area is represented by node points, as in the first of the electrical analogs described above, and an initial water table configuration is given. Gradients can be computed from the differences of water table elevation between adjacent node points. If the grid is rectangular, there will be four of these for each interior node point. The algebraic sum of these multiplied by the product of the transmissibility, the node spacing and selected time interval will give the flow of water into the square area tributary to the node point. This product divided by the horizontal area represented by the node point and divided by the voids ratio V will yield the rise of the water table during the selected time interval. When the water table elevations have all been corrected the process can be repeated.

With the analytical procedures there is no approximation in either space or time. The node-wise electric analog is step-wise in space but continuous in time. The digital computer procedure is step-wise in both space and time. Both of these latter two methods are, therefore, to some extent, approximations. Experience with analogs indicates, however, that they do give a satisfactory representation of the behavior of the aquifer prototype. Their local lack of resolution must be kept in mind, however, as will be explained later.

Design of analogs

For purposes of analog design, continuity of space and time will be assumed. The modifications needed to adapt the results to a node-wise analog will be made as the last step in the design procedure. A basic fact relating to the conductance of a uniform conducting sheet may as well be described now. The resistance to the flow of current between opposite sides of a square area is proportional to the length and inversely proportional to the width. This means that the resistance measured between opposite sides of a square area cut from the material is independent of the size of the square. The resistivity of the material can, therefore, be specified in terms of the resistance between opposite sides of a square area cut from the sheet. The resistivity of the sheet can, then, be specified in ohms. The conductance is the reciprocal of the resistance and can be expressed, if needed, in mhos.

The design can proceed by the following steps:

(1) Express the flow and continuity relationships for the aquifer.

(2) Express the flow and continuity relationships for the electrical conducting sheet.

(3) Write a series of correlation equations relating the coefficients of the aquifer and analog equations.

(4) Seek, by trial, a set of correlation equation coefficients that will adapt the analog to represent the prototype in a way which will be compatible with commercially available electrical components and the capabilities of the recording equipment.

If we are to deal with the design of a node-wise transient type analog and wish to accommodate its characteristics to work with a direct writing oscillograph the largest capacitor size conveniently available should be chosen. Capacitors of the "Milar" type with 1 microfarad capacities can be obtained. Larger values could be obtained with electrolytic types but these are vulnerable to being punctured by accidental reversed voltage and their use for analog elements is considered unwise. The 1 microfarad size has served well.

The pertinent equations are the following:

Aquifer	Analog
$q = - KD \frac{\partial s}{\partial x}$	$i = - \frac{1}{\rho} \frac{\partial E}{\partial \xi}$
$\frac{\partial q}{\partial x} = V \frac{\partial s}{\partial t}$	$\frac{\partial i}{\partial \xi} = C \frac{\partial E}{\partial \eta}$

(12-1)

The correlation equations are:

$$n_1 = \frac{q}{i} \qquad n_2 = \frac{s}{E} \qquad n_3 = \frac{x}{\xi} \qquad n_4 = \frac{t}{\eta} \tag{12-2}$$

By substitution into the first of the aquifer equations

$$n_1 i = - KD \frac{n_2 \partial E}{n_3 \partial \xi}$$

and, by comparison with the first of the analog equations

$$\frac{1}{\rho} = \frac{KDn_2}{n_1 n_3} \quad \text{or} \quad \rho = \frac{n_1 n_3}{KDn_2} \tag{12-3}$$

A substitution into the second of the aquifer relations yields

$$\frac{n_1}{n_3} \frac{\partial i}{\partial \xi} = V \frac{n_2 \partial E}{n_4 \partial \eta}$$

A comparison with the corresponding analog relationship will yield

$$C = \frac{n_2 n_3 V}{n_1 n_4} \tag{12-4}$$

An important relationship which will be required for analog operation is the current, I , which represents a flow Q in the prototype. If, for example, a well lifts water at the rate Q in the prototype, what current I will represent this flow in the analog? To develop this relationship suppose a closed loop to be drawn around the well in the prototype. Then if ℓ_1 represents a length measured along the loop and m_1 , a length measured normal to the loop

$$Q = \int_0 q d\ell_1 = KD \int_0 \frac{\partial s}{\partial m_1} d\ell_1$$

where the lower limit of the integral signifies that the integration is to be carried around the loop. In the analog the total current is, similarly,

$$I = \frac{1}{\rho} \int_0 \frac{\partial E}{\partial m_2} d\ell_2$$

By substitution from the correlation equations

$$Q = KD \int_0 \frac{n_2 \partial E \; n_3 \; d\ell_2}{n_3 \partial m_2} = n_2 \; KD\rho I \qquad (12\text{-}5)$$

But from formula 12-3 $KD\rho = \frac{n_1 n_3}{n_2}$ and, by substitution

$$Q = n_1 n_3 I \qquad (12\text{-}6)$$

Where transient conditions are being studied the loop must be shrunk until releases of storage between the loop and the well become negligible.

An example will illustrate the use of these relationships. The construction of an analog to represent the "Northern High Plains" area in eastern Colorado will be our goal. The aquifer is in the Ogallala and similar formations. These are water laid deposits which increase in thickness toward the east. Infiltration from precipitation sustains saturated thicknesses of up to 300 feet or more. The water table slopes toward the east and there is a ground water flow in this direction also. A part of this appears on the surface to sustain the base flow of streams but the bulk of the flow crosses the state line as underground flow. Pumping for irrigation and other uses has been developed. The flow pattern which is sustained by the precipitation will be considered as a basic steady state solution of the equations of ground water flow and the transient changes produced by pumping will be superimposed upon it. The configuration of the water table before much pumping was done is known from an early investigation (McGovern and Coffin, 1963). The analog will be used to evaluate the transient changes produced by pumping. The area to be represented is about 9000 square miles.

With a permeability of 0.001 (ft/sec) the transmissivities will be:

Saturated Thickness (feet)	Transmissivity (ft^2/sec)
0	0
50	0.050
100	0.100
200	0.200
300	0.300
400	0.400

A suitable available map is to the scale (1/250,000). This fixes the value of $n_3 = (x/\xi)$ making it $n_3 = 250{,}000$. The task of finding the remaining n values can now proceed. It is a trial process. The first set of trial values chosen will often be found to be hopeless but they will generally indicate what steps must be taken to make an improvement. After several trials the following values are taken:

$n_1 = 4$ $\qquad$ $n_3 = 250{,}000$

$n_2 = 5$ $\qquad$ $n_4 = 315{,}360{,}000$

The n_4 value so chosen will mean that one second of analog time will represent 10 years of prototype time. Then the resistivities can be evaluated from the relation

$$\rho = \frac{n_1 n_3}{KDn_2} = \frac{(4)\ (250,000)}{KD(5)} = \frac{200,000}{KD}$$

Saturated Depth (feet)	KD (ft^2/sec)	ρ (ohms)	Some Values Commercially Available in 1/2 Watt Resistors (ohms)
0	0	∞	--
50	0.050	4,000,000	3,900,000
100	0.100	2,000,000	2,200,000
200	0.200	1,000,000	1,000,000
300	0.300	667,000	680,000
400	0.400	500,000	470,000

On the basis that the effective porosity is $V = 0.15$ the value of distributed capacitance needed is

$$C = \frac{n_2 n_3 V}{n_1 n_4} = \frac{(5)(250,000)(0.15)}{(4)(315,360,000)} = 0.0001486\ \left(\frac{\text{farads}}{\text{ft}^2}\right)$$

Up to this point the analog quantities have been treated as being distributed but we must now convert them to lumped quantities. The arrangement of the analog elements is shown in figures 12-2 and 12-3. We choose to install node points at one inch centers. Since a rectangular grid is being used each resistor must represent the resistance across a one-inch square. Then, by the rule previously given, the resistors needed are of the values shown in the computation for ρ . If the capacitance needed is 0.0001486 farads per square foot of the analog area then the capacitance needed at each node point, which represents one square inch of the analog area, is;

$$\frac{0.0001486}{144} = 0.00000103 \text{ farads or, closely enough, one microfarad.}$$

This will be the capacitance needed at each node point. Capacitors and resistors with a 10 percent tolerance have been found to be satisfactory. It may be noted that the computed resistance often cannot be exactly matched with commercially available components anyway. Capacitors with a voltage rating of 200 volts have performed well.

Readout equipment

A direct writing oscillograph offers advantages for analog use because a permanent record is immediately available without the need for development, as would generally be the case with other types of oscillographs. The direct writing oscillograph used with the analog of figure 12-1 had a choice of paper speeds of 5, 10, 20, 50, 100 and 200 mm/sec. This would permit recording of a 100-year period on about eight inches of records at the 20 mm/sec speed. This proved suitable for the intended purposes. When many records are to be run it may be profitable to have rubber stamps made to show the horizontal and vertical scales. Other needed identification can be written directly on the record.

Personnel

A crew of two is generally needed to construct and operate an analog. These are a hydrographer and an electronics man. The electronics man should be able to construct the analog, to test the electrical equipment, to maintain the input and readout equipment and, when needed, to design and construct special instrumentation. If he is able to design and construct the servo devices mentioned previously, time and money can often be saved. The hydrographer will have the responsibility for setting up programs which will yield the information for which the analog was designed.

Test panel

Before the analog is constructed it is desirable to build and test a panel representing a circular area having elements appropriate for some selected part of the analog area. For the analog of the example a saturated depth of 200 feet could be chosen. A test panel with ten node points on a radius would serve very well. The ground wire is connected to all of the elements of the outer perimeter. There should be a node point at the center. Data from runs on this test panel should be compared with analytical solutions obtained by use of Table 6. Confirmation of the correctness of the design can be obtained from this test panel as well as some ideas about the resolution of the analog in the neighborhood of a node point. The center node point will represent a well of constant flow if the results are to conform to solutions obtained from Table 6. Near the center node point the analog and analytical solutions will not agree because the mesh of the analog is locally too coarse to have the needed resolution. The first node point out should conform fairly well and the second node point out from the center node point should give readings in good agreement with the analytical solution. If readings from the second node point out from the center are accepted and drawdowns close to the center node point are carried in from the second node point by use of the analytical development for steady state drawdowns around a well of constant flow, a good correlation with the solution obtained from Table 6 should be realized. If such a correlation is not realized an error in design of the analog is indicated.

Example of use of a steady-state analog

In the Northern High Plains area previously described it would be pertinent to inquire as to how much of the annual infiltration supply could be recovered. Under primitive conditions all of this supply moved eastward under the action of a regional gradient to appear as the base flow of streams. To recover the water for irrigation or other uses pumps are used. These must modify the gradients sufficiently to divert the flow to the well. About 50 feet of saturated depth is needed to sustain the flow of a well yielding about one cubic foot per second (USGS WSP 1819-1) which is considered to be about the smallest yield which would make the well economically viable (Colo GW Circular No. 8). Then recovery means that the well must compete with the pre-existing gradients for the water it gets. It must be expected that only a part of the total supply can be recovered.

The analog, constructed of Teledeltos paper, is shown on figure 12-4. The scale is 1/1,000,000. Pumping was assumed to be concentrated within an area of 25 townships covering, roughly, the area in Colorado within the 200 foot saturated depth contour. This area is outlined in conducting ink. Stream and river systems east of the state line are traced in conducting ink and connected to a ground line which crosses them from north to south near the east boundary of the analog sheet. A measurement of resistance between the boundary of the pumped area and the ground line gives 600 ohms. Test measurements made on a one inch square and between concentric circles give ρ = 3000 ohms as the resistance of the coating.

It will be convenient to choose $n_2 = 100$ (ft/volt) and to use formula 12-5.

$$Q = n_2 K D \rho I$$

For this area with $K = 0.00128$ (ft/sec) and $D = 200$ (feet) $KD = 0.256$ (ft^2/sec). If 50 feet of saturated depth must remain to supply the well then the permissible drawdown is 200 - 50 = 150 feet. With $n_2 = 100$ feet per volt the 150 foot drawdown would be represented by 1.5 volts applied between the pumped area boundary and the ground line. From Ohm's law and the measured resistance

$$I = \frac{1.5}{600} = 0.0025$$

then

$$Q = (100)\ (0.256)\ (3000)\ (0.0025) = 192\ (ft^3/sec)$$

This would be equivalent to 139,000 acre feet per year. A similar analog constructed for a square area of 25 townships in the Burlington area south of the Arickaree River and handled in the same way yields 44,000 acre feet per year as the recoverable yield of this area. Then the total estimated yield is 139,000 + 44,000 = 183,000 acre feet per year. If this figure is compared with the estimated 430,000 acre feet per year of total infiltration (Colo GW Circular No. 8) then the recoverable yield is about 43 percent of the supply. Estimates based on other considerations indicate that something around one-half of the supply can be recovered with present methods on a permanent basis. The method of operating the analogs accounts for the effects of pumping on the Colorado side of the state line only.

Check of the analog

On a piece of Teledeltos paper two concentric circles are drawn with radii 5 mm and 49 mm respectively. The circles are drawn with conducting ink and the resistance between them is found to be 1100 ohms. Then with KD = 0.256 (ft^2/sec) and one volt impressed between the circles the flow would be

$$Q = \frac{(100)(0.256)(3000)(1)}{1100} = 69.8\ (ft^3/sec)$$

For the prototype the radius of the outer boundary is

$$b_1 = \frac{(49)(1000000)}{304.8} = 160760 \text{ (feet)}$$

A similar computation yields $a_1 = 16404$ feet. The drawdown would be $s_a = n_2(1.0) = 100$ (feet). The flow would be, from the steady state flow formula,

$$Q = \frac{2\pi K D s_a}{\log_e(\frac{b_1}{a_1})} = \frac{(6.2832)(0.256)(100)}{2.28238} = 70.5 \text{ (ft}^3\text{/sec)}$$

This is close to the 69.8 (ft^3/sec) value found from the test analog and it can be concluded that the steady state high plains analog is properly designed.

If it should be desired to make an approximate estimate of the time required for water table readjustments to take place the pumped area could be idealized as being circular with a sink at the center. The flow of the wells in the pumped area must be obtained by depleting the flow of the streams which run eastwardly over the Ogallala formation. A source placed somewhere to the east of the state line could be substituted for the stream depletion if we knew where to put it. A circular area 16.93 miles in radius would have the same area as the 25 townships. The center of gravity of the pumped area lies 14.28 miles west of the state line. Then the idealization would be shown below

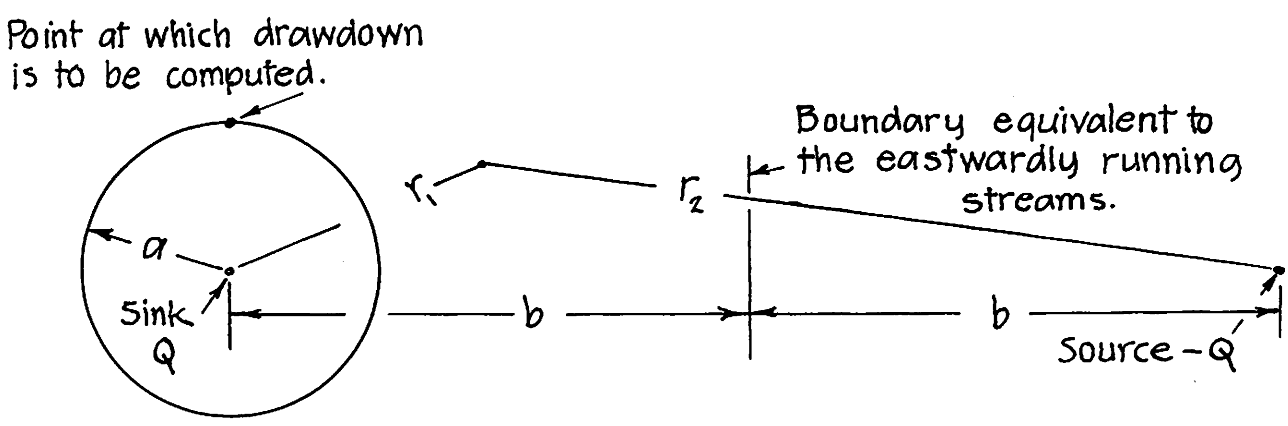


Fig 12-5 Source-sink combination.

The drawdown produced by a source sink combination, as shown, is

$$s = \frac{Q}{2\pi KD} \log_e\left(\frac{r_2}{r_1}\right) \qquad r_1 , r_2 < 0 \tag{12-7}$$

Along a line drawn midway between them, so that $r_1 = r_2$, the drawdown is zero. Suppose it is desired to estimate the drawdown at a point on the rim of the area north of the sink after pumping has continued for 100 years. It is first necessary to find the distance b . With $Q = 192$ (ft^3/sec) $KD = 0.256$ (ft^2/sec)

$$\frac{Q}{2\pi KD} = \frac{192}{(6.2832)(0.256)} = 119.4 \text{ (feet)} \quad \frac{s}{\left(\frac{Q}{2\pi KD}\right)} = \frac{150}{119.4} = 1.257$$

From tables with, approximately

$$\log_e \left(\frac{2b}{a}\right) = 1.257 \qquad \left(\frac{2b}{a}\right) = 3.515$$

$$b = \frac{(3.512)(16.93)}{2} = 29.73 \text{ miles} \qquad 2b = 59.50 \text{ miles}$$

The distance from the source to the point north of the sink is

$$r_2 = \sqrt{16.93^2 + 59.50^2} = 61.86 \text{ miles}$$

or 326,630. feet. 16.93 miles is 89,290. feet. (Note that here it is necessary to revert to consistent notation whereas before the ratio (r_2/r_1) would be the same whether the radii were expressed in miles or feet.) 100 years is 3,153,600,000. seconds. With

$$\sqrt{4\alpha t} = \sqrt{(4)(1.707)(3153.6)(10)^6} = 146700$$

$$\left(\frac{r_1}{\sqrt{4\alpha t}}\right) = \frac{89390}{146700} = 0.609 \qquad \left(\frac{r_2}{\sqrt{4\alpha t}}\right) = \frac{326431}{146700} = 2.226$$

From Table 1 for

$$\left(\frac{r_1}{\sqrt{4\alpha t}}\right) = 0.609 \qquad \int_{0.609}^{\infty} \frac{e^{-u^2} du}{u} = 0.37690$$

$$\left(\frac{r_2}{\sqrt{4\alpha t}}\right) = 2.226 \qquad \int_{2.226}^{\infty} \frac{e^{-u^2} du}{u} = 0.00061$$

$$\left(\frac{Q}{2\pi KD}\right) = \frac{192}{(6.2832)(0.256)} = 119.366$$

Then the drawdown at this time is

$$s = 119.366\ (0.37690 - 0.00061) = 44.92 \text{ feet}$$

Runs made on the transient state analog previously described indicated that something like 8000 years would be required to establish a new regimen in this area. Computations like those above will show that a drawdown of about 130 feet would be attained in 1000 years and that the full 150 feet would be substantially obtained in 8000 years. These results are only true if the extraction rate is one that can be permanently sustained. If the resource is over-encumbered, depletion can come quickly.

Combination methods

Each of the available methods for quantitatively evaluating the flow of ground water has its advantages and its limitations. These methods should be thought of as tools and when some specific task has to be done the best tool for the job should be selected to do it. Sometimes a combination of methods can be used which utilizes their possibilities to the best advantage and permits doing a specific task in a more effective way than would be possible if either were used alone. With a transient type analog, responses to unit inputs can be quickly obtained and if some sort of an operations study is to be made, these data can be fed into a digital computer whose prodigious ability to handle arithmetic can make short work of a job which, without it, would be laborious and time consuming. If a programmable input device were available the operations study could be done on the analog but so far as this writer is aware, no such device is now in existence. With digital computers at hand there seems to be no very good reason to try to develop one. A combination of analytical and digital methods is often very effective. A case has already been described where a simple steady state analog was used to give a boundary location which would permit a transient type analytical computation to be made.

Chapter 12

Problems

(12-1) A paper having a conducting coating has concentric circles of 100 mm and 10 mm diameter drawn on it with conducting ink. A measurement of the electrical resistance between the two circles yields 1040 ohms. Compute the resistivity ρ of the conducting coating.

Answer: 2838 ohms.

(12-2) A transient type analog is to be constructed to represent a 40 mile reach of river valley. The resistors are to be of around 100,000 ohms rating and the capacitors are to be of 1 microfarad size. The analog network is to be mounted on a 4 x 8 foot pegboard panel having holes at one inch centers. There is to be a node point at each half mile spacing, to scale. There are to be 1200 node points. The pegboard is to be mounted on a frame to support it at a convenient height. Estimate the cost of the completed network panel.

Answer: The cost should be in the neighborhood of one dollar per node point (at 1972 prices).

(12-3) What should be the scale of the maps to be mounted on the above panel?

Answer: 1/31680.

(12-4) Suppose that maps were available to a scale of 1/24000. What length of river valley could be accommodated on an 8-foot long pegboard sheet?

Answer: 36 miles.

(12-5) If available maps are reduced to the 1/31680 scale by photographic processes will it then be possible to locate node points at section corners and quarter corners?

Answer: It is not to be expected. The difficulties of mapping a spherical earth onto flat sheets does not permit perfect regularity of section lines representing mile squares.

Notation

The notation used herein has the following significance. Physical dimensions are specified in foot-second units but since consistent units are used everywhere in this text the formulas can be used without change in any consistent unit system. Basic quantities must, of course, be expressed in the chosen units.

a	represents the radius of a well	(ft)
$A_1, A_2 \ldots A_n$	Fourier series coefficients	
b	the radius to an outer boundary	(ft)
c	a constant	
C	capacitance	(farads)
C_1	capacitance in an analog	(farads/ft^2)
d	the vertical distance of a drain above the barrier	(ft)
D	an initial saturated depth	(ft)
D_a =	$(d + \frac{H}{2})$	
e	2.71828 + the base of the natural system of logarithms	(dimensionless)
E	a factor, which, when multiplied by the local saturated depth will give the flow to a unit length of drain, from one side	
E	electrical pressure	(volts)
$E_1(x) = -Ei(-x) = \int_x^\infty \frac{e^{-u}du}{u}$	the exponential integral	
$f = (K\gamma/V)$	the velocity of flow in an aquifer sustained by a natural gradient γ	(ft/sec)
f	a flow defined where used	
F	a flow defined where used	
$G(\frac{\sqrt{4\alpha t}}{a})$	a factor used to estimate the flow of an artesian well (see Table 4)	(dimensionless)

h	a drainable depth	(ft)
h_1	a pressure departure from a hydrostatic state, as used in Chapter 2	(ft)
h_1	a first approximation drainable depth as used in Chapter 6	(ft)
h_2	a saturated depth, as used in Chapter 2	(ft)
h_2	a drainable depth as used with Werner's treatment	
h_3	a drainable depth as used by Hauchild and Kruse	
h_c	a drainable depth midway between drains	(ft)
h_o	a depth over a tile drain acting to cause flow to the drain	(ft)
H_o	an initial saturated depth	
i	a gradient	(dimensionless)
i	an infiltration rate, reckoned as entire water	(ft/sec)
i	current in an analog	(amperes/ft)
I	a total current representing an input to, or output from, an analog	(amperes)
J_o and J_1	Bessel's functions of the zeroth and first orders	
k =	(a/b)	
K_o and K_1	modified Bessel's functions of the zeroth and first orders	
K	the permeability of an aquifer	(ft/sec)
l	an equivalent length	(ft)
L	a distance between drains	(ft)
L	a valley width	(ft)
m	a parameter which appears in Tables 13,14 and 15 (for definitions see Chapters 5 and 11)	
$m = 0.74930$	a number which appears in the development of formula (4-13)	
n	a term number in a Fourier series	
n_1, n_2, n_3 and n_4	numbers which appear in the correlation equations (12-2)	

p	the part of an original drainable volume which remains at the time t (see Table 11)	(dimensionless)
p	a permeability for vertical flow through a slowly permeable bed (see Figure 5-1)	(ft/sec)
p	a pressure measured in feet of water representing a departure from a hydrostatic state (refer to developments based on formula 8-14)	
p	a first derivative. Defined where used	
P	a total production volume	(ft^3)
q_1	a stream depletion caused by a well	(ft^3/sec)
q	a canal seepage rate as used in Chapter 7	(ft^2/sec)
q_2	a return flow rate used in the development of formula 10-2	(ft^2/sec)
Q	a rate of flow. A well flow	(ft^3/sec)
r	a radius	(ft)
$R(\frac{\alpha t}{L^2})$	a factor used to estimate a volume of return flow. (see Table 12)	(dimensionless)
R	a radius drawn from a well to a point where the drawdown is to be computed. It is used where a groundwater drift is present	(ft)
s	a drawdown	(ft)
t	time	(sec)
T	a time required to complete a cycle	(sec)
u	a generalized variable	(dimensionless)
$U_o(\beta_n r)$ =	$J_o(\beta_n a) \; Y_o(\beta_n r) - Y_o(\beta_n a) \; J_o(\beta_n r)$	
v	a generalized variable	(dimensionless)
V	the ratio of the drainable or fillable voids to the gross volume. It is used where a free water table is present	(dimensionless)
V_1	the volume yield of an artesian aquifer per unit of horizontal area per unit of pressure reduction	(dimensionless)
w	a variable of integration	(dimensionless)

W	a quantity of water assumed to be instantaneously removed	(ft^3)
x	a coordinate distance	(ft)
x_1	the distance of a well from a river	(ft)
y	a coordinate distance, defined where used	(ft)
y	a pressure reduction near an artesian well	(ft)
z	a coordinate distance	(ft)
z	a parameter, defined where used	
$\alpha = \frac{KD}{V}$	a constant for a given aquifer which defines the rapidity with which a transient change will take place	(ft^2/sec)
β_n	a root of a Bessel's function	(dimensionless)
γ	a naturally occurring water table gradient	(dimensionless)
η	time in an analog	(seconds)
λ	thickness of a semi-permeable confining bed	(ft)
μ	a viscosity	
ξ	length in an analog	(ft)
π =	3.1415926535+	(dimensionless)
ρ	resistance in an analog	(ohms)
σ =	$\frac{Q}{2\pi KD^2}$ (see formulas 3-2 and 3-3)	(dimensionless)

Notation used in the design of analogs

$C = \frac{n_2 n_3 V}{n_1 n_4}$	Capacitance per unit of area	$(\frac{\text{farads}}{\text{ft}^2})$ or $(\frac{\text{amp sec}}{\text{ft}^2\text{volt}})$
D	Saturated thickness in the prototype	(ft)
E	Electrical pressure in the analog	(volts)
i	Current flow in the analog	(amperes/ft)
K	Permeability in the prototype	(ft/sec)
KD	Transmissivity in the prototype	(ft^2/sec)
$n_1 = \frac{q}{i}$		(ft^3/sec amp)
$n_2 = \frac{s}{E}$		(ft/volt)
$n_3 = \frac{x}{\xi}$		(dimensionless)
$n_4 = \frac{t}{\eta}$		(dimensionless)
q	Flow in the prototype	(ft^2/sec)
s	Drawdown in the prototype	(ft)
t	Time in the prototype	(seconds)
V	Voids ratio in the prototype	(dimensionless)
x	Distance in the prototype	(ft)
η	Time in the analog	(seconds)
ξ	Length in the analog	(feet)
$\rho = \frac{n_1 n_3}{KDn_2}$	Resistance measured across a square area	(ohms) or (volts/amp)

Definition of terms

Aquifer

A water bearing formation.

Aquifer Constant

A number characteristic of an aquifer which denotes the speed with which transient changes will take place in it. In this text it is represented by the symbol α .

Artesian well

A well which taps a confined aquifer and which has a pressure sufficient to support a flowing well.

Bank storage

The water contained in an aquifer hydraulically connected with a stream or lake and capable of supplying water to the stream or lake following a lowering of the free water surface or of storing water flowing from the stream or lake on a rise of the free water surface.

Barrier

An impermeable formation in contact with an aquifer which confines the flow of groundwater to the aquifer.

Boundary conditions

Conditions imposed by boundaries.

Condition of continuity

The requirement that water volumes must be strictly accounted for.

Confined aquifer

An aquifer sandwiched between impermeable formations.

Consistent units

A consistent unit system is defined as being one that permits only one unit of a kind.

When data come to the computer expressed in units other than those of a chosen system the first task is to convert them to the chosen system. Factors for making this conversion for some commonly met units are given under the heading: Conversion factors, equivalents and useful values.

Conversion factors, equivalents and useful values

To convert	to	multiply by
Gallons per minute	cubic feet per second	0.002228
Meinzers unit (permeability)	feet per second	$1.5472(10)^{-6}$
Meinzers unit (transmissivity)	feet squared per second	$1.5472(10)^{-6}$
Acre feet	cubic feet	43560
Cubic feet per second	gallons per minute	448.8
One year (365 days)	seconds	31,536,000
One month (1/12 year)	seconds	2,628,000
One day	seconds	86,400

A township has an area of 23,040 acres or $1003.62(10)^6$ square feet.

A section has an area of 640 acres or $27.8784(10)^6$ square feet.

One cubic foot per second running for one day will deliver 1.983471 acre feet. One cubic foot per second running for 365 days will deliver 723.9669 acre feet.

Note that while a year of 365 days is assumed for computation purposes herein a year is 365.2422 days (Smithsonian Physical Tables). This is 31,556,930 seconds. A cubic foot per second running for one year will deliver 724.447 acre feet.

Darcy's law

A law discovered by Henry Philibert Gaspard Darcy (1803-1858). His experiments showed that the velocity of flow through porous media is proportional to the first power of the gradient.

Drainable depth

A depth of groundwater above the level of a system of drains or above the level of a river, in an aquifer hydraulically connected thereto.

Drawdown

The amount a water table has sunk from an initial stable configuration.

Drift

A term used to describe a flow of groundwater under the action of a naturally existing regional gradient.

Dupuit-Forchheimer idealization

An idealization whose use was pioneered by Arsene Jules Emile Juvenal Dupuit (1804-1866) and Philipp Forchheimer (1852-1933). Under this idealization the gradient of the water table is assumed to be effective throughout the saturated thickness of the aquifer. When the water table gradient is small compared to unity the postulated conditions are substantially realized.

Entire water

A term used to describe water which occupies volume to the exclusion of everything else. Water flowing in a canal or a river is Entire water. Interstitial water occupies only the interstices between grains in an aquifer.

Exponential Integral

A tabulated function of the variable x of the form

$$\int_x^\infty \frac{e^u du}{u} \qquad \text{or} \qquad \int_x^\infty \frac{e^{-u} du}{u}$$

Gradient

A slope of the water table tending to cause the flow of groundwater.

Image

A hypothetical well, source or sink used as a mathematical device to satisfy a boundary condition.

Impermeable

Not permeable.

Infiltration

Water moving into the ground from a surface supply such as precipitation or irrigation. Infiltration rates are reckoned on the basis that the water is entire water.

Initial conditions

The conditions that prevail at the time of initiation of a transient.

Interstitial water

Water which occu ies the interstices between grains in a permeable bed. (see Entire water)

Linearization

Many of the differential equations representing physical relationships are inherently nonlinear in form. Such relationships are generally difficult to handle and it is often desirable to replace them with approximations which have a more tractable linear form. This may be done in specific cases, by neglect of small quantities, by replacing a curve with its tangent in a range of interest or by other means. The process is called linearization.

Line source

A source uniformly distributed along a line.

Local convergence

A convergence of flow of groundwater as to a drain tile.

Meinzer unit

A unit of permeability used in the older publications of the U.S. Geological Survey. It is defined as the flow of water in gallons per day through a cross sectional area of one square foot under a gradient of one foot head of water per foot of length, measured in the direction of flow, at a temperature of 60°F.

Parallel drains

Drains of the type installed for drainage of agricultural land. They can be of the form of open ditches or buried tile lines.

Permeability

A term used to describe the ability of water or oil to move through a porous formation under the action of a gradient. The facility with which a fluid will move through a formation is greater for some than for others. For a given bed the permeability is expressed by a constant K representing the flow through unit area in unit time under the influence of a unit gradient. The flow is expressed in terms of Entire water.

Probability integral

A tabulated function of the variable x of the form

$$\frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du$$

Production

The total volume of well flow counted from the time of initiation of flow.

Radial flow

Flow converging toward a center.

Root

A value of an argument which will cause some given function of the argument to pass through zero.

Steady state cases

These are groundwater conditions which do not change with time.

Stream depletion

A depletion of stream flow caused by the operation of wells installed in an aquifer hydraulically connected to the stream.

Transient cases

These are groundwater conditions which are changing with time.

Voids ratio

The ratio of the volume of drainable or fillable voids to the gross volume.

Water table

The upper limit of the completely saturated material in an aquifer.

Linear interpolation

The tables presented herein are not extensive enough to insure that linear interpolation will give intermediate values with an accuracy comparable to the tabular values. Take, for example, these figures from Table 14 for m = 0.005.

x	Tabular value
0.40	0.70454
0.41	0.68358
0.42	0.66330

Interpolation between the extreme values yields 0.68392 for the intermediate value. This differs from the tabular value by 0.68392 - 0.68358 = 0.0034. If the true curve is approximated, in the interval of interest, with a parabola, then it can be inferred that the error in the middle of a tabular increment would be one-fourth of the error at the middle of two tabular increments, as above. On this basis there could be an error of eight units in the last place, due to linear interpolation, in certain parts of this table. Most of the uses to which these tables may be put will not require precisions which will exhaust the capabilities of the tables. However, if some usage would require such precision then the reader should be aware that linear interpolation may be inadequate.

Index to tables

		Page
(1)	$\int_x^\infty \frac{e^{-u^2}du}{u}$	212
(2)	$s/(\frac{Q}{2\pi KD})$	221
(3)	(y/y_0)	224
(4)	$G(\frac{\sqrt{4\alpha t}}{a})$	256
(5)	$H(\frac{\sqrt{4\alpha t}}{a})$	265
(6)	Maintained outer boundary	274
(7)	Impermeable outer boundary	277
(8)	$\frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2}du$	282
(9)	$\int_x^\infty \frac{e^{-u^2}du}{u^2}$	289
(10)	(h_c/H)	297
(11)	p	30(
(12)	R	304
(13)	$\int_x^\infty e^{-u^2 - \frac{m^2}{u^2}} du$	307
(14)	$\int_x^\infty \frac{e^{-u^2 - \frac{m^2}{u^2}} du}{u}$	338
(15)	$\int_x^\infty \frac{e^{-u^2 - \frac{m^2}{u^2}} du}{u^2}$	372

Table 1

Values of:

$$\int_x^\infty \frac{e^{-u^2}du}{u}$$

This table was computed by Simpson's rule integration. This integral can be converted to the exponential integral by the substitution $u^2 = v$. The relation is:

$$\int_x^\infty \frac{e^{-u^2}du}{u} = \frac{1}{2}\int_{x^2}^\infty \frac{e^{-v}dv}{v} = \frac{1}{2}E_1(x^2) .$$

The following comparison can be made:

x	$\frac{1}{2}\int_{x^2}^\infty \frac{e^{-v}dv}{v}$	Tabular Value
0.00001	11.224317	11.22432
0.0001	8.921732	8.92173
0.001	6.619147	6.61915
0.01	4.316612	4.31661
0.1	2.018964	2.01896
1.0	0.109691	0.10969
3.0	0.000006	0.00001

x	0	1	2	3	4	5	6	7	8	9
.0000		11.22432	10.53117	10.12571	9.83802	9.61488	9.43256	9.27841	9.14488	9.02709
.0001	8.92173	8.82642	8.73941	8.65937	8.58526	8.51627	8.45173	8.39110	8.33395	8.27988
.0002	8.22859	8.17980	8.13328	8.08882	8.04626	8.00544	7.96622	7.92848	7.89211	7.85702
.0003	7.82312	7.79033	7.75858	7.72781	7.69796	7.66897	7.64080	7.61340	7.58673	7.56076
.0004	7.53544	7.51075	7.48665	7.46312	7.44013	7.41765	7.39568	7.37417	7.35312	7.33250
.0005	7.31229	7.29249	7.27307	7.25403	7.23533	7.21698	7.19897	7.18127	7.16387	7.14678
.0006	7.12997	7.11344	7.09718	7.08118	7.06543	7.04993	7.03466	7.01962	7.00481	6.99021
.0007	6.97582	6.96164	6.94765	6.93386	6.92025	6.90683	6.89358	6.88051	6.86761	6.85487
.0008	6.84229	6.82987	6.81760	6.80548	6.79350	6.78167	6.76997	6.75841	6.74698	6.73568
.0009	6.72451	6.71346	6.70253	6.69172	6.68102	6.67044	6.65997	6.64961	6.63935	6.62920
.0010	6.61915	6.60920	6.59934	6.58959	6.57993	6.57036	6.56088	6.55149	6.54219	6.53297
.0011	6.52384	6.51479	6.50582	6.49693	6.48812	6.47939	6.47073	6.46214	6.45363	6.44519
.0012	6.43683	6.42853	6.42030	6.41213	6.40404	6.39600	6.38804	6.38013	6.37229	6.36451
.0013	6.35678	6.34912	6.34152	6.33397	6.32648	6.31904	6.31166	6.30434	6.29706	6.28984
.0014	6.28268	6.27556	6.26849	6.26147	6.25450	6.24758	6.24071	6.23388	6.22711	6.22037
.0015	6.21368	6.20704	6.20044	6.19388	6.18736	6.18089	6.17446	6.16807	6.16172	6.15541
.0016	6.14914	6.14291	6.13672	6.13057	6.12445	6.11837	6.11233	6.10632	6.10035	6.09442
.0017	6.08852	6.08265	6.07682	6.07103	6.06526	6.05953	6.05383	6.04817	6.04253	6.03693
.0018	6.03136	6.02582	6.02031	6.01483	6.00938	6.00396	5.99857	5.99321	5.98788	5.98257
.0019	5.97729	5.97204	5.96682	5.96163	5.95646	5.95132	5.94620	5.94111	5.93605	5.93101
.0020	5.92600	5.92101	5.91605	5.91111	5.90620	5.90131	5.89644	5.89160	5.88678	5.88198
.0021	5.87721	5.87246	5.86773	5.86303	5.85834	5.85368	5.84904	5.84442	5.83982	5.83525
.0022	5.83069	5.82615	5.82164	5.81715	5.81267	5.80822	5.80378	5.79937	5.79497	5.79060
.0023	5.78624	5.78190	5.77758	5.77328	5.76900	5.76473	5.76049	5.75626	5.75205	5.74785
.0024	5.74368	5.73952	5.73538	5.73126	5.72715	5.72306	5.71899	5.71493	5.71089	5.70686
.0025	5.70286	5.69886	5.69489	5.69093	5.68698	5.68305	5.67914	5.67524	5.67136	5.66749
.0026	5.66364	5.65980	5.65597	5.65216	5.64837	5.64459	5.64082	5.63707	5.63333	5.62961
.0027	5.62590	5.62220	5.61852	5.61485	5.61119	5.60755	5.60392	5.60030	5.59670	5.59311
.0028	5.58953	5.58596	5.58241	5.57887	5.57534	5.57183	5.56833	5.56484	5.56136	5.55789
.0029	5.55444	5.55099	5.54756	5.54414	5.54074	5.53734	5.53396	5.53059	5.52722	5.52387
.0030	5.52053	5.51721	5.51389	5.51058	5.50729	5.50401	5.50073	5.49747	5.49422	5.49098
.0031	5.48775	5.48452	5.48131	5.47811	5.47492	5.47174	5.46858	5.46542	5.46227	5.45913
.0032	5.45600	5.45288	5.44977	5.44667	5.44358	5.44050	5.43743	5.43436	5.43131	5.42827
.0033	5.42523	5.42220	5.41919	5.41618	5.41318	5.41019	5.40721	5.40424	5.40128	5.39832
.0034	5.39538	5.39244	5.38951	5.38659	5.38368	5.38078	5.37788	5.37500	5.37212	5.36925
.0035	5.36639	5.36354	5.36069	5.35786	5.35503	5.35221	5.34939	5.34659	5.34379	5.34100
.0036	5.33822	5.33545	5.33268	5.32992	5.32717	5.32443	5.32169	5.31896	5.31624	5.31353
.0037	5.31082	5.30812	5.30543	5.30275	5.30007	5.29740	5.29474	5.29208	5.28943	5.28679
.0038	5.28415	5.28153	5.27890	5.27629	5.27368	5.27108	5.26849	5.26590	5.26332	5.26075
.0039	5.25818	5.25562	5.25306	5.25052	5.24797	5.24544	5.24291	5.24039	5.23787	5.23536
.0040	5.23286	5.23036	5.22787	5.22539	5.22291	5.22044	5.21797	5.21551	5.21306	5.21061
.0041	5.20817	5.20573	5.20330	5.20088	5.19846	5.19605	5.19364	5.19124	5.18884	5.18646
.0042	5.18407	5.18169	5.17932	5.17695	5.17459	5.17224	5.16989	5.16754	5.16520	5.16287
.0043	5.16054	5.15822	5.15590	5.15359	5.15128	5.14898	5.14668	5.14439	5.14211	5.13983
.0044	5.13755	5.13528	5.13302	5.13076	5.12850	5.12625	5.12401	5.12177	5.11953	5.11730
.0045	5.11508	5.11286	5.11065	5.10844	5.10623	5.10403	5.10184	5.09964	5.09746	5.09528
.0046	5.09310	5.09093	5.08876	5.08660	5.08444	5.08229	5.08014	5.07800	5.07586	5.07373
.0047	5.07160	5.06947	5.06735	5.06523	5.06312	5.06101	5.05891	5.05681	5.05472	5.05263
.0048	5.05054	5.04846	5.04638	5.04431	5.04224	5.04018	5.03812	5.03607	5.03401	5.03197
0049	5.02992	5.02789	5.02585	5.02382	5.02179	5.01977	5.01775	5.01574	5.01373	5.01172
0050	5.00972	5.00772	5.00573	5.00374	5.00175	4.99977	4.99779	4.99582	4.99385	4.99188

x	0	1	2	3	4	5	6	7	8	9
.0051	4.98992	4.98796	4.98601	4.98405	4.98211	4.98016	4.97822	4.97629	4.97436	4.97243
.0052	4.97050	4.96858	4.96666	4.96475	4.96284	4.96093	4.95903	4.95713	4.95524	4.95334
.0053	4.95145	4.94957	4.94769	4.94581	4.94394	4.94207	4.94020	4.93833	4.93647	4.93462
.0054	4.93276	4.93091	4.92907	4.92722	4.92538	4.92355	4.92171	4.91988	4.91806	4.91623
.0055	4.91441	4.91260	4.91078	4.90897	4.90717	4.90536	4.90356	4.90177	4.89997	4.89818
.0056	4.89640	4.89461	4.89283	4.89105	4.88928	4.88751	4.88574	4.88397	4.88221	4.88045
.0057	4.87870	4.87694	4.87519	4.87345	4.87170	4.86996	4.86823	4.86649	4.86476	4.86303
.0058	4.86131	4.85958	4.85786	4.85615	4.85443	4.85272	4.85101	4.84931	4.84761	4.84591
.0059	4.84421	4.84252	4.84083	4.83914	4.83746	4.83577	4.83409	4.83242	4.83074	4.82907
.0060	4.82741	4.82574	4.82408	4.82242	4.82076	4.81911	4.81746	4.81581	4.81416	4.81252
.0061	4.81088	4.80924	4.80760	4.80597	4.80434	4.80271	4.80109	4.79947	4.79785	4.79623
.0062	4.79462	4.79301	4.79140	4.78979	4.78819	4.78659	4.78499	4.78339	4.78180	4.78021
.0063	4.77862	4.77703	4.77545	4.77387	4.77229	4.77071	4.76914	4.76757	4.76600	4.76443
.0064	4.76287	4.76131	4.75975	4.75819	4.75664	4.75509	4.75354	4.75199	4.75045	4.74891
.0065	4.74737	4.74583	4.74429	4.74276	4.74123	4.73970	4.73818	4.73666	4.73513	4.73362
.0066	4.73210	4.73059	4.72907	4.72756	4.72606	4.72455	4.72305	4.72155	4.72005	4.71856
.0067	4.71706	4.71557	4.71408	4.71259	4.71111	4.70963	4.70815	4.70667	4.70519	4.70372
.0068	4.70225	4.70078	4.69931	4.69785	4.69638	4.69492	4.69346	4.69201	4.69055	4.68910
.0069	4.68765	4.68620	4.68476	4.68331	4.68187	4.68043	4.67899	4.67756	4.67612	4.67469
.0070	4.67326	4.67183	4.67041	4.66899	4.66756	4.66614	4.66473	4.66331	4.66190	4.66049
.0071	4.65908	4.65767	4.65626	4.65486	4.65346	4.65206	4.65066	4.64927	4.64787	4.64648
.0072	4.64509	4.64370	4.64232	4.64093	4.63955	4.63817	4.63679	4.63542	4.63404	4.63267
.0073	4.63130	4.62993	4.62856	4.62720	4.62584	4.62447	4.62311	4.62176	4.62040	4.61905
.0074	4.61769	4.61634	4.61500	4.61365	4.61230	4.61096	4.60962	4.60828	4.60694	4.60561
.0075	4.60427	4.60294	4.60161	4.60028	4.59895	4.59763	4.59630	4.59498	4.59366	4.59234
.0076	4.59103	4.58971	4.58840	4.58709	4.58578	4.58447	4.58316	4.58186	4.58056	4.57926
.0077	4.57796	4.57666	4.57536	4.57407	4.57278	4.57148	4.57020	4.56891	4.56762	4.56634
.0078	4.56505	4.56377	4.56249	4.56122	4.55994	4.55866	4.55739	4.55612	4.55485	4.55358
.0079	4.55232	4.55105	4.54979	4.54853	4.54727	4.54601	4.54475	4.54349	4.54224	4.54099
.0080	4.53974	4.53849	4.53724	4.53599	4.53475	4.53351	4.53227	4.53103	4.52979	4.52855
.0081	4.52732	4.52608	4.52485	4.52362	4.52239	4.52116	4.51994	4.51871	4.51749	4.51627
.0082	4.51505	4.51383	4.51261	4.51140	4.51018	4.50897	4.50776	4.50655	4.50534	4.50413
.0083	4.50293	4.50172	4.50052	4.49932	4.49812	4.49692	4.49572	4.49453	4.49333	4.49214
.0084	4.49095	4.48976	4.48857	4.48739	4.48620	4.48502	4.48383	4.48265	4.48147	4.48029
.0085	4.47912	4.47794	4.47677	4.47559	4.47442	4.47325	4.47208	4.47092	4.46975	4.46859
.0086	4.46742	4.46626	4.46510	4.46394	4.46278	4.46163	4.46047	4.45932	4.45816	4.45701
.0087	4.45586	4.45471	4.45357	4.45242	4.45128	4.45013	4.44899	4.44785	4.44671	4.44557
.0088	4.44443	4.44330	4.44216	4.44103	4.43990	4.43877	4.43764	4.43651	4.43539	4.43426
.0089	4.43314	4.43201	4.43089	4.42977	4.42865	4.42753	4.42642	4.42530	4.42419	4.42307
.0090	4.42196	4.42085	4.41974	4.41864	4.41753	4.41642	4.41532	4.41422	4.41311	4.41201
.0091	4.41091	4.40982	4.40872	4.40762	4.40653	4.40544	4.40434	4.40325	4.40216	4.40107
.0092	4.39999	4.39890	4.39781	4.39673	4.39565	4.39457	4.39349	4.39241	4.39133	4.39025
.0093	4.38918	4.38810	4.38703	4.38596	4.38488	4.38381	4.38275	4.38168	4.38061	4.37955
.0094	4.37848	4.37742	4.37636	4.37530	4.37424	4.37318	4.37212	4.37106	4.37001	4.36895
.0095	4.36790	4.36685	4.36580	4.36475	4.36370	4.36265	4.36161	4.36056	4.35952	4.35847
.0096	4.35743	4.35639	4.35535	4.35431	4.35327	4.35224	4.35120	4.35017	4.34913	4.34810
.0097	4.34707	4.34604	4.34501	4.34398	4.34295	4.34193	4.34090	4.33988	4.33886	4.33783
.0098	4.33681	4.33579	4.33477	4.33376	4.33274	4.33172	4.33071	4.32970	4.32868	4.32767
.0099	4.32666	4.32565	4.32464	4.32364	4.32263	4.32162	4.32062	4.31962	4.31861	4.31761
.0100	4.31661									

x	0	1	2	3	4	5	6	7	8	9
.01	4.31661	4.22131	4.13431	4.05428	3.98019	3.91121	3.84669	3.78608	3.72894	3.67489
.02	3.62362	3.57485	3.52835	3.48392	3.44138	3.40058	3.36139	3.32368	3.28733	3.25227
.03	3.21840	3.18564	3.15392	3.12318	3.09336	3.06441	3.03628	3.00891	2.98228	2.95635
.04	2.93107	2.90642	2.88236	2.85887	2.83593	2.81350	2.79156	2.77010	2.74910	2.72853
.05	2.70837	2.68862	2.66925	2.65026	2.63162	2.61333	2.59536	2.57772	2.56039	2.54335
.06	2.52660	2.51013	2.49393	2.47800	2.46231	2.44687	2.43167	2.41670	2.40195	2.38742
.07	2.37310	2.35898	2.34507	2.33135	2.31782	2.30447	2.29130	2.27830	2.26548	2.25282
.08	2.24032	2.22797	2.21578	2.20375	2.19185	2.18010	2.16849	2.15702	2.14568	2.13446
.09	2.12338	2.11242	2.10158	2.09086	2.08026	2.06977	2.05940	2.04913	2.03897	2.02892
.10	2.01896	2.00911	1.99936	1.98971	1.98015	1.97068	1.96131	1.95203	1.94283	1.93372
.11	1.92470	1.91576	1.90690	1.89812	1.88943	1.88081	1.87226	1.86379	1.85540	1.84708
.12	1.83883	1.83065	1.82254	1.81450	1.80652	1.79862	1.79077	1.78299	1.77528	1.76762
.13	1.76003	1.75249	1.74502	1.73760	1.73025	1.72294	1.71570	1.70851	1.70137	1.69429
.14	1.68726	1.68028	1.67335	1.66648	1.65965	1.65287	1.64614	1.63946	1.63283	1.62624
.15	1.61970	1.61320	1.60675	1.60035	1.59398	1.58766	1.58139	1.57515	1.56896	1.56280
.16	1.55669	1.55062	1.54459	1.53859	1.53264	1.52672	1.52084	1.51500	1.50920	1.50343
.17	1.49770	1.49200	1.48634	1.48071	1.47512	1.46956	1.46403	1.45854	1.45308	1.44765
.18	1.44226	1.43690	1.43157	1.42627	1.42100	1.41576	1.41055	1.40537	1.40022	1.39510
.19	1.39001	1.38495	1.37992	1.37491	1.36993	1.36498	1.36006	1.35516	1.35029	1.34545
.20	1.34063	1.33584	1.33108	1.32634	1.32162	1.31693	1.31227	1.30763	1.30301	1.29842
.21	1.29385	1.28930	1.28478	1.28029	1.27581	1.27136	1.26693	1.26252	1.25814	1.25377
.22	1.24943	1.24511	1.24081	1.23653	1.23228	1.22804	1.22383	1.21963	1.21546	1.21131
.23	1.20717	1.20306	1.19896	1.19489	1.19083	1.18680	1.18278	1.17878	1.17480	1.17084
.24	1.16690	1.16297	1.15907	1.15518	1.15131	1.14746	1.14362	1.13980	1.13600	1.13222
.25	1.12845	1.12471	1.12097	1.11726	1.11356	1.10988	1.10621	1.10256	1.09892	1.09531
.26	1.09170	1.08812	1.08454	1.08099	1.07745	1.07392	1.07041	1.06692	1.06344	1.05997
.27	1.05652	1.05309	1.04966	1.04626	1.04286	1.03949	1.03612	1.03277	1.02943	1.02611
.28	1.02280	1.01951	1.01623	1.01296	1.00970	1.00646	1.00323	1.00002	.99681	.99363
.29	.99045	.98728	.98413	.98100	.97787	.97476	.97165	.96857	.96549	.96242
.30	.95937	.95633	.95330	.95029	.94728	.94429	.94131	.93834	.93538	.93243
.31	.92949	.92657	.92366	.92075	.91786	.91498	.91211	.90926	.90641	.90357
.32	.90074	.89793	.89512	.89233	.88955	.88677	.88401	.88126	.87851	.87578
.33	.87306	.87034	.86764	.86495	.86227	.85959	.85693	.85428	.85163	.84900
.34	.84637	.84376	.84115	.83856	.83597	.83339	.83082	.82826	.82571	.82317
.35	.82064	.81812	.81560	.81310	.81060	.80811	.80563	.80316	.80070	.79825
.36	.79580	.79337	.79094	.78852	.78611	.78371	.78131	.77893	.77655	.77418
.37	.77182	.76947	.76712	.76479	.76246	.76014	.75782	.75552	.75322	.75093
.38	.74865	.74638	.74411	.74185	.73960	.73736	.73512	.73289	.73067	.72846
.39	.72625	.72406	.72186	.71968	.71750	.71533	.71317	.71102	.70887	.70673
.40	.70459	.70247	.70035	.69823	.69613	.69403	.69194	.68985	.68777	.68570
.41	.68364	.68158	.67953	.67748	.67544	.67341	.67139	.66937	.66736	.66535
.42	.66335	.66136	.65937	.65739	.65542	.65345	.65149	.64954	.64759	.64564
.43	.64371	.64178	.63985	.63794	.63603	.63412	.63222	.63033	.62844	.62656
.44	.62468	.62281	.62095	.61909	.61724	.61539	.61355	.61172	.60989	.60806
.45	.60625	.60443	.60263	.60083	.59903	.59724	.59546	.59368	.59191	.59014
.46	.58838	.58662	.58487	.58312	.58138	.57965	.57792	.57619	.57447	.57276
.47	.57105	.56935	.56765	.56596	.56427	.56259	.56091	.55924	.55757	.55591
.48	.55425	.55260	.55095	.54931	.54767	.54604	.54441	.54279	.54117	.53956
.49	.53795	.53635	.53475	.53316	.53157	.52999	.52841	.52684	.52527	.52370
.50	.52214	.52059	.51904	.51749	.51595	.51441	.51288	.51135	.50983	.50831

x	0	1	2	3	4	5	6	7	8	9
.51	.50680	.50529	.50378	.50228	.50079	.49929	.49781	.49632	.49485	.49337
.52	.49190	.49044	.48898	.48752	.48607	.48462	.48317	.48174	.48030	.47887
.53	.47744	.47602	.47460	.47319	.47178	.47037	.46897	.46757	.46618	.46479
.54	.46340	.46202	.46064	.45927	.45790	.45654	.45517	.45382	.45246	.45111
.55	.44977	.44843	.44709	.44576	.44443	.44310	.44178	.44046	.43914	.43783
.56	.43653	.43522	.43392	.43263	.43134	.43005	.42876	.42748	.42621	.42493
.57	.42366	.42240	.42114	.41988	.41862	.41737	.41612	.41488	.41364	.41240
.58	.41117	.40994	.40871	.40749	.40627	.40505	.40384	.40263	.40143	.40023
.59	.39903	.39783	.39664	.39545	.39427	.39309	.39191	.39073	.38956	.38840
.60	.38723	.38607	.38491	.38376	.38261	.38146	.38031	.37917	.37803	.37690
.61	.37577	.37464	.37351	.37239	.37127	.37016	.36905	.36794	.36683	.36573
.62	.36463	.36353	.36244	.36135	.36026	.35918	.35810	.35702	.35594	.35487
.63	.35380	.35274	.35167	.35061	.34956	.34850	.34745	.34640	.34536	.34432
.64	.34328	.34224	.34121	.34018	.33915	.33813	.33711	.33609	.33507	.33406
.65	.33305	.33204	.33104	.33004	.32904	.32805	.32705	.32606	.32508	.32409
.66	.32311	.32213	.32116	.32018	.31921	.31824	.31728	.31632	.31536	.31440
.67	.31345	.31249	.31155	.31060	.30966	.30872	.30778	.30684	.30591	.30498
.68	.30405	.30313	.30221	.30129	.30037	.29945	.29854	.29763	.29673	.29582
.69	.29492	.29402	.29313	.29223	.29134	.29045	.28957	.28868	.28780	.28692
.70	.28604	.28517	.28430	.28343	.28256	.28170	.28084	.27998	.27912	.27827
.71	.27742	.27657	.27572	.27487	.27403	.27319	.27235	.27152	.27069	.26985
.72	.26903	.26820	.26738	.26656	.26574	.26492	.26411	.26329	.26248	.26168
.73	.26087	.26007	.25927	.25847	.25767	.25688	.25609	.25530	.25451	.25373
.74	.25295	.25216	.25139	.25061	.24984	.24906	.24830	.24753	.24676	.24600
.75	.24524	.24448	.24372	.24297	.24222	.24147	.24072	.23997	.23923	.23849
.76	.23775	.23701	.23628	.23554	.23481	.23408	.23336	.23263	.23191	.23119
.77	.23047	.22975	.22904	.22832	.22761	.22690	.22620	.22549	.22479	.22409
.78	.22339	.22269	.22200	.22131	.22062	.21993	.21924	.21855	.21787	.21719
.79	.21651	.21583	.21516	.21449	.21381	.21314	.21248	.21181	.21115	.21049
.80	.20983	.20917	.20851	.20786	.20720	.20655	.20590	.20526	.20461	.20397
.81	.20333	.20269	.20205	.20141	.20078	.20015	.19952	.19889	.19826	.19764
.82	.19701	.19639	.19577	.19515	.19454	.19392	.19331	.19270	.19209	.19148
.83	.19088	.19027	.18967	.18907	.18847	.18787	.18728	.18668	.18609	.18550
.84	.18491	.18432	.18374	.18316	.18257	.18199	.18141	.18084	.18026	.17969
.85	.17912	.17855	.17798	.17741	.17684	.17628	.17572	.17516	.17460	.17404
.86	.17349	.17293	.17238	.17183	.17128	.17073	.17018	.16964	.16910	.16855
.87	.16801	.16748	.16694	.16640	.16587	.16534	.16481	.16428	.16375	.16322
.88	.16270	.16218	.16166	.16114	.16062	.16010	.15958	.15907	.15856	.15805
.89	.15754	.15703	.15652	.15602	.15551	.15501	.15451	.15401	.15351	.15302
.90	.15252	.15203	.15154	.15104	.15056	.15007	.14958	.14910	.14861	.14813
.91	.14765	.14717	.14669	.14622	.14574	.14527	.14479	.14432	.14385	.14339
.92	.14292	.14245	.14199	.14153	.14106	.14060	.14015	.13969	.13923	.13878
.93	.13832	.13787	.13742	.13697	.13652	.13608	.13563	.13519	.13474	.13430
.94	.13386	.13342	.13298	.13255	.13211	.13168	.13125	.13082	.13039	.12996
.95	.12953	.12910	.12868	.12825	.12783	.12741	.12699	.12657	.12615	.12574
.96	.12532	.12491	.12450	.12408	.12367	.12327	.12286	.12245	.12205	.12164
.97	.12124	.12084	.12044	.12004	.11964	.11924	.11885	.11845	.11806	.11767
.98	.11727	.11688	.11650	.11611	.11572	.11534	.11495	.11457	.11419	.11381
.99	.11343	.11305	.11267	.11229	.11192	.11155	.11117	.11080	.11043	.11006
1.00	.10969	.10932	.10896	.10859	.10823	.10787	.10750	.10714	.10678	.10643

$$\int_x^\infty \frac{e^{-u^2}du}{u}$$

x	0	1	2	3	4	5	6	7	8	9
1.01	.10607	.10571	.10536	.10500	.10465	.10430	.10395	.10360	.10325	.10290
1.02	.10255	.10221	.10186	.10152	.10117	.10083	.10049	.10015	.09981	.09948
1.03	.09914	.09880	.09847	.09814	.09780	.09747	.09714	.09681	.09648	.09616
1.04	.09583	.09550	.09518	.09486	.09453	.09421	.09389	.09357	.09325	.09293
1.05	.09262	.09230	.09199	.09167	.09136	.09105	.09074	.09043	.09012	.08981
1.06	.08950	.08920	.08889	.08859	.08828	.08798	.08768	.08738	.08708	.08678
1.07	.08648	.08619	.08589	.08559	.08530	.08501	.08471	.08442	.08413	.08384
1.08	.08355	.08327	.08298	.08269	.08241	.08212	.08184	.08156	.08128	.08099
1.09	.08071	.08043	.08016	.07988	.07960	.07933	.07905	.07878	.07850	.07823
1.10	.07796	.07769	.07742	.07715	.07688	.07662	.07635	.07608	.07582	.07555
1.11	.07529	.07503	.07477	.07451	.07425	.07399	.07373	.07347	.07322	.07296
1.12	.07270	.07245	.07220	.07194	.07169	.07144	.07119	.07094	.07069	.07044
1.13	.07020	.06995	.06971	.06946	.06922	.06897	.06873	.06849	.06825	.06801
1.14	.06777	.06753	.06729	.06705	.06682	.06658	.06635	.06611	.06588	.06565
1.15	.06541	.06518	.06495	.06472	.06449	.06426	.06404	.06381	.06358	.06336
1.16	.06313	.06291	.06268	.06246	.06224	.06202	.06180	.06158	.06136	.06114
1.17	.06092	.06071	.06049	.06027	.06006	.05984	.05963	.05942	.05921	.05899
1.18	.05878	.05857	.05836	.05815	.05795	.05774	.05753	.05733	.05712	.05692
1.19	.05671	.05651	.05630	.05610	.05590	.05570	.05550	.05530	.05510	.05490
1.20	.05470	.05451	.05431	.05412	.05392	.05373	.05353	.05334	.05315	.05295
1.21	.05276	.05257	.05238	.05219	.05200	.05181	.05163	.05144	.05125	.05107
1.22	.05088	.05070	.05051	.05033	.05015	.04996	.04978	.04960	.04942	.04924
1.23	.04906	.04888	.04870	.04853	.04835	.04817	.04800	.04782	.04765	.04747
1.24	.04730	.04713	.04695	.04678	.04661	.04644	.04627	.04610	.04593	.04576
1.25	.04559	.04543	.04526	.04509	.04493	.04476	.04460	.04443	.04427	.04411
1.26	.04394	.04378	.04362	.04346	.04330	.04314	.04298	.04282	.04266	.04251
1.27	.04235	.04219	.04204	.04188	.04173	.04157	.04142	.04126	.04111	.04096
1.28	.04081	.04065	.04050	.04035	.04020	.04005	.03990	.03976	.03961	.03946
1.29	.03931	.03917	.03902	.03887	.03873	.03858	.03844	.03830	.03815	.03801
1.30	.03787	.03773	.03759	.03745	.03730	.03717	.03703	.03689	.03675	.03661
1.31	.03647	.03634	.03620	.03606	.03593	.03579	.03566	.03552	.03539	.03526
1.32	.03512	.03499	.03486	.03473	.03460	.03447	.03434	.03421	.03408	.03395
1.33	.03382	.03369	.03356	.03344	.03331	.03318	.03306	.03293	.03281	.03268
1.34	.03256	.03244	.03231	.03219	.03207	.03195	.03182	.03170	.03158	.03146
1.35	.03134	.03122	.03110	.03098	.03087	.03075	.03063	.03051	.03040	.03028
1.36	.03016	.03005	.02993	.02982	.02971	.02959	.02948	.02936	.02925	.02914
1.37	.02903	.02892	.02880	.02869	.02858	.02847	.02836	.02826	.02815	.02804
1.38	.02793	.02782	.02771	.02761	.02750	.02739	.02729	.02718	.02708	.02697
1.39	.02687	.02677	.02666	.02656	.02646	.02635	.02625	.02615	.02605	.02595
1.40	.02585	.02574	.02564	.02554	.02545	.02535	.02525	.02515	.02505	.02495
1.41	.02486	.02476	.02466	.02457	.02447	.02438	.02428	.02418	.02409	.02400
1.42	.02390	.02381	.02372	.02362	.02353	.02344	.02335	.02325	.02316	.02307
1.43	.02298	.02289	.02280	.02271	.02262	.02253	.02244	.02236	.02227	.02218
1.44	.02209	.02200	.02192	.02183	.02175	.02166	.02157	.02149	.02140	.02132
1.45	.02123	.02115	.02107	.02098	.02090	.02082	.02073	.02065	.02057	.02049
1.46	.02041	.02033	.02025	.02016	.02008	.02000	.01992	.01985	.01977	.01969
1.47	.01961	.01953	.01945	.01937	.01930	.01922	.01914	.01907	.01899	.01891
1.48	.01884	.01876	.01869	.01861	.01854	.01846	.01839	.01832	.01824	.01817
1.49	.01810	.01802	.01795	.01788	.01781	.01774	.01766	.01759	.01752	.01745
1.50	.01738	.01731	.01724	.01717	.01710	.01703	.01696	.01690	.01683	.01676

x	0	1	2	3	4	5	6	7	8	9
1.51	.01669	.01662	.01656	.01649	.01642	.01636	.01629	.01622	.01616	.01609
1.52	.01603	.01596	.01590	.01583	.01577	.01570	.01564	.01558	.01551	.01545
1.53	.01539	.01532	.01526	.01520	.01514	.01507	.01501	.01495	.01489	.01483
1.54	.01477	.01471	.01465	.01459	.01453	.01447	.01441	.01435	.01429	.01423
1.55	.01417	.01411	.01406	.01400	.01394	.01388	.01383	.01377	.01371	.01366
1.56	.01360	.01354	.01349	.01343	.01338	.01332	.01327	.01321	.01316	.01310
1.57	.01305	.01299	.01294	.01289	.01283	.01278	.01273	.01267	.01262	.01257
1.58	.01252	.01246	.01241	.01236	.01231	.01226	.01221	.01216	.01211	.01206
1.59	.01201	.01195	.01191	.01186	.01181	.01176	.01171	.01166	.01161	.01156
1.60	.01151	.01146	.01142	.01137	.01132	.01127	.01123	.01118	.01113	.01109
1.61	.01104	.01099	.01095	.01090	.01085	.01081	.01076	.01072	.01067	.01063
1.62	.01058	.01054	.01049	.01045	.01040	.01036	.01032	.01027	.01023	.01019
1.63	.01014	.01010	.01006	.01002	.00997	.00993	.00989	.00985	.00980	.00976
1.64	.00972	.00968	.00964	.00960	.00956	.00952	.00948	.00944	.00940	.00936
1.65	.00932	.00928	.00924	.00920	.00916	.00912	.00908	.00904	.00900	.00896
1.66	.00892	.00889	.00885	.00881	.00877	.00874	.00870	.00866	.00862	.00859
1.67	.00855	.00851	.00848	.00844	.00840	.00837	.00833	.00829	.00826	.00822
1.68	.00819	.00815	.00812	.00808	.00805	.00801	.00798	.00794	.00791	.00788
1.69	.00784	.00781	.00777	.00774	.00771	.00767	.00764	.00761	.00757	.00754
1.70	.00751	.00747	.00744	.00741	.00738	.00735	.00731	.00728	.00725	.00722
1.71	.00719	.00716	.00712	.00709	.00706	.00703	.00700	.00697	.00694	.00691
1.72	.00688	.00685	.00682	.00679	.00676	.00673	.00670	.00667	.00664	.00661
1.73	.00658	.00655	.00653	.00650	.00647	.00644	.00641	.00638	.00636	.00633
1.74	.00630	.00627	.00624	.00622	.00619	.00616	.00613	.00611	.00608	.00605
1.75	.00603	.00600	.00597	.00595	.00592	.00589	.00587	.00584	.00582	.00579
1.76	.00576	.00574	.00571	.00569	.00566	.00564	.00561	.00559	.00556	.00554
1.77	.00551	.00549	.00546	.00544	.00542	.00539	.00537	.00534	.00532	.00530
1.78	.00527	.00525	.00522	.00520	.00518	.00516	.00513	.00511	.00509	.00506
1.79	.00504	.00502	.00500	.00497	.00495	.00493	.00491	.00488	.00486	.00484
1.80	.00482	.00480	.00477	.00475	.00473	.00471	.00469	.00467	.00465	.00463
1.81	.00461	.00458	.00456	.00454	.00452	.00450	.00448	.00446	.00444	.00442
1.82	.00440	.00438	.00436	.00434	.00432	.00430	.00428	.00426	.00424	.00422
1.83	.00420	.00419	.00417	.00415	.00413	.00411	.00409	.00407	.00405	.00404
1.84	.00402	.00400	.00398	.00396	.00394	.00393	.00391	.00389	.00387	.00385
1.85	.00384	.00382	.00380	.00378	.00377	.00375	.00373	.00371	.00370	.00368
1.86	.00366	.00365	.00363	.00361	.00360	.00358	.00356	.00355	.00353	.00351
1.87	.00350	.00348	.00347	.00345	.00343	.00342	.00340	.00339	.00337	.00336
1.88	.00334	.00332	.00331	.00329	.00328	.00326	.00325	.00323	.00322	.00320
1.89	.00319	.00317	.00316	.00314	.00313	.00311	.00310	.00309	.00307	.00306
1.90	.00304	.00303	.00301	.00300	.00299	.00297	.00296	.00294	.00293	.00292
1.91	.00290	.00289	.00288	.00286	.00285	.00284	.00282	.00281	.00280	.00278
1.92	.00277	.00276	.00274	.00273	.00272	.00271	.00269	.00268	.00267	.00265
1.93	.00264	.00263	.00262	.00260	.00259	.00258	.00257	.00256	.00254	.00253
1.94	.00252	.00251	.00250	.00248	.00247	.00246	.00245	.00244	.00243	.00241
1.95	.00240	.00239	.00238	.00237	.00236	.00235	.00234	.00232	.00231	.00230
1.96	.00229	.00228	.00227	.00226	.00225	.00224	.00223	.00222	.00220	.00219
1.97	.00218	.00217	.00216	.00215	.00214	.00213	.00212	.00211	.00210	.00209
1.98	.00208	.00207	.00206	.00205	.00204	.00203	.00202	.00201	.00200	.00199
1.99	.00198	.00197	.00196	.00195	.00195	.00194	.00193	.00192	.00191	.00190
2.00	.00189	.00188	.00187	.00186	.00185	.00184	.00184	.00183	.00182	.00181

6.

x	0	1	2	3	4	5	6	7	8	9
2.01	.00180	.00179	.00178	.00177	.00177	.00176	.00175	.00174	.00173	.00172
2.02	.00171	.00171	.00170	.00169	.00168	.00167	.00167	.00166	.00165	.00164
2.03	.00163	.00162	.00162	.00161	.00160	.00159	.00159	.00158	.00157	.00156
2.04	.00155	.00155	.00154	.00153	.00152	.00152	.00151	.00150	.00149	.00149
2.05	.00148	.00147	.00147	.00146	.00145	.00144	.00144	.00143	.00142	.00142
2.06	.00141	.00140	.00139	.00139	.00138	.00137	.00137	.00136	.00135	.00135
2.07	.00134	.00133	.00133	.00132	.00131	.00131	.00130	.00129	.00129	.00128
2.08	.00128	.00127	.00126	.00126	.00125	.00124	.00124	.00123	.00123	.00122
2.09	.00121	.00121	.00120	.00120	.00119	.00118	.00118	.00117	.00117	.00116
2.10	.00115	.00115	.00114	.00114	.00113	.00113	.00112	.00111	.00111	.00110
2.11	.00110	.00109	.00109	.00108	.00108	.00107	.00106	.00106	.00105	.00105
2.12	.00104	.00104	.00103	.00103	.00102	.00102	.00101	.00101	.00100	.00100
2.13	.00099	.00099	.00098	.00098	.00097	.00097	.00096	.00096	.00095	.00095
2.14	.00094	.00094	.00093	.00093	.00092	.00092	.00091	.00091	.00091	.00090
2.15	.00090	.00089	.00089	.00088	.00088	.00087	.00087	.00086	.00086	.00086
2.16	.00085	.00085	.00084	.00084	.00083	.00083	.00083	.00082	.00082	.00081
2.17	.00081	.00080	.00080	.00080	.00079	.00079	.00078	.00078	.00078	.00077
2.18	.00077	.00076	.00076	.00076	.00075	.00075	.00075	.00074	.00074	.00073
2.19	.00073	.00073	.00072	.00072	.00071	.00071	.00071	.00070	.00070	.00070
2.20	.00069	.00069	.00069	.00068	.00068	.00068	.00067	.00067	.00066	.00066
2.21	.00066	.00065	.00065	.00065	.00064	.00064	.00064	.00063	.00063	.00063
2.22	.00062	.00062	.00062	.00061	.00061	.00061	.00061	.00060	.00060	.00060
2.23	.00059	.00059	.00059	.00058	.00058	.00058	.00057	.00057	.00057	.00057
2.24	.00056	.00056	.00056	.00055	.00055	.00055	.00054	.00054	.00054	.00054
2.25	.00053	.00053	.00053	.00053	.00052	.00052	.00052	.00051	.00051	.00051
2.26	.00051	.00050	.00050	.00050	.00050	.00049	.00049	.00049	.00049	.00048
2.27	.00048	.00048	.00047	.00047	.00047	.00047	.00046	.00046	.00046	.00046
2.28	.00046	.00045	.00045	.00045	.00045	.00044	.00044	.00044	.00044	.00043
2.29	.00043	.00043	.00043	.00042	.00042	.00042	.00042	.00042	.00041	.00041
2.30	.00041	.00041	.00040	.00040	.00040	.00040	.00040	.00039	.00039	.00039
2.31	.00039	.00039	.00038	.00038	.00038	.00038	.00038	.00037	.00037	.00037
2.32	.00037	.00037	.00036	.00036	.00036	.00036	.00036	.00035	.00035	.00035
2.33	.00035	.00035	.00034	.00034	.00034	.00034	.00034	.00034	.00033	.00033
2.34	.00033	.00033	.00033	.00032	.00032	.00032	.00032	.00032	.00032	.00031
2.35	.00031	.00031	.00031	.00031	.00031	.00030	.00030	.00030	.00030	.00030
2.36	.00030	.00029	.00029	.00029	.00029	.00029	.00029	.00028	.00028	.00028
2.37	.00028	.00028	.00028	.00028	.00027	.00027	.00027	.00027	.00027	.00027
2.38	.00026	.00026	.00026	.00026	.00026	.00026	.00026	.00025	.00025	.00025
2.39	.00025	.00025	.00025	.00025	.00025	.00024	.00024	.00024	.00024	.00024
2.40	.00024	.00024	.00023	.00023	.00023	.00023	.00023	.00023	.00023	.00023
2.41	.00022	.00022	.00022	.00022	.00022	.00022	.00022	.00022	.00021	.00021
2.42	.00021	.00021	.00021	.00021	.00021	.00021	.00021	.00020	.00020	.00020
2.43	.00020	.00020	.00020	.00020	.00020	.00020	.00019	.00019	.00019	.00019
2.44	.00019	.00019	.00019	.00019	.00019	.00018	.00018	.00018	.00018	.00018
2.45	.00018	.00018	.00018	.00018	.00018	.00017	.00017	.00017	.00017	.00017
2.46	.00017	.00017	.00017	.00017	.00017	.00016	.00016	.00016	.00016	.00016
2.47	.00016	.00016	.00016	.00016	.00016	.00016	.00016	.00015	.00015	.00015
2.48	.00015	.00015	.00015	.00015	.00015	.00015	.00015	.00015	.00014	.00014
2.49	.00014	.00014	.00014	.00014	.00014	.00014	.00014	.00014	.00014	.00014
2.50	.00014	.00013	.00013	.00013	.00013	.00013	.00013	.00013	.00013	.00013

Table 1.

$$\int_x^{\infty} \frac{e^{-u^2}\,du}{u}$$

x	0	1	2	3	4	5	6	7	8	9
2.51	.00013	.00013	.00013	.00013	.00012	.00012	.00012	.00012	.00012	.00012
2.52	.00012	.00012	.00012	.00012	.00012	.00012	.00012	.00012	.00012	.00011
2.53	.00011	.00011	.00011	.00011	.00011	.00011	.00011	.00011	.00011	.00011
2.54	.00011	.00011	.00011	.00011	.00011	.00010	.00010	.00010	.00010	.00010
2.55	.00010	.00010	.00010	.00010	.00010	.00010	.00010	.00010	.00010	.00010
2.56	.00010	.00010	.00009	.00009	.00009	.00009	.00009	.00009	.00009	.00009
2.57	.00009	.00009	.00009	.00009	.00009	.00009	.00009	.00009	.00009	.00009
2.58	.00009	.00008	.00008	.00008	.00008	.00008	.00008	.00008	.00008	.00008
2.59	.00008	.00008	.00008	.00008	.00008	.00008	.00008	.00008	.00008	.00008
2.60	.00008	.00008	.00007	.00007	.00007	.00007	.00007	.00007	.00007	.00007
2.61	.00007	.00007	.00007	.00007	.00007	.00007	.00007	.00007	.00007	.00007
2.62	.00007	.00007	.00007	.00007	.00007	.00007	.00006	.00006	.00006	.00006
2.63	.00006	.00006	.00006	.00006	.00006	.00006	.00006	.00006	.00006	.00006
2.64	.00006	.00006	.00006	.00006	.00006	.00006	.00006	.00006	.00006	.00006
2.65	.00006	.00006	.00006	.00006	.00005	.00005	.00005	.00005	.00005	.00005
2.66	.00005	.00005	.00005	.00005	.00005	.00005	.00005	.00005	.00005	.00005
2.67	.00005	.00005	.00005	.00005	.00005	.00005	.00005	.00005	.00005	.00005
2.68	.00005	.00005	.00005	.00005	.00005	.00005	.00005	.00005	.00004	.00004
2.69	.00004	.00004	.00004	.00004	.00004	.00004	.00004	.00004	.00004	.00004
2.70	.00004	.00004	.00004	.00004	.00004	.00004	.00004	.00004	.00004	.00004
2.71	.00004	.00004	.00004	.00004	.00004	.00004	.00004	.00004	.00004	.00004
2.72	.00004	.00004	.00004	.00004	.00004	.00004	.00004	.00004	.00004	.00003
2.73	.00003	.00003	.00003	.00003	.00003	.00003	.00003	.00003	.00003	.00003
2.74	.00003	.00003	.00003	.00003	.00003	.00003	.00003	.00003	.00003	.00003
2.75	.00003	.00003	.00003	.00003	.00003	.00003	.00003	.00003	.00003	.00003
2.76	.00003	.00003	.00003	.00003	.00003	.00003	.00003	.00003	.00003	.00003
2.77	.00003	.00003	.00003	.00003	.00003	.00003	.00003	.00003	.00003	.00003
2.78	.00003	.00003	.00003	.00003	.00002	.00002	.00002	.00002	.00002	.00002
2.79	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002
2.80	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002
2.81	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002
2.82	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002
2.83	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002
2.84	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002
2.85	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002
2.86	.00002	.00002	.00002	.00002	.00002	.00001	.00001	.00001	.00001	.00001
2.87	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.88	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.89	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.90	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.91	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.92	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.93	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.94	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.95	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.96	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.97	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.98	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.99	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
3.00	.00001									

Table 2

Values of:

$$s = \frac{Q}{2\pi KD}\frac{1}{\sigma}\left[1 - \sqrt{1 - 2\sigma\int_{\left(\frac{r}{\sqrt{4\alpha t}}\right)}^{\infty}\frac{e^{-u^2}du}{u}}\right]$$

where

$$\sigma = \frac{Q}{2\pi KD^2}$$

The following check values have been computed using values of

$$\int_{\left(\frac{r}{\sqrt{4\alpha t}}\right)}^{\infty}\frac{e^{-u^2}du}{u}$$

taken from Table 1.

$\left(\frac{r}{\sqrt{4\alpha t}}\right)$	$\int_{\left(\frac{r}{\sqrt{4\alpha t}}\right)}^{\infty}\frac{e^{-u^2}du}{u}$	σ	Computed Value of $\frac{s}{\left(\frac{Q}{2\pi KD}\right)}$	Tabular Value of $\frac{s}{\left(\frac{Q}{2\pi KD}\right)}$
0.00001	11.22432	0.01	11.93676	11.93675
0.00010	8.92173	0.02	9.90228	9.90228
0.00100	6.61915	0.03	7.45217	7.45217
0.00500	5.00972	0.04	5.64764	5.64764
0.01000	4.31661	0.05	4.92235	4.92235
0.05000	2.70837	0.06	2.97365	2.97365
0.10000	2.01896	0.07	2.18625	2.18625
0.50000	0.52214	0.08	0.53353	0.53353
0.70000	0.28604	0.09	0.28982	0.28982
1.00000	0.10969	0.10	0.11030	0.11030

The above formula and its derivation are described in the paper by Glover and Bittinger. Trans. ASCE, Vol. 126, Part III, 1961. Paper No. 3142.

This development relates to the drawdowns around a well pumping at the constant rate Q from an aquifer of unlimited extent where the drawdowns s are not small compared to the original saturated depth D.

$$\frac{s}{\left(\frac{Q}{2\pi KD}\right)} = \frac{1}{\sigma}\left[1 - \sqrt{1 - 2\sigma \int_{\left(\frac{r}{\sqrt{4\alpha t}}\right)}^{\infty} \frac{e^{-u^2}}{u}\,du}\right]$$

SIGMA σ

$\left(\frac{r}{\sqrt{4\alpha t}}\right)$	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10
.00001	11.93675	12.88439	14.28541	17.01350						
.00002	11.15313	11.96209	13.10877	15.07823						
.00003	10.69793	11.43279	12.45119	14.10437						
.00004	10.37637	11.06162	11.99691	13.46315	17.45460					
.00005	10.12773	10.77613	11.65110	12.98934	16.07510					
.00006	9.92510	10.54440	11.37260	12.61565	15.23579					
.00007	9.75412	10.34954	11.13985	12.30828	14.62750					
.00008	9.60628	10.18151	10.94019	12.04793	14.15150					
.00009	9.47607	10.03388	10.76555	11.82254	13.76171					
.00010	9.35976	9.90228	10.61046	11.62415	13.43260					
.00020	8.59823	9.04708	9.61543	10.38594	11.58236	14.79808				
.00030	8.15570	8.55500	9.05228	9.70804	10.66859	12.54270				
.00040	7.84300	8.20938	8.66050	9.24474	10.07113	11.50949				
.00050	7.60118	7.94325	8.36085	8.89456	9.63138	10.83275				
.00060	7.40407	7.72705	8.11866	8.61399	9.28547	10.33326	13.67898			
.00070	7.23775	7.54511	7.91570	8.38047	9.00150	9.93983	12.10113			
.00080	7.09391	7.38814	7.74118	8.18080	8.76130	9.61674	11.35525			
.00090	6.96722	7.25016	7.58823	8.00663	8.55362	9.34359	10.82843			
.00100	6.85404	7.12710	7.45217	7.85233	8.37098	9.10760	10.41749			
.00200	6.11283	6.32621	6.57433	6.86991	7.23442	7.70874	8.38933	9.65395		
.00300	5.68196	5.86445	6.07392	6.31917	6.61424	6.98369	7.47748	8.22956	10.22893	
.00400	5.37745	5.53975	5.72439	5.93808	6.19111	6.50059	6.89847	7.45733	8.43324	
.00500	5.14192	5.28951	5.45629	5.64764	5.87162	6.14112	6.47888	6.93161	7.62835	
.00600	4.94991	5.08609	5.23913	5.41353	5.61585	5.85629	6.15210	6.53637	7.08854	8.14208
.00700	4.78788	4.91482	5.05684	5.21776	5.40310	5.62120	5.88572	6.22159	6.68318	7.44368
.00800	4.64775	4.76698	4.89987	5.04973	5.22128	5.42153	5.66163	5.96115	6.35993	6.96599
.00900	4.52431	4.63698	4.76213	4.90269	5.06275	5.24830	5.46870	5.73975	6.09205	6.59989
.01000	4.41403	4.52101	4.63948	4.77206	4.92235	5.09555	5.29962	5.54768	5.86401	6.30301
.02000	3.69176	3.76540	3.84542	3.93298	4.02955	4.13708	4.25826	4.39694	4.55886	4.75332
.03000	3.27193	3.32924	3.39087	3.45748	3.52991	3.60919	3.69669	3.79425	3.90439	4.03075
.04000	2.97533	3.02242	3.07269	3.12658	3.18461	3.24744	3.31590	3.39103	3.47423	3.56738
.05000	2.74608	2.78599	2.82837	2.87352	2.92180	2.97365	3.02963	3.09040	3.15682	3.23003
.06000	2.55935	2.59388	2.63039	2.66908	2.71024	2.75416	2.80125	2.85194	2.90684	2.96665
.07000	2.40195	2.43226	2.46418	2.49789	2.53357	2.57147	2.61186	2.65508	2.70152	2.75169
.08000	2.26599	2.29289	2.32113	2.35085	2.38219	2.41533	2.45049	2.48790	2.52787	2.57075
.09000	2.14642	2.17049	2.19570	2.22214	2.24993	2.27923	2.31017	2.34296	2.37781	2.41499
.10000	2.03977	2.06146	2.08412	2.10782	2.13267	2.15877	2.18625	2.21526	2.24596	2.27856
.20000	1.34974	1.35910	1.36873	1.37865	1.38885	1.39938	1.41024	1.42145	1.43304	1.44504
.30000	.96402	.96876	.97359	.97852	.98356	.98870	.99395	.99932	1.00481	1.01042
.40000	.70709	.70963	.71220	.71481	.71746	.72015	.72288	.72566	.72847	.73134
.50000	.52351	.52490	.52630	.52771	.52914	.53059	.53205	.53353	.53502	.53653
.60000	.38798	.38874	.38951	.39028	.39105	.39184	.39263	.39342	.39422	.39503
.70000	.28645	.28687	.28728	.28770	.28812	.28854	.28897	.28939	.28982	.29026
.80000	.21005	.21027	.21049	.21071	.21094	.21116	.21139	.21162	.21185	.21207
.90000	.15264	.15275	.15287	.15299	.15311	.15323	.15334	.15346	.15358	.15370
1.00000	.10975	.10981	.10987	.10993	.10999	.11006	.11012	.11018	.11024	.11030

Table 3

Values of:

$$\frac{y}{y_0} = \left[1 - \sum_{n=1}^{n=\infty} A_n U_0 (\beta_n r) e^{-\frac{k^2(\beta_n b)^2}{4}\left(\frac{4\alpha t}{a^2}\right)} \right]$$

Where:

y represents the pressure drawdown produced by a flowing artesian well at the radius r and the time t . The drawdown is expressed in terms of an equivalent depth of water.

y_0 the value of y at the well where r = a .

The condition that $U_0 = 0$ when r = a is met by the Bessel function:

$$U_0(\beta_n r) = J_0(\beta_n a)\, Y_0(\beta_n r) - Y_0(\beta_n a)\, J_0(\beta_n r)$$

If:

$$k = (a/b)$$

$$A_n = \frac{\frac{2k}{(\beta_n b)} U_0'(\beta_n a)}{[U_0(\beta_n b)]^2 - k^2[U_0'(\beta_n a)]^2}$$

The β_n values are roots of

$$U_0'(\beta_n b) = 0 .$$

The development applies where a flowing artesian well, of radius a , taps a confined aquifer having an impermeable outer boundary at the finite radius b . When the well is shut down the pressure at the well rises toward the terminal value y_0 , expressed in terms of the depth of water which would exert the same pressure. The case where the outer boundary b is infinitely remote is approached by using a series of finite cases with successively greater outer radii. The finite case can represent the infinite case until the disturbance produced by the flow of the well reaches the outer boundary b . The sequence used here is as shown below:

(b/a)	Computation Range of (r/a)
25	3 to 6
50	6 to 12
100	12 to 25
200	25 to 50
400	50 to 100
800	100 to 200
1600	200 to 400
3200	400 to 800
6400	800 to 1600
12800	1600 to 3200
25600	3200 to 10000

This sequence provides an essentially exact solution until suitable approximation formulas can be used.

The β_n roots were computed by use of a root seeking digital computer program. When these roots were obtained the A_n values were computed and the computation of the tables was performed by means of the digital computer. An independent check is obtained by comparison with values for (b/a) = 100 given in Bulletin 3, Part VII of the Boulder Canyon Project Final Reports where an analogous heat flow case is treated. A comparison of independently computed and machine computed A_n values for (b/a) = 100 is shown below

n	Independently Computed A_n Values*	Digital Computer A_n Values**
1	0.399352	0.399352
2	0.092067	0.092067
3	0.069189	0.069189
4	0.059264	0.059264
5	0.053526	0.053522

n	Independently Computed A_n Values*	Digital Computer A_n Values**
6	0.049725	0.049725
7	0.047009	0.047009
8	0.044962	0.044962

* The first five values are as given in Bulletin 3. The values for $n = 6,7$ and 8 were computed by this writer.
**Last figure rounded.

Checks of the (y/y_0) ratio can be obtained by comparison with the approximation formulas of Chapter 4 . For example, for $(b/a) = 25600$ $(r/a) = 2000$ $(\sqrt{4\alpha t}/a) = 5000$, the tabular value of (y/y_0) is 0.08969. The approximation formula gives:

$$\frac{y}{y_0} \doteq \frac{\int_{\left(\frac{r}{\sqrt{4\alpha t}}\right)}^{\infty} \frac{e^{-u^2}du}{u}}{\int_{\left(\frac{a}{\sqrt{4\alpha t}}\right)}^{\infty} \frac{e^{-u^2}du}{u}} \doteq \frac{0.70459}{8.22859} \doteq 0.085627$$

where

$$\left(\frac{a}{\sqrt{4\alpha t}}\right) = \frac{1}{5000} = 0.0002 \qquad \left(\frac{r}{\sqrt{4\alpha t}}\right) = \frac{2000}{5000} = 0.4$$

A further check can be made by computing some (y/y_0) values using the independently derived A_n values. The following results are obtained:

For $(b/a) = 100$ $\left(\frac{r}{\sqrt{4\alpha t}}\right) = 25$

$(\frac{r}{a})$	Computed Values of (y/y_0)	Tabular Values of (y/y_0)
2.0	0.77655	0.77656
5.0	0.48248	0.48249
10.0	0.26768	0.26768
20.0	0.08802	0.08803
50.0	0.00103	0.00103
100.0	0.00000	0.00000

Table 3. (y/y_0)

(r/a)

(b/a)	$\left(\frac{\sqrt{4\kappa t}}{a}\right)$	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
25.	3.0	1.00000	.47392	.21228	.08370	.02817	.00795	.00186	.00036	.00006	.00000
	4.0	1.00000	.54433	.29964	.15698	.07640	.03410	.01382	.00506	.00167	.00050
	5.0	1.00000	.59017	.36253	.21926	.12785	.07107	.03740	.01855	.00863	.00376
	6.0	1.00000	.62265	.40935	.26961	.17451	.10991	.06695	.03928	.02213	.01195

(b/a)	$\left(\frac{\sqrt{4\kappa t}}{a}\right)$	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
50.	6.0	1.00000	.62266	.40935	.26961	.17451	.10991	.06695	.03929	.02213	.01195
	7.0	1.00000	.64706	.44552	.31037	.21484	.14637	.09774	.06365	.04033	.02482
	8.0	1.00000	.66620	.47438	.34384	.24934	.17935	.12733	.08891	.06092	.04089
	9.0	1.00000	.68169	.49800	.37178	.27892	.20859	.15475	.11353	.08219	.05862
	10.0	1.00000	.69454	.51776	.39545	.30447	.23446	.17975	.13679	.10312	.07689
	11.0	1.00000	.70542	.53458	.41580	.32674	.25740	.20241	.15842	.12317	.09499
	12.0	1.00000	.71478	.54911	.43351	.34630	.27784	.22291	.17837	.14208	.11250

(b/a)	$\left(\frac{\sqrt{4\kappa t}}{a}\right)$	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
100.	12.0	1.00000	.71478	.54911	.43351	.34631	.27784	.22291	.17837	.14208	.11239
	13.0	1.00000	.72294	.56182	.44909	.36365	.29613	.24151	.19673	.15977	.12917
	14.0	1.00000	.73014	.57306	.46292	.37914	.31260	.25841	.21361	.17627	.14500
	15.0	1.00000	.73654	.58309	.47530	.39308	.32751	.27382	.22915	.19161	.15990
	16.0	1.00000	.74230	.59210	.48646	.40570	.34108	.28794	.24348	.20589	.17390
	17.0	1.00000	.74749	.60026	.49659	.41718	.35348	.30091	.25674	.21919	.18703
	18.0	1.00000	.75223	.60770	.50583	.42769	.36487	.31287	.26903	.23159	.19936
	19.0	1.00000	.75656	.61451	.51431	.43736	.37537	.32395	.28045	.24317	.21094
	20.0	1.00000	.76054	.62078	.52213	.44628	.38510	.33423	.29109	.25400	.22183
	21.0	1.00000	.76422	.62657	.52937	.45456	.39413	.34380	.30103	.26417	.23208
	22.0	1.00000	.76763	.63195	.53609	.46226	.40255	.35275	.31035	.27371	.24174
	23.0	1.00000	.77081	.63696	.54236	.46944	.41042	.36113	.31909	.28270	.25087
	24.0	1.00000	.77378	.64164	.54822	.47617	.41780	.36900	.32732	.29118	.25951
	25.0	1.00000	.77656	.64603	.55371	.48249	.42474	.37641	.33508	.29919	.26768

1

Table 3. (y/y_o)

(b/a) 200.

$\left(\frac{\sqrt{4\alpha t}}{a}\right)$	$\left(\frac{r}{a}\right)$ 1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
25.0	1.00000	.77656	.64603	.55371	.48249	.42474	.37641	.33508	.29919	.26768
26.0	1.00000	.77917	.65016	.55889	.48843	.43127	.38340	.34241	.30678	.27544
27.0	1.00000	.78164	.65405	.56376	.49404	.43744	.39000	.34935	.31397	.28281
28.0	1.00000	.78396	.65772	.56837	.49934	.44329	.39626	.35594	.32080	.28982
29.0	1.00000	.78616	.66119	.57273	.50437	.44882	.40221	.36220	.32730	.29650
30.0	1.00000	.78825	.66449	.57687	.50914	.45409	.40785	.36815	.33349	.30288
31.0	1.00000	.79023	.66762	.58080	.51368	.45909	.41323	.37383	.33940	.30896
32.0	1.00000	.79211	.67060	.58455	.51800	.46386	.41836	.37924	.34505	.31479
33.0	1.00000	.79391	.67344	.58812	.52212	.46842	.42327	.38442	.35045	.32036
34.0	1.00000	.79563	.67616	.59153	.52606	.47277	.42795	.38938	.35562	.32571
35.0	1.00000	.79727	.67876	.59479	.52983	.47694	.43244	.39413	.36058	.33084
36.0	1.00000	.79884	.68124	.59792	.53344	.48094	.43675	.39869	.36535	.33577
37.0	1.00000	.80035	.68362	.60092	.53690	.48477	.44088	.40306	.36993	.34051
38.0	1.00000	.80180	.68591	.60379	.54023	.48845	.44485	.40727	.37433	.34508
39.0	1.00000	.80318	.68811	.60656	.54343	.49199	.44867	.41132	.37857	.34948
40.0	1.00000	.80452	.69023	.60922	.54651	.49541	.45235	.41523	.38266	.35372
41.0	1.00000	.80581	.69227	.61179	.54948	.49869	.45590	.41899	.38660	.35781
42.0	1.00000	.80705	.69423	.61426	.55234	.50186	.45932	.42262	.39041	.36177
43.0	1.00000	.80825	.69613	.61665	.55510	.50492	.46263	.42613	.39409	.36559
44.0	1.00000	.80941	.69796	.61895	.55776	.50788	.46582	.42953	.39765	.36929
45.0	1.00000	.81052	.69973	.62118	.56034	.51074	.46891	.43281	.40110	.37287
46.0	1.00000	.81160	.70144	.62334	.56284	.51351	.47190	.43599	.40443	.37634
47.0	1.00000	.81265	.70309	.62542	.56525	.51619	.47480	.43907	.40767	.37971
48.0	1.00000	.81367	.70470	.62745	.56760	.51878	.47761	.44205	.41080	.38297
49.0	1.00000	.81465	.70626	.62941	.56987	.52130	.48034	.44495	.41385	.38614
50.0	1.00000	.81560	.70777	.63131	.57207	.52375	.48298	.44776	.41680	.38922

(b/a) 3. 400.

$\left(\frac{\sqrt{4\alpha t}}{a}\right)$ \ $\left(\frac{r}{a}\right)$	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
50.0	1.00000	.81560	.70777	.63131	.57207	.52375	.48298	.44776	.41680	.38922
51.0	1.00000	.81653	.70923	.63316	.57421	.52612	.48555	.45050	.41968	.39221
52.0	1.00000	.81743	.71066	.63495	.57629	.52843	.48805	.45315	.42247	.39512
53.0	1.00000	.81830	.71204	.63669	.57831	.53067	.49047	.45573	.42518	.39795
54.0	1.00000	.81915	.71339	.63839	.58027	.53285	.49283	.45825	.42783	.40071
55.0	1.00000	.81998	.71470	.64004	.58219	.53498	.49513	.46070	.43040	.40339
56.0	1.00000	.82079	.71597	.64165	.58405	.53704	.49737	.46308	.43291	.40600
57.0	1.00000	.82157	.71722	.64321	.58586	.53906	.49955	.46540	.43535	.40855
58.0	1.00000	.82233	.71843	.64474	.58763	.54102	.50168	.46766	.43773	.41104
59.0	1.00000	.82308	.71961	.64623	.58935	.54294	.50375	.46987	.44006	.41346
60.0	1.00000	.82381	.72076	.64768	.59104	.54481	.50578	.47203	.44233	.41583
61.0	1.00000	.82452	.72188	.64909	.59268	.54663	.50775	.47413	.44454	.41814
62.0	1.00000	.82521	.72298	.65048	.59428	.54841	.50968	.47619	.44671	.42040
63.0	1.00000	.82589	.72405	.65183	.59584	.55015	.51156	.47819	.44882	.42261
64.0	1.00000	.82655	.72510	.65315	.59738	.55185	.51341	.48016	.45089	.42476
65.0	1.00000	.82719	.72612	.65444	.59887	.55351	.51521	.48208	.45291	.42687
66.0	1.00000	.82782	.72712	.65570	.60033	.55513	.51697	.48395	.45488	.42894
67.0	1.00000	.82844	.72810	.65693	.60176	.55672	.51869	.48579	.45682	.43096
68.0	1.00000	.82905	.72906	.65814	.60316	.55828	.52037	.48758	.45871	.43293
69.0	1.00000	.82964	.73000	.65932	.60453	.55980	.52202	.48934	.46056	.43487
70.0	1.00000	.83022	.73091	.66048	.60587	.56129	.52364	.49107	.46238	.43677
71.0	1.00000	.83078	.73181	.66161	.60719	.56275	.52522	.49276	.46416	.43863
72.0	1.00000	.83134	.73269	.66272	.60848	.56419	.52678	.49441	.46590	.44045
73.0	1.00000	.83188	.73356	.66381	.60974	.56559	.52830	.49603	.46761	.44223
74.0	1.00000	.83242	.73440	.66488	.61098	.56696	.52979	.49762	.46929	.44399
75.0	1.00000	.83294	.73523	.66592	.61219	.56831	.53125	.49918	.47093	.44570
76.0	1.00000	.83346	.73605	.66695	.61338	.56964	.53268	.50071	.47254	.44739
77.0	1.00000	.83396	.73684	.66796	.61455	.57093	.53409	.50221	.47413	.44904
78.0	1.00000	.83446	.73763	.66895	.61569	.57221	.53547	.50369	.47568	.45067
79.0	1.00000	.83494	.73840	.66992	.61682	.57346	.53683	.50513	.47721	.45226
80.0	1.00000	.83542	.73915	.67087	.61792	.57469	.53816	.50655	.47871	.45383
81.0	1.00000	.83589	.73990	.67180	.61901	.57590	.53947	.50795	.48018	.45537
82.0	1.00000	.83635	.74062	.67272	.62008	.57708	.54076	.50932	.48163	.45688
83.0	1.00000	.83680	.74134	.67363	.62112	.57825	.54202	.51067	.48305	.45837
84.0	1.00000	.83724	.74204	.67451	.62215	.57939	.54326	.51200	.48444	.45983
85.0	1.00000	.83768	.74274	.67539	.62316	.58052	.54448	.51330	.48582	.46127
86.0	1.00000	.83811	.74342	.67625	.62416	.58162	.54569	.51458	.48717	.46268
87.0	1.00000	.83853	.74409	.67709	.62514	.58271	.54687	.51584	.48850	.46407
88.0	1.00000	.83895	.74474	.67792	.62610	.58379	.54803	.51708	.48981	.46544
89.0	1.00000	.83936	.74539	.67874	.62705	.58484	.54917	.51830	.49109	.46678
90.0	1.00000	.83976	.74603	.67954	.62798	.58588	.55030	.51950	.49236	.46811
91.0	1.00000	.84015	.74666	.68033	.62890	.58690	.55140	.52068	.49361	.46941
92.0	1.00000	.84054	.74727	.68111	.62980	.58790	.55249	.52184	.49483	.47070
93.0	1.00000	.84093	.74788	.68188	.63069	.58889	.55357	.52299	.49604	.47196
94.0	1.00000	.84131	.74848	.68263	.63157	.58986	.55462	.52412	.49723	.47320
95.0	1.00000	.84168	.74907	.68337	.63243	.59082	.55566	.52523	.49840	.47443
96.0	1.00000	.84204	.74965	.68411	.63328	.59177	.55669	.52632	.49956	.47564
97.0	1.00000	.84240	.75022	.68483	.63412	.59270	.55770	.52740	.50070	.47683
98.0	1.00000	.84276	.75079	.68554	.63494	.59362	.55870	.52846	.50182	.47800
99.0	1.00000	.84311	.75134	.68624	.63576	.59452	.55968	.52951	.50292	.47916
100.0	1.00000	.84346	.75189	.68693	.63656	.59542	.56065	.53055	.50401	.48030

(b/a) 800.

$\left(\frac{\sqrt{4\alpha t}}{a}\right)$	(r/a) 1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
100.0	1.00000	.84346	.75189	.68693	.63656	.59542	.56065	.53055	.50402	.48030
110.0	1.00000	.84666	.75697	.69334	.64400	.60369	.56963	.54013	.51413	.49089
120.0	1.00000	.84948	.76144	.69898	.65054	.61097	.57752	.54856	.52303	.50020
130.0	1.00000	.85199	.76541	.70399	.65635	.61743	.58454	.55606	.53094	.50849
140.0	1.00000	.85424	.76897	.70848	.66157	.62324	.59084	.56279	.53805	.51593
150.0	1.00000	.85627	.77220	.71255	.66629	.62849	.59655	.56888	.54448	.52267
160.0	1.00000	.85812	.77514	.71626	.67059	.63328	.60174	.57443	.55035	.52881
170.0	1.00000	.85982	.77783	.71965	.67453	.63767	.60651	.57952	.55572	.53444
180.0	1.00000	.86139	.78031	.72279	.67817	.64172	.61090	.58422	.56068	.53963
190.0	1.00000	.86284	.78261	.72569	.68154	.64547	.61497	.58856	.56527	.54444
200.0	1.00000	.86419	.78475	.72838	.68467	.64895	.61876	.59261	.56954	.54891

(b/a) 1600.

	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
200.0	1.00000	.86419	.78475	.72839	.68467	.64895	.61876	.59261	.56954	.54891
210.0	1.00000	.86545	.78675	.73090	.68759	.65221	.62229	.59638	.57353	.55309
220.0	1.00000	.86663	.78862	.73326	.69033	.65525	.62560	.59992	.57726	.55700
230.0	1.00000	.86774	.79037	.73548	.69290	.65812	.62871	.60324	.58078	.56068
240.0	1.00000	.86878	.79203	.73757	.69533	.66082	.63164	.60637	.58408	.56415
250.0	1.00000	.86977	.79359	.73954	.69762	.66337	.63441	.60933	.58721	.56742
260.0	1.00000	.87071	.79507	.74141	.69979	.66578	.63703	.61213	.59017	.57052
270.0	1.00000	.87159	.79648	.74319	.70185	.66808	.63952	.61479	.59298	.57347
280.0	1.00000	.87244	.79782	.74487	.70381	.67026	.64189	.61732	.59565	.57627
290.0	1.00000	.87324	.79909	.74648	.70567	.67233	.64415	.61973	.59820	.57894
300.0	1.00000	.87401	.80031	.74801	.70746	.67432	.64630	.62203	.60063	.58148
310.0	1.00000	.87474	.80147	.74948	.70916	.67621	.64836	.62423	.60295	.58392
320.0	1.00000	.87544	.80258	.75089	.71079	.67803	.65033	.62634	.60518	.58625
330.0	1.00000	.87612	.80365	.75223	.71235	.67977	.65222	.62836	.60731	.58848
340.0	1.00000	.87676	.80467	.75352	.71385	.68144	.65403	.63029	.60936	.59063
350.0	1.00000	.87738	.80566	.75477	.71529	.68304	.65578	.63216	.61133	.59269
360.0	1.00000	.87798	.80660	.75596	.71668	.68459	.65745	.63395	.61322	.59468
370.0	1.00000	.87856	.80752	.75711	.71802	.68608	.65907	.63568	.61505	.59659
380.0	1.00000	.87911	.80840	.75823	.71931	.68751	.66063	.63735	.61681	.59843
390.0	1.00000	.87965	.80925	.75930	.72056	.68890	.66214	.63895	.61851	.60022
400.0	1.00000	.88017	.81007	.76034	.72176	.69024	.66359	.64051	.62015	.60194

4

Table 3. (y/y_0)

(b/a) 3200.

$\left(\frac{\sqrt{4\alpha t}}{a}\right)$	(r/a) 1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
400.0	1.00000	.88017	.81007	.76034	.72176	.69024	.66359	.64051	.62015	.60194
410.0	1.00000	.88067	.81087	.76134	.72292	.69154	.66500	.64201	.62174	.60360
420.0	1.00000	.88116	.81163	.76231	.72405	.69279	.66636	.64347	.62328	.60522
430.0	1.00000	.88163	.81238	.76325	.72514	.69401	.66768	.64488	.62477	.60678
440.0	1.00000	.88208	.81310	.76416	.72620	.69518	.66896	.64625	.62621	.60829
450.0	1.00000	.88252	.81380	.76505	.72723	.69633	.67020	.64757	.62761	.60976
460.0	1.00000	.88295	.81448	.76591	.72822	.69744	.67141	.64886	.62897	.61118
470.0	1.00000	.88337	.81515	.76674	.72919	.69852	.67258	.65011	.63030	.61257
480.0	1.00000	.88378	.81579	.76755	.73014	.69956	.67372	.65133	.63158	.61392
490.0	1.00000	.88417	.81641	.76834	.73105	.70058	.67483	.65251	.63283	.61523
500.0	1.00000	.88455	.81702	.76911	.73194	.70158	.67590	.65366	.63405	.61650
510.0	1.00000	.88493	.81761	.76986	.73281	.70254	.67695	.65479	.63523	.61774
520.0	1.00000	.88529	.81819	.77058	.73366	.70349	.67798	.65588	.63639	.61895
530.0	1.00000	.88565	.81876	.77130	.73448	.70440	.67897	.65694	.63751	.62013
540.0	1.00000	.88599	.81930	.77199	.73529	.70530	.67995	.65799	.63861	.62129
550.0	1.00000	.88633	.81984	.77266	.73607	.70617	.68090	.65900	.63969	.62241
560.0	1.00000	.88666	.82036	.77332	.73684	.70703	.68182	.65999	.64073	.62351
570.0	1.00000	.88699	.82088	.77397	.73759	.70786	.68273	.66096	.64175	.62458
580.0	1.00000	.88730	.82138	.77460	.73832	.70868	.68361	.66190	.64275	.62562
590.0	1.00000	.88761	.82186	.77522	.73903	.70947	.68448	.66283	.64373	.62665
600.0	1.00000	.88791	.82234	.77582	.73973	.71025	.68532	.66373	.64468	.62765
610.0	1.00000	.88820	.82281	.77641	.74042	.71101	.68615	.66461	.64562	.62863
620.0	1.00000	.88849	.82326	.77699	.74109	.71176	.68696	.66548	.64653	.62958
630.0	1.00000	.88878	.82371	.77755	.74174	.71249	.68775	.66633	.64743	.63052
640.0	1.00000	.88905	.82415	.77810	.74239	.71320	.68853	.66716	.64830	.63144
650.0	1.00000	.88932	.82458	.77865	.74302	.71390	.68929	.66797	.64916	.63234
660.0	1.00000	.88959	.82500	.77918	.74363	.71459	.69004	.66877	.65001	.63322
670.0	1.00000	.88985	.82541	.77970	.74424	.71526	.69077	.66955	.65083	.63409
680.0	1.00000	.89010	.82582	.78021	.74483	.71592	.69148	.67031	.65164	.63494
690.0	1.00000	.89035	.82622	.78071	.74541	.71657	.69219	.67106	.65243	.63577
700.0	1.00000	.89060	.82661	.78120	.74598	.71721	.69288	.67180	.65321	.63658
710.0	1.00000	.89084	.82699	.78168	.74654	.71783	.69355	.67253	.65398	.63739
720.0	1.00000	.89108	.82736	.78216	.74709	.71844	.69422	.67324	.65473	.63817
730.0	1.00000	.89131	.82773	.78262	.74763	.71904	.69487	.67393	.65547	.63894
740.0	1.00000	.89154	.82809	.78308	.74816	.71963	.69551	.67462	.65619	.63970
750.0	1.00000	.89176	.82845	.78353	.74868	.72021	.69614	.67529	.65690	.64045
760.0	1.00000	.89198	.82880	.78397	.74920	.72078	.69676	.67595	.65760	.64118
770.0	1.00000	.89220	.82914	.78440	.74970	.72134	.69737	.67660	.65829	.64190
780.0	1.00000	.89241	.82948	.78483	.75019	.72190	.69797	.67724	.65896	.64261
790.0	1.00000	.89262	.82981	.78525	.75068	.72244	.69856	.67787	.65963	.64331
800.0	1.00000	.89283	.83014	.78566	.75116	.72297	.69914	.67849	.66028	.64399

5

Table 3. (y/y_0)

$\left(\frac{\sqrt{4\alpha t}}{a}\right)$ $\left(\frac{r}{a}\right)$

6400.

$\sqrt{4\alpha t}/a$	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
800.0	1.00000	.89283	.83014	.78566	.75116	.72297	.69914	.67849	.66028	.64399
900.0	1.00000	.89473	.83314	.78945	.75556	.72787	.70445	.68417	.66629	.65028
1000.0	1.00000	.89636	.83574	.79273	.75936	.73210	.70906	.68909	.67148	.65573
1100.0	1.00000	.89780	.83802	.79561	.76271	.73583	.71310	.69341	.67604	.66051
1200.0	1.00000	.89908	.84005	.79817	.76568	.73914	.71669	.69725	.68010	.66476
1300.0	1.00000	.90023	.84187	.80047	.76835	.74211	.71992	.70070	.68375	.66858
1400.0	1.00000	.90128	.84353	.80255	.77077	.74480	.72285	.70383	.68705	.67205
1500.0	1.00000	.90223	.84503	.80445	.77298	.74726	.72552	.70668	.69007	.67520
1600.0	1.00000	.90310	.84642	.80620	.77501	.74952	.72797	.70930	.69283	.67811

(b/a) 12800.

	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
1600.0	1.00000	.90310	.84642	.80620	.77501	.74952	.72797	.70930	.69284	.67811
1700.0	1.00000	.90391	.84770	.80781	.77688	.75160	.73023	.71172	.69539	.68079
1800.0	1.00000	.90466	.84888	.80931	.77862	.75354	.73233	.71397	.69776	.68327
1900.0	1.00000	.90535	.84999	.81070	.78024	.75534	.73429	.71606	.69997	.68559
2000.0	1.00000	.90601	.85102	.81201	.78175	.75702	.73612	.71801	.70204	.68775
2100.0	1.00000	.90662	.85199	.81323	.78317	.75861	.73784	.71985	.70398	.68979
2200.0	1.00000	.90719	.85290	.81439	.78451	.76010	.73946	.72158	.70581	.69170
2300.0	1.00000	.90774	.85377	.81547	.78577	.76150	.74099	.72321	.70753	.69351
2400.0	1.00000	.90825	.85458	.81650	.78697	.76284	.74243	.72476	.70917	.69522
2500.0	1.00000	.90874	.85536	.81748	.78810	.76410	.74380	.72622	.71072	.69684
2600.0	1.00000	.90921	.85609	.81841	.78918	.76530	.74511	.72762	.71219	.69839
2700.0	1.00000	.90965	.85680	.81930	.79021	.76645	.74635	.72895	.71359	.69986
2800.0	1.00000	.91007	.85747	.82014	.79119	.76754	.74754	.73021	.71493	.70126
2900.0	1.00000	.91048	.85811	.82095	.79213	.76858	.74867	.73143	.71621	.70261
3000.0	1.00000	.91086	.85872	.82173	.79303	.76958	.74976	.73259	.71744	.70389
3100.0	1.00000	.91123	.85931	.82247	.79389	.77054	.75080	.73370	.71862	.70513
3200.0	1.00000	.91159	.85987	.82318	.79472	.77146	.75180	.73477	.71975	.70631

6.

(b/a) 25600. 7.

$\left(\frac{\sqrt{4\alpha t}}{a}\right)$	$\left(\frac{r}{a}\right)$ 1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
3200.0	1.00000	.91159	.85987	.82318	.79472	.77147	.75180	.73477	.71975	.70631
3300.0	1.00000	.91193	.86042	.82387	.79552	.77235	.75277	.73580	.72084	.70745
3400.0	1.00000	.91227	.86094	.82453	.79628	.77321	.75369	.73679	.72188	.70855
3500.0	1.00000	.91258	.86145	.82516	.79702	.77403	.75459	.73775	.72289	.70960
3600.0	1.00000	.91289	.86193	.82578	.79773	.77482	.75545	.73867	.72386	.71062
3700.0	1.00000	.91319	.86240	.82637	.79842	.77559	.75628	.73956	.72480	.71161
3800.0	1.00000	.91347	.86286	.82694	.79909	.77633	.75709	.74042	.72571	.71256
3900.0	1.00000	.91375	.86330	.82750	.79973	.77705	.75787	.74125	.72659	.71348
4000.0	1.00000	.91402	.86372	.82804	.80036	.77774	.75862	.74206	.72745	.71438
4100.0	1.00000	.91428	.86414	.82856	.80096	.77842	.75935	.74284	.72827	.71524
4200.0	1.00000	.91453	.86454	.82906	.80155	.77907	.76006	.74360	.72907	.71608
4300.0	1.00000	.91478	.86493	.82955	.80212	.77970	.76075	.74433	.72985	.71690
4400.0	1.00000	.91502	.86530	.83003	.80267	.78032	.76142	.74505	.73061	.71769
4500.0	1.00000	.91525	.86567	.83050	.80321	.78092	.76207	.74574	.73134	.71846
4600.0	1.00000	.91547	.86603	.83095	.80373	.78150	.76270	.74642	.73206	.71921
4700.0	1.00000	.91569	.86638	.83139	.80424	.78207	.76332	.74708	.73275	.71994
4800.0	1.00000	.91591	.86671	.83181	.80474	.78262	.76392	.74772	.73343	.72065
4900.0	1.00000	.91612	.86705	.83223	.80522	.78316	.76450	.74834	.73409	.72134
5000.0	1.00000	.91632	.86737	.83264	.80570	.78368	.76507	.74895	.73473	.72201
5100.0	1.00000	.91652	.86768	.83303	.80616	.78420	.76563	.74955	.73536	.72267
5200.0	1.00000	.91671	.86799	.83342	.80661	.78470	.76617	.75013	.73597	.72331
5300.0	1.00000	.91690	.86829	.83380	.80704	.78518	.76670	.75069	.73657	.72394
5400.0	1.00000	.91708	.86858	.83417	.80747	.78566	.76722	.75125	.73716	.72455
5500.0	1.00000	.91726	.86886	.83453	.80789	.78613	.76773	.75179	.73773	.72515
5600.0	1.00000	.91744	.86914	.83488	.80830	.78658	.76822	.75232	.73829	.72574
5700.0	1.00000	.91761	.86942	.83522	.80870	.78703	.76871	.75284	.73884	.72631
5800.0	1.00000	.91778	.86969	.83556	.80909	.78747	.76918	.75334	.73937	.72687
5900.0	1.00000	.91795	.86995	.83589	.80948	.78789	.76964	.75384	.73989	.72742
6000.0	1.00000	.91811	.87020	.83622	.80985	.78831	.77010	.75432	.74041	.72796
6100.0	1.00000	.91827	.87045	.83653	.81022	.78872	.77054	.75480	.74091	.72849
6200.0	1.00000	.91842	.87070	.83684	.81058	.78912	.77098	.75526	.74140	.72900
6300.0	1.00000	.91857	.87094	.83715	.81093	.78952	.77141	.75572	.74188	.72951
6400.0	1.00000	.91872	.87118	.83745	.81128	.78990	.77183	.75617	.74236	.73000
6500.0	1.00000	.91887	.87141	.83774	.81162	.79028	.77224	.75661	.74282	.73049
6600.0	1.00000	.91901	.87164	.83803	.81195	.79065	.77264	.75704	.74328	.73097
6700.0	1.00000	.91915	.87186	.83831	.81228	.79102	.77304	.75746	.74373	.73144
6800.0	1.00000	.91929	.87208	.83859	.81260	.79138	.77343	.75788	.74416	.73190
6900.0	1.00000	.91943	.87230	.83886	.81292	.79173	.77381	.75829	.74460	.73235
7000.0	1.00000	.91956	.87251	.83913	.81323	.79207	.77418	.75869	.74502	.73279
7100.0	1.00000	.91969	.87272	.83939	.81354	.79241	.77455	.75908	.74544	.73323
7200.0	1.00000	.91982	.87292	.83965	.81383	.79275	.77491	.75947	.74585	.73366
7300.0	1.00000	.91995	.87312	.83990	.81413	.79307	.77527	.75985	.74625	.73408
7400.0	1.00000	.92007	.87332	.84015	.81442	.79340	.77562	.76022	.74664	.73449
7500.0	1.00000	.92020	.87352	.84039	.81470	.79371	.77597	.76059	.74703	.73490
7600.0	1.00000	.92032	.87371	.84064	.81498	.79403	.77630	.76095	.74741	.73530
7700.0	1.00000	.92044	.87390	.84087	.81526	.79433	.77664	.76131	.74779	.73570
7800.0	1.00000	.92055	.87408	.84111	.81553	.79463	.77697	.76166	.74816	.73609
7900.0	1.00000	.92067	.87426	.84134	.81580	.79493	.77729	.76201	.74853	.73647
8000.0	1.00000	.92078	.87444	.84156	.81606	.79523	.77761	.76235	.74889	.73684
8100.0	1.00000	.92089	.87462	.84179	.81632	.79551	.77792	.76268	.74924	.73721
8200.0	1.00000	.92100	.87479	.84201	.81658	.79580	.77823	.76301	.74959	.73758
8300.0	1.00000	.92111	.87497	.84222	.81683	.79608	.77853	.76334	.74993	.73794

Table 3. (y/y_0)

(b/a) 25600.

8.

$\left(\frac{\sqrt{4\alpha t}}{a}\right)$	$\left(\frac{r}{a}\right)$ 1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
8400.0	1.00000	.92122	.87513	.84244	.81708	.79635	.77883	.76366	.75027	.73829
8500.0	1.00000	.92132	.87530	.84265	.81732	.79663	.77913	.76397	.75060	.73864
8600.0	1.00000	.92143	.87547	.84286	.81756	.79689	.77942	.76428	.75093	.73899
8700.0	1.00000	.92153	.87563	.84306	.81780	.79716	.77971	.76459	.75126	.73933
8800.0	1.00000	.92163	.87579	.84326	.81803	.79742	.77999	.76489	.75157	.73966
8900.0	1.00000	.92173	.87594	.84346	.81826	.79767	.78027	.76519	.75189	.73999
9000.0	1.00000	.92183	.87610	.84366	.81849	.79793	.78054	.76548	.75220	.74032
9100.0	1.00000	.92192	.87625	.84385	.81871	.79818	.78081	.76577	.75251	.74064
9200.0	1.00000	.92202	.87640	.84404	.81894	.79842	.78108	.76606	.75281	.74096
9300.0	1.00000	.92211	.87655	.84423	.81915	.79867	.78135	.76634	.75311	.74127
9400.0	1.00000	.92221	.87670	.84441	.81937	.79891	.78161	.76662	.75340	.74158
9500.0	1.00000	.92230	.87685	.84460	.81958	.79915	.78187	.76690	.75369	.74188
9600.0	1.00000	.92239	.87699	.84478	.81979	.79938	.78212	.76717	.75398	.74218
9700.0	1.00000	.92248	.87713	.84496	.82000	.79961	.78237	.76744	.75426	.74248
9800.0	1.00000	.92257	.87727	.84513	.82021	.79984	.78262	.76770	.75454	.74277
9900.0	1.00000	.92265	.87741	.84531	.82041	.80006	.78286	.76796	.75482	.74306
10000.0	1.00000	.92274	.87755	.84548	.82061	.80029	.78310	.76822	.75509	.74335

Table 3. (y/y_0)

(b/a)	$\left(\frac{\sqrt{4\alpha t}}{a}\right)$	(r/a) 10.	20.	30.	40.	50.	60.	70.	80.	90.	100.
25.	3.0	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
	4.0	.00050	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
	5.0	.00376	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
	6.0	.01195	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
50.	6.0	.01195	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
	7.0	.02482	.00003	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
	8.0	.04089	.00020	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
	9.0	.05862	.00072	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
	10.0	.07689	.00187	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000
	11.0	.09499	.00384	.00004	.00000	.00000	.00000	.00000	.00000	.00000	.00000
	12.0	.11250	.00674	.00013	.00000	.00000	.00000	.00000	.00000	.00000	.00000
100.	12.0	.11239	.00674	.00014	.00000	.00000	.00000	.00000	.00000	.00000	.00000
	13.0	.12917	.01054	.00035	.00000	.00000	.00000	.00000	.00000	.00000	.00000
	14.0	.14500	.01516	.00074	.00001	.00000	.00000	.00000	.00000	.00000	.00000
	15.0	.15990	.02049	.00139	.00004	.00000	.00000	.00000	.00000	.00000	.00000
	16.0	.17390	.02639	.00233	.00011	.00000	.00000	.00000	.00000	.00000	.00000
	17.0	.18703	.03271	.00361	.00023	.00001	.00000	.00000	.00000	.00000	.00000
	18.0	.19936	.03936	.00524	.00038	.00002	.00000	.00000	.00000	.00000	.00000
	19.0	.21094	.04621	.00722	.00071	.00005	.00000	.00000	.00000	.00000	.00000
	20.0	.22183	.05320	.00954	.00115	.00009	.00000	.00000	.00000	.00000	.00000
	21.0	.23208	.06024	.01216	.00174	.00017	.00001	.00000	.00000	.00000	.00000
	22.0	.24174	.06728	.01508	.00249	.00029	.00002	.00000	.00000	.00000	.00000
	23.0	.25087	.07429	.01824	.00341	.00047	.00005	.00000	.00000	.00000	.00000
	24.0	.25951	.08121	.02162	.00451	.00071	.00008	.00001	.00000	.00000	.00000
	25.0	.26768	.08803	.02518	.00578	.00103	.00014	.00001	.00000	.00000	.00000

9.

Table 3. (y/y_0)

(b/a) 200. 10.

$\left(\frac{\sqrt{4\alpha t}}{a}\right)$	(r/a) 10.	20.	30.	40.	50.	60.	70.	80.	90.	100.
25.0	.26768	.08803	.02518	.00578	.00103	.00014	.00000	.00000	.00000	.00000
26.0	.27544	.09473	.02890	.00724	.00144	.00022	.00000	.00000	.00000	.00000
27.0	.28281	.10129	.03275	.00885	.00194	.00034	.00000	.00000	.00000	.00000
28.0	.28982	.10772	.03669	.01063	.00254	.00049	.00000	.00000	.00000	.00000
29.0	.29650	.11398	.04072	.01256	.00325	.00069	.00001	.00000	.00000	.00000
30.0	.30288	.12010	.04479	.01462	.00406	.00094	.00010	.00003	.00000	.00000
31.0	.30896	.12606	.04891	.01682	.00498	.00125	.00021	.00005	.00001	.00000
32.0	.31479	.13186	.05304	.01912	.00600	.00161	.00033	.00007	.00001	.00000
33.0	.32036	.13751	.05719	.02152	.00713	.00205	.00048	.00011	.00002	.00000
34.0	.32571	.14301	.06132	.02401	.00835	.00255	.00065	.00015	.00003	.00000
35.0	.33084	.14836	.06545	.02658	.00968	.00311	.00086	.00022	.00005	.00001
36.0	.33577	.15357	.06955	.02921	.01109	.00375	.00111	.00029	.00007	.00001
37.0	.34051	.15864	.07362	.03190	.01258	.00446	.00140	.00039	.00010	.00002
38.0	.34508	.16356	.07765	.03464	.01416	.00523	.00173	.00051	.00013	.00003
39.0	.34948	.16836	.08164	.03741	.01581	.00608	.00211	.00066	.00018	.00005
40.0	.35372	.17302	.08559	.04022	.01753	.00699	.00253	.00082	.00024	.00006
41.0	.35781	.17756	.08949	.04305	.01931	.00797	.00300	.00102	.00032	.00009
42.0	.36177	.18198	.09333	.04590	.02114	.00901	.00352	.00125	.00041	.00012
43.0	.36559	.18629	.09712	.04876	.02303	.01011	.00409	.00152	.00052	.00016
44.0	.36929	.19048	.10086	.05162	.02496	.01128	.00471	.00181	.00064	.00021
45.0	.37287	.19457	.10454	.05449	.02694	.01249	.00538	.00214	.00079	.00027
46.0	.37634	.19855	.10817	.05735	.02894	.01375	.00609	.00251	.00096	.00034
47.0	.37971	.20243	.11173	.06021	.03098	.01506	.00685	.00291	.00115	.00042
48.0	.38297	.20621	.11524	.06306	.03305	.01641	.00766	.00335	.00136	.00052
49.0	.38614	.20990	.11869	.06589	.03514	.01781	.00851	.00382	.00160	.00063
50.0	.38922	.21350	.12209	.06871	.03725	.01924	.00940	.00433	.00187	.00076

Table 3. (y/y_0)

400. (b/a) 11.

$\left(\frac{\sqrt{4\alpha t}}{a}\right)$	$\left(\frac{r}{a}\right)$ 10.	20.	30.	40.	50.	60.	70.	80.	90.	100.
50.0	.38922	.21350	.12209	.06871	.03725	.01924	.00940	.00433	.00187	.00076
51.0	.39221	.21702	.12542	.07151	.03938	.02072	.01034	.00487	.00216	.00090
52.0	.39512	.22045	.12871	.07429	.04152	.02222	.01131	.00545	.00248	.00106
53.0	.39795	.22380	.13193	.07705	.04366	.02375	.01232	.00607	.00283	.00124
54.0	.40071	.22708	.13510	.07979	.04582	.02532	.01337	.00672	.00320	.00144
55.0	.40339	.23027	.13822	.08250	.04798	.02690	.01446	.00740	.00360	.00166
56.0	.40600	.23340	.14128	.08519	.05014	.02851	.01557	.00812	.00404	.00190
57.0	.40855	.23646	.14430	.08785	.05231	.03014	.01671	.00887	.00449	.00217
58.0	.41104	.23945	.14726	.09049	.05447	.03179	.01788	.00965	.00498	.00245
59.0	.41346	.24237	.15016	.09308	.05662	.03345	.01907	.01045	.00549	.00276
60.0	.41583	.24524	.15302	.09566	.05878	.03512	.02029	.01129	.00603	.00308
61.0	.41814	.24804	.15584	.09822	.06092	.03681	.02154	.01215	.00660	.00343
62.0	.42040	.25078	.15860	.10074	.06306	.03851	.02280	.01304	.00719	.00380
63.0	.42261	.25347	.16132	.10324	.06519	.04021	.02409	.01396	.00780	.00420
64.0	.42476	.25610	.16399	.10571	.06731	.04192	.02539	.01490	.00844	.00461
65.0	.42687	.25868	.16662	.10815	.06942	.04364	.02671	.01586	.00911	.00505
66.0	.42894	.26121	.16920	.11056	.07152	.04536	.02804	.01684	.00979	.00550
67.0	.43096	.26369	.17174	.11294	.07360	.04708	.02939	.01784	.01050	.00598
68.0	.43293	.26612	.17424	.11529	.07567	.04880	.03075	.01886	.01123	.00648
69.0	.43487	.26851	.17670	.11762	.07773	.05052	.03212	.01990	.01198	.00700
70.0	.43677	.27085	.17912	.11991	.07977	.05224	.03350	.02096	.01277	.00754
71.0	.43863	.27314	.18150	.12218	.08180	.05396	.03489	.02203	.01355	.00810
72.0	.44045	.27540	.18384	.12442	.08378	.05568	.03629	.02311	.01436	.00867
73.0	.44223	.27761	.18615	.12663	.08578	.05739	.03769	.02421	.01518	.00927
74.0	.44399	.27978	.18842	.12881	.08776	.05910	.03910	.02533	.01603	.00988
75.0	.44570	.28192	.19065	.13097	.08973	.06081	.04051	.02645	.01688	.01051
76.0	.44739	.28402	.19285	.13310	.09168	.06250	.04193	.02758	.01776	.01116
77.0	.44904	.28608	.19502	.13521	.09362	.06419	.04335	.02873	.01864	.01182
78.0	.45067	.28810	.19715	.13728	.09553	.06588	.04477	.02988	.01954	.01250
79.0	.45226	.29009	.19926	.13934	.09743	.06756	.04619	.03104	.02046	.01319
80.0	.45383	.29205	.20133	.14136	.09932	.06923	.04761	.03221	.02138	.01390
81.0	.45537	.29397	.20336	.14336	.10118	.07089	.04904	.03339	.02232	.01462
82.0	.45688	.29587	.20537	.14534	.10303	.07254	.05046	.03457	.02326	.01536
83.0	.45837	.29773	.20735	.14730	.10486	.07418	.05188	.03575	.02422	.01610
84.0	.45983	.29956	.20930	.14922	.10668	.07582	.05330	.03694	.02519	.01686
85.0	.46127	.30137	.21123	.15113	.10847	.07744	.05472	.03814	.02616	.01764
86.0	.46268	.30314	.21312	.15301	.11025	.07906	.05613	.03934	.02715	.01842
87.0	.46407	.30489	.21499	.15487	.11202	.08066	.05754	.04054	.02814	.01921
88.0	.46544	.30661	.21684	.15671	.11376	.08226	.05895	.04174	.02914	.02002
89.0	.46678	.30831	.21865	.15853	.11549	.08384	.06036	.04295	.03014	.02083
90.0	.46811	.30997	.22044	.16032	.11720	.08542	.06175	.04415	.03115	.02165
91.0	.46941	.31162	.22221	.16209	.11890	.08698	.06315	.04536	.03216	.02248
92.0	.47070	.31324	.22396	.16384	.12058	.08853	.06454	.04657	.03318	.02332
93.0	.47196	.31484	.22568	.16557	.12224	.09008	.06592	.04778	.03421	.02416
94.0	.47320	.31641	.22737	.16728	.12389	.09161	.06730	.04898	.03524	.02502
95.0	.47443	.31796	.22905	.16897	.12552	.09313	.06868	.05019	.03627	.02587
96.0	.47564	.31949	.23070	.17065	.12714	.09464	.07005	.05139	.03730	.02674
97.0	.47683	.32100	.23233	.17230	.12874	.09614	.07141	.05260	.03834	.02761
98.0	.47800	.32248	.23394	.17393	.13033	.09762	.07276	.05380	.03938	.02849
99.0	.47916	.32395	.23553	.17554	.13190	.09910	.07411	.05500	.04042	.02937
100.0	.48030	.32539	.23710	.17714	.13345	.10056	.07545	.05620	.04146	.03026

Table 3. (y/y_0)

(b/a) 800.

$(\sqrt{4\kappa t}/a)$	(r/a) 10.	20.	30.	40.	50.	60.	70.	80.	90.	100.
100.0	.48030	.32540	.23710	.17714	.13345	.10056	.07541	.05620	.04146	.03026
110.0	.49089	.33885	.25177	.19215	.14820	.11461	.08847	.06799	.05190	.03930
120.0	.50020	.35074	.26482	.20563	.16162	.12760	.10075	.07935	.06221	.04849
130.0	.50849	.36135	.27653	.21782	.17387	.13960	.11227	.09018	.07224	.05761
140.0	.51593	.37090	.28711	.22890	.18510	.15070	.12304	.10045	.08188	.06654
150.0	.52267	.37956	.29674	.23903	.19542	.16100	.13311	.11017	.09111	.07520
160.0	.52881	.38746	.30555	.24834	.20496	.17056	.14254	.11933	.09991	.08355
170.0	.53444	.39472	.31366	.25693	.21380	.17947	.15138	.12799	.10828	.09157
180.0	.53963	.40142	.32116	.26490	.22202	.18779	.15968	.13616	.11624	.09925
190.0	.54444	.40763	.32812	.27231	.22970	.19559	.16748	.14388	.12381	.10659
200.0	.54891	.41342	.33461	.27923	.23688	.20291	.17484	.15119	.13101	.11361

1600.

	10.	20.	30.	40.	50.	60.	70.	80.	90.	100.
200.0	.54891	.41342	.33461	.27924	.23688	.20291	.17484	.15119	.13101	.11361
210.0	.55309	.41882	.34068	.28572	.24362	.20980	.18179	.15812	.13785	.12032
220.0	.55700	.42388	.34638	.29181	.24997	.21630	.18835	.16469	.14437	.12674
230.0	.56068	.42864	.35173	.29756	.25596	.22244	.19458	.17094	.15058	.13287
240.0	.56415	.43313	.35679	.30298	.26163	.22826	.20049	.17688	.15651	.13874
250.0	.56742	.43738	.36157	.30811	.26700	.23379	.20611	.18254	.16217	.14437
260.0	.57052	.44140	.36611	.31298	.27210	.23904	.21146	.18794	.16758	.14975
270.0	.57347	.44521	.37041	.31761	.27695	.24405	.21657	.19311	.17276	.15492
280.0	.57627	.44885	.37451	.32202	.28158	.24883	.22145	.19804	.17773	.15988
290.0	.57894	.45231	.37842	.32623	.28599	.25339	.22611	.20278	.18249	.16465
300.0	.58148	.45561	.38215	.33024	.29021	.25776	.23058	.20731	.18707	.16924
310.0	.58392	.45877	.38572	.33409	.29426	.26194	.23487	.21167	.19146	.17365
320.0	.58625	.46179	.38914	.33778	.29813	.26596	.23899	.21586	.19569	.17791
330.0	.58848	.46469	.39242	.34131	.30186	.26982	.24294	.21988	.19977	.18201
340.0	.59063	.46748	.39557	.34471	.30543	.27353	.24675	.22377	.20370	.18596
350.0	.59269	.47016	.39860	.34798	.30888	.27710	.25043	.22751	.20749	.18979
360.0	.59468	.47274	.40152	.35113	.31219	.28055	.25397	.23112	.21115	.19348
370.0	.59659	.47522	.40433	.35416	.31539	.28387	.25738	.23461	.21469	.19705
380.0	.59843	.47762	.40704	.35709	.31848	.28708	.26068	.23798	.21811	.20051
390.0	.60022	.47993	.40966	.35992	.32146	.29018	.26388	.24124	.22143	.20386
400.0	.60194	.48217	.41219	.36265	.32435	.29318	.26696	.24440	.22464	.20711

12.

Table 3. (y/y_0)

(b/a) 3200.

$\left(\frac{\sqrt{4\alpha t}}{a}\right)$	$\left(\frac{r}{a}\right)$ 10.	20.	30.	40.	50.	60.	70.	80.	90.	100.
400.0	.60194	.48217	.41220	.36266	.32435	.29318	.26697	.24440	.22464	.20711
410.0	.60360	.48433	.41464	.36530	.32714	.29609	.26996	.24746	.22775	.21026
420.0	.60522	.48642	.41701	.36786	.32985	.29890	.27286	.25043	.23077	.21332
430.0	.60678	.48845	.41931	.37035	.33247	.30163	.27567	.25331	.23370	.21629
440.0	.60829	.49042	.42154	.37276	.33501	.30428	.27840	.25610	.23655	.21918
450.0	.60976	.49233	.42370	.37509	.33748	.30685	.28105	.25882	.23931	.22198
460.0	.61118	.49418	.42580	.37736	.33988	.30935	.28363	.26146	.24201	.22472
470.0	.61257	.49598	.42784	.37957	.34221	.31178	.28614	.26403	.24462	.22737
480.0	.61392	.49773	.42983	.38172	.34448	.31414	.28858	.26653	.24717	.22996
490.0	.61523	.49943	.43176	.38380	.34668	.31644	.29095	.26896	.24966	.23248
500.0	.61650	.50109	.43363	.38584	.34883	.31868	.29326	.27133	.25208	.23494
510.0	.61774	.50271	.43546	.38782	.35093	.32086	.29552	.27364	.25444	.23734
520.0	.61895	.50428	.43725	.38975	.35297	.32299	.29771	.27590	.25674	.23968
530.0	.62013	.50581	.43899	.39163	.35496	.32506	.29986	.27810	.25898	.24196
540.0	.62129	.50731	.44068	.39346	.35690	.32709	.30195	.28025	.26118	.24419
550.0	.62241	.50877	.44234	.39525	.35879	.32906	.30399	.28234	.26332	.24637
560.0	.62351	.51020	.44395	.39701	.36064	.33099	.30599	.28439	.26541	.24850
570.0	.62458	.51159	.44553	.39871	.36245	.33288	.30794	.28639	.26746	.25059
580.0	.62562	.51295	.44708	.40038	.36422	.33472	.30984	.28835	.26946	.25262
590.0	.62665	.51428	.44858	.40202	.36595	.33653	.31171	.29027	.27142	.25462
600.0	.62765	.51558	.45006	.40361	.36764	.33829	.31353	.29214	.27333	.25657
610.0	.62863	.51685	.45150	.40518	.36929	.34002	.31532	.29398	.27521	.25848
620.0	.62958	.51810	.45292	.40671	.37091	.34171	.31707	.29577	.27705	.26035
630.0	.63052	.51932	.45430	.40821	.37249	.34336	.31878	.29753	.27885	.26218
640.0	.63144	.52051	.45565	.40967	.37405	.34498	.32045	.29926	.28061	.26398
650.0	.63234	.52168	.45698	.41111	.37557	.34657	.32210	.30095	.28234	.26574
660.0	.63322	.52283	.45828	.41252	.37706	.34813	.32371	.30260	.28403	.26747
670.0	.63409	.52395	.45956	.41390	.37852	.34966	.32529	.30423	.28570	.26916
680.0	.63494	.52506	.46081	.41526	.37996	.35115	.32684	.30582	.28733	.27083
690.0	.63577	.52614	.46204	.41658	.38136	.35262	.32836	.30739	.28893	.27246
700.0	.63658	.52720	.46324	.41789	.38274	.35407	.32986	.30892	.29050	.27406
710.0	.63739	.52824	.46442	.41917	.38410	.35548	.33132	.31043	.29204	.27564
720.0	.63817	.52927	.46558	.42043	.38543	.35687	.33276	.31191	.29356	.27718
730.0	.63894	.53027	.46672	.42166	.38674	.35824	.33417	.31337	.29505	.27870
740.0	.63970	.53126	.46784	.42287	.38802	.35958	.33556	.31479	.29651	.28019
750.0	.64045	.53223	.46894	.42407	.38928	.36090	.33693	.31620	.29795	.28166
760.0	.64118	.53318	.47002	.42524	.39052	.36219	.33827	.31758	.29936	.28310
770.0	.64190	.53412	.47109	.42639	.39174	.36347	.33959	.31894	.30075	.28452
780.0	.64261	.53504	.47213	.42752	.39294	.36472	.34089	.32027	.30212	.28592
790.0	.64331	.53594	.47316	.42863	.39412	.36595	.34216	.32159	.30347	.28729
800.0	.64399	.53683	.47417	.42973	.39528	.36716	.34342	.32288	.30479	.28864

13.

(b/a) = 6400.

$\left(\frac{\sqrt{4\alpha t}}{a}\right)$ \ $\left(\frac{r}{a}\right)$	10.	20.	30.	40.	50.	60.	70.	80.	90.	100.
800.0	.64399	.53684	.47417	.42973	.39528	.36717	.34342	.32288	.30479	.28864
900.0	.65028	.54502	.48345	.43979	.40594	.37830	.35496	.33476	.31696	.30107
1000.0	.65573	.55210	.49149	.44850	.41517	.38795	.36496	.34505	.32752	.31185
1100.0	.66051	.55832	.49855	.45615	.42328	.39643	.37375	.35411	.33680	.32134
1200.0	.66476	.56385	.50483	.46296	.43049	.40398	.38157	.36217	.34507	.32979
1300.0	.66858	.56882	.51047	.46908	.43698	.41076	.38860	.36942	.35250	.33739
1400.0	.67205	.57333	.51558	.47462	.44285	.41690	.39497	.37598	.35924	.34427
1500.0	.67520	.57743	.52025	.47968	.44821	.42251	.40079	.38198	.36539	.35057
1600.0	.67811	.58121	.52453	.48432	.45314	.42766	.40613	.38749	.37105	.35635

(b/a) = 12800.

$\left(\frac{\sqrt{4\alpha t}}{a}\right)$ \ $\left(\frac{r}{a}\right)$	10.	20.	30.	40.	50.	60.	70.	80.	90.	100.
1600.0	.67811	.58121	.52453	.48432	.45314	.42767	.40613	.38749	.37105	.35635
1700.0	.68079	.58469	.52849	.48861	.45769	.43242	.41107	.39258	.37627	.36169
1800.0	.68327	.58793	.53216	.49259	.46191	.43684	.41565	.39730	.38111	.36664
1900.0	.68559	.59094	.53558	.49630	.46584	.44095	.41992	.40170	.38563	.37126
2000.0	.68775	.59376	.53878	.49977	.46952	.44480	.42391	.40582	.38986	.37559
2100.0	.68979	.59640	.54178	.50303	.47297	.44842	.42766	.40968	.39383	.37965
2200.0	.69170	.59890	.54461	.50610	.47622	.45182	.43119	.41332	.39756	.38347
2300.0	.69351	.60125	.54728	.50899	.47929	.45503	.43452	.41676	.40109	.38708
2400.0	.69522	.60347	.54981	.51173	.48220	.45807	.43768	.42001	.40443	.39049
2500.0	.69684	.60559	.55221	.51433	.48496	.46096	.44067	.42310	.40760	.39374
2600.0	.69839	.60759	.55449	.51681	.48758	.46370	.44352	.42603	.41061	.39682
2700.0	.69986	.60951	.55666	.51916	.49008	.46632	.44623	.42883	.41348	.39976
2800.0	.70126	.61134	.55873	.52141	.49246	.46881	.44882	.43150	.41623	.40257
2900.0	.70261	.61308	.56072	.52356	.49474	.47120	.45129	.43405	.41885	.40525
3000.0	.70389	.61476	.56261	.52562	.49693	.47349	.45367	.43650	.42136	.40782
3100.0	.70513	.61636	.56444	.52760	.49902	.47568	.45594	.43884	.42377	.41028
3200.0	.70631	.61790	.56619	.52949	.50103	.47778	.45812	.44110	.42608	.41265

14.

Table 3 (y/y_0)

(b/a) 25600.

15.

$\left(\frac{\sqrt{4\alpha t}}{a}\right)$	10.	20.	30.	40.	50. ($\frac{r}{a}$)	60.	70.	80.	90.	100.
3200.0	.70631	.61790	.56619	.52949	.50104	.47778	.45813	.44110	.42608	.41265
3300.0	.70745	.61938	.56787	.53132	.50297	.47981	.46023	.44326	.42830	.41492
3400.0	.70855	.62081	.56949	.53308	.50483	.48176	.46225	.44535	.43045	.41712
3500.0	.70960	.62219	.57105	.53477	.50663	.48364	.46420	.44736	.43251	.41923
3600.0	.71062	.62351	.57256	.53640	.50836	.48545	.46608	.44930	.43450	.42127
3700.0	.71161	.62479	.57401	.53798	.51003	.48720	.46790	.45118	.43643	.42324
3800.0	.71256	.62603	.57542	.53951	.51165	.48890	.46965	.45299	.43829	.42514
3900.0	.71348	.62723	.57678	.54098	.51322	.49053	.47135	.45474	.44009	.42698
4000.0	.71438	.62839	.57810	.54241	.51474	.49212	.47300	.45644	.44183	.42877
4100.0	.71524	.62952	.57938	.54380	.51621	.49366	.47460	.45809	.44352	.43050
4200.0	.71608	.63061	.58062	.54515	.51763	.49515	.47615	.45969	.44516	.43218
4300.0	.71690	.63167	.58182	.54645	.51902	.49660	.47765	.46124	.44676	.43381
4400.0	.71769	.63270	.58299	.54772	.52036	.49801	.47911	.46274	.44831	.43539
4500.0	.71846	.63371	.58413	.54895	.52167	.49938	.48053	.46421	.44981	.43693
4600.0	.71921	.63468	.58524	.55015	.52294	.50071	.48191	.46563	.45127	.43843
4700.0	.71994	.63563	.58631	.55132	.52418	.50201	.48326	.46702	.45270	.43988
4800.0	.72065	.63655	.58736	.55246	.52539	.50327	.48457	.46837	.45408	.44130
4900.0	.72134	.63745	.58838	.55357	.52656	.50450	.48585	.46969	.45544	.44269
5000.0	.72201	.63833	.58938	.55465	.52771	.50570	.48709	.47097	.45675	.44404
5100.0	.72267	.63919	.59035	.55570	.52883	.50687	.48831	.47222	.45804	.44535
5200.0	.72331	.64002	.59130	.55673	.52992	.50801	.48949	.47345	.45930	.44664
5300.0	.72394	.64084	.59223	.55774	.53099	.50913	.49065	.47464	.46052	.44789
5400.0	.72455	.64164	.59313	.55872	.53203	.51022	.49178	.47581	.46172	.44912
5500.0	.72515	.64242	.59402	.55968	.53305	.51128	.49288	.47695	.46289	.45032
5600.0	.72574	.64318	.59488	.56062	.53404	.51233	.49397	.47806	.46403	.45149
5700.0	.72631	.64392	.59573	.56154	.53501	.51334	.49502	.47915	.46515	.45263
5800.0	.72687	.64465	.59656	.56243	.53597	.51434	.49606	.48022	.46625	.45375
5900.0	.72742	.64537	.59737	.56331	.53690	.51532	.49707	.48126	.46732	.45485
6000.0	.72796	.64607	.59816	.56417	.53781	.51627	.49806	.48229	.46837	.45592
6100.0	.72849	.64675	.59894	.56502	.53871	.51721	.49903	.48329	.46940	.45698
6200.0	.72900	.64742	.59970	.56585	.53958	.51813	.49998	.48427	.47041	.45801
6300.0	.72951	.64808	.60045	.56666	.54044	.51903	.50092	.48523	.47140	.45902
6400.0	.73000	.64873	.60118	.56745	.54129	.51991	.50183	.48618	.47237	.46001
6500.0	.73049	.64936	.60190	.56823	.54211	.52077	.50273	.48710	.47332	.46099
6600.0	.73097	.64998	.60261	.56900	.54292	.52162	.50361	.48801	.47425	.46194
6700.0	.73144	.65059	.60330	.56975	.54372	.52246	.50448	.48890	.47517	.46288
6800.0	.73190	.65119	.60398	.57048	.54450	.52327	.50533	.48978	.47607	.46380
6900.0	.73235	.65178	.60465	.57121	.54527	.52408	.50616	.49064	.47695	.46470
7000.0	.73279	.65236	.60530	.57192	.54602	.52487	.50698	.49148	.47782	.46559
7100.0	.73323	.65292	.60595	.57262	.54677	.52564	.50778	.49231	.47867	.46646
7200.0	.73366	.65348	.60658	.57330	.54749	.52641	.50857	.49313	.47951	.46732
7300.0	.73408	.65403	.60720	.57398	.54821	.52715	.50935	.49393	.48033	.46816
7400.0	.73449	.65457	.60782	.57464	.54891	.52789	.51012	.49472	.48114	.46899
7500.0	.73490	.65510	.60842	.57530	.54961	.52862	.51087	.49550	.48194	.46981
7600.0	.73530	.65562	.60901	.57594	.55029	.52933	.51161	.49626	.48272	.47061
7700.0	.73570	.65613	.60959	.57657	.55096	.53003	.51234	.49701	.48349	.47140
7800.0	.73609	.65664	.61017	.57719	.55162	.53072	.51305	.49775	.48425	.47217
7900.0	.73647	.65714	.61073	.57781	.55227	.53140	.51376	.49848	.48500	.47294
8000.0	.73684	.65763	.61129	.57841	.55291	.53207	.51445	.49919	.48573	.47369
8100.0	.73721	.65811	.61183	.57900	.55354	.53273	.51514	.49990	.48646	.47443
8200.0	.73758	.65858	.61237	.57959	.55416	.53338	.51581	.50059	.48717	.47516
8300.0	.73794	.65905	.61291	.58016	.55477	.53402	.51647	.50128	.48787	.47588

Table 3. (y/y_0)

(b/a) 25600.

$\left(\frac{\sqrt{4\alpha t}}{a}\right)$	(r/a) 10.	20.	30.	40.	50.	60.	70.	80.	90.	100.
8400.0	.73829	.65951	.61343	.58073	.55537	.53465	.51713	.50195	.48857	.47659
8500.0	.73864	.65997	.61395	.58129	.55596	.53527	.51777	.50262	.48925	.47729
8600.0	.73899	.66042	.61445	.58184	.55655	.53588	.51841	.50327	.48992	.47798
8700.0	.73933	.66086	.61496	.58239	.55713	.53649	.51904	.50392	.49059	.47866
8800.0	.73966	.66129	.61545	.58292	.55769	.53708	.51965	.50456	.49124	.47933
8900.0	.73999	.66172	.61594	.58345	.55826	.53767	.52026	.50518	.49188	.47999
9000.0	.74032	.66215	.61642	.58397	.55881	.53825	.52086	.50580	.49252	.48064
9100.0	.74064	.66256	.61689	.58449	.55935	.53882	.52146	.50641	.49315	.48128
9200.0	.74096	.66298	.61736	.58500	.55989	.53938	.52204	.50702	.49377	.48191
9300.0	.74127	.66338	.61782	.58550	.56042	.53994	.52262	.50761	.49438	.48254
9400.0	.74158	.66378	.61828	.58599	.56095	.54049	.52319	.50820	.49498	.48316
9500.0	.74188	.66418	.61873	.58648	.56147	.54103	.52375	.50878	.49558	.48377
9600.0	.74218	.66457	.61917	.58696	.56198	.54156	.52430	.50935	.49616	.48437
9700.0	.74248	.66496	.61961	.58744	.56248	.54209	.52485	.50992	.49675	.48496
9800.0	.74277	.66534	.62004	.58791	.56298	.54261	.52539	.51048	.49732	.48555
9900.0	.74306	.66572	.62047	.58837	.56347	.54313	.52593	.51103	.49788	.48613
10000.0	.74335	.66609	.62090	.58883	.56396	.54364	.52645	.51157	.49844	.48670

16.

Table 3. (y/y_0)

(b/a) 200.

$\left(\frac{\sqrt{4\alpha t}}{a}\right)$	(r/a) 100.	200.	300.	400.	500.	600.	700.	800.	900.	1000.
25.0	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
26.0	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
27.0	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
28.0	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
29.0	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
30.0	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
31.0	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
32.0	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
33.0	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
34.0	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
35.0	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
36.0	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
37.0	.00002	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
38.0	.00003	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
39.0	.00005	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
40.0	.00006	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
41.0	.00009	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
42.0	.00012	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
43.0	.00016	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
44.0	.00021	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
45.0	.00027	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
46.0	.00034	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
47.0	.00042	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
48.0	.00052	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
49.0	.00063	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
50.0	.00076	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000

$\left(\frac{\sqrt{4\alpha t}}{a}\right)$ \ $\left(\frac{r}{a}\right)$	100.	200.	300.	400.	500.	600.	700.	800.	900.	1000.
50.0	.00076	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
51.0	.00090	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
52.0	.00106	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
53.0	.00124	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
54.0	.00144	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
55.0	.00166	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
56.0	.00190	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
57.0	.00217	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
58.0	.00245	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
59.0	.00276	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
60.0	.00308	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
61.0	.00343	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
62.0	.00380	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
63.0	.00420	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
64.0	.00461	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
65.0	.00505	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
66.0	.00550	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
67.0	.00598	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
68.0	.00648	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
69.0	.00700	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
70.0	.00754	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
71.0	.00810	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
72.0	.00867	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
73.0	.00927	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
74.0	.00988	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
75.0	.01051	.00002	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
76.0	.01116	.00002	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
77.0	.01182	.00003	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
78.0	.01250	.00003	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
79.0	.01319	.00004	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
80.0	.01390	.00005	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
81.0	.01462	.00006	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
82.0	.01536	.00007	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
83.0	.01610	.00008	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
84.0	.01686	.00009	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
85.0	.01764	.00011	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
86.0	.01842	.00012	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
87.0	.01921	.00014	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
88.0	.02002	.00016	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
89.0	.02083	.00018	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
90.0	.02165	.00021	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
91.0	.02248	.00023	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
92.0	.02332	.00026	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
93.0	.02416	.00029	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
94.0	.02502	.00033	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
95.0	.02587	.00036	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
96.0	.02674	.00040	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
97.0	.02761	.00045	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
98.0	.02849	.00049	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
99.0	.02937	.00054	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
100.0	.03026	.00059	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000

400. (b/a)

18.

Table 3. (y/y_0)

(r/a)

$\left(\frac{\sqrt{4\alpha t}}{a}\right)$	100.	200.	300.	400.	500.	600.	700.	800.	900.	1000.
100.0	.03026	.00059	.00004	.00000	.00000	.00000	.00000	.00000	.00000	.00000
110.0	.03930	.00132	.00003	.00000	.00000	.00000	.00000	.00000	.00000	.00000
120.0	.04849	.00248	.00005	.00000	.00000	.00000	.00000	.00000	.00000	.00000
130.0	.05761	.00410	.00012	.00000	.00000	.00000	.00000	.00000	.00000	.00000
140.0	.06654	.00616	.00027	.00000	.00000	.00000	.00000	.00000	.00000	.00000
150.0	.07520	.00865	.00053	.00001	.00000	.00000	.00000	.00000	.00000	.00000
160.0	.08355	.01150	.00092	.00004	.00000	.00000	.00000	.00000	.00000	.00000
170.0	.09157	.01466	.00149	.00009	.00000	.00000	.00000	.00000	.00000	.00000
180.0	.09925	.01807	.00222	.00017	.00001	.00000	.00000	.00000	.00000	.00000
190.0	.10659	.02169	.00315	.00030	.00002	.00000	.00000	.00000	.00000	.00000
200.0	.11361	.02545	.00427	.00049	.00004	.00000	.00000	.00000	.00000	.00000

800.

(b/a)

	100.	200.	300.	400.	500.	600.	700.	800.	900.	1000.
200.0	.11361	.02545	.00427	.00047	.00004	.00004	.00000	.00000	.00000	.00000
210.0	.12032	.02931	.00557	.00075	.00007	.00002	.00000	.00000	.00000	.00000
220.0	.12674	.03325	.00704	.00110	.00012	.00002	.00000	.00000	.00000	.00000
230.0	.13287	.03724	.00867	.00154	.00020	.00003	.00000	.00000	.00000	.00000
240.0	.13874	.04123	.01044	.00207	.00031	.00004	.00000	.00000	.00000	.00000
250.0	.14437	.04523	.01235	.00271	.00046	.00006	.00001	.00000	.00000	.00000
260.0	.14975	.04920	.01436	.00344	.00065	.00010	.00001	.00000	.00000	.00000
270.0	.15492	.05314	.01648	.00428	.00090	.00015	.00002	.00000	.00000	.00000
280.0	.15988	.05704	.01868	.00521	.00120	.00022	.00003	.00000	.00000	.00000
290.0	.16465	.06088	.02095	.00623	.00155	.00032	.00005	.00001	.00000	.00000
300.0	.16924	.06467	.02327	.00734	.00197	.00044	.00008	.00001	.00000	.00000
310.0	.17365	.06840	.02565	.00853	.00245	.00059	.00012	.00002	.00000	.00000
320.0	.17791	.07206	.02806	.00981	.00298	.00078	.00017	.00003	.00000	.00000
330.0	.18201	.07565	.03050	.01114	.00358	.00100	.00024	.00005	.00001	.00000
340.0	.18596	.07917	.03296	.01254	.00424	.00126	.00032	.00007	.00001	.00000
350.0	.18979	.08263	.03543	.01400	.00496	.00155	.00043	.00010	.00002	.00000
360.0	.19348	.08601	.03791	.01550	.00573	.00189	.00055	.00014	.00003	.00001
370.0	.19705	.08932	.04038	.01706	.00656	.00227	.00070	.00019	.00005	.00001
380.0	.20051	.09257	.04286	.01865	.00745	.00269	.00087	.00025	.00006	.00001
390.0	.20386	.09575	.04532	.02028	.00838	.00315	.00107	.00033	.00009	.00002
400.0	.20711	.09886	.04777	.02194	.00935	.00365	.00129	.00041	.00012	.00003

1600.

19.

Table 3. (y/y_0)

$\left(\frac{r}{a}\right)$

$\left(\frac{\sqrt{4\alpha t}}{a}\right)$	100.	200.	300.	400.	500.	600.	700.	800.	900.	1000.
400.0	.20711	.09886	.04777	.02194	.00936	.00365	.00129	.00040	.00012	.00003
410.0	.21026	.10190	.05021	.02363	.01038	.00419	.00155	.00051	.00016	.00005
420.0	.21332	.10488	.05263	.02534	.01144	.00477	.00183	.00063	.00020	.00006
430.0	.21629	.10780	.05503	.02706	.01253	.00539	.00214	.00078	.00026	.00008
440.0	.21918	.11065	.05741	.02880	.01366	.00605	.00248	.00094	.00033	.00010
450.0	.22198	.11344	.05976	.03056	.01483	.00674	.00285	.00112	.00039	.00013
460.0	.22472	.11618	.06207	.03232	.01602	.00747	.00325	.00132	.00049	.00017
470.0	.22737	.11886	.06438	.03408	.01723	.00823	.00368	.00154	.00059	.00022
480.0	.22996	.12148	.06667	.03585	.01847	.00902	.00414	.00178	.00071	.00027
490.0	.23248	.12405	.06892	.03762	.01973	.00984	.00463	.00204	.00084	.00033
500.0	.23494	.12657	.07115	.03939	.02101	.01069	.00514	.00233	.00099	.00040
510.0	.23734	.12904	.07335	.04115	.02231	.01156	.00568	.00264	.00115	.00047
520.0	.23968	.13145	.07552	.04291	.02362	.01245	.00625	.00297	.00133	.00056
530.0	.24196	.13382	.07766	.04467	.02494	.01337	.00684	.00332	.00153	.00066
540.0	.24419	.13615	.07977	.04642	.02627	.01431	.00745	.00370	.00174	.00077
550.0	.24637	.13842	.08186	.04816	.02761	.01527	.00810	.00409	.00197	.00089
560.0	.24850	.14066	.08391	.04989	.02896	.01625	.00876	.00451	.00221	.00103
570.0	.25059	.14285	.08594	.05161	.03032	.01724	.00944	.00494	.00247	.00118
580.0	.25262	.14500	.08794	.05332	.03168	.01825	.01014	.00540	.00275	.00134
590.0	.25462	.14711	.08992	.05502	.03304	.01927	.01086	.00588	.00305	.00151
600.0	.25657	.14918	.09186	.05671	.03440	.02031	.01159	.00639	.00337	.00170
610.0	.25848	.15122	.09378	.05838	.03577	.02135	.01235	.00690	.00370	.00190
620.0	.26035	.15321	.09567	.06004	.03713	.02241	.01312	.00743	.00404	.00212
630.0	.26218	.15517	.09754	.06169	.03850	.02347	.01391	.00798	.00441	.00235
640.0	.26398	.15710	.09938	.06332	.03986	.02455	.01470	.00854	.00479	.00259
650.0	.26574	.15899	.10120	.06494	.04122	.02563	.01552	.00912	.00518	.00284
660.0	.26747	.16085	.10299	.06654	.04257	.02671	.01634	.00972	.00559	.00311
670.0	.26916	.16268	.10475	.06813	.04392	.02780	.01718	.01033	.00602	.00339
680.0	.27083	.16448	.10649	.06971	.04527	.02890	.01803	.01095	.00646	.00369
690.0	.27246	.16625	.10821	.07127	.04662	.02999	.01888	.01159	.00691	.00400
700.0	.27406	.16799	.10991	.07282	.04795	.03110	.01975	.01224	.00738	.00432
710.0	.27564	.16970	.11158	.07435	.04928	.03220	.02062	.01290	.00786	.00465
720.0	.27718	.17138	.11323	.07586	.05061	.03330	.02150	.01357	.00835	.00500
730.0	.27870	.17304	.11485	.07737	.05193	.03441	.02239	.01425	.00886	.00536
740.0	.28019	.17467	.11646	.07885	.05324	.03551	.02328	.01495	.00937	.00573
750.0	.28166	.17627	.11804	.08033	.05454	.03662	.02418	.01565	.00990	.00611
760.0	.28310	.17785	.11960	.08178	.05584	.03772	.02508	.01636	.01044	.00651
770.0	.28452	.17940	.12115	.08323	.05712	.03882	.02599	.01708	.01099	.00691
780.0	.28592	.18094	.12267	.08465	.05840	.03993	.02690	.01781	.01155	.00733
790.0	.28729	.18244	.12417	.08607	.05968	.04103	.02782	.01854	.01212	.00776
800.0	.28864	.18393	.12565	.08747	.06094	.04212	.02874	.01929	.01270	.00819

3200.

20.

Table 3 (y/y)

(b/a) 6400.

$\left(\frac{\sqrt{4\alpha t}}{a}\right)$ \ $\left(\frac{r}{a}\right)$	100.	200.	300.	400.	500.	600.	700.	800.	900.	1000.
800.0	.28864	.18393	.12565	.08747	.06094	.04212	.02874	.01929	.01270	.00820
900.0	.30107	.19767	.13949	.10070	.07308	.05289	.03797	.02696	.01890	.01305
1000.0	.31185	.20970	.15176	.11265	.08433	.06316	.04711	.03488	.02558	.01856
1100.0	.32134	.22034	.16273	.12348	.09471	.07285	.05594	.04276	.03248	.02447
1200.0	.32979	.22985	.17261	.13334	.10429	.08194	.06439	.05048	.03939	.03056
1300.0	.33739	.23844	.18157	.14236	.11313	.09044	.07242	.05793	.04621	.03670
1400.0	.34427	.24624	.18976	.15065	.12132	.09839	.08001	.06508	.05285	.04279
1500.0	.35057	.25338	.19728	.15829	.12893	.10583	.08719	.07191	.05926	.04875
1600.0	.35635	.25996	.20421	.16538	.13602	.11281	.09397	.07842	.06544	.05456

12800.

	100.	200.	300.	400.	500.	600.	700.	800.	900.	1000.
1600.0	.35635	.25996	.20421	.16538	.13602	.11281	.09397	.07842	.06544	.05456
1700.0	.36169	.26604	.21065	.17198	.14265	.11937	.10038	.08461	.07137	.06018
1800.0	.36664	.27169	.21664	.17813	.14885	.12554	.10644	.09051	.07706	.06561
1900.0	.37126	.27696	.22224	.18390	.15468	.13136	.11218	.09613	.08250	.07085
2000.0	.37559	.28190	.22749	.18932	.16018	.13685	.11763	.10147	.08771	.07589
2100.0	.37965	.28654	.23242	.19442	.16536	.14206	.12280	.10657	.09270	.08073
2200.0	.38347	.29091	.23708	.19924	.17027	.14700	.12772	.11144	.09748	.08539
2300.0	.38708	.29504	.24148	.20381	.17493	.15169	.13241	.11608	.10205	.08987
2400.0	.39049	.29895	.24566	.20814	.17935	.15616	.13688	.12053	.10644	.09418
2500.0	.39374	.30267	.24963	.21226	.18357	.16042	.14116	.12478	.11065	.09833
2600.0	.39682	.30620	.25340	.21619	.18758	.16449	.14524	.12886	.11470	.10232
2700.0	.39976	.30957	.25700	.21993	.19142	.16838	.14916	.13278	.11859	.10617
2800.0	.40257	.31278	.26044	.22352	.19510	.17211	.15291	.13653	.12233	.10988
2900.0	.40525	.31586	.26373	.22694	.19862	.17568	.15652	.14015	.12594	.11345
3000.0	.40782	.31880	.26689	.23023	.20199	.17912	.15999	.14363	.12941	.11691
3100.0	.41028	.32163	.26992	.23339	.20524	.18242	.16332	.14698	.13276	.12024
3200.0	.41265	.32434	.27283	.23643	.20836	.18560	.16654	.15021	.13600	.12347

12

Table 3. (y/y_0)

(b/a) 25600.

$\left(\frac{\sqrt{4\alpha t}}{a}\right)$	$\left(\frac{r}{a}\right)$ 100.	200.	300.	400.	500.	600.	700.	800.	900.	1000.
3200.0	.41265	.32435	.27283	.23643	.20836	.18560	.16654	.15021	.13600	.12347
3300.0	.41492	.32696	.27563	.23935	.21137	.18866	.16964	.15333	.13912	.12659
3400.0	.41712	.32947	.27832	.24217	.21426	.19162	.17263	.15635	.14215	.12961
3500.0	.41923	.33190	.28092	.24488	.21706	.19447	.17552	.15926	.14507	.13253
3600.0	.42127	.33424	.28343	.24750	.21976	.19723	.17832	.16208	.14791	.13537
3700.0	.42324	.33650	.28586	.25004	.22238	.19990	.18103	.16482	.15065	.13812
3800.0	.42514	.33869	.28820	.25249	.22490	.20248	.18365	.16746	.15331	.14079
3900.0	.42698	.34080	.29047	.25487	.22735	.20499	.18619	.17003	.15590	.14338
4000.0	.42877	.34285	.29268	.25717	.22973	.20741	.18866	.17252	.15841	.14590
4100.0	.43050	.34484	.29481	.25940	.23203	.20977	.19105	.17495	.16085	.14835
4200.0	.43218	.34677	.29688	.26157	.23427	.21206	.19338	.17730	.16322	.15073
4300.0	.43381	.34864	.29889	.26367	.23644	.21428	.19564	.17959	.16553	.15305
4400.0	.43539	.35046	.30084	.26571	.23855	.21644	.19784	.18181	.16777	.15531
4500.0	.43693	.35223	.30274	.26770	.24060	.21854	.19998	.18398	.16996	.15751
4600.0	.43843	.35395	.30459	.26964	.24260	.22059	.20206	.18609	.17209	.15966
4700.0	.43988	.35562	.30639	.27152	.24455	.22258	.20409	.18815	.17417	.16175
4800.0	.44130	.35725	.30814	.27336	.24644	.22453	.20607	.19016	.17620	.16379
4900.0	.44269	.35884	.30985	.27515	.24829	.22642	.20800	.19211	.17818	.16579
5000.0	.44404	.36039	.31152	.27689	.25010	.22827	.20988	.19403	.18011	.16773
5100.0	.44535	.36191	.31314	.27859	.25186	.23008	.21172	.19589	.18199	.16963
5200.0	.44664	.36338	.31473	.28026	.25358	.23184	.21352	.19771	.18384	.17149
5300.0	.44789	.36482	.31628	.28188	.25526	.23356	.21527	.19950	.18564	.17331
5400.0	.44912	.36623	.31779	.28347	.25690	.23524	.21699	.20124	.18740	.17509
5500.0	.45032	.36761	.31927	.28502	.25850	.23689	.21867	.20294	.18913	.17683
5600.0	.45149	.36896	.32072	.28654	.26007	.23850	.22031	.20461	.19082	.17853
5700.0	.45263	.37027	.32213	.28802	.26161	.24007	.22192	.20624	.19247	.18020
5800.0	.45375	.37156	.32352	.28947	.26311	.24161	.22349	.20784	.19409	.18184
5900.0	.45485	.37282	.32487	.29089	.26458	.24312	.22503	.20940	.19567	.18344
6000.0	.45592	.37406	.32620	.29229	.26602	.24460	.22654	.21094	.19722	.18501
6100.0	.45698	.37527	.32750	.29365	.26743	.24605	.22802	.21244	.19875	.18655
6200.0	.45801	.37646	.32878	.29499	.26882	.24747	.22947	.21392	.20024	.18806
6300.0	.45902	.37762	.33003	.29630	.27017	.24887	.23089	.21536	.20171	.18954
6400.0	.46001	.37876	.33126	.29759	.27151	.25023	.23229	.21678	.20315	.19099
6500.0	.46099	.37988	.33246	.29885	.27281	.25157	.23366	.21817	.20456	.19242
6600.0	.46194	.38098	.33364	.30009	.27409	.25289	.23500	.21954	.20594	.19382
6700.0	.46288	.38205	.33480	.30130	.27535	.25418	.23632	.22088	.20730	.19519
6800.0	.46380	.38311	.33594	.30250	.27659	.25545	.23762	.22220	.20864	.19655
6900.0	.46470	.38415	.33706	.30367	.27780	.25670	.23889	.22350	.20995	.19787
7000.0	.46559	.38517	.33815	.30482	.27899	.25792	.24014	.22477	.21124	.19918
7100.0	.46646	.38617	.33923	.30595	.28017	.25913	.24137	.22602	.21251	.20046
7200.0	.46732	.38716	.34029	.30706	.28132	.26031	.24258	.22725	.21376	.20173
7300.0	.46816	.38813	.34133	.30816	.28245	.26147	.24377	.22846	.21499	.20297
7400.0	.46899	.38908	.34236	.30923	.28356	.26262	.24494	.22965	.21620	.20419
7500.0	.46981	.39002	.34337	.31029	.28466	.26374	.24609	.23082	.21738	.20539
7600.0	.47061	.39094	.34436	.31133	.28574	.26485	.24722	.23197	.21855	.20658
7700.0	.47140	.39185	.34534	.31236	.28680	.26594	.24833	.23311	.21970	.20774
7800.0	.47217	.39274	.34630	.31336	.28784	.26701	.24943	.23422	.22084	.20889
7900.0	.47294	.39362	.34724	.31436	.28887	.26807	.25051	.23532	.22195	.21002
8000.0	.47369	.39449	.34817	.31533	.28988	.26911	.25157	.23640	.22305	.21113
8100.0	.47443	.39534	.34909	.31630	.29088	.27013	.25262	.23747	.22413	.21223
8200.0	.47516	.39618	.34999	.31724	.29186	.27114	.25365	.23852	.22520	.21331
8300.0	.47588	.39701	.35088	.31818	.29283	.27214	.25467	.23955	.22625	.21437

22.

Table 3. (y/y_0)

(b/a) 25600.

$\left(\frac{\sqrt{4\alpha t}}{a}\right)$	$(\frac{r}{a})$ 100.	200.	300.	400.	500.	600.	700.	800.	900.	1000.
8400.0	.47659	.39782	.35176	.31910	.29378	.27312	.25567	.24058	.22728	.21542
8500.0	.47729	.39863	.35263	.32001	.29472	.27408	.25666	.24158	.22831	.21645
8600.0	.47798	.39942	.35348	.32090	.29565	.27504	.25763	.24257	.22931	.21747
8700.0	.47866	.40020	.35432	.32178	.29656	.27598	.25859	.24355	.23030	.21848
8800.0	.47933	.40097	.35515	.32265	.29746	.27690	.25954	.24451	.23128	.21947
8900.0	.47999	.40173	.35596	.32351	.29835	.27781	.26047	.24546	.23225	.22045
9000.0	.48064	.40248	.35677	.32435	.29923	.27872	.26139	.24640	.23320	.22141
9100.0	.48128	.40322	.35756	.32519	.30009	.27960	.26230	.24733	.23414	.22237
9200.0	.48191	.40394	.35835	.32601	.30095	.28048	.26320	.24824	.23507	.22331
9300.0	.48254	.40466	.35912	.32682	.30179	.28135	.26408	.24914	.23599	.22423
9400.0	.48316	.40537	.35989	.32763	.30262	.28220	.26496	.25003	.23689	.22515
9500.0	.48377	.40608	.36064	.32842	.30344	.28304	.26582	.25091	.23778	.22605
9600.0	.48437	.40677	.36138	.32920	.30425	.28388	.26667	.25178	.23866	.22695
9700.0	.48496	.40745	.36212	.32997	.30505	.28470	.26751	.25264	.23953	.22783
9800.0	.48555	.40812	.36285	.33073	.30584	.28551	.26834	.25348	.24039	.22870
9900.0	.48613	.40879	.36356	.33148	.30662	.28631	.26916	.25432	.24124	.22956
10000.0	.48670	.40945	.36427	.33223	.30739	.28710	.26997	.25514	.24208	.23041

Table 3. (y/y_0)

$\left(\frac{r}{a}\right)$

$\left(\frac{\sqrt{4\alpha t}}{a}\right)$	1000.	2000.	3000.	4000.	5000.	6000.	7000.	8000.	9000.	10000.
200.0	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
210.0	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
220.0	.00002	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
230.0	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
240.0	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
250.0	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
260.0	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
270.0	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
280.0	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
290.0	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
300.0	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
310.0	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
320.0	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
330.0	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
340.0	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
350.0	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
360.0	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
370.0	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
380.0	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
390.0	.00002	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
400.0	.00003	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000

(b/a) 1600.

24.

Table 3 (y/y_0)

(b/a) 3200.

25.

$\left(\frac{\sqrt{4\alpha t}}{a}\right)$ \ $\left(\frac{r}{a}\right)$	1000.	2000.	3000.	4000.	5000.	6000.	7000.	8000.	9000.	10000.
400.0	.00003	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
410.0	.00005	.00003	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
420.0	.00006	.00002	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
430.0	.00008	.00002	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
440.0	.00010	.00002	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
450.0	.00013	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
460.0	.00017	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
470.0	.00022	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
480.0	.00027	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
490.0	.00033	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
500.0	.00040	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
510.0	.00047	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
520.0	.00056	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
530.0	.00066	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
540.0	.00077	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
550.0	.00089	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
560.0	.00103	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
570.0	.00118	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
580.0	.00134	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
590.0	.00151	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
600.0	.00170	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
610.0	.00190	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
620.0	.00212	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
630.0	.00235	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
640.0	.00259	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
650.0	.00284	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
660.0	.00311	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
670.0	.00339	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
680.0	.00369	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
690.0	.00400	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
700.0	.00432	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
710.0	.00465	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
720.0	.00500	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
730.0	.00536	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
740.0	.00573	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
750.0	.00611	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
760.0	.00651	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
770.0	.00691	.00002	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
780.0	.00733	.00002	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
790.0	.00776	.00002	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
800.0	.00819	.00003	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000

Table 3. (y/y_0)

(b/a) 6400.

$\left(\frac{\sqrt{4\alpha t}}{a}\right)$ \ (r/a)	1000.	2000.	3000.	4000.	5000.	6000.	7000.	8000.	9000.	10000.
800.0	.00820	.00003	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
900.0	.01305	.00012	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
1000.0	.01856	.00035	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
1100.0	.02447	.00079	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000
1200.0	.03056	.00150	.00003	.00000	.00000	.00000	.00000	.00000	.00000	.00000
1300.0	.03670	.00251	.00007	.00000	.00000	.00000	.00000	.00000	.00000	.00000
1400.0	.04279	.00363	.00016	.00000	.00000	.00000	.00000	.00000	.00000	.00000
1500.0	.04875	.00543	.00032	.00001	.00000	.00000	.00000	.00000	.00000	.00000
1600.0	.05456	.00729	.00057	.00002	.00000	.00000	.00000	.00000	.00000	.00000

(b/a) 12800.

$\left(\frac{\sqrt{4\alpha t}}{a}\right)$ \ (r/a)	1000.	2000.	3000.	4000.	5000.	6000.	7000.	8000.	9000.	10000.
1600.0	.05456	.00729	.00057	.00003	.00000	.00000	.00000	.00000	.00000	.00000
1700.0	.06018	.00938	.00093	.00005	.00000	.00000	.00000	.00000	.00000	.00000
1800.0	.06561	.01165	.00140	.00010	.00000	.00000	.00000	.00000	.00000	.00000
1900.0	.07085	.01407	.00200	.00019	.00001	.00000	.00000	.00000	.00000	.00000
2000.0	.07589	.01662	.00273	.00031	.00002	.00000	.00000	.00000	.00000	.00000
2100.0	.08073	.01926	.00359	.00047	.00004	.00000	.00000	.00000	.00000	.00000
2200.0	.08539	.02196	.00456	.00070	.00008	.00001	.00000	.00000	.00000	.00000
2300.0	.08987	.02471	.00565	.00099	.00013	.00002	.00000	.00000	.00000	.00000
2400.0	.09418	.02749	.00685	.00134	.00019	.00002	.00000	.00000	.00000	.00000
2500.0	.09833	.03028	.00814	.00176	.00029	.00004	.00000	.00000	.00000	.00000
2600.0	.10232	.03308	.00951	.00225	.00042	.00006	.00001	.00000	.00000	.00000
2700.0	.10617	.03586	.01096	.00281	.00058	.00010	.00001	.00000	.00000	.00000
2800.0	.10988	.03862	.01247	.00343	.00078	.00014	.00002	.00000	.00000	.00000
2900.0	.11345	.04136	.01404	.00413	.00102	.00021	.00003	.00000	.00000	.00000
3000.0	.11691	.04407	.01566	.00488	.00129	.00029	.00005	.00001	.00000	.00000
3100.0	.12024	.04673	.01732	.00569	.00161	.00039	.00008	.00001	.00000	.00000
3200.0	.12347	.04937	.01901	.00656	.00198	.00051	.00011	.00002	.00000	.00000

Table 3. (y/y_0).

$\left(\frac{\sqrt{4\alpha t}}{a}\right)$ \ $\left(\frac{r}{a}\right)$	1000.	2000.	3000.	4000.	5000.	6000.	7000.	8000.	9000.	10000.
3200.0	.12347	.04938	.01901	.00657	.00206	.00051	.00009	.00002	.00000	.00000
3300.0	.12659	.05197	.02072	.00749	.00244	.00066	.00014	.00003	.00001	.00000
3400.0	.12961	.05453	.02246	.00846	.00288	.00083	.00020	.00005	.00001	.00000
3500.0	.13253	.05705	.02421	.00947	.00335	.00103	.00027	.00007	.00002	.00000
3600.0	.13537	.05952	.02597	.01052	.00388	.00126	.00036	.00009	.00002	.00000
3700.0	.13812	.06195	.02773	.01161	.00444	.00152	.00046	.00013	.00003	.00001
3800.0	.14079	.06433	.02950	.01273	.00505	.00180	.00058	.00017	.00004	.00001
3900.0	.14338	.06667	.03127	.01387	.00569	.00212	.00071	.00022	.00006	.00002
4000.0	.14590	.06897	.03303	.01504	.00637	.00246	.00087	.00028	.00008	.00002
4100.0	.14835	.07122	.03479	.01624	.00708	.00284	.00104	.00034	.00010	.00003
4200.0	.15073	.07343	.03654	.01745	.00782	.00324	.00123	.00043	.00014	.00004
4300.0	.15305	.07560	.03828	.01868	.00859	.00367	.00145	.00052	.00017	.00005
4400.0	.15531	.07773	.04001	.01949	.00938	.00412	.00168	.00063	.00022	.00007
4500.0	.15751	.07982	.04172	.02081	.01020	.00461	.00194	.00075	.00027	.00009
4600.0	.15966	.08187	.04342	.02212	.01104	.00512	.00221	.00089	.00033	.00011
4700.0	.16175	.08388	.04511	.02344	.01191	.00565	.00251	.00104	.00040	.00014
4800.0	.16379	.08586	.04678	.02476	.01279	.00620	.00283	.00121	.00048	.00018
4900.0	.16579	.08779	.04844	.02607	.01369	.00678	.00317	.00139	.00057	.00021
5000.0	.16773	.08969	.05007	.02738	.01460	.00738	.00353	.00159	.00067	.00026
5100.0	.16963	.09156	.05169	.02868	.01553	.00800	.00391	.00180	.00078	.00032
5200.0	.17149	.09339	.05330	.02999	.01647	.00863	.00431	.00204	.00091	.00038
5300.0	.17331	.09519	.05488	.03128	.01742	.00928	.00472	.00228	.00104	.00045
5400.0	.17509	.09696	.05645	.03257	.01838	.00995	.00516	.00254	.00119	.00053
5500.0	.17683	.09869	.05800	.03386	.01934	.01064	.00561	.00282	.00135	.00061
5600.0	.17853	.10040	.05953	.03514	.02032	.01134	.00608	.00311	.00152	.00071
5700.0	.18020	.10207	.06105	.03641	.02130	.01205	.00656	.00342	.00170	.00081
5800.0	.18184	.10372	.06254	.03767	.02228	.01277	.00706	.00374	.00190	.00092
5900.0	.18344	.10534	.06402	.03893	.02327	.01351	.00757	.00408	.00211	.00104
6000.0	.18501	.10693	.06548	.04018	.02427	.01425	.00810	.00443	.00233	.00117
6100.0	.18655	.10849	.06692	.04142	.02526	.01501	.00864	.00480	.00256	.00131
6200.0	.18806	.11003	.06834	.04265	.02626	.01577	.00919	.00518	.00281	.00146
6300.0	.18954	.11154	.06974	.04387	.02726	.01654	.00975	.00557	.00306	.00162
6400.0	.19099	.11303	.07113	.04509	.02825	.01732	.01033	.00597	.00333	.00179
6500.0	.19242	.11449	.07250	.04629	.02925	.01810	.01092	.00638	.00361	.00198
6600.0	.19382	.11593	.07386	.04749	.03025	.01889	.01151	.00681	.00391	.00217
6700.0	.19519	.11735	.07519	.04867	.03124	.01969	.01211	.00725	.00421	.00236
6800.0	.19655	.11874	.07651	.04985	.03224	.02049	.01273	.00770	.00452	.00257
6900.0	.19787	.12012	.07782	.05102	.03323	.02129	.01335	.00816	.00485	.00279
7000.0	.19918	.12147	.07910	.05217	.03421	.02209	.01397	.00863	.00518	.00302
7100.0	.20046	.12280	.08037	.05332	.03520	.02290	.01461	.00910	.00553	.00326
7200.0	.20173	.12412	.08163	.05446	.03618	.02371	.01525	.00959	.00588	.00351
7300.0	.20297	.12541	.08287	.05559	.03716	.02452	.01590	.01008	.00624	.00376
7400.0	.20419	.12668	.08410	.05671	.03813	.02534	.01655	.01059	.00662	.00403
7500.0	.20539	.12794	.08531	.05781	.03910	.02615	.01721	.01110	.00700	.00430
7600.0	.20658	.12917	.08650	.05891	.04006	.02697	.01787	.01161	.00739	.00459
7700.0	.20774	.13039	.08768	.06000	.04102	.02778	.01853	.01214	.00778	.00488
7800.0	.20889	.13160	.08885	.06108	.04198	.02859	.01920	.01267	.00819	.00518
7900.0	.21002	.13278	.09000	.06215	.04293	.02941	.01987	.01320	.00860	.00549
8000.0	.21113	.13395	.09114	.06320	.04387	.03022	.02055	.01374	.00902	.00580
8100.0	.21223	.13510	.09227	.06425	.04481	.03103	.02122	.01429	.00945	.00612
8200.0	.21331	.13624	.09338	.06529	.04575	.03184	.02190	.01484	.00988	.00645
8300.0	.21437	.13736	.09448	.06632	.04667	.03265	.02258	.01539	.01032	.00679

25600. (b/a)

Table 3. (y/y_0)

(b/a) 25600.

$\left(\frac{r}{a}\right)$

$\left(\frac{\sqrt{4\alpha t}}{a}\right)$	1000.	2000.	3000.	4000.	5000.	6000.	7000.	8000.	9000.	10000.
8400.0	.21542	.13847	.09557	.06734	.04759	.03345	.02326	.01595	.01076	.00713
8500.0	.21645	.13956	.09664	.06835	.04851	.03426	.02395	.01652	.01121	.00748
8600.0	.21747	.14064	.09771	.06936	.04942	.03506	.02463	.01708	.01167	.00784
8700.0	.21848	.14170	.09876	.07035	.05033	.03586	.02532	.01765	.01213	.00820
8800.0	.21947	.14275	.09979	.07133	.05123	.03666	.02600	.01823	.01260	.00857
8900.0	.22045	.14379	.10082	.07231	.05212	.03745	.02669	.01880	.01307	.00894
9000.0	.22141	.14482	.10184	.07327	.05301	.03824	.02737	.01938	.01354	.00932
9100.0	.22237	.14583	.10284	.07423	.05389	.03903	.02806	.01996	.01402	.00971
9200.0	.22331	.14683	.10383	.07517	.05477	.03982	.02874	.02054	.01450	.01010
9300.0	.22423	.14781	.10481	.07611	.05563	.04060	.02943	.02113	.01499	.01049
9400.0	.22515	.14879	.10579	.07704	.05650	.04138	.03011	.02171	.01548	.01089
9500.0	.22605	.14975	.10675	.07796	.05735	.04215	.03080	.02230	.01597	.01129
9600.0	.22695	.15070	.10770	.07888	.05821	.04292	.03148	.02289	.01647	.01170
9700.0	.22783	.15164	.10864	.07978	.05905	.04369	.03216	.02348	.01696	.01211
9800.0	.22870	.15257	.10957	.08068	.05989	.04445	.03284	.02407	.01746	.01253
9900.0	.22956	.15349	.11049	.08157	.06072	.04521	.03352	.02466	.01797	.01295
10000.0	.23041	.15440	.11140	.08245	.06155	.04597	.03419	.02525	.01847	.01337

Table 4

Values of:

$$G\left(\frac{\sqrt{4\alpha t}}{a}\right) = \sum_{n=1}^{n=\infty} A_n(\beta_n a)\, U_0'(\beta_n a)\, e^{-\left(\frac{\beta_n a}{4}\right)\left(\frac{4\alpha t}{a^2}\right)}$$

An independent check computation can be made by use of values given in Bulletin 3, Part VII of the Boulder Canyon Project Final Reports. These apply for (b/a) = 100. A computation for the parameter $(\sqrt{4\alpha t}/a) = 25$ is shown below:

n	$A_n(\beta_n a)\ U_0'(\beta_n a)$	$e^{-\left(\frac{\beta_n a}{4}\right)\left(\frac{4\alpha t}{a^2}\right)}$	Term
1	0.254236	0.992006	0.252204
2	0.058612	0.750094	0.043965
3	0.044047	0.410733	0.018092
4	0.037729	0.163466	0.006167
5	0.034078	0.047347	0.001613
6	0.03166*	0.009988	0.000316
7	0.02993*	0.001535	0.000046
8	0.02862*	0.000172	0.000005
		Total	.322408

The tabular value is 0.32241. The agreement is satisfactory.

*Supplied by this writer.

Table 4. $G\left(\frac{\sqrt{4\alpha t}}{a}\right)$

	$\left(\frac{\sqrt{4\alpha t}}{a}\right)$	0	1	2	3	4	5	6	7	8	9
	3.	.77456	.76063	.74750	.73511	.72339	.71228	.70173	.69171	.68217	.67308
	4.	.66440	.65610	.64816	.64055	.63326	.62626	.61953	.61306	.60683	.60082
	5.	.59504	.58945	.58405	.57884	.57380	.56892	.56419	.55961	.55517	.55087
25.	6.	.54668									
	6.	.54668	.54262	.53868	.53484	.53111	.52748	.52394	.52050	.51715	.51388
	7.	.51068	.50757	.50454	.50157	.49868	.49585	.49308	.49038	.48774	.48515
	8.	.48262	.48015	.47772	.47535	.47302	.47074	.46851	.46631	.46417	.46206
	9.	.45999	.45796	.45597	.45402	.45210	.45021	.44836	.44654	.44475	.44299
	10.	.44126	.43956	.43789	.43624	.43462	.43303	.43146	.42992	.42840	.42691
50.	11.	.42543	.42398	.42255	.42114	.41976	.41839	.41704	.41571	.41440	.41311
	12.	.41184									
	12.	.41184	.41058	.40934	.40812	.40691	.40572	.40455	.40339	.40224	.40111
	13.	.40000	.39889	.39781	.39673	.39567	.39462	.39359	.39256	.39155	.39055
	14.	.38956	.38859	.38762	.38667	.38573	.38479	.38387	.38296	.38206	.38116
	15.	.38028	.37941	.37855	.37769	.37685	.37601	.37518	.37436	.37355	.37275
	16.	.37196	.37117	.37039	.36962	.36886	.36810	.36735	.36661	.36588	.36515
	17.	.36443	.36372	.36301	.36231	.36162	.36093	.36025	.35957	.35890	.35824
	18.	.35758	.35693	.35629	.35565	.35501	.35438	.35376	.35314	.35253	.35192
	19.	.35132	.35072	.35013	.34954	.34896	.34838	.34781	.34724	.34667	.34611
100.	20.	.34556	.34501	.34446	.34392	.34338	.34285	.34232	.34179	.34127	.34075
	21.	.34024	.33973	.33922	.33872	.33822	.33773	.33724	.33675	.33626	.33578
	22.	.33531	.33483	.33436	.33389	.33343	.33297	.33251	.33206	.33161	.33116
	23.	.33071	.33027	.32983	.32940	.32896	.32853	.32811	.32768	.32726	.32684
	24.	.32643	.32601	.32560	.32519	.32479	.32438	.32398	.32359	.32319	.32280
	25.	.32241									
	25.	.32240	.32202	.32163	.32125	.32087	.32049	.32011	.31974	.31937	.31900
	26.	.31863	.31827	.31790	.31754	.31718	.31683	.31647	.31612	.31577	.31542
(b/a)	27.	.31507	.31473	.31439	.31405	.31371	.31337	.31304	.31270	.31237	.31204
	28.	.31172	.31139	.31107	.31075	.31043	.31011	.30979	.30948	.30916	.30885
	29.	.30854	.30823	.30792	.30762	.30732	.30701	.30671	.30642	.30612	.30582
	30.	.30553	.30524	.30494	.30465	.30437	.30408	.30379	.30351	.30323	.30295
	31.	.30267	.30239	.30211	.30184	.30156	.30129	.30102	.30075	.30048	.30021
	32.	.29994	.29968	.29941	.29915	.29889	.29863	.29837	.29811	.29786	.29760
	33.	.29735	.29709	.29684	.29659	.29634	.29609	.29585	.29560	.29536	.29511
	34.	.29487	.29463	.29439	.29415	.29391	.29367	.29343	.29320	.29296	.29273
	35.	.29250	.29227	.29204	.29181	.29158	.29135	.29113	.29090	.29068	.29045
	36.	.29023	.29001	.28979	.28957	.28935	.28913	.28891	.28870	.28848	.28827
200.	37.	.28806	.28784	.28763	.28742	.28721	.28700	.28679	.28659	.28638	.28617
	38.	.28597	.28576	.28556	.28536	.28516	.28495	.28475	.28455	.28436	.28416
	39.	.28396	.28376	.28357	.28337	.28318	.28299	.28279	.28260	.28241	.28222
	40.	.28203	.28184	.28165	.28147	.28128	.28109	.28091	.28072	.28054	.28036
	41.	.28017	.27999	.27981	.27963	.27945	.27927	.27909	.27891	.27873	.27856
	42.	.27838	.27821	.27803	.27786	.27768	.27751	.27734	.27716	.27699	.27682
	43.	.27665	.27648	.27631	.27615	.27598	.27581	.27564	.27548	.27531	.27515
	44.	.27498	.27482	.27466	.27449	.27433	.27417	.27401	.27385	.27369	.27353
	45.	.27337	.27321	.27305	.27290	.27274	.27258	.27243	.27227	.27212	.27196
1.	46.	.27181	.27166	.27150	.27135	.27120	.27105	.27090	.27075	.27060	.27045
	47.	.27030	.27015	.27000	.26985	.26971	.26956	.26941	.26927	.26912	.26898
	48.	.26883	.26869	.26855	.26840	.26826	.26812	.26798	.26784	.26770	.26755
	49.	.26741	.26727	.26714	.26700	.26686	.26672	.26658	.26645	.26631	.26617
	50.	.26604									

Table 4. $G\left(\frac{\sqrt{4\alpha t}}{a}\right)$

$\left(\frac{\sqrt{4\alpha t}}{a}\right)$	0	1	2	3	4	5	6	7	8	9
50.	.26604	.26590	.26577	.26563	.26550	.26536	.26523	.26510	.26497	.26483
51.	.26470	.26457	.26444	.26431	.26418	.26405	.26392	.26379	.26366	.26353
52.	.26340	.26327	.26315	.26302	.26289	.26277	.26264	.26252	.26239	.26227
53.	.26214	.26202	.26189	.26177	.26165	.26152	.26140	.26128	.26116	.26104
54.	.26091	.26079	.26067	.26055	.26043	.26031	.26019	.26008	.25996	.25984
55.	.25972	.25960	.25949	.25937	.25925	.25914	.25902	.25890	.25879	.25867
56.	.25856	.25844	.25833	.25822	.25810	.25799	.25787	.25776	.25765	.25754
57.	.25743	.25731	.25720	.25709	.25698	.25687	.25676	.25665	.25654	.25643
58.	.25632	.25621	.25611	.25600	.25589	.25578	.25567	.25557	.25546	.25535
59.	.25525	.25514	.25504	.25493	.25483	.25472	.25462	.25451	.25441	.25430
60.	.25420	.25410	.25399	.25389	.25379	.25368	.25358	.25348	.25338	.25328
61.	.25318	.25307	.25297	.25287	.25277	.25267	.25257	.25247	.25237	.25227
62.	.25218	.25208	.25198	.25188	.25178	.25168	.25159	.25149	.25139	.25130
63.	.25120	.25110	.25101	.25091	.25082	.25072	.25063	.25053	.25044	.25034
64.	.25025	.25015	.25006	.24996	.24987	.24978	.24968	.24959	.24950	.24941
65.	.24931	.24922	.24913	.24904	.24895	.24886	.24877	.24867	.24858	.24849
66.	.24840	.24831	.24822	.24813	.24804	.24796	.24787	.24778	.24769	.24760
67.	.24751	.24742	.24734	.24725	.24716	.24707	.24699	.24690	.24681	.24673
68.	.24664	.24655	.24647	.24638	.24630	.24621	.24612	.24604	.24595	.24587
69.	.24579	.24570	.24562	.24553	.24545	.24537	.24528	.24520	.24512	.24503
70.	.24495	.24487	.24478	.24470	.24462	.24454	.24446	.24437	.24429	.24421
71.	.24413	.24405	.24397	.24389	.24381	.24373	.24365	.24357	.24349	.24341
72.	.24333	.24325	.24317	.24309	.24301	.24293	.24286	.24278	.24270	.24262
73.	.24254	.24246	.24239	.24231	.24223	.24216	.24208	.24200	.24192	.24185
74.	.24177	.24170	.24162	.24154	.24147	.24139	.24132	.24124	.24117	.24109
75.	.24102	.24094	.24087	.24079	.24072	.24064	.24057	.24049	.24042	.24035
76.	.24027	.24020	.24013	.24005	.23998	.23991	.23984	.23976	.23969	.23962
77.	.23955	.23947	.23940	.23933	.23926	.23919	.23912	.23905	.23897	.23890
78.	.23883	.23876	.23869	.23862	.23855	.23848	.23841	.23834	.23827	.23820
79.	.23813	.23806	.23799	.23792	.23786	.23779	.23772	.23765	.23758	.23751
80.	.23744	.23738	.23731	.23724	.23717	.23711	.23704	.23697	.23690	.23684
81.	.23677	.23670	.23664	.23657	.23650	.23644	.23637	.23630	.23624	.23617
82.	.23611	.23604	.23597	.23591	.23584	.23578	.23571	.23565	.23558	.23552
83.	.23545	.23539	.23532	.23526	.23520	.23513	.23507	.23500	.23494	.23488
84.	.23481	.23475	.23468	.23462	.23456	.23449	.23443	.23437	.23431	.23424
85.	.23418	.23412	.23406	.23399	.23393	.23387	.23381	.23375	.23368	.23362
86.	.23356	.23350	.23344	.23338	.23332	.23326	.23319	.23313	.23307	.23301
87.	.23295	.23289	.23283	.23277	.23271	.23265	.23259	.23253	.23247	.23241
88.	.23235	.23229	.23223	.23217	.23212	.23206	.23200	.23194	.23188	.23182
89.	.23176	.23170	.23165	.23159	.23153	.23147	.23141	.23136	.23130	.23124
90.	.23118	.23113	.23107	.23101	.23095	.23090	.23084	.23078	.23073	.23067
91.	.23061	.23056	.23050	.23044	.23039	.23033	.23027	.23022	.23016	.23011
92.	.23005	.22999	.22994	.22988	.22983	.22977	.22972	.22966	.22961	.22955
93.	.22950	.22944	.22939	.22933	.22928	.22922	.22917	.22911	.22906	.22901
94.	.22895	.22890	.22884	.22879	.22874	.22868	.22863	.22857	.22852	.22847
95.	.22841	.22836	.22831	.22825	.22820	.22815	.22810	.22804	.22799	.22794
96.	.22789	.22783	.22778	.22773	.22768	.22762	.22757	.22752	.22747	.22742
97.	.22736	.22731	.22726	.22721	.22716	.22711	.22705	.22700	.22695	.22690
98.	.22685	.22680	.22675	.22670	.22665	.22660	.22655	.22650	.22645	.22639
99.	.22634	.22629	.22624	.22619	.22614	.22609	.22604	.22599	.22594	.22590
100.	.22585									

Table 4. $G\left(\frac{\sqrt{4\alpha t}}{a}\right)$

$\left(\frac{\sqrt{4\alpha t}}{a}\right)$	0	1	2	3	4	5	6	7	8	9
100.	.22585	.22535	.22487	.22439	.22392	.22345	.22299	.22254	.22209	.22165
110.	.22122	.22079	.22036	.21994	.21953	.21912	.21872	.21832	.21793	.21754
120.	.21715	.21677	.21639	.21602	.21566	.21529	.21493	.21458	.21423	.21388
130.	.21354	.21320	.21286	.21253	.21220	.21187	.21155	.21123	.21092	.21060
140.	.21029	.20999	.20968	.20938	.20908	.20879	.20850	.20821	.20792	.20764
150.	.20736	.20708	.20680	.20653	.20626	.20599	.20572	.20546	.20520	.20494
160.	.20468	.20443	.20418	.20393	.20368	.20343	.20319	.20294	.20270	.20247
170.	.20223	.20200	.20176	.20153	.20131	.20108	.20085	.20063	.20041	.20019
180.	.19997	.19975	.19954	.19933	.19912	.19891	.19870	.19849	.19828	.19808
190.	.19788	.19768	.19748	.19728	.19708	.19689	.19669	.19650	.19631	.19612
200.	.19593									
200.	.19593	.19574	.19556	.19537	.19519	.19501	.19483	.19465	.19447	.19429
210.	.19411	.19394	.19376	.19359	.19342	.19325	.19308	.19291	.19274	.19258
220.	.19241	.19225	.19208	.19192	.19176	.19160	.19144	.19128	.19112	.19097
230.	.19081	.19066	.19050	.19035	.19020	.19005	.18990	.18975	.18960	.18945
240.	.18930	.18916	.18901	.18887	.18873	.18858	.18844	.18830	.18816	.18802
250.	.18788	.18774	.18761	.18747	.18733	.18720	.18706	.18693	.18680	.18667
260.	.18653	.18640	.18627	.18614	.18601	.18588	.18576	.18563	.18550	.18538
270.	.18525	.18513	.18501	.18488	.18476	.18464	.18452	.18440	.18428	.18416
280.	.18404	.18392	.18380	.18368	.18357	.18345	.18333	.18322	.18310	.18299
290.	.18288	.18276	.18265	.18254	.18243	.18232	.18221	.18210	.18199	.18188
300.	.18177	.18166	.18155	.18145	.18134	.18124	.18113	.18102	.18092	.18082
310.	.18071	.18061	.18051	.18040	.18030	.18020	.18010	.18000	.17990	.17980
320.	.17970	.17960	.17950	.17940	.17931	.17921	.17911	.17901	.17892	.17882
330.	.17873	.17863	.17854	.17844	.17835	.17826	.17816	.17807	.17798	.17789
340.	.17780	.17770	.17761	.17752	.17743	.17734	.17725	.17716	.17708	.17699
350.	.17690	.17681	.17672	.17664	.17655	.17646	.17638	.17629	.17621	.17612
360.	.17604	.17595	.17587	.17578	.17570	.17562	.17553	.17545	.17537	.17529
370.	.17521	.17512	.17504	.17496	.17488	.17480	.17472	.17464	.17456	.17448
380.	.17440	.17432	.17425	.17417	.17409	.17401	.17394	.17386	.17378	.17371
390.	.17363	.17355	.17348	.17340	.17333	.17325	.17318	.17310	.17303	.17296
400.	.17288									

800.

(b/a)

1600

3.

Table 4. $G\left(\frac{\sqrt{4\alpha t}}{a}\right)$

$\left(\frac{\sqrt{4\alpha t}}{a}\right)$	0	1	2	3	4	5	6	7	8	9
400.	.17288	.17281	.17273	.17266	.17259	.17252	.17244	.17237	.17230	.17223
410.	.17216	.17209	.17202	.17195	.17187	.17180	.17173	.17167	.17160	.17153
420.	.17146	.17139	.17132	.17125	.17118	.17111	.17105	.17098	.17091	.17085
430.	.17078	.17071	.17065	.17058	.17051	.17045	.17038	.17032	.17025	.17019
440.	.17012	.17006	.16999	.16993	.16986	.16980	.16974	.16967	.16961	.16955
450.	.16948	.16942	.16936	.16930	.16923	.16917	.16911	.16905	.16899	.16893
460.	.16886	.16880	.16874	.16868	.16862	.16856	.16850	.16844	.16838	.16832
470.	.16826	.16820	.16814	.16808	.16803	.16797	.16791	.16785	.16779	.16773
480.	.16768	.16762	.16756	.16750	.16745	.16739	.16733	.16728	.16722	.16716
490.	.16711	.16705	.16700	.16694	.16688	.16683	.16677	.16672	.16666	.16661
500.	.16655	.16650	.16644	.16639	.16634	.16628	.16623	.16617	.16612	.16607
510.	.16601	.16596	.16591	.16585	.16580	.16575	.16570	.16564	.16559	.16554
520.	.16549	.16544	.16538	.16533	.16528	.16523	.16518	.16513	.16508	.16503
530.	.16498	.16493	.16487	.16482	.16477	.16472	.16467	.16462	.16457	.16453
540.	.16448	.16443	.16438	.16433	.16428	.16423	.16418	.16413	.16408	.16404
550.	.16399	.16394	.16389	.16384	.16380	.16375	.16370	.16365	.16361	.16356
560.	.16351	.16346	.16342	.16337	.16332	.16328	.16323	.16318	.16314	.16309
570.	.16305	.16300	.16295	.16291	.16286	.16282	.16277	.16273	.16268	.16264
580.	.16259	.16255	.16250	.16246	.16241	.16237	.16232	.16228	.16224	.16219
590.	.16215	.16210	.16206	.16202	.16197	.16193	.16189	.16184	.16180	.16176
600.	.16171	.16167	.16163	.16158	.16154	.16150	.16146	.16141	.16137	.16133
610.	.16129	.16125	.16120	.16116	.16112	.16108	.16104	.16100	.16095	.16091
620.	.16087	.16083	.16079	.16075	.16071	.16067	.16063	.16059	.16054	.16050
630.	.16046	.16042	.16038	.16034	.16030	.16026	.16022	.16018	.16014	.16010
640.	.16006	.16002	.15999	.15995	.15991	.15987	.15983	.15979	.15975	.15971
650.	.15967	.15963	.15960	.15956	.15952	.15948	.15944	.15940	.15937	.15933
660.	.15929	.15925	.15921	.15918	.15914	.15910	.15906	.15903	.15899	.15895
670.	.15891	.15888	.15884	.15880	.15877	.15873	.15869	.15866	.15862	.15858
680.	.15855	.15851	.15847	.15844	.15840	.15837	.15833	.15829	.15826	.15822
690.	.15819	.15815	.15811	.15808	.15804	.15801	.15797	.15794	.15790	.15787
700.	.15783	.15780	.15776	.15773	.15769	.15766	.15762	.15759	.15755	.15752
710.	.15748	.15745	.15741	.15738	.15735	.15731	.15728	.15724	.15721	.15718
720.	.15714	.15711	.15707	.15704	.15701	.15697	.15694	.15691	.15687	.15684
730.	.15681	.15677	.15674	.15671	.15667	.15664	.15661	.15657	.15654	.15651
740.	.15648	.15644	.15641	.15638	.15635	.15631	.15628	.15625	.15622	.15618
750.	.15615	.15612	.15609	.15606	.15602	.15599	.15596	.15593	.15590	.15587
760.	.15583	.15580	.15577	.15574	.15571	.15568	.15565	.15561	.15558	.15555
770.	.15552	.15549	.15546	.15543	.15540	.15537	.15534	.15531	.15527	.15524
780.	.15521	.15518	.15515	.15512	.15509	.15506	.15503	.15500	.15497	.15494
790.	.15491	.15488	.15485	.15482	.15479	.15476	.15473	.15470	.15467	.15464
800.	.15461									

3200. (b/a) 4.

Table 4. $G\left(\frac{\sqrt{4\alpha t}}{a}\right)$

$\left(\frac{\sqrt{4\alpha t}}{a}\right)$	0	1	2	3	4	5	6	7	8	9
800.	.15461	.15458	.15455	.15452	.15449	.15447	.15444	.15441	.15438	.15435
810.	.15432	.15429	.15426	.15423	.15420	.15418	.15415	.15412	.15409	.15406
820.	.15403	.15400	.15397	.15395	.15392	.15389	.15386	.15383	.15380	.15378
830.	.15375	.15372	.15369	.15366	.15363	.15361	.15358	.15355	.15352	.15350
840.	.15347	.15344	.15341	.15339	.15336	.15333	.15330	.15328	.15325	.15322
850.	.15319	.15317	.15314	.15311	.15308	.15306	.15303	.15300	.15298	.15295
860.	.15292	.15290	.15287	.15284	.15282	.15279	.15276	.15274	.15271	.15268
870.	.15266	.15263	.15260	.15258	.15255	.15253	.15250	.15247	.15245	.15242
880.	.15239	.15237	.15234	.15232	.15229	.15226	.15224	.15221	.15219	.15216
890.	.15214	.15211	.15208	.15206	.15203	.15201	.15198	.15196	.15193	.15191
900.	.15188	.15186	.15183	.15180	.15178	.15175	.15173	.15170	.15168	.15165
910.	.15163	.15160	.15158	.15155	.15153	.15150	.15148	.15145	.15143	.15140
920.	.15138	.15136	.15133	.15131	.15128	.15126	.15123	.15121	.15118	.15116
930.	.15114	.15111	.15109	.15106	.15104	.15102	.15099	.15097	.15094	.15092
940.	.15090	.15087	.15085	.15082	.15080	.15078	.15075	.15073	.15070	.15068
950.	.15066	.15063	.15061	.15059	.15056	.15054	.15052	.15049	.15047	.15045
960.	.15042	.15040	.15038	.15035	.15033	.15031	.15028	.15026	.15024	.15022
970.	.15019	.15017	.15015	.15012	.15010	.15008	.15005	.15003	.15001	.14999
980.	.14996	.14994	.14992	.14990	.14987	.14985	.14983	.14981	.14978	.14976
990.	.14974	.14972	.14969	.14967	.14965	.14963	.14960	.14958	.14956	.14954
1000.	.14951	.14949	.14947	.14945	.14943	.14940	.14938	.14936	.14934	.14932
1010.	.14930	.14927	.14925	.14923	.14921	.14919	.14917	.14914	.14912	.14910
1020.	.14908	.14906	.14904	.14901	.14899	.14897	.14895	.14893	.14891	.14889
1030.	.14887	.14884	.14882	.14880	.14878	.14876	.14874	.14872	.14870	.14867
1040.	.14865	.14863	.14861	.14859	.14857	.14855	.14853	.14851	.14849	.14847
1050.	.14845	.14842	.14840	.14838	.14836	.14834	.14832	.14830	.14828	.14826
1060.	.14824	.14822	.14820	.14818	.14816	.14814	.14812	.14810	.14808	.14806
1070.	.14804	.14802	.14800	.14797	.14795	.14793	.14791	.14789	.14787	.14785
1080.	.14783	.14781	.14779	.14777	.14775	.14773	.14771	.14769	.14767	.14766
1090.	.14764	.14762	.14760	.14758	.14756	.14754	.14752	.14750	.14748	.14746
1100.	.14744	.14742	.14740	.14738	.14736	.14734	.14732	.14730	.14728	.14726
1110.	.14724	.14723	.14720	.14719	.14717	.14715	.14713	.14711	.14709	.14707
1120.	.14705	.14703	.14701	.14699	.14698	.14696	.14694	.14692	.14690	.14688
1130.	.14686	.14684	.14682	.14681	.14679	.14677	.14675	.14673	.14671	.14669
1140.	.14667	.14666	.14664	.14662	.14660	.14658	.14656	.14654	.14653	.14651
1150.	.14649	.14647	.14645	.14643	.14642	.14640	.14638	.14636	.14634	.14632
1160.	.14631	.14629	.14627	.14625	.14623	.14621	.14620	.14618	.14616	.14614
1170.	.14612	.14611	.14609	.14607	.14605	.14603	.14602	.14600	.14598	.14596
1180.	.14594	.14593	.14591	.14589	.14587	.14586	.14584	.14582	.14580	.14578
1190.	.14577	.14575	.14573	.14571	.14570	.14568	.14566	.14564	.14563	.14561
1200.	.14559	.14557	.14556	.14554	.14552	.14550	.14549	.14547	.14545	.14544
1210.	.14542	.14540	.14538	.14537	.14535	.14533	.14531	.14530	.14528	.14526
1220.	.14525	.14523	.14521	.14519	.14518	.14516	.14514	.14513	.14511	.14509
1230.	.14508	.14506	.14504	.14503	.14501	.14499	.14497	.14496	.14494	.14492
1240.	.14491	.14489	.14487	.14486	.14484	.14482	.14481	.14479	.14477	.14476
1250.	.14474	.14472	.14471	.14469	.14467	.14466	.14464	.14462	.14461	.14459
1260.	.14457	.14456	.14454	.14453	.14451	.14449	.14448	.14446	.14444	.14443
1270.	.14441	.14440	.14438	.14436	.14435	.14433	.14431	.14430	.14428	.14427
1280.	.14425	.14423	.14422	.14420	.14419	.14417	.14415	.14414	.14412	.14411
1290.	.14409	.14407	.14406	.14404	.14403	.14401	.14400	.14398	.14396	.14395
1300.	.14393	.14392	.14390	.14388	.14387	.14385	.14384	.14382	.14381	.14379

6400. (b/a) B.

Table 4. $G\left(\frac{\sqrt{4\alpha t}}{a}\right)$

$\left(\frac{\sqrt{4\alpha t}}{a}\right)$	0	1	2	3	4	5	6	7	8	9
1310.	.14378	.14376	.14374	.14373	.14371	.14370	.14368	.14367	.14365	.14364
1320.	.14362	.14360	.14359	.14357	.14356	.14354	.14353	.14351	.14350	.14348
1330.	.14347	.14345	.14344	.14342	.14340	.14339	.14337	.14336	.14334	.14333
1340.	.14331	.14330	.14328	.14327	.14325	.14324	.14322	.14321	.14319	.14318
1350.	.14316	.14315	.14313	.14312	.14310	.14309	.14307	.14306	.14304	.14303
1360.	.14301	.14300	.14298	.14297	.14295	.14294	.14292	.14291	.14289	.14288
1370.	.14287	.14285	.14284	.14282	.14281	.14279	.14278	.14276	.14275	.14273
1380.	.14272	.14270	.14269	.14267	.14266	.14265	.14263	.14262	.14260	.14259
1390.	.14257	.14256	.14254	.14253	.14252	.14250	.14249	.14247	.14246	.14244
1400.	.14243	.14241	.14240	.14239	.14237	.14236	.14234	.14233	.14231	.14230
1410.	.14229	.14227	.14226	.14224	.14223	.14222	.14220	.14219	.14217	.14216
1420.	.14214	.14213	.14212	.14210	.14209	.14207	.14206	.14205	.14203	.14202
1430.	.14200	.14199	.14198	.14196	.14195	.14193	.14192	.14191	.14189	.14188
1440.	.14187	.14185	.14184	.14182	.14181	.14180	.14178	.14177	.14175	.14174
1450.	.14173	.14171	.14170	.14169	.14167	.14166	.14165	.14163	.14162	.14160
1460.	.14159	.14158	.14156	.14155	.14154	.14152	.14151	.14150	.14148	.14147
1470.	.14146	.14144	.14143	.14142	.14140	.14139	.14138	.14136	.14135	.14133
1480.	.14132	.14131	.14129	.14128	.14127	.14125	.14124	.14123	.14122	.14120
1490.	.14119	.14118	.14116	.14115	.14114	.14112	.14111	.14110	.14108	.14107
1500.	.14106	.14104	.14103	.14102	.14100	.14099	.14098	.14097	.14095	.14094
1510.	.14093	.14091	.14090	.14089	.14087	.14086	.14085	.14084	.14082	.14081
1520.	.14080	.14078	.14077	.14076	.14074	.14073	.14072	.14071	.14069	.14068
1530.	.14067	.14066	.14064	.14063	.14062	.14060	.14059	.14058	.14057	.14055
1540.	.14054	.14053	.14052	.14050	.14049	.14048	.14046	.14045	.14044	.14043
1550.	.14041	.14040	.14039	.14038	.14036	.14035	.14034	.14033	.14031	.14030
1560.	.14029	.14028	.14026	.14025	.14024	.14023	.14021	.14020	.14019	.14018
1570.	.14016	.14015	.14014	.14013	.14011	.14010	.14009	.14008	.14007	.14005
1580.	.14004	.14003	.14002	.14000	.13999	.13998	.13997	.13996	.13994	.13993
1590.	.13992	.13991	.13989	.13988	.13987	.13986	.13985	.13983	.13982	.13981
1600.	.13980									
1600.	.13980	.13968	.13956	.13944	.13932	.13920	.13909	.13897	.13886	.13874
1700.	.13863	.13852	.13841	.13830	.13819	.13808	.13798	.13787	.13776	.13766
1800.	.13755	.13745	.13735	.13724	.13714	.13704	.13694	.13684	.13674	.13665
1900.	.13655	.13645	.13635	.13626	.13616	.13607	.13598	.13588	.13579	.13570
2000.	.13561	.13552	.13543	.13534	.13525	.13516	.13507	.13498	.13490	.13481
2100.	.13472	.13464	.13455	.13447	.13439	.13430	.13422	.13414	.13405	.13397
2200.	.13389	.13381	.13373	.13365	.13357	.13349	.13342	.13334	.13326	.13318
2300.	.13311	.13303	.13296	.13288	.13280	.13273	.13266	.13258	.13251	.13244
2400.	.13236	.13229	.13222	.13215	.13208	.13201	.13194	.13187	.13180	.13173
2500.	.13166	.13159	.13152	.13145	.13139	.13132	.13125	.13119	.13112	.13105
2600.	.13099	.13092	.13086	.13079	.13073	.13066	.13060	.13054	.13047	.13041
2700.	.13035	.13029	.13023	.13016	.13010	.13004	.12998	.12992	.12986	.12980
2800.	.12974	.12968	.12962	.12956	.12950	.12944	.12939	.12933	.12927	.12921
2900.	.12916	.12910	.12904	.12899	.12893	.12887	.12882	.12876	.12871	.12865
3000.	.12860	.12854	.12849	.12843	.12838	.12833	.12827	.12822	.12817	.12812
3100.	.12806	.12801	.12796	.12791	.12785	.12780	.12775	.12770	.12765	.12760
3200.	.12755									

6400.

(b/a)

12800.

Table 4. $G\left(\frac{\sqrt{4\alpha t}}{a}\right)$.

25600. (b/a) 7.

$\left(\frac{\sqrt{4\alpha t}}{a}\right)$	0	1	2	3	4	5	6	7	8	9
3200.	.12755	.12750	.12745	.12740	.12735	.12730	.12725	.12720	.12715	.12710
3300.	.12705	.12700	.12696	.12691	.12686	.12681	.12676	.12672	.12667	.12662
3400.	.12658	.12653	.12648	.12644	.12639	.12634	.12630	.12625	.12621	.12616
3500.	.12612	.12607	.12603	.12598	.12594	.12589	.12585	.12580	.12576	.12572
3600.	.12567	.12563	.12559	.12554	.12550	.12546	.12542	.12537	.12533	.12529
3700.	.12525	.12520	.12516	.12512	.12508	.12504	.12500	.12496	.12491	.12487
3800.	.12483	.12479	.12475	.12471	.12467	.12463	.12459	.12455	.12451	.12447
3900.	.12443	.12439	.12435	.12431	.12428	.12424	.12420	.12416	.12412	.12408
4000.	.12404	.12401	.12397	.12393	.12389	.12386	.12382	.12378	.12374	.12371
4100.	.12367	.12363	.12359	.12356	.12352	.12349	.12345	.12341	.12338	.12334
4200.	.12330	.12327	.12323	.12320	.12316	.12313	.12309	.12306	.12302	.12299
4300.	.12295	.12291	.12288	.12285	.12281	.12278	.12274	.12271	.12267	.12264
4400.	.12261	.12257	.12254	.12250	.12247	.12244	.12240	.12237	.12234	.12230
4500.	.12227	.12224	.12221	.12217	.12214	.12211	.12208	.12204	.12201	.12198
4600.	.12195	.12191	.12188	.12185	.12182	.12179	.12176	.12172	.12169	.12166
4700.	.12163	.12160	.12157	.12154	.12151	.12147	.12144	.12141	.12138	.12135
4800.	.12132	.12129	.12126	.12123	.12120	.12117	.12114	.12111	.12108	.12105
4900.	.12102	.12099	.12096	.12093	.12090	.12087	.12084	.12081	.12079	.12076
5000.	.12073	.12070	.12067	.12064	.12061	.12058	.12056	.12053	.12050	.12047
5100.	.12044	.12041	.12039	.12036	.12033	.12030	.12027	.12025	.12022	.12019
5200.	.12016	.12014	.12011	.12008	.12005	.12003	.12000	.11997	.11994	.11992
5300.	.11989	.11986	.11984	.11981	.11978	.11976	.11973	.11970	.11968	.11965
5400.	.11962	.11960	.11957	.11955	.11952	.11949	.11947	.11944	.11942	.11939
5500.	.11936	.11934	.11931	.11929	.11926	.11924	.11921	.11919	.11916	.11913
5600.	.11911	.11908	.11906	.11903	.11901	.11898	.11896	.11893	.11891	.11889
5700.	.11886	.11884	.11881	.11879	.11876	.11874	.11871	.11869	.11867	.11864
5800.	.11862	.11859	.11857	.11855	.11852	.11850	.11847	.11845	.11843	.11840
5900.	.11838	.11836	.11833	.11831	.11829	.11826	.11824	.11822	.11819	.11817
6000.	.11815	.11812	.11810	.11808	.11805	.11803	.11801	.11799	.11796	.11794
6100.	.11792	.11789	.11787	.11785	.11783	.11780	.11778	.11776	.11774	.11772
6200.	.11769	.11767	.11765	.11763	.11760	.11758	.11756	.11754	.11752	.11750
6300.	.11747	.11745	.11743	.11741	.11739	.11737	.11734	.11732	.11730	.11728
6400.	.11726	.11724	.11722	.11719	.11717	.11715	.11713	.11711	.11709	.11707
6500.	.11705	.11703	.11700	.11698	.11696	.11694	.11692	.11690	.11688	.11686

Table 4. $G\left(\frac{\sqrt{4\alpha t}}{a}\right)$.

$\left(\frac{\sqrt{4\alpha t}}{a}\right)$	0	1	2	3	4	5	6	7	8	9
6600.	.11684	.11682	.11680	.11678	.11676	.11674	.11672	.11670	.11668	.11666
6700.	.11664	.11662	.11659	.11657	.11655	.11653	.11651	.11650	.11648	.11646
6800.	.11644	.11642	.11640	.11638	.11636	.11634	.11632	.11630	.11628	.11626
6900.	.11624	.11622	.11620	.11618	.11616	.11614	.11612	.11610	.11608	.11607
7000.	.11605	.11603	.11601	.11599	.11597	.11595	.11593	.11591	.11589	.11588
7100.	.11586	.11584	.11582	.11580	.11578	.11576	.11574	.11573	.11571	.11569
7200.	.11567	.11565	.11563	.11562	.11560	.11558	.11556	.11554	.11552	.11551
7300.	.11549	.11547	.11545	.11543	.11542	.11540	.11538	.11536	.11534	.11533
7400.	.11531	.11529	.11527	.11525	.11524	.11522	.11520	.11518	.11517	.11515
7500.	.11513	.11511	.11510	.11508	.11506	.11504	.11503	.11501	.11499	.11497
7600.	.11496	.11494	.11492	.11490	.11489	.11487	.11485	.11484	.11482	.11480
7700.	.11479	.11477	.11475	.11473	.11472	.11470	.11468	.11467	.11465	.11463
7800.	.11462	.11460	.11458	.11457	.11455	.11453	.11452	.11450	.11448	.11447
7900.	.11445	.11443	.11442	.11440	.11438	.11437	.11435	.11434	.11432	.11430
8000.	.11429	.11427	.11425	.11424	.11422	.11421	.11419	.11417	.11416	.11414
8100.	.11413	.11411	.11409	.11408	.11406	.11405	.11403	.11401	.11400	.11398
8200.	.11397	.11395	.11394	.11392	.11390	.11389	.11387	.11386	.11384	.11383
8300.	.11381	.11380	.11378	.11376	.11375	.11373	.11372	.11370	.11369	.11367
8400.	.11366	.11364	.11363	.11361	.11360	.11358	.11357	.11355	.11354	.11352
8500.	.11351	.11349	.11348	.11346	.11345	.11343	.11342	.11340	.11339	.11337
8600.	.11336	.11334	.11333	.11331	.11330	.11328	.11327	.11325	.11324	.11322
8700.	.11321	.11319	.11318	.11316	.11315	.11314	.11312	.11311	.11309	.11308
8800.	.11306	.11305	.11303	.11302	.11301	.11299	.11298	.11296	.11295	.11293
8900.	.11292	.11291	.11289	.11288	.11286	.11285	.11283	.11282	.11281	.11279
9000.	.11278	.11276	.11275	.11274	.11272	.11271	.11269	.11268	.11267	.11265
9100.	.11264	.11263	.11261	.11260	.11258	.11257	.11256	.11254	.11253	.11252
9200.	.11250	.11249	.11247	.11246	.11245	.11243	.11242	.11241	.11239	.11238
9300.	.11237	.11235	.11234	.11233	.11231	.11230	.11228	.11227	.11226	.11224
9400.	.11223	.11222	.11220	.11219	.11218	.11217	.11215	.11214	.11213	.11211
9500.	.11210	.11209	.11207	.11206	.11205	.11203	.11202	.11201	.11199	.11198
9600.	.11197	.11196	.11194	.11193	.11192	.11190	.11189	.11188	.11187	.11185
9700.	.11184	.11183	.11181	.11180	.11179	.11178	.11176	.11175	.11174	.11172
9800.	.11171	.11170	.11169	.11167	.11166	.11165	.11164	.11162	.11161	.11160
9900.	.11159	.11157	.11156	.11155	.11154	.11152	.11151	.11150	.11149	.11147
10000.	.11146									

25600.

(b/a)

8.

Table 5

Values of:

$$H\left(\frac{\sqrt{4\alpha t}}{a}\right) = \left(\frac{a^2}{4\alpha t}\right)\left[\left(\frac{b^2-a^2}{2a^2}\right) - \sum_{n=1}^{n=\infty} \frac{A_n U_0'(\beta_n a)\, e^{-\left(\frac{\beta_n a}{4}\right)\left(\frac{4\alpha t}{a^2}\right)}}{(\beta_n a)}\right]$$

The whole volume produced by the artesian well out of the time t is:

$$P = 8\pi KDy_0 t \; H\left(\frac{\sqrt{4\alpha t}}{a}\right)$$

A spot check computation for the point $(\sqrt{4\alpha t}/a) = 25$ can be made by using independently computed values for the $(b/a) = 100$ case.

$$\left(\frac{b^2-a^2}{2a^2}\right) = 4999.500 \qquad \left(\frac{4\alpha t}{a^2}\right) = 625$$

n	$\frac{A_n U_0'(\beta_n a)}{(\beta_n a)}$	$e^{-\left(\frac{\beta_n a}{4}\right)\left(\frac{4\alpha t}{a^2}\right)}$	Term of Series
1	4949.625	0.992006	4910.058
2	31.847	0.750097	23.888
3	7.734	0.410731	3.177
4	3.254	0.163467	0.532
5	1.745	0.047347	0.082
6	1.074	0.009988	0.011
7	0.722	0.001535	0.001
Total	4996.001	Total	4937.749

$$4999.500 - 4937.749 = 61.751 \qquad \frac{61.751}{625} = 0.098802$$

Tabular value is .098799. The correspondence is close enough to show that the program was operating properly.

A further confirmation of the validity of this table is obtained from a comparison of tabular values with those obtained from an independently derived approximate formula. A comparison for $(\sqrt{4\alpha t}/a) = 100$ is made in the text.

The comparable H values are

From the table	0.040671
From the approximation	0.04061

Table 5. $H\left(\frac{\sqrt{4\alpha t}}{a}\right)$

$\left(\frac{\sqrt{4\alpha t}}{a}\right)$	0	1	2	3	4	5	6	7	8	9
3.	.293577	.287121	.281053	.275338	.269945	.264848	.260020	.255442	.251094	.246958
4.	.243018	.239261	.235674	.232245	.228963	.225819	.222804	.219910	.217130	.214456
5.	.211883	.209404	.207015	.204710	.202485	.200336	.198258	.196248	.194303	.192419
6.	.190594									
6.	.190593	.188823	.187106	.185440	.183821	.182249	.180720	.179234	.177788	.176380
7.	.175010	.173675	.172374	.171105	.169869	.168662	.167484	.166335	.165212	.164115
8.	.163043	.161995	.160971	.159969	.158988	.158029	.157089	.156170	.155269	.154386
9.	.153522	.152674	.151843	.151028	.150229	.149445	.148676	.147920	.147179	.146451
10.	.145736	.145034	.144344	.143667	.143000	.142345	.141702	.141069	.140446	.139833
11.	.139231	.138638	.138055	.137480	.136915	.136358	.135810	.135270	.134738	.134215
12.	.133699									
12.	.133698	.133190	.132688	.132194	.131707	.131227	.130754	.130287	.129826	.129372
13.	.128924	.128482	.128046	.127616	.127191	.126772	.126358	.125949	.125546	.125148
14.	.124754	.124366	.123982	.123603	.123229	.122859	.122493	.122132	.121775	.121422
15.	.121073	.120729	.120388	.120051	.119718	.119389	.119063	.118741	.118422	.118107
16.	.117795	.117487	.117182	.116880	.116581	.116285	.115993	.115703	.115417	.115133
17.	.114852	.114574	.114299	.114027	.113757	.113490	.113225	.112963	.112704	.112447
18.	.112193	.111940	.111691	.111443	.111198	.110955	.110715	.110476	.110240	.110006
19.	.109774	.109544	.109316	.109090	.108866	.108644	.108424	.108206	.107990	.107775
20.	.107563	.107352	.107143	.106935	.106730	.106526	.106324	.106123	.105924	.105727
21.	.105531	.105337	.105144	.104953	.104764	.104575	.104389	.104204	.104020	.103837
22.	.103656	.103477	.103299	.103122	.102946	.102772	.102599	.102427	.102257	.102088
23.	.101920	.101753	.101588	.101423	.101260	.101098	.100937	.100778	.100619	.100462
24.	.100305	.100150	.099996	.099843	.099691	.099539	.099389	.099240	.099092	.098945
25.	.098799									
25.	.098798	.098653	.098509	.098366	.098224	.098082	.097942	.097802	.097664	.097526
26.	.097389	.097253	.097118	.096984	.096851	.096718	.096586	.096455	.096325	.096196
27.	.096067	.095940	.095813	.095686	.095561	.095436	.095312	.095189	.095067	.094945
28.	.094824	.094704	.094584	.094465	.094347	.094229	.094112	.093996	.093881	.093766
29.	.093652	.093538	.093425	.093313	.093201	.093090	.092979	.092870	.092760	.092652
30.	.092544	.092436	.092329	.092223	.092117	.092012	.091908	.091804	.091700	.091597
31.	.091495	.091393	.091292	.091191	.091091	.090991	.090892	.090793	.090695	.090597
32.	.090500	.090403	.090307	.090211	.090116	.090021	.089927	.089833	.089740	.089647
33.	.089554	.089462	.089371	.089280	.089189	.089099	.089009	.088920	.088831	.088742
34.	.088654	.088566	.088479	.088392	.088306	.088220	.088134	.088049	.087964	.087880
35.	.087796	.087712	.087629	.087546	.087464	.087382	.087300	.087218	.087137	.087057
36.	.086976	.086897	.086817	.086738	.086659	.086580	.086502	.086424	.086347	.086270
37.	.086193	.086116	.086040	.085964	.085889	.085814	.085739	.085664	.085590	.085516
38.	.085443	.085370	.085297	.085224	.085152	.085079	.085008	.084936	.084865	.084794
39.	.084724	.084653	.084583	.084514	.084444	.084375	.084306	.084238	.084169	.084101
40.	.084034	.083966	.083899	.083832	.083765	.083699	.083633	.083567	.083501	.083436
41.	.083371	.083306	.083241	.083177	.083112	.083049	.082985	.082922	.082858	.082795
42.	.082733	.082670	.082608	.082546	.082484	.082423	.082362	.082301	.082240	.082179
43.	.082119	.082059	.081999	.081939	.081880	.081820	.081761	.081702	.081644	.081585
44.	.081527	.081469	.081411	.081354	.081296	.081239	.081182	.081126	.081069	.081013
45.	.080956	.080901	.080845	.080789	.080734	.080679	.080624	.080569	.080514	.080460
46.	.080406	.080351	.080298	.080244	.080190	.080137	.080084	.080031	.079978	.079926
47.	.079873	.079821	.079769	.079717	.079665	.079614	.079562	.079511	.079460	.079409
48.	.079358	.079308	.079258	.079207	.079157	.079107	.079058	.079008	.078959	.078909
49.	.078860	.078811	.078763	.078714	.078666	.078617	.078569	.078521	.078473	.078426
50.	.078378									

(b/a): 25. 50. 100. 200. 1.

Table 5. $H\left(\frac{\sqrt{4\alpha t}}{a}\right)$

$\left(\frac{\sqrt{4\alpha t}}{a}\right)$	0	1	2	3	4	5	6	7	8	9
50.	.078379	.078331	.078284	.078237	.078190	.078143	.078097	.078050	.078004	.077957
51.	.077911	.077865	.077820	.077774	.077728	.077683	.077638	.077593	.077548	.077503
52.	.077458	.077414	.077369	.077325	.077281	.077237	.077193	.077149	.077105	.077062
53.	.077018	.076975	.076932	.076889	.076846	.076803	.076761	.076718	.076676	.076633
54.	.076591	.076549	.076507	.076466	.076424	.076382	.076341	.076300	.076258	.076217
55.	.076176	.076136	.076095	.076054	.076014	.075973	.075933	.075893	.075853	.075813
56.	.075773	.075733	.075694	.075654	.075615	.075576	.075537	.075498	.075459	.075420
57.	.075381	.075342	.075304	.075265	.075227	.075189	.075151	.075113	.075075	.075037
58.	.074999	.074962	.074924	.074887	.074850	.074812	.074775	.074738	.074701	.074665
59.	.074628	.074591	.074555	.074518	.074482	.074446	.074410	.074374	.074338	.074302
60.	.074266	.074230	.074195	.074159	.074124	.074089	.074053	.074018	.073983	.073948
61.	.073913	.073879	.073844	.073809	.073775	.073740	.073706	.073672	.073638	.073604
62.	.073570	.073536	.073502	.073468	.073434	.073401	.073367	.073334	.073301	.073267
63.	.073234	.073201	.073168	.073135	.073102	.073070	.073037	.073004	.072972	.072939
64.	.072907	.072875	.072842	.072810	.072778	.072746	.072714	.072683	.072651	.072619
65.	.072587	.072556	.072525	.072493	.072462	.072431	.072399	.072368	.072337	.072306
66.	.072276	.072245	.072214	.072183	.072153	.072122	.072092	.072061	.072031	.072001
67.	.071971	.071941	.071911	.071881	.071851	.071821	.071791	.071762	.071732	.071702
68.	.071673	.071643	.071614	.071585	.071556	.071526	.071497	.071468	.071439	.071410
69.	.071382	.071353	.071324	.071296	.071267	.071238	.071210	.071182	.071153	.071125
70.	.071097	.071069	.071041	.071013	.070985	.070957	.070929	.070901	.070873	.070846
71.	.070818	.070791	.070763	.070736	.070708	.070681	.070654	.070627	.070600	.070572
72.	.070545	.070519	.070492	.070465	.070438	.070411	.070385	.070358	.070331	.070305
73.	.070279	.070252	.070226	.070199	.070173	.070147	.070121	.070095	.070069	.070043
74.	.070017	.069991	.069965	.069940	.069914	.069888	.069863	.069837	.069812	.069786
75.	.069761	.069736	.069710	.069685	.069660	.069635	.069610	.069585	.069560	.069535
76.	.069510	.069485	.069460	.069436	.069411	.069386	.069362	.069337	.069313	.069288
77.	.069264	.069240	.069215	.069191	.069167	.069143	.069119	.069095	.069071	.069047
78.	.069023	.068999	.068975	.068951	.068928	.068904	.068880	.068857	.068833	.068810
79.	.068786	.068763	.068739	.068716	.068693	.068670	.068647	.068623	.068600	.068577
80.	.068554	.068531	.068508	.068485	.068463	.068440	.068417	.068394	.068372	.068349
81.	.068327	.068304	.068282	.068259	.068237	.068214	.068192	.068170	.068147	.068125
82.	.068103	.068081	.068059	.068037	.068015	.067993	.067971	.067949	.067927	.067905
83.	.067884	.067862	.067840	.067819	.067797	.067776	.067754	.067733	.067711	.067690
84.	.067668	.067647	.067626	.067604	.067583	.067562	.067541	.067520	.067499	.067478
85.	.067457	.067436	.067415	.067394	.067373	.067352	.067332	.067311	.067290	.067270
86.	.067249	.067228	.067208	.067187	.067167	.067146	.067126	.067106	.067085	.067065
87.	.067045	.067024	.067004	.066984	.066964	.066944	.066924	.066904	.066884	.066864
88.	.066844	.066824	.066804	.066784	.066765	.066745	.066725	.066706	.066686	.066666
89.	.066647	.066627	.066608	.066588	.066569	.066549	.066530	.066511	.066491	.066472
90.	.066453	.066433	.066414	.066395	.066376	.066357	.066338	.066319	.066300	.066281
91.	.066262	.066243	.066224	.066205	.066186	.066168	.066149	.066130	.066112	.066093
92.	.066074	.066056	.066037	.066019	.066000	.065982	.065963	.065945	.065926	.065908
93.	.065890	.065871	.065853	.065835	.065817	.065798	.065780	.065762	.065744	.065726
94.	.065708	.065690	.065672	.065654	.065636	.065618	.065600	.065583	.065565	.065547
95.	.065529	.065511	.065494	.065476	.065458	.065441	.065423	.065406	.065388	.065371
96.	.065353	.065336	.065318	.065301	.065284	.065266	.065249	.065232	.065214	.065197
97.	.065180	.065163	.065146	.065128	.065111	.065094	.065077	.065060	.065043	.065026
98.	.065009	.064992	.064975	.064959	.064942	.064925	.064908	.064891	.064875	.064858
99.	.064841	.064825	.064808	.064791	.064775	.064758	.064742	.064725	.064709	.064692
100.	.064676									

400. (b/a)

2.

	$\left(\frac{\sqrt{4\alpha t}}{a}\right)$	0	1	2	3	4	5	6	7	8	9
	100.	.064675	.064512	.064351	.064193	.064037	.063883	.063731	.063582	.063434	.063289
	110.	.063145	.063003	.062864	.062726	.062590	.062455	.062323	.062192	.062063	.061935
	120.	.061809	.061684	.061561	.061440	.061319	.061201	.061083	.060967	.060853	.060739
	130.	.060627	.060516	.060407	.060299	.060191	.060085	.059980	.059877	.059774	.059673
800	140.	.059572	.059473	.059374	.059277	.059180	.059085	.058990	.058897	.058804	.058712
	150.	.058621	.058531	.058442	.058354	.058266	.058180	.058094	.058009	.057925	.057841
	160.	.057759	.057677	.057595	.057515	.057435	.057356	.057277	.057200	.057123	.057046
	170.	.056970	.056895	.056821	.056747	.056674	.056601	.056529	.056458	.056387	.056316
	180.	.056247	.056177	.056109	.056041	.055973	.055906	.055839	.055773	.055708	.055643
	190.	.055578	.055514	.055451	.055387	.055325	.055263	.055201	.055140	.055079	.055018
	200.	.054958									
	200.	.054958	.054899	.054839	.054781	.054722	.054665	.054607	.054550	.054493	.054437
	210.	.054381	.054326	.054270	.054216	.054161	.054107	.054053	.054000	.053947	.053894
	220.	.053842	.053790	.053738	.053687	.053636	.053585	.053535	.053485	.053435	.053386
	230.	.053336	.053288	.053239	.053191	.053143	.053095	.053048	.053001	.052954	.052907
	240.	.052861	.052815	.052769	.052724	.052679	.052634	.052589	.052545	.052501	.052457
	250.	.052413	.052370	.052326	.052284	.052241	.052198	.052156	.052114	.052072	.052031
	260.	.051990	.051948	.051908	.051867	.051827	.051786	.051746	.051706	.051667	.051628
	270.	.051588	.051549	.051511	.051472	.051434	.051395	.051358	.051320	.051282	.051245
	280.	.051207	.051170	.051134	.051097	.051060	.051024	.050988	.050952	.050916	.050881
	290.	.050845	.050810	.050775	.050740	.050705	.050671	.050636	.050602	.050568	.050534
1600	300.	.050500	.050466	.050433	.050400	.050366	.050333	.050300	.050268	.050235	.050203
	310.	.050170	.050138	.050106	.050074	.050043	.050011	.049980	.049948	.049917	.049886
	320.	.049855	.049825	.049794	.049764	.049733	.049703	.049673	.049643	.049613	.049583
	330.	.049554	.049524	.049495	.049466	.049437	.049408	.049379	.049350	.049321	.049293
	340.	.049265	.049236	.049208	.049180	.049152	.049124	.049097	.049069	.049042	.049014
	350.	.048987	.048960	.048933	.048906	.048879	.048852	.048826	.048799	.048773	.048746
	360.	.048720	.048694	.048668	.048642	.048616	.048590	.048565	.048539	.048514	.048489
	370.	.048463	.048438	.048413	.048388	.048363	.048338	.048314	.048289	.048265	.048240
	380.	.048216	.048192	.048167	.048143	.048119	.048096	.048072	.048048	.048024	.048001
	390.	.047977	.047954	.047931	.047907	.047884	.047861	.047838	.047815	.047792	.047770
	400.	.047747									
	400.	.047748	.047726	.047703	.047681	.047658	.047636	.047614	.047592	.047570	.047548
	410.	.047526	.047504	.047482	.047460	.047439	.047417	.047396	.047374	.047353	.047332
	420.	.047310	.047289	.047268	.047247	.047226	.047205	.047185	.047164	.047143	.047123
	430.	.047102	.047082	.047061	.047041	.047021	.047000	.046980	.046960	.046940	.046920
	440.	.046900	.046881	.046861	.046841	.046821	.046802	.046782	.046763	.046743	.046724
	450.	.046705	.046686	.046666	.046647	.046628	.046609	.046590	.046571	.046553	.046534
	460.	.046515	.046496	.046478	.046459	.046441	.046422	.046404	.046386	.046367	.046349
	470.	.046331	.046313	.046295	.046277	.046259	.046241	.046223	.046205	.046187	.046170
3200.	480.	.046152	.046134	.046117	.046099	.046082	.046064	.046047	.046030	.046012	.045995
	490.	.045978	.045961	.045944	.045927	.045910	.045893	.045876	.045859	.045842	.045826
	500.	.045809	.045792	.045776	.045759	.045743	.045726	.045710	.045693	.045677	.045661
	510.	.045644	.045628	.045612	.045596	.045580	.045564	.045548	.045532	.045516	.045500
3.	520.	.045484	.045468	.045453	.045437	.045421	.045406	.045390	.045374	.045359	.045343
	530.	.045328	.045313	.045297	.045282	.045267	.045251	.045236	.045221	.045206	.045191
	540.	.045176	.045161	.045146	.045131	.045116	.045101	.045086	.045072	.045057	.045042

Table 5. $H\left(\frac{\sqrt{4\alpha t}}{a}\right)$

(b/a) = 3200. / 6400.

$\left(\frac{\sqrt{4\alpha t}}{a}\right)$	0	1	2	3	4	5	6	7	8	9
550.	.045027	.045013	.044998	.044984	.044969	.044955	.044940	.044926	.044911	.044897
560.	.044883	.044868	.044854	.044840	.044826	.044812	.044797	.044783	.044769	.044755
570.	.044741	.044727	.044713	.044700	.044686	.044672	.044658	.044644	.044631	.044617
580.	.044603	.044590	.044576	.044563	.044549	.044536	.044522	.044509	.044495	.044482
590.	.044469	.044455	.044442	.044429	.044415	.044402	.044389	.044376	.044363	.044350
600.	.044337	.044324	.044311	.044298	.044285	.044272	.044259	.044246	.044233	.044221
610.	.044208	.044195	.044183	.044170	.044157	.044145	.044132	.044119	.044107	.044094
620.	.044082	.044069	.044057	.044045	.044032	.044020	.044008	.043995	.043983	.043971
630.	.043959	.043946	.043934	.043922	.043910	.043898	.043886	.043874	.043862	.043850
640.	.043838	.043826	.043814	.043802	.043790	.043779	.043767	.043755	.043743	.043731
650.	.043720	.043708	.043696	.043685	.043673	.043662	.043650	.043638	.043627	.043615
660.	.043604	.043593	.043581	.043570	.043558	.043547	.043536	.043524	.043513	.043502
670.	.043491	.043479	.043468	.043457	.043446	.043435	.043424	.043412	.043401	.043390
680.	.043379	.043368	.043357	.043346	.043335	.043325	.043314	.043303	.043292	.043281
690.	.043270	.043260	.043249	.043238	.043227	.043217	.043206	.043195	.043185	.043174
700.	.043163	.043153	.043142	.043132	.043121	.043111	.043100	.043090	.043079	.043069
710.	.043059	.043048	.043038	.043027	.043017	.043007	.042997	.042986	.042976	.042966
720.	.042956	.042945	.042935	.042925	.042915	.042905	.042895	.042885	.042875	.042865
730.	.042855	.042845	.042835	.042825	.042815	.042805	.042795	.042785	.042775	.042765
740.	.042755	.042746	.042736	.042726	.042716	.042707	.042697	.042687	.042677	.042668
750.	.042658	.042648	.042639	.042629	.042620	.042610	.042600	.042591	.042581	.042572
760.	.042562	.042553	.042543	.042534	.042525	.042515	.042506	.042496	.042487	.042478
770.	.042468	.042459	.042450	.042440	.042431	.042422	.042413	.042404	.042394	.042385
780.	.042376	.042367	.042358	.042349	.042339	.042330	.042321	.042312	.042303	.042294
790.	.042285	.042276	.042267	.042258	.042249	.042240	.042231	.042222	.042214	.042205
800.	.042196									
800.	.042195	.042186	.042177	.042168	.042160	.042151	.042142	.042133	.042125	.042116
810.	.042107	.042098	.042090	.042081	.042072	.042064	.042055	.042047	.042038	.042029
820.	.042021	.042012	.042004	.041995	.041987	.041978	.041970	.041961	.041953	.041944
830.	.041936	.041927	.041919	.041911	.041902	.041894	.041885	.041877	.041869	.041860
840.	.041852	.041844	.041836	.041827	.041819	.041811	.041803	.041794	.041786	.041778
850.	.041770	.041762	.041754	.041745	.041737	.041729	.041721	.041713	.041705	.041697
860.	.041689	.041681	.041673	.041665	.041657	.041649	.041641	.041633	.041625	.041617
870.	.041609	.041601	.041593	.041585	.041577	.041570	.041562	.041554	.041546	.041538
880.	.041530	.041523	.041515	.041507	.041499	.041492	.041484	.041476	.041468	.041461
890.	.041453	.041445	.041438	.041430	.041422	.041415	.041407	.041400	.041392	.041384
900.	.041377	.041369	.041362	.041354	.041347	.041339	.041332	.041324	.041317	.041309
910.	.041302	.041294	.041287	.041279	.041272	.041264	.041257	.041250	.041242	.041235
920.	.041228	.041220	.041213	.041206	.041198	.041191	.041184	.041176	.041169	.041162
930.	.041155	.041147	.041140	.041133	.041126	.041118	.041111	.041104	.041097	.041090
940.	.041083	.041075	.041068	.041061	.041054	.041047	.041040	.041033	.041026	.041019
950.	.041012	.041005	.040998	.040991	.040984	.040977	.040970	.040963	.040956	.040949
960.	.040942	.040935	.040928	.040921	.040914	.040907	.040900	.040893	.040886	.040880
970.	.040873	.040866	.040859	.040852	.040845	.040838	.040832	.040825	.040818	.040811
980.	.040805	.040798	.040791	.040784	.040778	.040771	.040764	.040757	.040751	.040744
990.	.040737	.040731	.040724	.040717	.040711	.040704	.040698	.040691	.040684	.040678

$\left(\frac{\sqrt{4\alpha t}}{a}\right)$	0	1	2	3	4	5	6	7	8	9
1000.	.040671	.040665	.040658	.040651	.040645	.040638	.040632	.040625	.040619	.040612
1010.	.040606	.040599	.040593	.040586	.040580	.040573	.040567	.040560	.040554	.040548
1020.	.040541	.040535	.040528	.040522	.040516	.040509	.040503	.040496	.040490	.040484
1030.	.040477	.040471	.040465	.040458	.040452	.040446	.040440	.040433	.040427	.040421
1040.	.040415	.040408	.040402	.040396	.040390	.040383	.040377	.040371	.040365	.040359
1050.	.040352	.040346	.040340	.040334	.040328	.040322	.040315	.040309	.040303	.040297
1060.	.040291	.040285	.040279	.040273	.040267	.040261	.040255	.040249	.040243	.040236
1070.	.040230	.040224	.040218	.040212	.040206	.040200	.040194	.040188	.040183	.040177
1080.	.040171	.040165	.040159	.040153	.040147	.040141	.040135	.040129	.040123	.040117
1090.	.040111	.040106	.040100	.040094	.040088	.040082	.040076	.040071	.040065	.040059
1100.	.040053	.040047	.040041	.040036	.040030	.040024	.040018	.040013	.040007	.040001
1110.	.039995	.039990	.039984	.039978	.039972	.039967	.039961	.039955	.039950	.039944
1120.	.039938	.039933	.039927	.039921	.039916	.039910	.039904	.039899	.039893	.039888
1130.	.039882	.039876	.039871	.039865	.039860	.039854	.039848	.039843	.039837	.039832
1140.	.039826	.039821	.039815	.039810	.039804	.039799	.039793	.039788	.039782	.039777
1150.	.039771	.039766	.039760	.039755	.039749	.039744	.039738	.039733	.039728	.039722
1160.	.039717	.039711	.039706	.039701	.039695	.039690	.039684	.039679	.039674	.039668
1170.	.039663	.039658	.039652	.039647	.039642	.039636	.039631	.039626	.039620	.039615
1180.	.039610	.039604	.039599	.039594	.039589	.039583	.039578	.039573	.039567	.039562
1190.	.039557	.039552	.039547	.039541	.039536	.039531	.039526	.039520	.039515	.039510
1200.	.039505	.039500	.039495	.039489	.039484	.039479	.039474	.039469	.039464	.039459
1210.	.039453	.039448	.039443	.039438	.039433	.039428	.039423	.039418	.039413	.039408
1220.	.039402	.039397	.039392	.039387	.039382	.039377	.039372	.039367	.039362	.039357
1230.	.039352	.039347	.039342	.039337	.039332	.039327	.039322	.039317	.039312	.039307
1240.	.039302	.039297	.039292	.039287	.039282	.039277	.039273	.039268	.039263	.039258
1250.	.039253	.039248	.039243	.039238	.039233	.039228	.039224	.039219	.039214	.039209
1260.	.039204	.039199	.039194	.039189	.039185	.039180	.039175	.039170	.039165	.039161
1270.	.039156	.039151	.039146	.039141	.039137	.039132	.039127	.039122	.039117	.039113
1280.	.039108	.039103	.039098	.039094	.039089	.039084	.039079	.039075	.039070	.039065
1290.	.039061	.039056	.039051	.039046	.039042	.039037	.039032	.039028	.039023	.039018
1300.	.039014	.039009	.039004	.039000	.038995	.038990	.038986	.038981	.038977	.038972
1310.	.038967	.038963	.038958	.038953	.038949	.038944	.038940	.038935	.038931	.038926
1320.	.038921	.038917	.038912	.038908	.038903	.038899	.038894	.038890	.038885	.038880
1330.	.038876	.038871	.038867	.038862	.038858	.038853	.038849	.038844	.038840	.038835
1340.	.038831	.038826	.038822	.038817	.038813	.038809	.038804	.038800	.038795	.038791
1350.	.038786	.038782	.038777	.038773	.038769	.038764	.038760	.038755	.038751	.038747
1360.	.038742	.038738	.038733	.038729	.038725	.038720	.038716	.038711	.038707	.038703
1370.	.038698	.038694	.038690	.038685	.038681	.038677	.038672	.038668	.038664	.038659
1380.	.038655	.038651	.038646	.038642	.038638	.038634	.038629	.038625	.038621	.038616
1390.	.038612	.038608	.038604	.038599	.038595	.038591	.038587	.038582	.038578	.038574
1400.	.038570	.038565	.038561	.038557	.038553	.038548	.038544	.038540	.038536	.038532
1410.	.038527	.038523	.038519	.038515	.038511	.038507	.038502	.038498	.038494	.038490
1420.	.038486	.038482	.038477	.038473	.038469	.038465	.038461	.038457	.038453	.038449
1430.	.038444	.038440	.038436	.038432	.038428	.038424	.038420	.038416	.038412	.038408
1440.	.038403	.038399	.038395	.038391	.038387	.038383	.038379	.038375	.038371	.038367
1450.	.038363	.038359	.038355	.038351	.038347	.038343	.038339	.038335	.038331	.038327
1460.	.038323	.038319	.038315	.038311	.038307	.038303	.038299	.038295	.038291	.038287
1470.	.038283	.038279	.038275	.038271	.038267	.038263	.038259	.038255	.038251	.038247
1480.	.038243	.038239	.038235	.038231	.038227	.038224	.038220	.038216	.038212	.038208
1490.	.038204	.038200	.038196	.038192	.038188	.038184	.038181	.038177	.038173	.038169

Table 5. $H\left(\frac{\sqrt{4\alpha t}}{a}\right)$

$\left(\frac{\sqrt{4\alpha t}}{a}\right)$	0	1	2	3	4	5	6	7	8	9
1500.	.038165	.038161	.038157	.038154	.038150	.038146	.038142	.038138	.038134	.038130
1510.	.038127	.038123	.038119	.038115	.038111	.038107	.038104	.038100	.038096	.038092
1520.	.038088	.038085	.038081	.038077	.038073	.038069	.038066	.038062	.038058	.038054
1530.	.038051	.038047	.038043	.038039	.038036	.038032	.038028	.038024	.038021	.038017
1540.	.038013	.038009	.038006	.038002	.037998	.037994	.037991	.037987	.037983	.037980
1550.	.037976	.037972	.037968	.037965	.037961	.037957	.037954	.037950	.037946	.037943
1560.	.037939	.037935	.037932	.037928	.037924	.037921	.037917	.037913	.037910	.037906
1570.	.037902	.037899	.037895	.037891	.037888	.037884	.037880	.037877	.037873	.037870
1580.	.037866	.037862	.037859	.037855	.037852	.037848	.037844	.037841	.037837	.037834
1590.	.037830	.037826	.037823	.037819	.037816	.037812	.037809	.037805	.037801	.037798
1600.	.037794									
1600.	.037793	.037758	.037723	.037688	.037653	.037619	.037585	.037551	.037518	.037485
1700.	.037452	.037419	.037386	.037354	.037322	.037290	.037259	.037228	.037197	.037166
1800.	.037135	.037105	.037074	.037044	.037015	.036985	.036956	.036927	.036898	.036869
1900.	.036840	.036812	.036784	.036756	.036728	.036700	.036673	.036646	.036619	.036592
2000.	.036565	.036539	.036512	.036486	.036460	.036434	.036408	.036383	.036357	.036332
2100.	.036307	.036282	.036257	.036233	.036208	.036184	.036160	.036136	.036112	.036088
2200.	.036064	.036041	.036018	.035994	.035971	.035948	.035925	.035903	.035880	.035858
2300.	.035835	.035813	.035791	.035769	.035747	.035726	.035704	.035683	.035661	.035640
2400.	.035619	.035598	.035577	.035556	.035536	.035515	.035495	.035474	.035454	.035434
2500.	.035414	.035394	.035374	.035354	.035335	.035315	.035296	.035276	.035257	.035238
2600.	.035219	.035200	.035181	.035162	.035144	.035125	.035106	.035088	.035070	.035051
2700.	.035033	.035015	.034997	.034979	.034961	.034944	.034926	.034908	.034891	.034874
2800.	.034856	.034839	.034822	.034805	.034788	.034771	.034754	.034737	.034720	.034704
2900.	.034687	.034671	.034654	.034638	.034622	.034605	.034589	.034573	.034557	.034541
3000.	.034525	.034509	.034494	.034478	.034462	.034447	.034431	.034416	.034401	.034385
3100.	.034370	.034355	.034340	.034325	.034310	.034295	.034280	.034265	.034251	.034236
3200.	.034221									
3200.	.034223	.034208	.034194	.034179	.034165	.034151	.034136	.034122	.034108	.034094
3300.	.034080	.034066	.034052	.034038	.034024	.034010	.033996	.033983	.033969	.033956
3400.	.033942	.033929	.033915	.033902	.033888	.033875	.033862	.033849	.033835	.033822
3500.	.033809	.033796	.033783	.033770	.033758	.033745	.033732	.033719	.033707	.033694
3600.	.033681	.033669	.033656	.033644	.033631	.033619	.033607	.033594	.033582	.033570
3700.	.033558	.033546	.033534	.033522	.033510	.033498	.033486	.033474	.033462	.033450
3800.	.033438	.033427	.033415	.033403	.033392	.033380	.033369	.033357	.033346	.033334
3900.	.033323	.033312	.033300	.033289	.033278	.033267	.033256	.033244	.033233	.033222
4000.	.033211	.033200	.033189	.033178	.033168	.033157	.033146	.033135	.033124	.033114
4100.	.033103	.033092	.033082	.033071	.033061	.033050	.033040	.033029	.033019	.033008
4200.	.032998	.032988	.032977	.032967	.032957	.032947	.032936	.032926	.032916	.032906
4300.	.032896	.032886	.032876	.032866	.032856	.032846	.032836	.032826	.032817	.032807
4400.	.032797	.032787	.032778	.032768	.032758	.032749	.032739	.032730	.032720	.032710
4500.	.032701	.032692	.032682	.032673	.032663	.032654	.032645	.032635	.032626	.032617
4600.	.032607	.032598	.032589	.032580	.032571	.032562	.032553	.032544	.032535	.032525
4700.	.032517	.032508	.032499	.032490	.032481	.032472	.032463	.032454	.032445	.032437
4800.	.032428	.032419	.032411	.032402	.032393	.032385	.032376	.032367	.032359	.032350
4900.	.032342	.032333	.032325	.032316	.032308	.032299	.032291	.032283	.032274	.032266

6400.

12800.

(b/a)

25600.

6.

Table 5. $H\left(\frac{\sqrt{4\alpha t}}{a}\right)$

$\left(\frac{\sqrt{4\alpha t}}{a}\right)$	0	1	2	3	4	5	6	7	8	9
5000.	.032258	.032249	.032241	.032233	.032225	.032216	.032208	.032200	.032192	.032184
5100.	.032176	.032168	.032159	.032151	.032143	.032135	.032127	.032119	.032111	.032104
5200.	.032096	.032088	.032080	.032072	.032064	.032056	.032049	.032041	.032033	.032025
5300.	.032018	.032010	.032002	.031994	.031987	.031979	.031972	.031964	.031956	.031949
5400.	.031941	.031934	.031926	.031919	.031911	.031904	.031896	.031889	.031882	.031874
5500.	.031867	.031859	.031852	.031845	.031837	.031830	.031823	.031816	.031808	.031801
5600.	.031794	.031787	.031780	.031772	.031765	.031758	.031751	.031744	.031737	.031730
5700.	.031723	.031716	.031709	.031702	.031695	.031688	.031681	.031674	.031667	.031660
5800.	.031653	.031646	.031639	.031632	.031626	.031619	.031612	.031605	.031598	.031592
5900.	.031585	.031578	.031571	.031565	.031558	.031551	.031545	.031538	.031531	.031525
6000.	.031518	.031511	.031505	.031498	.031492	.031485	.031479	.031472	.031466	.031459
6100.	.031453	.031446	.031440	.031433	.031427	.031421	.031414	.031408	.031401	.031395
6200.	.031389	.031382	.031376	.031370	.031363	.031357	.031351	.031345	.031338	.031332
6300.	.031326	.031320	.031314	.031307	.031301	.031295	.031289	.031283	.031277	.031270
6400.	.031264	.031258	.031252	.031246	.031240	.031234	.031228	.031222	.031216	.031210
6500.	.031204	.031198	.031192	.031186	.031180	.031174	.031168	.031162	.031157	.031151
6600.	.031145	.031139	.031133	.031127	.031121	.031116	.031110	.031104	.031098	.031092
6700.	.031087	.031081	.031075	.031070	.031064	.031058	.031052	.031047	.031041	.031035
6800.	.031030	.031024	.031018	.031013	.031007	.031002	.030996	.030990	.030985	.030979
6900.	.030974	.030968	.030963	.030957	.030952	.030946	.030941	.030935	.030930	.030924
7000.	.030919	.030913	.030908	.030902	.030897	.030892	.030886	.030881	.030875	.030870
7100.	.030865	.030859	.030854	.030849	.030843	.030838	.030833	.030828	.030822	.030817
7200.	.030812	.030806	.030801	.030796	.030791	.030785	.030780	.030775	.030770	.030765
7300.	.030760	.030754	.030749	.030744	.030739	.030734	.030729	.030724	.030718	.030713
7400.	.030708	.030703	.030698	.030693	.030688	.030683	.030678	.030673	.030668	.030663
7500.	.030658	.030653	.030648	.030643	.030638	.030633	.030628	.030623	.030618	.030613
7600.	.030608	.030603	.030598	.030594	.030589	.030584	.030579	.030574	.030569	.030564
7700.	.030559	.030555	.030550	.030545	.030540	.030535	.030531	.030526	.030521	.030516
7800.	.030511	.030507	.030502	.030497	.030492	.030488	.030483	.030478	.030474	.030469
7900.	.030464	.030460	.030455	.030450	.030445	.030441	.030436	.030432	.030427	.030422
8000.	.030418	.030413	.030408	.030404	.030399	.030395	.030390	.030386	.030381	.030376
8100.	.030372	.030367	.030363	.030358	.030354	.030349	.030345	.030340	.030336	.030331
8200.	.030327	.030322	.030318	.030313	.030309	.030304	.030300	.030296	.030291	.030287
8300.	.030282	.030278	.030274	.030269	.030265	.030260	.030256	.030252	.030247	.030243
8400.	.030239	.030234	.030230	.030226	.030221	.030217	.030213	.030208	.030204	.030200
8500.	.030195	.030191	.030187	.030183	.030178	.030174	.030170	.030166	.030161	.030157
8600.	.030153	.030149	.030145	.030140	.030136	.030132	.030128	.030124	.030119	.030115
8700.	.030111	.030107	.030103	.030099	.030095	.030090	.030086	.030082	.030078	.030074
8800.	.030070	.030066	.030062	.030058	.030053	.030049	.030045	.030041	.030037	.030033
8900.	.030029	.030025	.030021	.030017	.030013	.030009	.030005	.030001	.029997	.029993
9000.	.029989	.029985	.029981	.029977	.029973	.029969	.029965	.029961	.029957	.029953
9100.	.029949	.029945	.029942	.029938	.029934	.029930	.029926	.029922	.029918	.029914
9200.	.029910	.029906	.029903	.029899	.029895	.029891	.029887	.029883	.029879	.029876
9300.	.029872	.029868	.029864	.029860	.029856	.029853	.029849	.029845	.029841	.029837
9400.	.029834	.029830	.029826	.029822	.029819	.029815	.029811	.029807	.029804	.029800
9500.	.029796	.029792	.029789	.029785	.029781	.029778	.029774	.029770	.029766	.029763
9600.	.029759	.029755	.029752	.029748	.029744	.029741	.029737	.029733	.029730	.029726
9700.	.029723	.029719	.029715	.029712	.029708	.029704	.029701	.029697	.029694	.029690
9800.	.029686	.029683	.029679	.029676	.029672	.029669	.029665	.029661	.029658	.029654
9900.	.029651	.029647	.029644	.029640	.029637	.029633	.029630	.029626	.029623	.029619
10000.	.029616									

(b/a). 25600.

7.

Table 6

Values of:

$$\frac{s}{\left(\frac{Q}{2\pi KD}\right)} = \left[\log_e\left(\frac{b}{r}\right) - \sum_{n=1}^{n=\infty} \frac{2\, J_0(\beta_n r)\, e^{-(\beta_n b)^2 \left(\frac{\alpha t}{b^2}\right)}}{(\beta_n b)^2\, J_1^2\, (\beta_n b)} \right]$$

This solution applies where a well at the center of a circular aquifer delivers water at the steady rate Q . The water table level is maintained at the outer boundary $r = b$. The $(\beta_n b)$ values are roots of $J_0(\beta_n b) = 0$.

To provide a check, the value of $s/\left(\frac{Q}{2\pi KD}\right)$ will be computed for $\left(\frac{\alpha t}{b^2}\right) = 0.20$ $\left(\frac{r}{b}\right) = 0.5$ $\log_e\left(\frac{b}{r}\right) = 0.693147$

n	$(\beta_n b)$	$J_1(\beta_n b)$	$J_0(\beta_n r)$	$e^{-(\beta_n b)^2\left(\frac{\alpha t}{b^2}\right)}$	Term
1	2.404826	+0.519147	+0.669930	0.314542	+0.270390
2	5.520078	-0.340265	-0.168401	0.002226	-0.000212
3	8.653728	+0.271452	-0.356278	0.000000	--
					0.270178

$$\frac{s}{\left(\frac{Q}{2\pi KD}\right)} = 0.693147 - 0.270178 = 0.422969$$

The tabular value is 0.42297. This is a satisfactory check.

Table 6. Water table maintained at r=b.

$\left(\frac{\alpha t}{b^2}\right)$	$\left(\frac{r}{b}\right)$ 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
.01	.52213	.10969	.01738	.00189	.00014	.00001	.00000	.00000	.00000	.00000
.02	.81171	.27989	.08931	.02445	.00557	.00104	.00016	.00002	.00000	.00000
.03	.99466	.41444	.17017	.06433	.02180	.00652	.00171	.00039	.00007	.00000
.04	1.12845	.52214	.24524	.10969	.04559	.01738	.00603	.00188	.00050	.00000
.05	1.23395	.61132	.31267	.15530	.07321	.03235	.01329	.00501	.00159	.00000
.06	1.32104	.68726	.37310	.19920	.10233	.04999	.02301	.00979	.00347	.00000
.07	1.39519	.75330	.42755	.24076	.13169	.06915	.03451	.01597	.00606	.00000
.08	1.45976	.81171	.47696	.27985	.16058	.08906	.04718	.02318	.00923	.00000
.09	1.51693	.86403	.52210	.31653	.18863	.10917	.06053	.03108	.01280	.00000
.10	1.56822	.91141	.56358	.35096	.21564	.12910	.07417	.03937	.01663	.00000
.11	1.61471	.95466	.60191	.38328	.24150	.14861	.08782	.04784	.02059	.00000
.12	1.65719	.99441	.63747	.41365	.26617	.16753	.10128	.05632	.02459	.00000
.13	1.69627	1.03115	.67058	.44220	.28965	.18577	.11442	.06467	.02857	.00000
.14	1.73241	1.06524	.70150	.46907	.31194	.20326	.12714	.07283	.03246	.00000
.15	1.76598	1.09700	.73043	.49438	.33309	.21998	.13937	.08072	.03625	.00000
.16	1.79725	1.12667	.75755	.51821	.35311	.23590	.15110	.08831	.03991	.00000
.17	1.82647	1.15444	.78301	.54067	.37207	.25104	.16229	.09558	.04342	.00000
.18	1.85384	1.18048	.80695	.56185	.39000	.26541	.17295	.10253	.04678	.00000
.19	1.87950	1.20493	.82946	.58182	.40695	.27903	.18308	.10914	.04998	.00000
.20	1.90361	1.22792	.85066	.60065	.42297	.29193	.19269	.11543	.05302	.00000
.21	1.92627	1.24955	.87062	.61841	.43811	.30414	.20180	.12139	.05591	.00000
.22	1.94760	1.26991	.88944	.63517	.45241	.31568	.21042	.12704	.05865	.00000
.23	1.96768	1.28909	.90717	.65098	.46591	.32660	.21858	.13239	.06125	.00000
.24	1.98659	1.30717	.92389	.66590	.47866	.33691	.22630	.13746	.06371	.00000
.25	2.00442	1.32421	.93966	.67998	.49070	.34666	.23359	.14224	.06604	.00000
.26	2.02123	1.34028	.95454	.69327	.50207	.35586	.24049	.14677	.06824	.00000
.27	2.03708	1.35543	.96857	.70580	.51280	.36456	.24700	.15105	.07031	.00000
.28	2.05203	1.36973	.98182	.71763	.52293	.37277	.25315	.15509	.07228	.00000
.29	2.06613	1.38321	.99431	.72880	.53249	.38052	.25896	.15890	.07413	.00000
.30	2.07943	1.39594	1.00610	.73934	.54151	.38783	.26444	.16251	.07588	.00000
.31	2.09198	1.40794	1.01722	.74928	.55003	.39474	.26962	.16591	.07754	.00000
.32	2.10382	1.41927	1.02772	.75867	.55807	.40126	.27451	.16912	.07910	.00000
.33	2.11500	1.42996	1.03763	.76752	.56566	.40741	.27912	.17216	.08057	.00000
.34	2.12554	1.44005	1.04698	.77588	.57282	.41322	.28348	.17502	.08197	.00000
.35	2.13550	1.44957	1.05580	.78377	.57958	.41871	.28759	.17772	.08328	.00000
.36	2.14489	1.45856	1.06413	.79122	.58596	.42388	.29147	.18027	.08452	.00000
.37	2.15375	1.46704	1.07199	.79825	.59199	.42877	.29513	.18268	.08569	.00000
.38	2.16211	1.47504	1.07940	.80488	.59767	.43338	.29859	.18496	.08680	.00000
.39	2.17001	1.48259	1.08640	.81114	.60304	.43773	.30185	.18710	.08784	.00000
.40	2.17746	1.48972	1.09301	.81705	.60810	.44183	.30493	.18913	.08882	.00000
.41	2.18449	1.49645	1.09924	.82263	.61288	.44571	.30784	.19104	.08975	.00000
.42	2.19113	1.50280	1.10513	.82789	.61739	.44937	.31058	.19284	.09063	.00000
.43	2.19739	1.50879	1.11068	.83286	.62165	.45282	.31317	.19454	.09146	.00000
.44	2.20330	1.51445	1.11593	.83755	.62566	.45608	.31562	.19615	.09224	.00000
.45	2.20888	1.51978	1.12087	.84197	.62945	.45916	.31793	.19767	.09298	.00000
.46	2.21415	1.52482	1.12554	.84615	.63303	.46206	.32010	.19910	.09367	.00000
.47	2.21912	1.52958	1.12995	.85009	.63641	.46480	.32216	.20045	.09433	.00000
.48	2.22381	1.53406	1.13411	.85381	.63960	.46739	.32410	.20173	.09495	.00000
.49	2.22823	1.53830	1.13804	.85732	.64261	.46983	.32593	.20293	.09553	.00000
.50	2.23241	1.54230	1.14174	.86063	.64545	.47213	.32766	.20406	.09609	.00000

Table 6. Water table maintained at r=b.

$(\frac{\alpha t}{b^2})$	$(\frac{r}{b})$ 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
.51	2.23635	1.54607	1.14524	.86376	.64813	.47431	.32929	.20514	.09661	.00000
.52	2.24008	1.54963	1.14854	.86671	.65066	.47636	.33083	.20615	.09710	.00000
.53	2.24359	1.55299	1.15165	.86950	.65305	.47830	.33228	.20710	.09756	.00000
.54	2.24690	1.55616	1.15459	.87213	.65530	.48012	.33365	.20801	.09800	.00000
.55	2.25003	1.55916	1.15737	.87461	.65743	.48185	.33494	.20886	.09841	.00000
.56	2.25299	1.56198	1.15999	.87695	.65943	.48348	.33616	.20966	.09880	.00000
.57	2.25577	1.56465	1.16246	.87916	.66133	.48501	.33732	.21042	.09917	.00000
.58	2.25840	1.56717	1.16479	.88125	.66312	.48646	.33840	.21113	.09952	.00000
.59	2.26089	1.56954	1.16699	.88322	.66480	.48783	.33943	.21181	.09985	.00000
.60	2.26323	1.57178	1.16907	.88508	.66640	.48912	.34040	.21244	.10016	.00000
.61	2.26544	1.57390	1.17103	.88683	.66790	.49034	.34131	.21304	.10045	.00000
.62	2.26753	1.57590	1.17288	.88849	.66932	.49150	.34218	.21361	.10073	.00000
.63	2.26950	1.57778	1.17463	.89005	.67066	.49258	.34299	.21415	.10099	.00000
.64	2.27136	1.57956	1.17628	.89152	.67192	.49361	.34376	.21465	.10123	.00000
.65	2.27311	1.58124	1.17783	.89291	.67311	.49457	.34449	.21513	.10146	.00000
.66	2.27477	1.58282	1.17930	.89423	.67424	.49549	.34517	.21558	.10168	.00000
.67	2.27633	1.58432	1.18069	.89547	.67530	.49635	.34582	.21601	.10189	.00000
.68	2.27781	1.58573	1.18200	.89664	.67630	.49716	.34643	.21641	.10209	.00000
.69	2.27920	1.58706	1.18323	.89774	.67725	.49793	.34700	.21679	.10227	.00000
.70	2.28051	1.58832	1.18440	.89878	.67814	.49865	.34755	.21714	.10244	.00000
.71	2.28175	1.58951	1.18550	.89977	.67899	.49934	.34806	.21748	.10261	.00000
.72	2.28292	1.59063	1.18654	.90070	.67978	.49998	.34854	.21780	.10276	.00000
.73	2.28403	1.59168	1.18752	.90157	.68053	.50059	.34900	.21810	.10291	.00000
.74	2.28507	1.59268	1.18844	.90240	.68124	.50117	.34943	.21838	.10305	.00000
.75	2.28606	1.59362	1.18931	.90318	.68191	.50171	.34984	.21865	.10318	.00000
.76	2.28698	1.59451	1.19014	.90392	.68254	.50222	.35022	.21890	.10330	.00000
.77	2.28786	1.59535	1.19091	.90461	.68314	.50271	.35059	.21914	.10341	.00000
.78	2.28869	1.59614	1.19165	.90527	.68370	.50316	.35093	.21937	.10352	.00000
.79	2.28947	1.59689	1.19234	.90589	.68423	.50359	.35125	.21958	.10363	.00000
.80	2.29021	1.59759	1.19299	.90647	.68473	.50400	.35156	.21978	.10372	.00000
.81	2.29090	1.59826	1.19361	.90702	.68521	.50438	.35184	.21997	.10382	.00000
.82	2.29156	1.59889	1.19419	.90754	.68565	.50475	.35211	.22015	.10390	.00000
.83	2.29218	1.59948	1.19474	.90804	.68607	.50509	.35237	.22031	.10398	.00000
.84	2.29276	1.60004	1.19526	.90850	.68647	.50541	.35261	.22047	.10406	.00000
.85	2.29331	1.60057	1.19575	.90894	.68685	.50571	.35284	.22062	.10414	.00000
.86	2.29384	1.60107	1.19621	.90935	.68720	.50600	.35306	.22076	.10420	.00000
.87	2.29433	1.60154	1.19665	.90974	.68753	.50627	.35326	.22090	.10427	.00000
.88	2.29479	1.60198	1.19706	.91011	.68785	.50653	.35345	.22102	.10433	.00000
.89	2.29523	1.60240	1.19745	.91046	.68815	.50677	.35363	.22114	.10439	.00000
.90	2.29564	1.60280	1.19782	.91078	.68843	.50700	.35380	.22126	.10444	.00000
.91	2.29603	1.60317	1.19816	.91109	.68869	.50721	.35397	.22136	.10449	.00000
.92	2.29640	1.60352	1.19849	.91139	.68894	.50742	.35412	.22146	.10454	.00000
.93	2.29675	1.60385	1.19880	.91166	.68918	.50761	.35426	.22156	.10459	.00000
.94	2.29708	1.60417	1.19909	.91192	.68940	.50779	.35440	.22165	.10463	.00000
.95	2.29739	1.60446	1.19936	.91217	.68961	.50796	.35452	.22173	.10467	.00000
.96	2.29768	1.60474	1.19962	.91240	.68981	.50812	.35465	.22181	.10471	.00000
.97	2.29795	1.60501	1.19987	.91262	.69000	.50827	.35476	.22188	.10475	.00000
.98	2.29821	1.60526	1.20010	.91282	.69018	.50842	.35487	.22196	.10478	.00000
.99	2.29846	1.60549	1.20031	.91302	.69034	.50855	.35497	.22202	.10482	.00000
1.00	2.29869	1.60571	1.20052	.91320	.69050	.50868	.35506	.22208	.10485	.00000

2.

Table 7

Values of:

$$\frac{y}{(\frac{Q}{2\pi KD})} = \left[\log_e(\frac{b}{r}) + \frac{r^2}{2b^2} - \frac{3}{4} + \frac{2\alpha_1 t}{b^2} - \sum_{n=1}^{n=\infty} \frac{2\, J_0(\beta_n r)\, e^{-(\beta_n b)^2 (\frac{\alpha_1 t}{b^2})}}{(\beta_n b)^2\, J_0^2\, (\beta_n b)} \right]$$

With $J_1\ (\beta_n b) = 0$.

This solution applies where a well at the center of a circular aquifer, of outer radius b , lifts water at the steady rate Q . An impermeable boundary is assumed to be present at the radius b .

To provide a check the value of $y/(Q/2\pi KD)$ will be computed for $(\frac{\alpha_1 t}{b^2}) = 0.20$ where $(r/b) = 0.5$.

$$\log_e(\frac{b}{r}) = 0.693147 \quad \frac{r^2}{2b^2} = 0.125000 \quad 2\ (\frac{\alpha_1 t}{b^2}) = 0.40$$

The terms preceding the summation total 0.468147.

n	$(\beta_n b)$	$J_0(\beta_n b)$	$J_0(\beta_n r)$	$e^{-(\beta_n b)^2(\frac{\alpha_1 t}{b^2})}$	Term
1	3.831706	-0.402759	+0.272608	0.053057	+0.012146
2	7.015587	+0.300116	-0.381185	0.000053	-0.000009
3	10.173468	-0.249705	-0.148799	0.000000	--
				Total	+0.012137

$$y/(\frac{Q}{2\pi KD}) = 0.468147 - 0.012137 = 0.45601$$

The tabular value is 0.45601. This is a satisfactory check.

Table 7. Impermeable outer boundary.

$\left(\frac{\alpha t}{b^2}\right)$	$\left(\frac{r}{b}\right)$ .000001	.000005	.000010	.000050	.000100	.000500	.001000	.005000	.010000	.050000
.01	11.91746	10.30803	9.61488	8.00544	7.31229	5.70286	5.00972	3.40058	2.70837	1.12845
.02	12.26404	10.65460	9.96145	8.35202	7.65887	6.04943	5.35629	3.74700	3.05432	1.45976
.03	12.46677	10.85733	10.16419	8.55475	7.86160	6.25216	5.55902	3.94968	3.25685	1.65736
.04	12.61061	11.00117	10.30803	8.69859	8.00544	6.39600	5.70286	4.09350	3.40058	1.79862
.05	12.72218	11.11275	10.41960	8.81016	8.11701	6.50758	5.81443	4.20505	3.51209	1.90864
.06	12.81334	11.20391	10.51076	8.90132	8.20817	6.59874	5.90559	4.29620	3.60321	1.99876
.07	12.89042	11.28098	10.58784	8.97840	8.28525	6.67581	5.98267	4.37327	3.68026	2.07510
.08	12.95719	11.34775	10.65460	9.04516	8.35202	6.74258	6.04943	4.44003	3.74700	2.14131
.09	13.01609	11.40665	10.71350	9.10406	8.41092	6.80148	6.10833	4.49893	3.80588	2.19977
.10	13.06878	11.45935	10.76620	9.15676	8.46361	6.85418	6.16103	4.55162	3.85857	2.25213
.11	13.11648	11.50704	10.81390	9.20446	8.51131	6.90187	6.20873	4.59932	3.90625	2.29954
.12	13.16007	11.55063	10.85748	9.24805	8.55490	6.94546	6.25232	4.64290	3.94983	2.34290
.13	13.20024	11.59080	10.89766	9.28822	8.59507	6.98563	6.29249	4.68307	3.99000	2.38288
.14	13.23754	11.62810	10.93495	9.32551	8.63237	7.02293	6.32978	4.72036	4.02729	2.42001
.15	13.27240	11.66296	10.96981	9.36037	8.66723	7.05779	6.36464	4.75522	4.06214	2.45473
.16	13.30518	11.69574	11.00260	9.39316	8.70001	7.09057	6.39743	4.78801	4.09492	2.48739
.17	13.33619	11.72676	11.03361	9.42417	8.73102	7.12159	6.42844	4.81902	4.12593	2.51830
.18	13.36569	11.75625	11.06310	9.45366	8.76051	7.15108	6.45793	4.84851	4.15542	2.54771
.19	13.39387	11.78443	11.09129	9.48185	8.78870	7.17926	6.48612	4.87669	4.18360	2.57582
.20	13.42093	11.81149	11.11835	9.50891	8.81576	7.20632	6.51318	4.90376	4.21066	2.60281
.21	13.44703	11.83759	11.14444	9.53500	8.84185	7.23242	6.53927	4.92985	4.23675	2.62885
.22	13.47228	11.86285	11.16970	9.56026	8.86711	7.25768	6.56453	4.95511	4.26201	2.65406
.23	13.49682	11.88739	11.19424	9.58480	8.89165	7.28222	6.58907	4.97965	4.28654	2.67856
.24	13.52074	11.91130	11.21816	9.60872	8.91557	7.30613	6.61299	5.00356	4.31046	2.70244
.25	13.54412	11.93469	11.24154	9.63210	8.93895	7.32952	6.63637	5.02695	4.33384	2.72579
.26	13.56705	11.95761	11.26446	9.65502	8.96188	7.35244	6.65929	5.04987	4.35676	2.74869
.27	13.58957	11.98013	11.28698	9.67754	8.98440	7.37496	6.68181	5.07239	4.37928	2.77119
.28	13.61174	12.00231	11.30916	9.69972	9.00657	7.39714	6.70399	5.09457	4.40146	2.79334
.29	13.63362	12.02419	11.33104	9.72160	9.02845	7.41902	6.72587	5.11644	4.42334	2.81521
.30	13.65525	12.04581	11.35266	9.74322	9.05008	7.44064	6.74749	5.13807	4.44496	2.83681
.31	13.67665	12.06721	11.37406	9.76463	9.07148	7.46204	6.76889	5.15947	4.46636	2.85820
.32	13.69786	12.08842	11.39527	9.78584	9.09269	7.48325	6.79010	5.18068	4.48757	2.87940
.33	13.71890	12.10947	11.41632	9.80688	9.11373	7.50430	6.81115	5.20172	4.50862	2.90044
.34	13.73981	12.13037	11.43722	9.82778	9.13464	7.52520	6.83205	5.22263	4.52952	2.92133
.35	13.76059	12.15115	11.45800	9.84856	9.15541	7.54598	6.85283	5.24340	4.55030	2.94210
.36	13.78126	12.17182	11.47867	9.86923	9.17609	7.56665	6.87350	5.26408	4.57097	2.96277
.37	13.80184	12.19240	11.49925	9.88982	9.19667	7.58723	6.89408	5.28466	4.59155	2.98334
.38	13.82234	12.21290	11.51975	9.91032	9.21717	7.60773	6.91459	5.30516	4.61205	3.00384
.39	13.84277	12.23333	11.54019	9.93075	9.23760	7.62816	6.93502	5.32559	4.63248	3.02427
.40	13.86315	12.25371	11.56056	9.95112	9.25798	7.64854	6.95539	5.34597	4.65286	3.04464
.41	13.88347	12.27403	11.58088	9.97145	9.27830	7.66886	6.97571	5.36629	4.67318	3.06496
.42	13.90375	12.29431	11.60116	9.99173	9.29858	7.68914	6.99599	5.38657	4.69346	3.08524
.43	13.92399	12.31455	11.62140	10.01197	9.31882	7.70938	7.01623	5.40681	4.71370	3.10547
.44	13.94420	12.33476	11.64161	10.03217	9.33903	7.72959	7.03644	5.42702	4.73391	3.12568
.45	13.96438	12.35494	11.66179	10.05235	9.35921	7.74977	7.05662	5.44720	4.75409	3.14586
.46	13.98453	12.37509	11.68195	10.07251	9.37936	7.76992	7.07678	5.46735	4.77424	3.16601
.47	14.00466	12.39523	11.70208	10.09264	9.39949	7.79006	7.09691	5.48748	4.79437	3.18614
.48	14.02478	12.41534	11.72220	10.11276	9.41961	7.81017	7.11703	5.50760	4.81449	3.20626
.49	14.04488	12.43544	11.74229	10.13286	9.43971	7.83027	7.13713	5.52770	4.83459	3.22636
.50	14.06497	12.45553	11.76238	10.15294	9.45980	7.85036	7.15721	5.54779	4.85468	3.24644

1.

$(\frac{\alpha t}{b^2})$	.000001	.000005	.000010	.000050	.000100	.000500	$(\frac{r}{b})$ .001000	.005000	.010000	.050000
.51	14.08504	12.47560	11.78246	10.17302	9.47987	7.87043	7.17729	5.56786	4.87475	3.26652
.52	14.10510	12.49567	11.80252	10.19308	9.49993	7.89050	7.19735	5.58792	4.89481	3.28658
.53	14.12516	12.51572	11.82257	10.21314	9.51999	7.91055	7.21741	5.60798	4.91487	3.30663
.54	14.14521	12.53577	11.84262	10.23318	9.54004	7.93060	7.23745	5.62803	4.93492	3.32668
.55	14.16525	12.55581	11.86266	10.25323	9.56008	7.95064	7.25749	5.64807	4.95496	3.34672
.56	14.18528	12.57585	11.88270	10.27326	9.58011	7.97068	7.27753	5.66810	4.97499	3.36676
.57	14.20532	12.59588	11.90273	10.29329	9.60015	7.99071	7.29756	5.68814	4.99503	3.38679
.58	14.22534	12.61590	11.92276	10.31332	9.62017	8.01073	7.31759	5.70816	5.01505	3.40682
.59	14.24537	12.63593	11.94278	10.33334	9.64020	8.03076	7.33761	5.72818	5.03507	3.42684
.60	14.26539	12.65595	11.96280	10.35336	9.66021	8.05078	7.35763	5.74820	5.05509	3.44686
.61	14.28540	12.67596	11.98282	10.37338	9.68023	8.07079	7.37765	5.76822	5.07511	3.46687
.62	14.30542	12.69598	12.00283	10.39339	9.70025	8.09081	7.39766	5.78824	5.09513	3.48689
.63	14.32543	12.71599	12.02284	10.41341	9.72026	8.11082	7.41768	5.80825	5.11514	3.50690
.64	14.34544	12.73600	12.04286	10.43342	9.74027	8.13083	7.43769	5.82826	5.13515	3.52691
.65	14.36545	12.75601	12.06287	10.45343	9.76028	8.15084	7.45770	5.84827	5.15516	3.54692
.66	14.38546	12.77602	12.08287	10.47344	9.78029	8.17085	7.47770	5.86828	5.17517	3.56693
.67	14.40547	12.79603	12.10288	10.49344	9.80030	8.19086	7.49771	5.88828	5.19518	3.58694
.68	14.42547	12.81603	12.12289	10.51345	9.82030	8.21086	7.51772	5.90829	5.21518	3.60694
.69	14.44548	12.83604	12.14289	10.53345	9.84031	8.23087	7.53772	5.92830	5.23519	3.62695
.70	14.46548	12.85604	12.16290	10.55346	9.86031	8.25087	7.55773	5.94830	5.25519	3.64695
.71	14.48549	12.87605	12.18290	10.57346	9.88032	8.27088	7.57773	5.96830	5.27520	3.66696
.72	14.50549	12.89605	12.20290	10.59347	9.90032	8.29088	7.59773	5.98831	5.29520	3.68696
.73	14.52549	12.91605	12.22291	10.61347	9.92032	8.31088	7.61774	6.00831	5.31520	3.70696
.74	14.54549	12.93606	12.24291	10.63347	9.94032	8.33089	7.63774	6.02831	5.33520	3.72697
.75	14.56550	12.95606	12.26291	10.65347	9.96033	8.35089	7.65774	6.04832	5.35521	3.74697
.76	14.58550	12.97606	12.28291	10.67348	9.98033	8.37089	7.67774	6.06832	5.37521	3.76697
.77	14.60550	12.99606	12.30292	10.69348	10.00033	8.39089	7.69775	6.08832	5.39521	3.78697
.78	14.62550	13.01606	12.32292	10.71348	10.02033	8.41089	7.71775	6.10832	5.41521	3.80697
.79	14.64550	13.03606	12.34292	10.73348	10.04033	8.43089	7.73775	6.12832	5.43521	3.82697
.80	14.66550	13.05607	12.36292	10.75348	10.06033	8.45090	7.75775	6.14832	5.45521	3.84698
.81	14.68550	13.07607	12.38292	10.77348	10.08033	8.47090	7.77775	6.16832	5.47521	3.86698
.82	14.70551	13.09607	12.40292	10.79348	10.10034	8.49090	7.79775	6.18832	5.49522	3.88698
.83	14.72551	13.11607	12.42292	10.81348	10.12034	8.51090	7.81775	6.20833	5.51522	3.90698
.84	14.74551	13.13607	12.44292	10.83348	10.14034	8.53090	7.83775	6.22833	5.53522	3.92698
.85	14.76551	13.15607	12.46292	10.85348	10.16034	8.55090	7.85775	6.24833	5.55522	3.94698
.86	14.78551	13.17607	12.48292	10.87348	10.18034	8.57090	7.87775	6.26833	5.57522	3.96698
.87	14.80551	13.19607	12.50292	10.89349	10.20034	8.59090	7.89775	6.28833	5.59522	3.98698
.88	14.82551	13.21607	12.52292	10.91349	10.22034	8.61090	7.91775	6.30833	5.61522	4.00698
.89	14.84551	13.23607	12.54292	10.93349	10.24034	8.63090	7.93775	6.32833	5.63522	4.02698
.90	14.86551	13.25607	12.56292	10.95349	10.26034	8.65090	7.95775	6.34833	5.65522	4.04698
.91	14.88551	13.27607	12.58292	10.97349	10.28034	8.67090	7.97775	6.36833	5.67522	4.06698
.92	14.90551	13.29607	12.60292	10.99349	10.30034	8.69090	7.99775	6.38833	5.69522	4.08698
.93	14.92551	13.31607	12.62292	11.01349	10.32034	8.71090	8.01775	6.40833	5.71522	4.10698
.94	14.94551	13.33607	12.64292	11.03349	10.34034	8.73090	8.03775	6.42833	5.73522	4.12698
.95	14.96551	13.35607	12.66292	11.05349	10.36034	8.75090	8.05776	6.44833	5.75522	4.14698
.96	14.98551	13.37607	12.68292	11.07349	10.38034	8.77090	8.07776	6.46833	5.77522	4.16698
.97	15.00551	13.39607	12.70292	11.09349	10.40034	8.79090	8.09776	6.48833	5.79522	4.18698
.98	15.02551	13.41607	12.72292	11.11349	10.42034	8.81090	8.11776	6.50833	5.81522	4.20698
.99	15.04551	13.43607	12.74293	11.13349	10.44034	8.83090	8.13776	6.52833	5.83522	4.22698
1.00	15.06551	13.45607	12.76293	11.15349	10.46034	8.85090	8.15776	6.54833	5.85522	4.24698

2.

$\left(\frac{\alpha t}{b^2}\right)$	.001	.01	.1	.2	.3	.4	$\left(\frac{r}{b}\right)$ .5	.6	.7	.8	.9	1.0
.01	5.00972	2.70837	.52214	.10969	.01738	.00189	.00014	.00001	.00000	-.00000	.00000	-.00000
.02	5.35629	3.05432	.81171	.27989	.08930	.02445	.00557	.00104	.00016	.00002	.00000	.00000
.03	5.55902	3.25685	.99466	.41444	.17017	.06433	.02180	.00652	.00171	.00039	.00008	.00003
.04	5.70286	3.40058	1.12845	.52214	.24524	.10969	.04559	.01738	.00603	.00190	.00057	.00028
.05	5.81443	3.51209	1.23395	.61133	.31267	.15530	.07321	.03236	.01332	.00513	.00198	.00120
.06	5.90559	3.60321	1.32104	.68726	.37310	.19921	.10234	.05004	.02317	.01026	.00473	.00325
.07	5.98267	3.68026	1.39519	.75330	.42756	.24078	.13175	.06934	.03500	.01722	.00906	.00676
.08	6.04943	3.74700	1.45977	.81172	.47699	.27993	.16078	.08957	.04837	.02587	.01502	.01187
.09	6.10833	3.80588	1.51696	.86408	.52219	.31675	.18913	.11028	.06292	.03601	.02259	.01862
.10	6.16103	3.85857	1.56829	.91152	.56382	.35145	.21667	.13121	.07837	.04747	.03167	.02692
.11	6.20873	3.90625	1.61487	.95492	.60241	.38424	.24338	.15221	.09456	.06008	.04212	.03667
.12	6.25232	3.94983	1.65754	.99494	.63841	.41535	.26930	.17320	.11133	.07369	.05380	.04771
.13	6.29249	3.99000	1.69693	1.03212	.67219	.44498	.29449	.19413	.12858	.08816	.06657	.05993
.14	6.32978	4.02729	1.73358	1.06688	.70407	.47332	.31901	.21497	.14623	.10339	.08031	.07317
.15	6.36464	4.06214	1.76788	1.09958	.73432	.50053	.34295	.23573	.16422	.11926	.09488	.08731
.16	6.39743	4.09492	1.80019	1.13053	.76316	.52676	.36636	.25640	.18249	.13570	.11018	.10223
.17	6.42844	4.12593	1.83080	1.15996	.79079	.55215	.38932	.27698	.20101	.15262	.12612	.11784
.18	6.45793	4.15542	1.85994	1.18810	.81737	.57681	.41188	.29750	.21973	.16996	.14261	.13405
.19	6.48612	4.18360	1.88783	1.21512	.84305	.60083	.43410	.31795	.23863	.18766	.15957	.15077
.20	6.51318	4.21066	1.91463	1.24117	.86796	.62431	.45601	.33834	.25767	.20568	.17695	.16794
.21	6.53927	4.23675	1.94050	1.26640	.89220	.64731	.47767	.35867	.27685	.22397	.19469	.18549
.22	6.56453	4.26201	1.96557	1.29092	.91585	.66990	.49910	.37897	.29615	.24249	.21273	.20338
.23	6.58907	4.28654	1.98994	1.31481	.93901	.69214	.52033	.39922	.31553	.26121	.23104	.22155
.24	6.61299	4.31046	2.01372	1.33818	.96174	.71407	.54140	.41944	.33501	.28011	.24958	.23997
.25	6.63637	4.33384	2.03698	1.36108	.98409	.73574	.56232	.43963	.35455	.29916	.26833	.25861
.26	6.65929	4.35676	2.05979	1.38359	1.00612	.75718	.58311	.45979	.37416	.31834	.28724	.27744
.27	6.68181	4.37928	2.08222	1.40575	1.02788	.77842	.60380	.47993	.39382	.33763	.30630	.29642
.28	6.70399	4.40146	2.10432	1.42762	1.04939	.79950	.62439	.50005	.41353	.35702	.32549	.31554
.29	6.72587	4.42334	2.12613	1.44923	1.07070	.82042	.64491	.52016	.43327	.37649	.34479	.33479
.30	6.74749	4.44496	2.14770	1.47063	1.09183	.84123	.66535	.54025	.45306	.39603	.36418	.35413
.31	6.76889	4.46636	2.16905	1.49183	1.11281	.86192	.68573	.56033	.47287	.41564	.38366	.37357
.32	6.79010	4.48757	2.19021	1.51287	1.13365	.88251	.70606	.58040	.49270	.43530	.40321	.39308
.33	6.81115	4.50862	2.21122	1.53377	1.15437	.90303	.72635	.60046	.51256	.45500	.42282	.41266
.34	6.83205	4.52952	2.23209	1.55454	1.17500	.92348	.74659	.62051	.53244	.47475	.44249	.43230
.35	6.85283	4.55030	2.25284	1.57521	1.19554	.94386	.76680	.64055	.55234	.49453	.46220	.45198
.36	6.87350	4.57097	2.27349	1.59579	1.21601	.96419	.78699	.66059	.57225	.51434	.48194	.47171
.37	6.89408	4.59155	2.29405	1.61629	1.23642	.98448	.80715	.68062	.59217	.53418	.50173	.49148
.38	6.91459	4.61205	2.31453	1.63672	1.25677	1.00473	.82728	.70065	.61210	.55404	.52154	.51128
.39	6.93502	4.63248	2.33495	1.65709	1.27707	1.02494	.84740	.72067	.63204	.57391	.54138	.53110
.40	6.95539	4.65286	2.35531	1.67741	1.29733	1.04512	.86750	.74069	.65199	.59381	.56124	.55095
.41	6.97571	4.67318	2.37562	1.69769	1.31755	1.06528	.88759	.76071	.67195	.61372	.58112	.57082
.42	6.99599	4.69346	2.39589	1.71792	1.33775	1.08542	.90767	.78073	.69191	.63364	.60102	.59071
.43	7.01623	4.71370	2.41612	1.73813	1.35791	1.10554	.92773	.80074	.71188	.65357	.62093	.61061
.44	7.03644	4.73391	2.43632	1.75831	1.37806	1.12564	.94779	.82075	.73185	.67351	.64085	.63053
.45	7.05662	4.75409	2.45649	1.77846	1.39818	1.14573	.96784	.84076	.75183	.69346	.66078	.65046
.46	7.07678	4.77424	2.47664	1.79860	1.41829	1.16581	.98788	.86077	.77181	.71342	.68073	.67039
.47	7.09691	4.79437	2.49677	1.81871	1.43838	1.18587	1.00792	.88078	.79179	.73338	.70068	.69034
.48	7.11703	4.81449	2.51688	1.83881	1.45846	1.20593	1.02795	.90078	.81177	.75335	.72063	.71029
.49	7.13713	4.83459	2.53698	1.85890	1.47853	1.22598	1.04798	.92079	.83176	.77332	.74060	.73025
.50	7.15721	4.85468	2.55706	1.87897	1.49859	1.24602	1.06800	.94080	.85175	.79330	.76056	.75022

3.

$\left(\frac{\alpha t}{b^2}\right)$	$\left(\frac{r}{b}\right)$ .001	.01	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0
.51	7.17729	4.87475	2.57713	1.89903	1.51865	1.26606	1.08802	.96080	.87174	.81328	.78054	.77019
.52	7.19735	4.89481	2.59719	1.91909	1.53869	1.28609	1.10804	.98080	.89173	.83326	.80051	.79016
.53	7.21741	4.91487	2.61725	1.93914	1.55873	1.30612	1.12805	1.00081	.91172	.85324	.82049	.81014
.54	7.23745	4.93492	2.63729	1.95918	1.57876	1.32614	1.14806	1.02081	.93172	.87323	.84047	.83012
.55	7.25749	4.95496	2.65733	1.97921	1.59879	1.34616	1.16808	1.04081	.95171	.89322	.86046	.85011
.56	7.27753	4.97499	2.67737	1.99924	1.61882	1.36618	1.18809	1.06081	.97171	.91321	.88044	.87009
.57	7.29756	4.99503	2.69740	2.01927	1.63884	1.38619	1.20809	1.08081	.99170	.93320	.90043	.89008
.58	7.31759	5.01505	2.71742	2.03929	1.65886	1.40621	1.22810	1.10082	1.01170	.95319	.92042	.91007
.59	7.33761	5.03507	2.73745	2.05931	1.67887	1.42622	1.24811	1.12082	1.03169	.97318	.94041	.93006
.60	7.35763	5.05509	2.75746	2.07933	1.69889	1.44623	1.26811	1.14082	1.05169	.99318	.96041	.95005
.61	7.37765	5.07511	2.77748	2.09934	1.71890	1.46624	1.28812	1.16082	1.07169	1.01317	.98040	.97004
.62	7.39766	5.09513	2.79749	2.11936	1.73891	1.48624	1.30812	1.18082	1.09169	1.03317	1.00040	.99004
.63	7.41768	5.11514	2.81751	2.13937	1.75892	1.50625	1.32813	1.20082	1.11169	1.05317	1.02039	1.01003
.64	7.43769	5.13515	2.83752	2.15938	1.77892	1.52626	1.34813	1.22082	1.13168	1.07316	1.04039	1.03003
.65	7.45770	5.15516	2.85753	2.17939	1.79893	1.54626	1.36813	1.24082	1.15168	1.09316	1.06038	1.05002
.66	7.47770	5.17517	2.87754	2.19939	1.81894	1.56627	1.38813	1.26082	1.17168	1.11316	1.08038	1.07002
.67	7.49771	5.19518	2.89754	2.21940	1.83894	1.58627	1.40813	1.28082	1.19168	1.13316	1.10038	1.09002
.68	7.51772	5.21518	2.91755	2.23940	1.85895	1.60627	1.42814	1.30082	1.21168	1.15315	1.12037	1.11002
.69	7.53772	5.23519	2.93755	2.25941	1.87895	1.62627	1.44814	1.32082	1.23168	1.17315	1.14037	1.13001
.70	7.55773	5.25519	2.95756	2.27941	1.89895	1.64628	1.46814	1.34082	1.25168	1.19315	1.16037	1.15001
.71	7.57773	5.27520	2.97756	2.29942	1.91896	1.66628	1.48814	1.36082	1.27168	1.21315	1.18037	1.17001
.72	7.59773	5.29520	2.99756	2.31942	1.93896	1.68628	1.50814	1.38082	1.29168	1.23315	1.20037	1.19001
.73	7.61774	5.31520	3.01757	2.33942	1.95896	1.70628	1.52814	1.40082	1.31168	1.25315	1.22037	1.21001
.74	7.63774	5.33520	3.03757	2.35942	1.97896	1.72628	1.54814	1.42082	1.33168	1.27315	1.24037	1.23001
.75	7.65774	5.35521	3.05757	2.37943	1.99896	1.74628	1.56814	1.44082	1.35168	1.29315	1.26037	1.25001
.76	7.67774	5.37521	3.07757	2.39943	2.01896	1.76628	1.58814	1.46082	1.37168	1.31315	1.28036	1.27000
.77	7.69775	5.39521	3.09758	2.41943	2.03897	1.78629	1.60814	1.48083	1.39168	1.33315	1.30036	1.29000
.78	7.71775	5.41521	3.11758	2.43943	2.05897	1.80629	1.62814	1.50083	1.41168	1.35315	1.32036	1.31000
.79	7.73775	5.43521	3.13758	2.45943	2.07897	1.82629	1.64815	1.52083	1.43168	1.37315	1.34036	1.33000
.80	7.75775	5.45521	3.15758	2.47943	2.09897	1.84629	1.66815	1.54083	1.45168	1.39315	1.36036	1.35000
.81	7.77775	5.47521	3.17758	2.49943	2.11897	1.86629	1.68815	1.56083	1.47168	1.41315	1.38036	1.37000
.82	7.79775	5.49522	3.19758	2.51943	2.13897	1.88629	1.70815	1.58083	1.49168	1.43314	1.40036	1.39000
.83	7.81775	5.51522	3.21758	2.53943	2.15897	1.90629	1.72815	1.60083	1.51168	1.45314	1.42036	1.41000
.84	7.83775	5.53522	3.23758	2.55943	2.17897	1.92629	1.74815	1.62083	1.53168	1.47314	1.44036	1.43000
.85	7.85775	5.55522	3.25758	2.57944	2.19897	1.94629	1.76815	1.64083	1.55168	1.49314	1.46036	1.45000
.86	7.87775	5.57522	3.27758	2.59944	2.21897	1.96629	1.78815	1.66083	1.57168	1.51314	1.48036	1.47000
.87	7.89775	5.59522	3.29758	2.61944	2.23897	1.98629	1.80815	1.68083	1.59168	1.53314	1.50036	1.49000
.88	7.91775	5.61522	3.31758	2.63944	2.25897	2.00629	1.82815	1.70083	1.61168	1.55314	1.52036	1.51000
.89	7.93775	5.63522	3.33758	2.65944	2.27897	2.02629	1.84815	1.72083	1.63168	1.57314	1.54036	1.53000
.90	7.95775	5.65522	3.35758	2.67944	2.29897	2.04629	1.86815	1.74083	1.65168	1.59314	1.56036	1.55000
.91	7.97775	5.67522	3.37758	2.69944	2.31897	2.06629	1.88815	1.76083	1.67168	1.61314	1.58036	1.57000
.92	7.99775	5.69522	3.39758	2.71944	2.33897	2.08629	1.90815	1.78083	1.69168	1.63314	1.60036	1.59000
.93	8.01775	5.71522	3.41758	2.73944	2.35897	2.10629	1.92815	1.80083	1.71168	1.65314	1.62036	1.61000
.94	8.03775	5.73522	3.43758	2.75944	2.37897	2.12629	1.94815	1.82083	1.73168	1.67314	1.64036	1.63000
.95	8.05776	5.75522	3.45758	2.77944	2.39897	2.14629	1.96815	1.84083	1.75168	1.69314	1.66036	1.65000
.96	8.07776	5.77522	3.47758	2.79944	2.41897	2.16629	1.98815	1.86083	1.77168	1.71314	1.68036	1.67000
.97	8.09776	5.79522	3.49758	2.81944	2.43897	2.18629	2.00815	1.88083	1.79168	1.73314	1.70036	1.69000
.98	8.11776	5.81522	3.51758	2.83944	2.45897	2.20629	2.02815	1.90083	1.81168	1.75314	1.72036	1.71000
.99	8.13776	5.83522	3.53758	2.85944	2.47897	2.22629	2.04815	1.92083	1.83167	1.77314	1.74036	1.73000
1.00	8.15776	5.85522	3.55758	2.87944	2.49897	2.24629	2.06815	1.94083	1.85167	1.79314	1.76036	1.75000

Table 8

Values of:

$$\frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du$$

This is the "Probability integral."

This integral table was made by Simpson's rule procedures. The table was prepared in this way to make it fit into the format of the other tables and to avoid the tedious and expensive typesetting and checking operations which would have been required if it were to be made up from values to be found in the excellent existing tables.

The following spot comparisons are made with values from the U.S. Department of Commerce Applied Mathematics Series No. 41.

x	These Tables	Vol. 41*
0	0	0
0.5	0.52050	0.520499
1.0	0.84270	0.842700
1.5	0.96611	0.966105
2.0	0.99532	0.995322
2.5	0.99959	0.999593
3.0	0.99998	0.999977

*Last figure not rounded.

Table 8. $\frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}\,du$

x	0	1	2	3	4	5	6	7	8	9
.00	0	.00113	.00226	.00339	.00451	.00564	.00677	.00790	.00903	.01016
.01	.01128	.01241	.01354	.01467	.01580	.01692	.01805	.01918	.02031	.02144
.02	.02256	.02369	.02482	.02595	.02708	.02820	.02933	.03046	.03159	.03271
.03	.03384	.03497	.03610	.03722	.03835	.03948	.04060	.04173	.04286	.04398
.04	.04511	.04624	.04736	.04849	.04962	.05074	.05187	.05299	.05412	.05525
.05	.05637	.05750	.05862	.05975	.06087	.06200	.06312	.06425	.06537	.06650
.06	.06762	.06875	.06987	.07099	.07212	.07324	.07437	.07549	.07661	.07773
.07	.07886	.07998	.08110	.08223	.08335	.08447	.08559	.08671	.08784	.08896
.08	.09008	.09120	.09232	.09344	.09456	.09568	.09680	.09792	.09904	.10016
.09	.10128	.10240	.10352	.10464	.10576	.10687	.10799	.10911	.11023	.11135
.10	.11246	.11358	.11470	.11581	.11693	.11805	.11916	.12028	.12139	.12251
.11	.12362	.12474	.12585	.12697	.12808	.12919	.13031	.13142	.13253	.13365
.12	.13476	.13587	.13698	.13809	.13921	.14032	.14143	.14254	.14365	.14476
.13	.14587	.14698	.14809	.14919	.15030	.15141	.15252	.15363	.15473	.15584
.14	.15695	.15805	.15916	.16027	.16137	.16248	.16358	.16468	.16579	.16689
.15	.16800	.16910	.17020	.17130	.17241	.17351	.17461	.17571	.17681	.17791
.16	.17901	.18011	.18121	.18231	.18341	.18451	.18560	.18670	.18780	.18890
.17	.18999	.19109	.19218	.19328	.19437	.19547	.19656	.19766	.19875	.19984
.18	.20094	.20203	.20312	.20421	.20530	.20639	.20748	.20857	.20966	.21075
.19	.21184	.21293	.21402	.21510	.21619	.21728	.21836	.21945	.22053	.22162
.20	.22270	.22379	.22487	.22595	.22704	.22812	.22920	.23028	.23136	.23244
.21	.23352	.23460	.23568	.23676	.23784	.23891	.23999	.24107	.24214	.24322
.22	.24430	.24537	.24645	.24752	.24859	.24967	.25074	.25181	.25288	.25395
.23	.25502	.25609	.25716	.25823	.25930	.26037	.26144	.26250	.26357	.26463
.24	.26570	.26677	.26783	.26889	.26996	.27102	.27208	.27314	.27421	.27527
.25	.27633	.27739	.27845	.27950	.28056	.28162	.28268	.28373	.28479	.28584
.26	.28690	.28795	.28901	.29006	.29111	.29217	.29322	.29427	.29532	.29637
.27	.29742	.29847	.29952	.30056	.30161	.30266	.30370	.30475	.30579	.30684
.28	.30788	.30892	.30997	.31101	.31205	.31309	.31413	.31517	.31621	.31725
.29	.31828	.31932	.32036	.32139	.32243	.32346	.32450	.32553	.32656	.32760
.30	.32863	.32966	.33069	.33172	.33275	.33378	.33480	.33583	.33686	.33788
.31	.33891	.33993	.34096	.34198	.34300	.34403	.34505	.34607	.34709	.34811
.32	.34913	.35014	.35116	.35218	.35319	.35421	.35523	.35624	.35725	.35827
.33	.35928	.36029	.36130	.36231	.36332	.36433	.36534	.36635	.36735	.36836
.34	.36936	.37037	.37137	.37238	.37338	.37438	.37538	.37638	.37738	.37838
.35	.37938	.38038	.38138	.38237	.38337	.38436	.38536	.38635	.38735	.38834
.36	.38933	.39032	.39131	.39230	.39329	.39428	.39526	.39625	.39724	.39822
.37	.39921	.40019	.40117	.40215	.40314	.40412	.40510	.40608	.40705	.40803
.38	.40901	.40999	.41096	.41194	.41291	.41388	.41486	.41583	.41680	.41777
.39	.41874	.41971	.42068	.42164	.42261	.42358	.42454	.42550	.42647	.42743
.40	.42839	.42935	.43031	.43127	.43223	.43319	.43415	.43510	.43606	.43701
.41	.43797	.43892	.43988	.44083	.44178	.44273	.44368	.44463	.44557	.44652
.42	.44747	.44841	.44936	.45030	.45124	.45219	.45313	.45407	.45501	.45595
.43	.45689	.45782	.45876	.45970	.46063	.46157	.46250	.46343	.46436	.46529
.44	.46623	.46715	.46808	.46901	.46994	.47086	.47179	.47271	.47364	.47456
.45	.47548	.47640	.47732	.47824	.47916	.48008	.48100	.48191	.48283	.48374
.46	.48466	.48557	.48648	.48739	.48830	.48921	.49012	.49103	.49193	.49284
.47	.49375	.49465	.49555	.49646	.49736	.49826	.49916	.50006	.50096	.50185
.48	.50275	.50365	.50454	.50543	.50633	.50722	.50811	.50900	.50989	.51078
.49	.51167	.51256	.51344	.51433	.51521	.51609	.51698	.51786	.51874	.51962
.50	.52050	.52138	.52226	.52313	.52401	.52488	.52576	.52663	.52750	.52837

1.

Table 8

x	0	1	2	3	4	5	6	7	8	9
.51	.52924	.53011	.53098	.53185	.53272	.53358	.53445	.53531	.53617	.53704
.52	.53790	.53876	.53962	.54048	.54134	.54219	.54305	.54390	.54476	.54561
.53	.54646	.54732	.54817	.54902	.54987	.55071	.55156	.55241	.55325	.55410
.54	.55494	.55578	.55662	.55746	.55830	.55914	.55998	.56082	.56165	.56249
.55	.56332	.56416	.56499	.56582	.56665	.56748	.56831	.56914	.56996	.57079
.56	.57162	.57244	.57326	.57409	.57491	.57573	.57655	.57737	.57818	.57900
.57	.57982	.58063	.58144	.58226	.58307	.58388	.58469	.58550	.58631	.58712
.58	.58792	.58873	.58953	.59034	.59114	.59194	.59274	.59354	.59434	.59514
.59	.59594	.59673	.59753	.59832	.59912	.59991	.60070	.60149	.60228	.60307
.60	.60386	.60464	.60543	.60621	.60700	.60778	.60856	.60934	.61012	.61090
.61	.61168	.61246	.61323	.61401	.61478	.61556	.61633	.61710	.61787	.61864
.62	.61941	.62018	.62095	.62171	.62248	.62324	.62400	.62477	.62553	.62629
.63	.62705	.62780	.62856	.62932	.63007	.63083	.63158	.63233	.63309	.63384
.64	.63459	.63533	.63608	.63683	.63757	.63832	.63906	.63981	.64055	.64129
.65	.64203	.64277	.64351	.64424	.64498	.64572	.64645	.64718	.64791	.64865
.66	.64938	.65011	.65083	.65156	.65229	.65301	.65374	.65446	.65519	.65591
.67	.65663	.65735	.65807	.65878	.65950	.66022	.66093	.66165	.66236	.66307
.68	.66378	.66449	.66520	.66591	.66662	.66732	.66803	.66873	.66944	.67014
.69	.67084	.67154	.67224	.67294	.67364	.67433	.67503	.67572	.67642	.67711
.70	.67780	.67849	.67918	.67987	.68056	.68125	.68193	.68262	.68330	.68398
.71	.68467	.68535	.68603	.68671	.68738	.68806	.68874	.68941	.69009	.69076
.72	.69143	.69210	.69278	.69344	.69411	.69478	.69545	.69611	.69678	.69744
.73	.69810	.69877	.69943	.70009	.70075	.70140	.70206	.70272	.70337	.70403
.74	.70468	.70533	.70598	.70663	.70728	.70793	.70858	.70922	.70987	.71051
.75	.71116	.71180	.71244	.71308	.71372	.71436	.71500	.71563	.71627	.71690
.76	.71754	.71817	.71880	.71943	.72006	.72069	.72132	.72195	.72257	.72320
.77	.72382	.72444	.72507	.72569	.72631	.72693	.72755	.72816	.72878	.72940
.78	.73001	.73062	.73124	.73185	.73246	.73307	.73368	.73429	.73489	.73550
.79	.73610	.73671	.73731	.73791	.73851	.73911	.73971	.74031	.74091	.74151
.80	.74210	.74270	.74329	.74388	.74447	.74506	.74565	.74624	.74683	.74742
.81	.74800	.74859	.74917	.74976	.75034	.75092	.75150	.75208	.75266	.75323
.82	.75381	.75439	.75496	.75553	.75611	.75668	.75725	.75782	.75839	.75896
.83	.75952	.76009	.76066	.76122	.76178	.76234	.76291	.76347	.76403	.76459
.84	.76514	.76570	.76626	.76681	.76736	.76792	.76847	.76902	.76957	.77012
.85	.77067	.77122	.77176	.77231	.77285	.77340	.77394	.77448	.77502	.77556
.86	.77610	.77664	.77718	.77771	.77825	.77878	.77932	.77985	.78038	.78091
.87	.78144	.78197	.78250	.78302	.78355	.78408	.78460	.78512	.78565	.78617
.88	.78669	.78721	.78773	.78824	.78876	.78928	.78979	.79031	.79082	.79133
.89	.79184	.79235	.79286	.79337	.79388	.79439	.79489	.79540	.79590	.79641
.90	.79691	.79741	.79791	.79841	.79891	.79941	.79990	.80040	.80090	.80139
.91	.80188	.80238	.80287	.80336	.80385	.80434	.80482	.80531	.80580	.80628
.92	.80677	.80725	.80773	.80822	.80870	.80918	.80966	.81013	.81061	.81109
.93	.81156	.81204	.81251	.81299	.81346	.81393	.81440	.81487	.81534	.81580
.94	.81627	.81674	.81720	.81767	.81813	.81859	.81905	.81951	.81997	.82043
.95	.82089	.82135	.82180	.82226	.82271	.82317	.82362	.82407	.82452	.82497
.96	.82542	.82587	.82632	.82677	.82721	.82766	.82810	.82855	.82899	.82943
.97	.82987	.83031	.83075	.83119	.83162	.83206	.83250	.83293	.83337	.83380
.98	.83423	.83466	.83509	.83552	.83595	.83638	.83681	.83723	.83766	.83808
.99	.83851	.83893	.83935	.83977	.84020	.84061	.84103	.84145	.84187	.84229
1.00	.84270	.84312	.84353	.84394	.84435	.84477	.84518	.84559	.84600	.84640

2.

Table 8. $\frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}\,du$

x	0	1	2	3	4	5	6	7	8	9
1.01	.84681	.84722	.84762	.84803	.84843	.84883	.84924	.84964	.85004	.85044
1.02	.85084	.85124	.85163	.85203	.85243	.85282	.85322	.85361	.85400	.85439
1.03	.85478	.85517	.85556	.85595	.85634	.85673	.85711	.85750	.85788	.85827
1.04	.85865	.85903	.85941	.85979	.86017	.86055	.86093	.86131	.86169	.86206
1.05	.86244	.86281	.86318	.86356	.86393	.86430	.86467	.86504	.86541	.86578
1.06	.86614	.86651	.86688	.86724	.86760	.86797	.86833	.86869	.86905	.86941
1.07	.86977	.87013	.87049	.87085	.87120	.87156	.87191	.87227	.87262	.87297
1.08	.87333	.87368	.87403	.87438	.87473	.87507	.87542	.87577	.87611	.87646
1.09	.87680	.87715	.87749	.87783	.87817	.87851	.87885	.87919	.87953	.87987
1.10	.88021	.88054	.88088	.88121	.88155	.88188	.88221	.88254	.88287	.88320
1.11	.88353	.88386	.88419	.88452	.88484	.88517	.88549	.88582	.88614	.88647
1.12	.88679	.88711	.88743	.88775	.88807	.88839	.88871	.88902	.88934	.88966
1.13	.88997	.89029	.89060	.89091	.89122	.89154	.89185	.89216	.89247	.89277
1.14	.89308	.89339	.89370	.89400	.89431	.89461	.89492	.89522	.89552	.89582
1.15	.89612	.89642	.89672	.89702	.89732	.89762	.89792	.89821	.89851	.89880
1.16	.89910	.89939	.89968	.89997	.90027	.90056	.90085	.90114	.90142	.90171
1.17	.90200	.90229	.90257	.90286	.90314	.90343	.90371	.90399	.90428	.90456
1.18	.90484	.90512	.90540	.90568	.90595	.90623	.90651	.90678	.90706	.90733
1.19	.90761	.90788	.90815	.90843	.90870	.90897	.90924	.90951	.90978	.91005
1.20	.91031	.91058	.91085	.91111	.91138	.91164	.91191	.91217	.91243	.91269
1.21	.91296	.91322	.91348	.91374	.91399	.91425	.91451	.91477	.91502	.91528
1.22	.91553	.91579	.91604	.91630	.91655	.91680	.91705	.91730	.91755	.91780
1.23	.91805	.91830	.91855	.91879	.91904	.91929	.91953	.91978	.92002	.92026
1.24	.92051	.92075	.92099	.92123	.92147	.92171	.92195	.92219	.92243	.92266
1.25	.92290	.92314	.92337	.92361	.92384	.92408	.92431	.92454	.92477	.92500
1.26	.92524	.92547	.92570	.92593	.92615	.92638	.92661	.92684	.92706	.92729
1.27	.92751	.92774	.92796	.92819	.92841	.92863	.92885	.92907	.92929	.92951
1.28	.92973	.92995	.93017	.93039	.93061	.93082	.93104	.93126	.93147	.93168
1.29	.93190	.93211	.93232	.93254	.93275	.93296	.93317	.93338	.93359	.93380
1.30	.93401	.93422	.93442	.93463	.93484	.93504	.93525	.93545	.93566	.93586
1.31	.93606	.93627	.93647	.93667	.93687	.93707	.93727	.93747	.93767	.93787
1.32	.93807	.93826	.93846	.93866	.93885	.93905	.93924	.93944	.93963	.93982
1.33	.94002	.94021	.94040	.94059	.94078	.94097	.94116	.94135	.94154	.94173
1.34	.94191	.94210	.94229	.94247	.94266	.94284	.94303	.94321	.94340	.94358
1.35	.94376	.94394	.94413	.94431	.94449	.94467	.94485	.94503	.94521	.94538
1.36	.94556	.94574	.94592	.94609	.94627	.94644	.94662	.94679	.94697	.94714
1.37	.94731	.94748	.94766	.94783	.94800	.94817	.94834	.94851	.94868	.94885
1.38	.94902	.94918	.94935	.94952	.94968	.94985	.95002	.95018	.95035	.95051
1.39	.95067	.95084	.95100	.95116	.95132	.95148	.95165	.95181	.95197	.95213
1.40	.95229	.95244	.95260	.95276	.95292	.95307	.95323	.95339	.95354	.95370
1.41	.95385	.95401	.95416	.95431	.95447	.95462	.95477	.95492	.95507	.95523
1.42	.95538	.95553	.95568	.95582	.95597	.95612	.95627	.95642	.95656	.95671
1.43	.95686	.95700	.95715	.95729	.95744	.95758	.95773	.95787	.95801	.95815
1.44	.95830	.95844	.95858	.95872	.95886	.95900	.95914	.95928	.95942	.95956
1.45	.95970	.95983	.95997	.96011	.96024	.96038	.96051	.96065	.96078	.96092
1.46	.96105	.96119	.96132	.96145	.96159	.96172	.96185	.96198	.96211	.96224
1.47	.96237	.96250	.96263	.96276	.96289	.96302	.96315	.96327	.96340	.96353
1.48	.96365	.96378	.96391	.96403	.96416	.96428	.96440	.96453	.96465	.96478
1.49	.96490	.96502	.96514	.96526	.96539	.96551	.96563	.96575	.96587	.96599
1.50	.96611	.96622	.96634	.96646	.96658	.96670	.96681	.96693	.96705	.96716

3.

x	0	1	2	3	4	5	6	7	8	9
1.51	.96728	.96739	.96751	.96762	.96774	.96785	.96796	.96808	.96819	.96830
1.52	.96841	.96853	.96864	.96875	.96886	.96897	.96908	.96919	.96930	.96941
1.53	.96952	.96962	.96973	.96984	.96995	.97006	.97016	.97027	.97037	.97048
1.54	.97059	.97069	.97080	.97090	.97100	.97111	.97121	.97131	.97142	.97152
1.55	.97162	.97172	.97183	.97193	.97203	.97213	.97223	.97233	.97243	.97253
1.56	.97263	.97273	.97283	.97292	.97302	.97312	.97322	.97331	.97341	.97351
1.57	.97360	.97370	.97379	.97389	.97398	.97408	.97417	.97427	.97436	.97445
1.58	.97455	.97464	.97473	.97482	.97492	.97501	.97510	.97519	.97528	.97537
1.59	.97546	.97555	.97564	.97573	.97582	.97591	.97600	.97609	.97617	.97626
1.60	.97635	.97644	.97652	.97661	.97670	.97678	.97687	.97695	.97704	.97712
1.61	.97721	.97729	.97738	.97746	.97754	.97763	.97771	.97779	.97787	.97796
1.62	.97804	.97812	.97820	.97828	.97836	.97844	.97852	.97860	.97868	.97876
1.63	.97884	.97892	.97900	.97908	.97916	.97924	.97931	.97939	.97947	.97955
1.64	.97962	.97970	.97977	.97985	.97993	.98000	.98008	.98015	.98023	.98030
1.65	.98038	.98045	.98052	.98060	.98067	.98074	.98082	.98089	.98096	.98103
1.66	.98110	.98118	.98125	.98132	.98139	.98146	.98153	.98160	.98167	.98174
1.67	.98181	.98188	.98195	.98202	.98209	.98215	.98222	.98229	.98236	.98243
1.68	.98249	.98256	.98263	.98269	.98276	.98283	.98289	.98296	.98302	.98309
1.69	.98315	.98322	.98328	.98335	.98341	.98347	.98354	.98360	.98366	.98373
1.70	.98379	.98385	.98392	.98398	.98404	.98410	.98416	.98422	.98429	.98435
1.71	.98441	.98447	.98453	.98459	.98465	.98471	.98477	.98483	.98489	.98494
1.72	.98500	.98506	.98512	.98518	.98524	.98529	.98535	.98541	.98546	.98552
1.73	.98558	.98563	.98569	.98575	.98580	.98586	.98591	.98597	.98602	.98608
1.74	.98613	.98619	.98624	.98630	.98635	.98641	.98646	.98651	.98657	.98662
1.75	.98667	.98672	.98678	.98683	.98688	.98693	.98699	.98704	.98709	.98714
1.76	.98719	.98724	.98729	.98734	.98739	.98744	.98749	.98754	.98759	.98764
1.77	.98769	.98774	.98779	.98784	.98789	.98793	.98798	.98803	.98808	.98813
1.78	.98817	.98822	.98827	.98832	.98836	.98841	.98846	.98850	.98855	.98859
1.79	.98864	.98869	.98873	.98878	.98882	.98887	.98891	.98896	.98900	.98905
1.80	.98909	.98913	.98918	.98922	.98927	.98931	.98935	.98940	.98944	.98948
1.81	.98952	.98957	.98961	.98965	.98969	.98974	.98978	.98982	.98986	.98990
1.82	.98994	.98998	.99003	.99007	.99011	.99015	.99019	.99023	.99027	.99031
1.83	.99035	.99039	.99043	.99047	.99050	.99054	.99058	.99062	.99066	.99070
1.84	.99074	.99077	.99081	.99085	.99089	.99093	.99096	.99100	.99104	.99107
1.85	.99111	.99115	.99118	.99122	.99126	.99129	.99133	.99137	.99140	.99144
1.86	.99147	.99151	.99154	.99158	.99161	.99165	.99168	.99172	.99175	.99179
1.87	.99182	.99185	.99189	.99192	.99196	.99199	.99202	.99206	.99209	.99212
1.88	.99216	.99219	.99222	.99225	.99229	.99232	.99235	.99238	.99242	.99245
1.89	.99248	.99251	.99254	.99257	.99261	.99264	.99267	.99270	.99273	.99276
1.90	.99279	.99282	.99285	.99288	.99291	.99294	.99297	.99300	.99303	.99306
1.91	.99309	.99312	.99315	.99318	.99321	.99324	.99326	.99329	.99332	.99335
1.92	.99338	.99341	.99343	.99346	.99349	.99352	.99355	.99357	.99360	.99363
1.93	.99366	.99368	.99371	.99374	.99376	.99379	.99382	.99384	.99387	.99390
1.94	.99392	.99395	.99397	.99400	.99403	.99405	.99408	.99410	.99413	.99415
1.95	.99418	.99420	.99423	.99425	.99428	.99430	.99433	.99435	.99438	.99440
1.96	.99443	.99445	.99447	.99450	.99452	.99455	.99457	.99459	.99462	.99464
1.97	.99466	.99469	.99471	.99473	.99476	.99478	.99480	.99482	.99485	.99487
1.98	.99489	.99491	.99494	.99496	.99498	.99500	.99502	.99505	.99507	.99509
1.99	.99511	.99513	.99515	.99518	.99520	.99522	.99524	.99526	.99528	.99530
2.00	.99532	.99534	.99536	.99538	.99540	.99542	.99544	.99546	.99548	.99550

4.

x	0	1	2	3	4	5	6	7	8	9
2.01	.99552	.99554	.99556	.99558	.99560	.99562	.99564	.99566	.99568	.99570
2.02	.99572	.99574	.99576	.99578	.99580	.99581	.99583	.99585	.99587	.99589
2.03	.99591	.99592	.99594	.99596	.99598	.99600	.99601	.99603	.99605	.99607
2.04	.99609	.99610	.99612	.99614	.99616	.99617	.99619	.99621	.99622	.99624
2.05	.99626	.99627	.99629	.99631	.99633	.99634	.99636	.99637	.99639	.99641
2.06	.99642	.99644	.99646	.99647	.99649	.99650	.99652	.99654	.99655	.99657
2.07	.99658	.99660	.99661	.99663	.99664	.99666	.99667	.99669	.99670	.99672
2.08	.99673	.99675	.99676	.99678	.99679	.99681	.99682	.99684	.99685	.99687
2.09	.99688	.99689	.99691	.99692	.99694	.99695	.99697	.99698	.99699	.99701
2.10	.99702	.99703	.99705	.99706	.99707	.99709	.99710	.99712	.99713	.99714
2.11	.99715	.99717	.99718	.99719	.99721	.99722	.99723	.99725	.99726	.99727
2.12	.99728	.99730	.99731	.99732	.99733	.99735	.99736	.99737	.99738	.99739
2.13	.99741	.99742	.99743	.99744	.99745	.99747	.99748	.99749	.99750	.99751
2.14	.99753	.99754	.99755	.99756	.99757	.99758	.99759	.99761	.99762	.99763
2.15	.99764	.99765	.99766	.99767	.99768	.99769	.99770	.99772	.99773	.99774
2.16	.99775	.99776	.99777	.99778	.99779	.99780	.99781	.99782	.99783	.99784
2.17	.99785	.99786	.99787	.99788	.99789	.99790	.99791	.99792	.99793	.99794
2.18	.99795	.99796	.99797	.99798	.99799	.99800	.99801	.99802	.99803	.99804
2.19	.99805	.99806	.99806	.99807	.99808	.99809	.99810	.99811	.99812	.99813
2.20	.99814	.99815	.99815	.99816	.99817	.99818	.99819	.99820	.99821	.99822
2.21	.99822	.99823	.99824	.99825	.99826	.99827	.99827	.99828	.99829	.99830
2.22	.99831	.99832	.99832	.99833	.99834	.99835	.99836	.99836	.99837	.99838
2.23	.99839	.99840	.99840	.99841	.99842	.99843	.99843	.99844	.99845	.99846
2.24	.99846	.99847	.99848	.99849	.99849	.99850	.99851	.99852	.99852	.99853
2.25	.99854	.99854	.99855	.99856	.99857	.99857	.99858	.99859	.99859	.99860
2.26	.99861	.99861	.99862	.99863	.99863	.99864	.99865	.99865	.99866	.99867
2.27	.99867	.99868	.99869	.99869	.99870	.99871	.99871	.99872	.99873	.99873
2.28	.99874	.99874	.99875	.99876	.99876	.99877	.99877	.99878	.99879	.99879
2.29	.99880	.99880	.99881	.99882	.99882	.99883	.99883	.99884	.99885	.99885
2.30	.99886	.99886	.99887	.99887	.99888	.99888	.99889	.99890	.99890	.99891
2.31	.99891	.99892	.99892	.99893	.99893	.99894	.99894	.99895	.99896	.99896
2.32	.99897	.99897	.99898	.99898	.99899	.99899	.99900	.99900	.99901	.99901
2.33	.99902	.99902	.99903	.99903	.99904	.99904	.99905	.99905	.99906	.99906
2.34	.99906	.99907	.99907	.99908	.99908	.99909	.99909	.99910	.99910	.99911
2.35	.99911	.99912	.99912	.99912	.99913	.99913	.99914	.99914	.99915	.99915
2.36	.99915	.99916	.99916	.99917	.99917	.99918	.99918	.99918	.99919	.99919
2.37	.99920	.99920	.99920	.99921	.99921	.99922	.99922	.99923	.99923	.99923
2.38	.99924	.99924	.99924	.99925	.99925	.99926	.99926	.99926	.99927	.99927
2.39	.99928	.99928	.99928	.99929	.99929	.99929	.99930	.99930	.99930	.99931
2.40	.99931	.99932	.99932	.99932	.99933	.99933	.99933	.99934	.99934	.99934
2.41	.99935	.99935	.99935	.99936	.99936	.99936	.99937	.99937	.99937	.99938
2.42	.99938	.99938	.99939	.99939	.99939	.99940	.99940	.99940	.99940	.99941
2.43	.99941	.99941	.99942	.99942	.99942	.99943	.99943	.99943	.99943	.99944
2.44	.99944	.99944	.99945	.99945	.99945	.99946	.99946	.99946	.99946	.99947
2.45	.99947	.99947	.99947	.99948	.99948	.99948	.99949	.99949	.99949	.99949
2.46	.99950	.99950	.99950	.99950	.99951	.99951	.99951	.99951	.99952	.99952
2.47	.99952	.99953	.99953	.99953	.99953	.99954	.99954	.99954	.99954	.99954
2.48	.99955	.99955	.99955	.99955	.99956	.99956	.99956	.99956	.99957	.99957
2.49	.99957	.99957	.99958	.99958	.99958	.99958	.99958	.99959	.99959	.99959
2.50	.99959	.99960	.99960	.99960	.99960	.99960	.99961	.99961	.99961	.99961

5.

Table 8.

x	0	1	2	3	4	5	6	7	8	9
2.51	.99961	.99962	.99962	.99962	.99962	.99962	.99963	.99963	.99963	.99963
2.52	.99963	.99964	.99964	.99964	.99964	.99964	.99965	.99965	.99965	.99965
2.53	.99965	.99966	.99966	.99966	.99966	.99966	.99966	.99967	.99967	.99967
2.54	.99967	.99967	.99968	.99968	.99968	.99968	.99968	.99968	.99969	.99969
2.55	.99969	.99969	.99969	.99969	.99970	.99970	.99970	.99970	.99970	.99970
2.56	.99971	.99971	.99971	.99971	.99971	.99971	.99972	.99972	.99972	.99972
2.57	.99972	.99972	.99972	.99973	.99973	.99973	.99973	.99973	.99973	.99973
2.58	.99974	.99974	.99974	.99974	.99974	.99974	.99974	.99975	.99975	.99975
2.59	.99975	.99975	.99975	.99975	.99976	.99976	.99976	.99976	.99976	.99976
2.60	.99976	.99977	.99977	.99977	.99977	.99977	.99977	.99977	.99977	.99978
2.61	.99978	.99978	.99978	.99978	.99978	.99978	.99978	.99979	.99979	.99979
2.62	.99979	.99979	.99979	.99979	.99979	.99979	.99980	.99980	.99980	.99980
2.63	.99980	.99980	.99980	.99980	.99980	.99981	.99981	.99981	.99981	.99981
2.64	.99981	.99981	.99981	.99981	.99982	.99982	.99982	.99982	.99982	.99982
2.65	.99982	.99982	.99982	.99982	.99983	.99983	.99983	.99983	.99983	.99983
2.66	.99983	.99983	.99983	.99983	.99984	.99984	.99984	.99984	.99984	.99984
2.67	.99984	.99984	.99984	.99984	.99984	.99985	.99985	.99985	.99985	.99985
2.68	.99985	.99985	.99985	.99985	.99985	.99985	.99985	.99986	.99986	.99986
2.69	.99986	.99986	.99986	.99986	.99986	.99986	.99986	.99986	.99986	.99986
2.70	.99987	.99987	.99987	.99987	.99987	.99987	.99987	.99987	.99987	.99987
2.71	.99987	.99987	.99987	.99988	.99988	.99988	.99988	.99988	.99988	.99988
2.72	.99988	.99988	.99988	.99988	.99988	.99988	.99988	.99989	.99989	.99989
2.73	.99989	.99989	.99989	.99989	.99989	.99989	.99989	.99989	.99989	.99989
2.74	.99989	.99989	.99989	.99990	.99990	.99990	.99990	.99990	.99990	.99990
2.75	.99990	.99990	.99990	.99990	.99990	.99990	.99990	.99990	.99990	.99990
2.76	.99991	.99991	.99991	.99991	.99991	.99991	.99991	.99991	.99991	.99991
2.77	.99991	.99991	.99991	.99991	.99991	.99991	.99991	.99991	.99991	.99992
2.78	.99992	.99992	.99992	.99992	.99992	.99992	.99992	.99992	.99992	.99992
2.79	.99992	.99992	.99992	.99992	.99992	.99992	.99992	.99992	.99992	.99992
2.80	.99992	.99993	.99993	.99993	.99993	.99993	.99993	.99993	.99993	.99993
2.81	.99993	.99993	.99993	.99993	.99993	.99993	.99993	.99993	.99993	.99993
2.82	.99993	.99993	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99994
2.83	.99994	.99994	.99994	.99994	.99994	.99994	.99994	.99994	.99994	.99994
2.84	.99994	.99994	.99994	.99994	.99994	.99994	.99994	.99994	.99994	.99994
2.85	.99994	.99994	.99995	.99995	.99995	.99995	.99995	.99995	.99995	.99995
2.86	.99995	.99995	.99995	.99995	.99995	.99995	.99995	.99995	.99995	.99995
2.87	.99995	.99995	.99995	.99995	.99995	.99995	.99995	.99995	.99995	.99995
2.88	.99995	.99995	.99995	.99995	.99995	.99995	.99996	.99996	.99996	.99996
2.89	.99996	.99996	.99996	.99996	.99996	.99996	.99996	.99996	.99996	.99996
2.90	.99996	.99996	.99996	.99996	.99996	.99996	.99996	.99996	.99996	.99996
2.91	.99996	.99996	.99996	.99996	.99996	.99996	.99996	.99996	.99996	.99996
2.92	.99996	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997	.99997
2.93	.99997	.99997	.99997	.99997	.99997	.99997	.99997	.99997	.99997	.99997
2.94	.99997	.99997	.99997	.99997	.99997	.99997	.99997	.99997	.99997	.99997
2.95	.99997	.99997	.99997	.99997	.99997	.99997	.99997	.99997	.99997	.99997
2.96	.99997	.99997	.99997	.99997	.99997	.99997	.99997	.99997	.99997	.99997
2.97	.99997	.99997	.99997	.99997	.99997	.99997	.99997	.99997	.99997	.99997
2.98	.99997	.99998	.99998	.99998	.99998	.99998	.99998	.99998	.99998	.99998
2.99	.99998	.99998	.99998	.99998	.99998	.99998	.99998	.99998	.99998	.99998
3.00	.99998									

Table 9

Values of:

$$\int_x^\infty \frac{e^{-u^2} du}{u^2}$$

This integral can be evaluated as

$$\int_x^\infty \frac{e^{-u^2} du}{u^2} = \sqrt{\pi} \left[\frac{e^{-x^2}}{x\sqrt{\pi}} - 1 + \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du \right]$$

A comparison of independent evaluations made, by use of this expression, and tabular values is as shown below:

x	Computed Value	Tabular Value
0.0001	9998.2276	9998.228
0.001	998.2285	998.229
0.01	98.2377	98.238
0.1	8.32738	8.32738
1.0	0.08907	0.08907

A graphical determination for x = 1.0 yields 0.090. This compares with the tabular value 0.08907.

Table 9. $\int_x^\infty \frac{e^{-u^2}du}{u^2}$

x	0	1	2	3	4	5	6	7	8	9
.0001	9998.228	9089.137	8331.561	7690.535	7141.085	6664.894	6248.228	5880.581	5553.783	5261.336
.0002	4998.228	4760.133	4543.682	4346.054	4164.894	3998.228	3844.382	3701.932	3569.656	3446.504
.0003	3331.561	3224.034	3123.228	3028.531	2939.404	2855.371	2776.006	2700.931	2629.807	2562.331
.0004	2498.228	2437.252	2379.180	2323.809	2270.955	2220.450	2172.141	2125.868	2081.561	2039.944
.0005	1998.228	1959.012	1921.305	1885.021	1850.080	1816.410	1783.942	1752.614	1722.366	1693.143
.0006	1664.895	1637.572	1611.131	1585.530	1560.728	1536.690	1513.380	1490.766	1468.816	1447.504
.0007	1426.800	1406.679	1387.117	1368.091	1349.580	1331.562	1314.018	1296.930	1280.290	1264.051
.0008	1248.228	1232.796	1217.741	1203.048	1188.705	1174.699	1161.019	1147.654	1134.592	1121.324
.0009	1109.340	1097.130	1085.185	1073.497	1062.058	1050.860	1039.895	1029.156	1018.637	1008.330
.0010	998.229									

.001	998.229	907.338	831.580	767.478	712.533	664.914	623.247	586.483	553.803	524.563
.002	498.248	474.438	452.793	433.031	414.915	398.248	382.864	368.619	355.391	343.076
.003	331.582	320.829	310.749	301.279	292.367	283.964	276.027	268.520	261.407	254.660
.004	248.250	242.152	236.345	230.808	225.523	220.472	215.642	211.016	206.584	202.332
.005	198.251	194.329	190.559	186.930	183.436	180.069	176.823	173.690	170.665	167.743
.006	164.918	162.186	159.542	156.982	154.502	152.098	149.767	147.506	145.311	143.180
.007	141.110	139.098	137.142	135.239	133.388	131.587	129.832	128.124	126.459	124.836
.008	123.254	121.711	120.205	118.736	117.302	115.901	114.533	113.197	111.391	110.614
.009	109.366	108.145	106.951	105.782	104.638	103.518	102.422	101.348	100.260	99.230
.010	98.238									

x	0	1	2	3	4	5	6	7	8	9
.01	98.23755	89.14764	81.57288	75.16362	69.67012	64.90921	60.74355	57.06807	53.80110	50.87812
.02	48.24754	45.86759	43.70409	41.72880	39.91821	38.25254	36.71508	35.29158	33.96983	32.73930
.03	31.59087	30.51661	29.50954	28.56357	27.67330	26.83397	26.04132	25.29156	24.58133	23.90756
.04	23.26754	22.65878	22.07906	21.52635	20.99880	20.49475	20.01266	19.55112	19.10886	18.68469
.05	18.27753	17.88637	17.51029	17.14845	16.80004	16.46434	16.14066	15.82837	15.52689	15.23566
.06	14.95418	14.68195	14.41854	14.16352	13.91650	13.67712	13.44501	13.21987	13.00138	12.78925
.07	12.58320	12.38299	12.18837	11.99911	11.81499	11.63581	11.46137	11.29148	11.12598	10.96469
.08	10.80746	10.65414	10.50458	10.35864	10.21621	10.07715	9.94135	9.80869	9.67907	9.55238
.09	9.42854	9.30743	9.18898	9.07310	8.95971	8.84872	8.74007	8.63367	8.52947	8.42739
.10	8.32738	8.22936	8.13329	8.03910	7.94674	7.85616	7.76731	7.68014	7.59460	7.51064
.11	7.42823	7.34733	7.26788	7.18986	7.11323	7.03795	6.96398	6.89129	6.81985	6.74963
.12	6.68059	6.61271	6.54597	6.48032	6.41575	6.35222	6.28972	6.22822	6.16770	6.10813
.13	6.04949	5.99176	5.93492	5.87895	5.82383	5.76954	5.71607	5.66339	5.61149	5.56034
.14	5.50995	5.46028	5.41132	5.36307	5.31549	5.26859	5.22234	5.17674	5.13176	5.08741
.15	5.04365	5.00049	4.95791	4.91590	4.87445	4.83354	4.79317	4.75333	4.71401	4.67519
.16	4.63687	4.59903	4.56168	4.52480	4.48838	4.45241	4.41688	4.38180	4.34714	4.31291
.17	4.27908	4.24567	4.21266	4.18004	4.14780	4.11594	4.08446	4.05335	4.02259	3.99219
.18	3.96214	3.93243	3.90305	3.87401	3.84530	3.81690	3.78883	3.76106	3.73360	3.70643
.19	3.67957	3.65300	3.62671	3.60070	3.57498	3.54952	3.52434	3.49942	3.47477	3.45037
.20	3.42622	3.40233	3.37868	3.35527	3.33210	3.30917	3.28647	3.26400	3.24175	3.21973
.21	3.19792	3.17633	3.15495	3.13379	3.11282	3.09207	3.07151	3.05115	3.03099	3.01102
.22	2.99124	2.97165	2.95224	2.93302	2.91398	2.89511	2.87642	2.85790	2.83956	2.82138
.23	2.80337	2.78552	2.76783	2.75031	2.73294	2.71573	2.69867	2.68176	2.66501	2.64840
.24	2.63194	2.61562	2.59944	2.58341	2.56751	2.55176	2.53614	2.52065	2.50529	2.49007
.25	2.47497	2.46001	2.44517	2.43045	2.41586	2.40139	2.38704	2.37280	2.35869	2.34469
.26	2.33081	2.31704	2.30338	2.28984	2.27640	2.26307	2.24985	2.23674	2.22373	2.21082
.27	2.19802	2.18531	2.17271	2.16021	2.14780	2.13550	2.12328	2.11117	2.09914	2.08721
.28	2.07537	2.06362	2.05197	2.04040	2.02892	2.01752	2.00621	1.99499	1.98385	1.97280
.29	1.96182	1.95093	1.94012	1.92939	1.91874	1.90817	1.89768	1.88726	1.87692	1.86665
.30	1.85646	1.84634	1.83630	1.82632	1.81642	1.80659	1.79683	1.78714	1.77752	1.76797
.31	1.75848	1.74906	1.73971	1.73042	1.72120	1.71204	1.70295	1.69392	1.68495	1.67604
.32	1.66719	1.65841	1.64969	1.64102	1.63241	1.62387	1.61538	1.60694	1.59857	1.59025
.33	1.58199	1.57378	1.56563	1.55753	1.54948	1.54149	1.53355	1.52567	1.51783	1.51005
.34	1.50232	1.49464	1.48701	1.47943	1.47190	1.46441	1.45698	1.44959	1.44225	1.43496
.35	1.42771	1.42052	1.41336	1.40625	1.39919	1.39217	1.38520	1.37827	1.37139	1.36454
.36	1.35774	1.35099	1.34427	1.33760	1.33097	1.32438	1.31783	1.31132	1.30485	1.29842
.37	1.29203	1.28568	1.27937	1.27310	1.26686	1.26066	1.25451	1.24838	1.24230	1.23625
.38	1.23024	1.22426	1.21832	1.21242	1.20655	1.20071	1.19491	1.18915	1.18342	1.17772
.39	1.17205	1.16642	1.16083	1.15526	1.14973	1.14423	1.13876	1.13333	1.12793	1.12255
.40	1.11721	1.11190	1.10662	1.10137	1.09615	1.09096	1.08580	1.08067	1.07557	1.07050
.41	1.06546	1.06044	1.05546	1.05050	1.04557	1.04067	1.03580	1.03095	1.02613	1.02134
.42	1.01657	1.01183	1.00712	1.00244	.99778	.99314	.98853	.98395	.97939	.97486
.43	.97035	.96587	.96141	.95698	.95257	.94818	.94382	.93949	.93517	.93088
.44	.92661	.92236	.91814	.91394	.90977	.90561	.90148	.89737	.89328	.88922
.45	.88517	.88115	.87715	.87317	.86921	.86528	.86136	.85746	.85359	.84973
.46	.84590	.84208	.83829	.83451	.83076	.82702	.82331	.81961	.81593	.81228
.47	.80864	.80502	.80142	.79783	.79427	.79072	.78719	.78368	.78019	.77672
.48	.77326	.76982	.76640	.76300	.75961	.75625	.75289	.74956	.74624	.74294
.49	.73966	.73639	.73314	.72990	.72668	.72348	.72030	.71713	.71397	.71083
.50	.70771	.70460	.70151	.69843	.69537	.69233	.68930	.68628	.68323	.68029

Table 9. $\int_x^\infty \frac{e^{-u^2}\,du}{u^2}$

x	0	1	2	3	4	5	6	7	8	9
.51	.67732	.67436	.67142	.66849	.66558	.66268	.65980	.65693	.65407	.65123
.52	.64840	.64558	.64278	.63999	.63722	.63446	.63171	.62898	.62626	.62355
.53	.62085	.61817	.61550	.61285	.61020	.60757	.60495	.60235	.59976	.59718
.54	.59461	.59205	.58951	.58693	.58446	.58195	.57945	.57697	.57450	.57204
.55	.56959	.56715	.56473	.56231	.55991	.55752	.55514	.55277	.55041	.54806
.56	.54573	.54340	.54109	.53879	.53649	.53421	.53194	.52968	.52743	.52519
.57	.52296	.52074	.51853	.51634	.51415	.51197	.50980	.50764	.50550	.50336
.58	.50123	.49911	.49700	.49490	.49281	.49073	.48866	.48660	.48455	.48251
.59	.48047	.47845	.47644	.47443	.47243	.47045	.46847	.46650	.46454	.46259
.60	.46065	.45871	.45679	.45487	.45296	.45107	.44918	.44729	.44542	.44356
.61	.44170	.43985	.43801	.43618	.43436	.43254	.43073	.42893	.42714	.42536
.62	.42358	.42182	.42006	.41831	.41656	.41483	.41310	.41138	.40966	.40796
.63	.40626	.40457	.40289	.40121	.39954	.39788	.39623	.39458	.39294	.39131
.64	.38969	.38807	.38646	.38486	.38326	.38167	.38009	.37852	.37695	.37539
.65	.37383	.37228	.37074	.36921	.36768	.36616	.36464	.36314	.36164	.36014
.66	.35865	.35717	.35569	.35423	.35276	.35131	.34986	.34841	.34698	.34554
.67	.34412	.34270	.34129	.33988	.33848	.33709	.33570	.33431	.33294	.33157
.68	.33020	.32884	.32749	.32614	.32480	.32347	.32214	.32081	.31949	.31818
.69	.31687	.31557	.31427	.31298	.31170	.31042	.30914	.30787	.30661	.30535
.70	.30410	.30285	.30161	.30037	.29914	.29791	.29669	.29548	.29426	.29306
.71	.29186	.29066	.28947	.28829	.28710	.28593	.28476	.28359	.28243	.28128
.72	.28012	.27898	.27784	.27670	.27557	.27444	.27332	.27220	.27109	.26998
.73	.26888	.26778	.26668	.26559	.26451	.26343	.26235	.26128	.26021	.25915
.74	.25809	.25704	.25599	.25494	.25390	.25287	.25183	.25081	.24978	.24876
.75	.24775	.24674	.24573	.24473	.24373	.24273	.24174	.24076	.23978	.23880
.76	.23783	.23686	.23589	.23493	.23397	.23302	.23207	.23112	.23018	.22924
.77	.22831	.22738	.22645	.22553	.22461	.22369	.22278	.22188	.22097	.22007
.78	.21918	.21828	.21739	.21651	.21563	.21475	.21387	.21300	.21214	.21127
.79	.21041	.20956	.20870	.20785	.20701	.20616	.20532	.20449	.20366	.20283
.80	.20200	.20118	.20036	.19954	.19873	.19792	.19712	.19632	.19552	.19472
.81	.19393	.19314	.19235	.19157	.19079	.19001	.18924	.18847	.18770	.18694
.82	.18618	.18542	.18467	.18392	.18317	.18242	.18168	.18094	.18020	.17947
.83	.17874	.17801	.17729	.17657	.17585	.17513	.17442	.17371	.17300	.17230
.84	.17160	.17090	.17020	.16951	.16882	.16813	.16745	.16677	.16609	.16541
.85	.16474	.16407	.16340	.16274	.16207	.16141	.16076	.16010	.15945	.15880
.86	.15815	.15751	.15687	.15623	.15559	.15496	.15433	.15370	.15307	.15245
.87	.15183	.15121	.15059	.14998	.14937	.14876	.14815	.14755	.14695	.14635
.88	.14575	.14516	.14457	.14398	.14339	.14281	.14223	.14165	.14107	.14049
.89	.13992	.13935	.13878	.13822	.13765	.13709	.13653	.13597	.13542	.13487
.90	.13432	.13377	.13322	.13268	.13214	.13160	.13106	.13053	.12999	.12946
.91	.12893	.12841	.12788	.12736	.12684	.12632	.12581	.12529	.12478	.12427
.92	.12376	.12326	.12275	.12225	.12175	.12125	.12076	.12026	.11977	.11928
.93	.11879	.11831	.11782	.11734	.11686	.11638	.11591	.11543	.11496	.11449
.94	.11402	.11355	.11309	.11263	.11217	.11171	.11125	.11079	.11034	.10989
.95	.10944	.10899	.10854	.10810	.10765	.10721	.10677	.10633	.10590	.10546
.96	.10503	.10460	.10417	.10374	.10332	.10289	.10247	.10205	.10163	.10122
.97	.10080	.10039	.09997	.09956	.09915	.09875	.09834	.09794	.09753	.09713
.98	.09673	.09634	.09594	.09555	.09515	.09476	.09437	.09398	.09360	.09321
.99	.09283	.09245	.09206	.09169	.09131	.09093	.09056	.09018	.08981	.08944
1.00	.08907	.08871	.08834	.08798	.08761	.08725	.08689	.08653	.08618	.08582

3.

Table 9.

x	0	1	2	3	4	5	6	7	8	9
1.01	.08547	.08511	.08476	.08441	.08407	.08372	.08337	.08303	.08268	.08234
1.02	.08200	.08166	.08133	.08099	.08066	.08032	.07999	.07966	.07933	.07900
1.03	.07867	.07835	.07802	.07770	.07738	.07706	.07674	.07642	.07611	.07579
1.04	.07548	.07516	.07485	.07454	.07423	.07392	.07362	.07331	.07301	.07270
1.05	.07240	.07210	.07180	.07150	.07121	.07091	.07062	.07032	.07003	.06974
1.06	.06945	.06916	.06887	.06859	.06830	.06802	.06774	.06745	.06717	.06689
1.07	.06661	.06634	.06606	.06579	.06551	.06524	.06497	.06470	.06443	.06416
1.08	.06389	.06362	.06336	.06309	.06283	.06257	.06231	.06205	.06179	.06153
1.09	.06127	.06102	.06076	.06051	.06025	.06000	.05975	.05950	.05925	.05900
1.10	.05876	.05851	.05827	.05802	.05778	.05754	.05730	.05706	.05682	.05658
1.11	.05634	.05611	.05587	.05564	.05540	.05517	.05494	.05471	.05448	.05425
1.12	.05402	.05379	.05357	.05334	.05312	.05290	.05267	.05245	.05223	.05201
1.13	.05179	.05157	.05136	.05114	.05093	.05071	.05050	.05029	.05007	.04986
1.14	.04965	.04944	.04923	.04903	.04882	.04861	.04841	.04820	.04800	.04780
1.15	.04760	.04739	.04719	.04700	.04680	.04660	.04640	.04621	.04601	.04582
1.16	.04562	.04543	.04524	.04504	.04485	.04466	.04447	.04429	.04410	.04391
1.17	.04372	.04354	.04335	.04317	.04299	.04280	.04262	.04244	.04226	.04208
1.18	.04190	.04173	.04155	.04137	.04120	.04102	.04085	.04067	.04050	.04033
1.19	.04015	.03998	.03981	.03964	.03947	.03931	.03914	.03897	.03881	.03864
1.20	.03848	.03831	.03815	.03798	.03782	.03766	.03750	.03734	.03718	.03702
1.21	.03686	.03671	.03655	.03639	.03624	.03608	.03593	.03577	.03562	.03547
1.22	.03532	.03516	.03501	.03486	.03471	.03456	.03442	.03427	.03412	.03398
1.23	.03383	.03368	.03354	.03339	.03325	.03311	.03297	.03282	.03268	.03254
1.24	.03240	.03226	.03212	.03199	.03185	.03171	.03157	.03144	.03130	.03117
1.25	.03103	.03090	.03077	.03063	.03050	.03037	.03024	.03011	.02998	.02985
1.26	.02972	.02959	.02946	.02933	.02921	.02908	.02896	.02883	.02871	.02858
1.27	.02846	.02833	.02821	.02809	.02797	.02785	.02772	.02760	.02748	.02737
1.28	.02725	.02713	.02701	.02689	.02678	.02666	.02654	.02643	.02631	.02620
1.29	.02608	.02597	.02586	.02575	.02563	.02552	.02541	.02530	.02519	.02508
1.30	.02497	.02486	.02475	.02464	.02454	.02443	.02432	.02422	.02411	.02401
1.31	.02390	.02380	.02369	.02359	.02349	.02338	.02328	.02318	.02308	.02298
1.32	.02287	.02277	.02267	.02257	.02248	.02238	.02228	.02218	.02208	.02199
1.33	.02189	.02179	.02170	.02160	.02151	.02141	.02132	.02123	.02113	.02104
1.34	.02095	.02085	.02076	.02067	.02058	.02049	.02040	.02031	.02022	.02013
1.35	.02004	.01995	.01986	.01978	.01969	.01960	.01952	.01943	.01934	.01926
1.36	.01917	.01909	.01900	.01892	.01883	.01875	.01867	.01859	.01850	.01842
1.37	.01834	.01826	.01818	.01810	.01802	.01794	.01786	.01778	.01770	.01762
1.38	.01754	.01746	.01738	.01731	.01723	.01715	.01708	.01700	.01693	.01685
1.39	.01677	.01670	.01663	.01655	.01648	.01640	.01633	.01626	.01619	.01611
1.40	.01604	.01597	.01590	.01583	.01576	.01569	.01561	.01555	.01548	.01541
1.41	.01534	.01527	.01520	.01513	.01506	.01500	.01493	.01486	.01480	.01473
1.42	.01466	.01460	.01453	.01447	.01440	.01434	.01427	.01421	.01414	.01408
1.43	.01402	.01395	.01389	.01383	.01377	.01370	.01364	.01358	.01352	.01346
1.44	.01340	.01334	.01328	.01322	.01316	.01310	.01304	.01298	.01292	.01286
1.45	.01280	.01275	.01269	.01263	.01257	.01252	.01246	.01240	.01235	.01229
1.46	.01223	.01218	.01212	.01207	.01201	.01196	.01190	.01185	.01180	.01174
1.47	.01169	.01164	.01158	.01153	.01148	.01143	.01137	.01132	.01127	.01122
1.48	.01117	.01112	.01107	.01102	.01097	.01092	.01087	.01082	.01077	.01072
1.49	.01067	.01062	.01057	.01052	.01047	.01043	.01038	.01033	.01029	.01024
1.50	.01019	.01014	.01010	.01005	.01000	.00996	.00991	.00987	.00982	.00978

4.

Table 9. $\int_x^\infty \frac{e^{-u^2}du}{u^2}$

x	0	1	2	3	4	5	6	7	8	9
1.51	.00973	.00969	.00964	.00960	.00955	.00951	.00947	.00942	.0093[illegible]	.00933
1.52	.00929	.00925	.00921	.00916	.00912	.00908	.00904	.00900	.00895	.00891
1.53	.00887	.00883	.00879	.00875	.00871	.00867	.00863	.00859	.00855	.00851
1.54	.00847	.00843	.00839	.00835	.00831	.00827	.00824	.00820	.00816	.00812
1.55	.00808	.00805	.00801	.00797	.00793	.00790	.00786	.00782	.00779	.00775
1.56	.00772	.00768	.00764	.00761	.00757	.00754	.00750	.00747	.00743	.00740
1.57	.00736	.00733	.00729	.00726	.00723	.00719	.00716	.00713	.00709	.00706
1.58	.00703	.00699	.00696	.00693	.00689	.00686	.00683	.00680	.00677	.00673
1.59	.00670	.00667	.00664	.00661	.00658	.00655	.00652	.00649	.00645	.00642
1.60	.00639	.00636	.00633	.00630	.00627	.00624	.00622	.00619	.00616	.00613
1.61	.00610	.00607	.00604	.00601	.00598	.00596	.00593	.00590	.00587	.00584
1.62	.00582	.00579	.00576	.00573	.00571	.00568	.00565	.00563	.00560	.00557
1.63	.00555	.00552	.00549	.00547	.00544	.00542	.00539	.00536	.00534	.00531
1.64	.00529	.00526	.00524	.00521	.00519	.00516	.00514	.00511	.00509	.00507
1.65	.00504	.00502	.00499	.00497	.00495	.00492	.00490	.00487	.00485	.00483
1.66	.00481	.00478	.00476	.00474	.00471	.00469	.00467	.00465	.00462	.00460
1.67	.00458	.00456	.00454	.00451	.00449	.00447	.00445	.00443	.00441	.00439
1.68	.00436	.00434	.00432	.00430	.00428	.00426	.00424	.00422	.00420	.00418
1.69	.00416	.00414	.00412	.00410	.00408	.00406	.00404	.00402	.00400	.00398
1.70	.00396	.00394	.00392	.00390	.00389	.00387	.00385	.00383	.00381	.00379
1.71	.00377	.00375	.00374	.00372	.00370	.00368	.00366	.00365	.00363	.00361
1.72	.00359	.00358	.00356	.00354	.00352	.00351	.00349	.00347	.00346	.00344
1.73	.00342	.00341	.00339	.00337	.00336	.00334	.00332	.00331	.00329	.00327
1.74	.00326	.00324	.00323	.00321	.00320	.00318	.00316	.00315	.00313	.00312
1.75	.00310	.00309	.00307	.00306	.00304	.00303	.00301	.00300	.00298	.00297
1.76	.00295	.00294	.00292	.00291	.00290	.00288	.00287	.00285	.00284	.00282
1.77	.00281	.00280	.00278	.00277	.00276	.00274	.00273	.00271	.00270	.00269
1.78	.00267	.00266	.00265	.00264	.00262	.00261	.00260	.00258	.00257	.00256
1.79	.00254	.00253	.00252	.00251	.00249	.00248	.00247	.00246	.00245	.00243
1.80	.00242	.00241	.00240	.00239	.00237	.00236	.00235	.00234	.00233	.00231
1.81	.00230	.00229	.00228	.00227	.00226	.00225	.00223	.00222	.00221	.00220
1.82	.00219	.00218	.00217	.00216	.00215	.00214	.00213	.00211	.00210	.00209
1.83	.00208	.00207	.00206	.00205	.00204	.00203	.00202	.00201	.00200	.00199
1.84	.00198	.00197	.00196	.00195	.00194	.00193	.00192	.00191	.00190	.00189
1.85	.00188	.00187	.00186	.00185	.00185	.00184	.00183	.00182	.00181	.00180
1.86	.00179	.00178	.00177	.00176	.00175	.00174	.00174	.00173	.00172	.00171
1.87	.00170	.00169	.00168	.00168	.00167	.00166	.00165	.00164	.00163	.00162
1.88	.00162	.00161	.00160	.00159	.00158	.00158	.00157	.00156	.00155	.00154
1.89	.00154	.00153	.00152	.00151	.00150	.00150	.00149	.00148	.00147	.00147
1.90	.00146	.00145	.00144	.00144	.00143	.00142	.00141	.00141	.00140	.00139
1.91	.00139	.00138	.00137	.00136	.00136	.00135	.00134	.00134	.00133	.00132
1.92	.00132	.00131	.00130	.00130	.00129	.00128	.00128	.00127	.00126	.00126
1.93	.00125	.00124	.00124	.00123	.00122	.00122	.00121	.00121	.00120	.00119
1.94	.00119	.00118	.00117	.00117	.00116	.00116	.00115	.00114	.00114	.00113
1.95	.00113	.00112	.00112	.00111	.00110	.00110	.00109	.00109	.0010[illegible]	.00108
1.96	.00107	.00106	.00106	.00105	.00105	.00104	.00104	.00103	.00103	.00102
1.97	.00101	.00101	.00100	.00100	.00099	.00099	.00098	.00098	.00097	.00097
1.98	.00096	.00096	.00095	.00095	.00094	.00094	.00093	.00093	.00092	.00092
1.99	.00091	.00091	.00090	.00090	.00089	.00089	.00089	.00088	.00088	.00087
2.00	.00087	.00086	.00086	.00085	.00085	.00084	.00084	.00084	.00083	.00083

5.

x	0	1	2	3	4	5	6	7	8	9
2.01	.00082	.00082	.00081	.00081	.00080	.00080	.00080	.00079	.00079	.00078
2.02	.00078	.00078	.00077	.00077	.00076	.00076	.00076	.00075	.00075	.00074
2.03	.00074	.00074	.00073	.00073	.00072	.00072	.00072	.00071	.00071	.00070
2.04	.00070	.00070	.00069	.00069	.00069	.00068	.00068	.00068	.00067	.00067
2.05	.00066	.00066	.00066	.00065	.00065	.00065	.00064	.00064	.00064	.00063
2.06	.00063	.00063	.00062	.00062	.00062	.00061	.00061	.00061	.00060	.00060
2.07	.00060	.00059	.00059	.00059	.00058	.00058	.00058	.00057	.00057	.00057
2.08	.00057	.00056	.00056	.00056	.00055	.00055	.00055	.00054	.00054	.00054
2.09	.00054	.00053	.00053	.00053	.00052	.00052	.00052	.00052	.00051	.00051
2.10	.00051	.00050	.00050	.00050	.00050	.00049	.00049	.00049	.00049	.00048
2.11	.00048	.00048	.00048	.00047	.00047	.00047	.00046	.00046	.00046	.00046
2.12	.00045	.00045	.00045	.00045	.00044	.00044	.00044	.00044	.00044	.00043
2.13	.00043	.00043	.00043	.00042	.00042	.00042	.00042	.00041	.00041	.00041
2.14	.00041	.00041	.00040	.00040	.00040	.00040	.00039	.00039	.00039	.00039
2.15	.00039	.00038	.00038	.00038	.00038	.00038	.00037	.00037	.00037	.00037
2.16	.00037	.00036	.00036	.00036	.00036	.00036	.00035	.00035	.00035	.00035
2.17	.00035	.00034	.00034	.00034	.00034	.00034	.00033	.00033	.00033	.00033
2.18	.00033	.00032	.00032	.00032	.00032	.00032	.00032	.00031	.00031	.00031
2.19	.00031	.00031	.00031	.00030	.00030	.00030	.00030	.00030	.00030	.00029
2.20	.00029	.00029	.00029	.00029	.00029	.00028	.00028	.00028	.00028	.00028
2.21	.00028	.00027	.00027	.00027	.00027	.00027	.00027	.00027	.00026	.00026
2.22	.00026	.00026	.00026	.00026	.00026	.00025	.00025	.00025	.00025	.00025
2.23	.00025	.00025	.00024	.00024	.00024	.00024	.00024	.00024	.00024	.00023
2.24	.00023	.00023	.00023	.00023	.00023	.00023	.00023	.00022	.00022	.00022
2.25	.00022	.00022	.00022	.00022	.00022	.00021	.00021	.00021	.00021	.00021
2.26	.00021	.00021	.00021	.00020	.00020	.00020	.00020	.00020	.00020	.00020
2.27	.00020	.00020	.00019	.00019	.00019	.00019	.00019	.00019	.00019	.00019
2.28	.00019	.00018	.00018	.00018	.00018	.00018	.00018	.00018	.00018	.00018
2.29	.00018	.00017	.00017	.00017	.00017	.00017	.00017	.00017	.00017	.00017
2.30	.00017	.00016	.00016	.00016	.00016	.00016	.00016	.00016	.00016	.00016
2.31	.00016	.00016	.00015	.00015	.00015	.00015	.00015	.00015	.00015	.00015
2.32	.00015	.00015	.00015	.00015	.00014	.00014	.00014	.00014	.00014	.00014
2.33	.00014	.00014	.00014	.00014	.00014	.00014	.00013	.00013	.00013	.00013
2.34	.00013	.00013	.00013	.00013	.00013	.00013	.00013	.00013	.00013	.00012
2.35	.00012	.00012	.00012	.00012	.00012	.00012	.00012	.00012	.00012	.00012
2.36	.00012	.00012	.00012	.00012	.00011	.00011	.00011	.00011	.00011	.00011
2.37	.00011	.00011	.00011	.00011	.00011	.00011	.00011	.00011	.00011	.00010
2.38	.00010	.00010	.00010	.00010	.00010	.00010	.00010	.00010	.00010	.00010
2.39	.00010	.00010	.00010	.00010	.00010	.00010	.00009	.00009	.00009	.00009
2.40	.00009	.00009	.00009	.00009	.00009	.00009	.00009	.00009	.00009	.00009
2.41	.00009	.00009	.00009	.00009	.00009	.00008	.00008	.00008	.00008	.00008
2.42	.00008	.00008	.00008	.00008	.00008	.00008	.00008	.00008	.00008	.00008
2.43	.00008	.00008	.00008	.00008	.00008	.00008	.00007	.00007	.00007	.00007
2.44	.00007	.00007	.00007	.00007	.00007	.00007	.00007	.00007	.00007	.00007
2.45	.00007	.00007	.00007	.00007	.00007	.00007	.00007	.00007	.00007	.00007
2.46	.00006	.00006	.00006	.00006	.00006	.00006	.00006	.00006	.00006	.00006
2.47	.00006	.00006	.00006	.00006	.00006	.00006	.00006	.00006	.00006	.00006
2.48	.00006	.00006	.00006	.00006	.00006	.00006	.00006	.00006	.00005	.00005
2.49	.00005	.00005	.00005	.00005	.00005	.00005	.00005	.00005	.00005	.00005
2.50	.00005	.00005	.00005	.00005	.00005	.00005	.00005	.00005	.00005	.00005

6.

Table 9. $\int_x^\infty \frac{e^{-u^2}}{u^2}\,du$

x	0	1	2	3	4	5	6	7	8	9
2.51	.00005	.00005	.00005	.00005	.00005	.00005	.00005	.00005	.00005	.00005
2.52	.00005	.00004	.00004	.00004	.00004	.00004	.00004	.00004	.00004	.00004
2.53	.00004	.00004	.00004	.00004	.00004	.00004	.00004	.00004	.00004	.00004
2.54	.00004	.00004	.00004	.00004	.00004	.00004	.00004	.00004	.00004	.00004
2.55	.00004	.00004	.00004	.00004	.00004	.00004	.00004	.00004	.00004	.00004
2.56	.00004	.00004	.00003	.00003	.00003	.00003	.00003	.00003	.00003	.00003
2.57	.00003	.00003	.00003	.00003	.00003	.00003	.00003	.00003	.00003	.00003
2.58	.00003	.00003	.00003	.00003	.00003	.00003	.00003	.00003	.00003	.00003
2.59	.00003	.00003	.00003	.00003	.00003	.00003	.00003	.00003	.00003	.00003
2.60	.00003	.00003	.00003	.00003	.00003	.00003	.00003	.00003	.00003	.00003
2.61	.00003	.00003	.00003	.00003	.00003	.00003	.00002	.00002	.00002	.00002
2.62	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002
2.63	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002
2.64	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002
2.65	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002
2.66	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002
2.67	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002
2.68	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002
2.69	.00002	.00002	.00002	.00002	.00002	.00002	.00001	.00001	.00001	.00001
2.70	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.71	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.72	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.73	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.74	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.75	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.76	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.77	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.78	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.79	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.80	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.81	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.82	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.83	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.84	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.85	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.86	.00001	.00001	.00001	.00001	.00000	.00000	.00000	.00000	.00000	.00000
2.87	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.88	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.89	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.90	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.91	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.92	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.93	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.94	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.95	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.96	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.97	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.98	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.99	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.00	.00000									

Table 10

Values of:

$$\frac{h_c}{H} = \frac{4}{\pi} \sum_{n=1,3,5...}^{n=\infty} \frac{e^{-n^2\pi^2(\frac{\alpha t}{L^2})} \sin\left(\frac{n\pi}{2}\right)}{n}$$

The drainable depth at the point midway between drains can be obtained by use of this table.

Table 10. (h_c/H).

$\left(\frac{\alpha t}{L^2}\right)$	0	1	2	3	4	5	6	7	8	9
0.0000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99999	.99995	.99985	.99961
.0100	.99919	.99850	.99750	.99614	.99439	.99222	.98962	.98661	.98318	.97936
.0200	.97516	.97061	.96572	.96052	.95504	.94931	.94333	.93715	.93078	.92424
.0300	.91755	.91072	.90379	.89675	.88963	.88244	.87519	.86789	.86055	.85318
.0400	.84580	.83840	.83100	.82361	.81622	.80884	.80148	.79415	.78684	.77956
.0500	.77231	.76510	.75793	.75080	.74372	.73668	.72968	.72274	.71584	.70899
.0600	.70220	.69546	.68877	.68214	.67555	.66903	.66256	.65614	.64978	.64347
.0700	.63722	.63103	.62489	.61881	.61278	.60680	.60089	.59502	.58921	.58346
.0800	.57775	.57211	.56651	.56097	.55548	.55004	.54466	.53932	.53404	.52881
.0900	.52363	.51850	.51341	.50838	.50340	.49846	.49357	.48873	.48394	.47919
.1000	.47449	.46983	.46522	.46066	.45614	.45166	.44723	.44284	.43849	.43419
.1100	.42992	.42570	.42152	.41739	.41329	.40923	.40521	.40123	.39729	.39339
.1200	.38953	.38571	.38192	.37817	.37445	.37078	.36714	.36353	.35996	.35643
.1300	.35293	.34946	.34603	.34263	.33927	.33593	.33264	.32937	.32613	.32293
.1400	.31976	.31662	.31351	.31043	.30738	.30436	.30137	.29841	.29548	.29258
.1500	.28971	.28686	.28405	.28126	.27849	.27576	.27305	.27037	.26771	.26509
.1600	.26248	.25990	.25735	.25482	.25232	.24984	.24739	.24496	.24255	.24017
.1700	.23781	.23548	.23317	.23088	.22861	.22636	.22414	.22194	.21976	.21760
.1800	.21546	.21335	.21125	.20918	.20712	.20509	.20307	.20108	.19911	.19715
.1900	.19521	.19330	.19140	.18952	.18766	.18581	.18399	.18218	.18039	.17862
.2000	.17687	.17513	.17341	.17171	.17002	.16835	.16670	.16506	.16344	.16183
.2100	.16024	.15867	.15711	.15557	.15404	.15253	.15103	.14955	.14808	.14662
.2200	.14518	.14376	.14235	.14095	.13956	.13819	.13684	.13549	.13416	.13284
.2300	.13154	.13025	.12897	.12770	.12645	.12521	.12398	.12276	.12155	.12036
.2400	.11918	.11801	.11685	.11570	.11456	.11344	.11233	.11122	.11013	.10905
.2500	.10798	.10692	.10587	.10483	.10380	.10278	.10177	.10077	.09978	.09880
.2600	.09783	.09687	.09592	.09498	.09404	.09312	.09220	.09130	.09040	.08951
.2700	.08863	.08776	.08690	.08605	.08520	.08437	.08354	.08272	.08191	.08110
.2800	.08030	.07952	.07874	.07796	.07720	.07644	.07569	.07494	.07421	.07348
.2900	.07276	.07204	.07134	.07063	.06994	.06925	.06857	.06790	.06723	.06657
.3000	.06592	.06527	.06463	.06400	.06337	.06275	.06213	.06152	.06092	.06032
.3100	.05972	.05914	.05856	.05798	.05741	.05685	.05629	.05574	.05519	.05465
.3200	.05411	.05358	.05305	.05253	.05202	.05151	.05100	.05050	.05000	.04951
.3300	.04903	.04854	.04807	.04760	.04713	.04667	.04621	.04575	.04530	.04486
.3400	.04442	.04398	.04355	.04312	.04270	.04228	.04186	.04145	.04105	.04064
.3500	.04024	.03985	.03946	.03907	.03869	.03831	.03793	.03756	.03719	.03682
.3600	.03646	.03610	.03575	.03540	.03505	.03471	.03437	.03403	.03369	.03336
.3700	.03303	.03271	.03239	.03207	.03176	.03144	.03114	.03083	.03053	.03023
.3800	.02993	.02964	.02935	.02906	.02877	.02849	.02821	.02793	.02766	.02739
.3900	.02712	.02685	.02659	.02633	.02607	.02581	.02556	.02531	.02506	.02481
.4000	.02457	.02433	.02409	.02385	.02362	.02339	.02316	.02293	.02270	.02248
.4100	.02226	.02204	.02182	.02161	.02140	.02119	.02098	.02077	.02057	.02037
.4200	.02017	.01997	.01977	.01958	.01939	.01920	.01901	.01882	.01864	.01845
.4300	.01827	.01809	.01792	.01774	.01757	.01739	.01722	.01705	.01689	.01672
.4400	.01656	.01639	.01623	.01607	.01591	.01576	.01560	.01545	.01530	.01515
.4500	.01500	.01485	.01471	.01456	.01442	.01428	.01414	.01400	.01386	.01372
.4600	.01359	.01346	.01332	.01319	.01306	.01294	.01281	.01268	.01256	.01243
.4700	.01231	.01219	.01207	.01195	.01184	.01172	.01160	.01149	.01138	.01127
.4800	.01116	.01105	.01094	.01083	.01072	.01062	.01051	.01041	.01031	.01021
.4900	.01011	.01001	.00991	.00981	.00972	.00962	.00953	.00943	.00934	.00925
.5000	.00916	.00907	.00898	.00889	.00880	.00872	.00863	.00855	.00846	.00838

$\left(\frac{\alpha t}{L^2}\right)$	0	1	2	3	4	5	6	7	8	9
.5100	.00830	.00821	.00813	.00805	.00798	.00790	.00782	.00774	.00767	.00759
.5200	.00752	.00744	.00737	.00730	.00723	.00715	.00708	.00701	.00695	.00688
.5300	.00681	.00674	.00668	.00661	.00655	.00648	.00642	.00636	.00629	.00623
.5400	.00617	.00611	.00605	.00599	.00593	.00587	.00582	.00576	.00570	.00565
.5500	.00559	.00554	.00548	.00543	.00537	.00532	.00527	.00522	.00517	.00512
.5600	.00506	.00502	.00497	.00492	.00487	.00482	.00477	.00473	.00468	.00463
.5700	.00459	.00454	.00450	.00446	.00441	.00437	.00433	.00428	.00424	.00420
.5800	.00416	.00412	.00408	.00404	.00400	.00396	.00392	.00388	.00384	.00380
.5900	.00377	.00373	.00369	.00366	.00362	.00359	.00355	.00352	.00348	.00345
.6000	.00341	.00338	.00335	.00331	.00328	.00325	.00322	.00319	.00315	.00312
.6100	.00309	.00306	.00303	.00300	.00297	.00294	.00291	.00289	.00286	.00283
.6200	.00280	.00277	.00275	.00272	.00269	.00267	.00264	.00261	.00259	.00256
.6300	.00254	.00251	.00249	.00246	.00244	.00242	.00239	.00237	.00235	.00232
.6400	.00230	.00228	.00225	.00223	.00221	.00219	.00217	.00215	.00213	.00210
.6500	.00208	.00206	.00204	.00202	.00200	.00198	.00196	.00194	.00193	.00191
.6600	.00189	.00187	.00185	.00183	.00181	.00180	.00178	.00176	.00174	.00173
.6700	.00171	.00169	.00168	.00166	.00164	.00163	.00161	.00160	.00158	.00156
.6800	.00155	.00153	.00152	.00150	.00149	.00147	.00146	.00145	.00143	.00142
.6900	.00140	.00139	.00138	.00136	.00135	.00134	.00132	.00131	.00130	.00128
.7000	.00127	.00126	.00125	.00123	.00122	.00121	.00120	.00119	.00118	.00116
.7100	.00115	.00114	.00113	.00112	.00111	.00110	.00109	.00108	.00106	.00105
.7200	.00104	.00103	.00102	.00101	.00100	.00099	.00098	.00097	.00096	.00096
.7300	.00095	.00094	.00093	.00092	.00091	.00090	.00089	.00088	.00087	.00087
.7400	.00086	.00085	.00084	.00083	.00082	.00082	.00081	.00080	.00079	.00078
.7500	.00078	.00077	.00076	.00075	.00075	.00074	.00073	.00072	.00072	.00071
.7600	.00070	.00070	.00069	.00068	.00068	.00067	.00066	.00066	.00065	.00064
.7700	.00064	.00063	.00062	.00062	.00061	.00061	.00060	.00059	.00059	.00058
.7800	.00058	.00057	.00057	.00056	.00056	.00055	.00054	.00054	.00053	.00053
.7900	.00052	.00052	.00051	.00051	.00050	.00050	.00049	.00049	.00048	.00048
.8000	.00047	.00047	.00046	.00046	.00046	.00045	.00045	.00044	.00044	.00043
.8100	.00043	.00043	.00042	.00042	.00041	.00041	.00040	.00040	.00040	.00039
.8200	.00039	.00039	.00038	.00038	.00037	.00037	.00037	.00036	.00036	.00036
.8300	.00035	.00035	.00035	.00034	.00034	.00034	.00033	.00033	.00033	.00032
.8400	.00032	.00032	.00031	.00031	.00031	.00030	.00030	.00030	.00030	.00029
.8500	.00029	.00029	.00028	.00028	.00028	.00028	.00027	.00027	.00027	.00026
.8600	.00026	.00026	.00026	.00025	.00025	.00025	.00025	.00024	.00024	.00024
.8700	.00024	.00024	.00023	.00023	.00023	.00023	.00022	.00022	.00022	.00022
.8800	.00022	.00021	.00021	.00021	.00021	.00020	.00020	.00020	.00020	.00020
.8900	.00020	.00019	.00019	.00019	.00019	.00019	.00018	.00018	.00018	.00018
.9000	.00018	.00017	.00017	.00017	.00017	.00017	.00017	.00016	.00016	.00016
.9100	.00016	.00016	.00016	.00016	.00015	.00015	.00015	.00015	.00015	.00015
.9200	.00015	.00014	.00014	.00014	.00014	.00014	.00014	.00014	.00013	.00013
.9300	.00013	.00013	.00013	.00013	.00013	.00013	.00012	.00012	.00012	.00012
.9400	.00012	.00012	.00012	.00012	.00011	.00011	.00011	.00011	.00011	.00011
.9500	.00011	.00011	.00011	.00010	.00010	.00010	.00010	.00010	.00010	.00010
.9600	.00010	.00010	.00010	.00009	.00009	.00009	.00009	.00009	.00009	.00009
.9700	.00009	.00009	.00009	.00009	.00009	.00008	.00008	.00008	.00008	.00008
.9800	.00008	.00008	.00008	.00008	.00008	.00008	.00008	.00007	.00007	.00007
.9900	.00007	.00007	.00007	.00007	.00007	.00007	.00007	.00007	.00007	.00007
1.0000	.00007									

Table 11

Values of:

$$p = \frac{8}{\pi^2} \sum_{n=1,3,5...}^{n=\infty} \frac{e^{-n^2\pi^2(\frac{\alpha t}{L^2})}}{n^2}$$

The part of the original drainable volume which remains at the time t, of water moving to drains at the spacing L, where the aquifer constant is α, can be obtained by use of this table.

$\left(\frac{\alpha t}{L^2}\right)$	0	1	2	3	4	5	6	7	8	9
0.0000	1.0000000	.9774323	.9680846	.9609118	.9548648	.9495373	.9447209	.9402918	.9361692	.9322972
.0010	.9286350	.9251518	.9218236	.9186314	.9155598	.9125961	.9097297	.9069515	.9042538	.9016302
.0020	.8990747	.8965823	.8941486	.8917697	.8894419	.8871621	.8849275	.8827354	.8805836	.8784698
.0030	.8763923	.8743490	.8723385	.8703591	.8684095	.8664884	.8645945	.8627267	.8608841	.8590655
.0040	.8572701	.8554970	.8537453	.8520145	.8503036	.8486120	.8469392	.8452844	.8436472	.8420269
.0050	.8404231	.8388352	.8372628	.8357055	.8341628	.8326343	.8311197	.8296185	.8281304	.8266551
.0060	.8251923	.8237415	.8223027	.8208754	.8194593	.8180543	.8166601	.8152764	.8139029	.8125396
.0070	.8111861	.8098422	.8085077	.8071825	.8058663	.8045590	.8032604	.8019703	.8006885	.7994149
.0080	.7981494	.7968917	.7956418	.7943995	.7931647	.7919372	.7907168	.7895036	.7882973	.7870978
.0090	.7859051	.7847190	.7835393	.7823661	.7811992	.7800384	.7788837	.7777351	.7765923	.7754554
.0100	.7743242	.7633092	.7527845	.7426900	.7329768	.7236047	.7145401	.7057547	.6972241	.6889274
.0200	.6808463	.6729648	.6652689	.6577462	.6503854	.6431766	.6361109	.6291801	.6223770	.6156948
.0300	.6091277	.6026699	.5963166	.5900630	.5839049	.5778385	.5718600	.5659662	.5601540	.5544205
.0400	.5487632	.5431795	.5376672	.5322242	.5268485	.5215383	.5162917	.5111073	.5059835	.5009189
.0500	.4959122	.4909620	.4860672	.4812267	.4764395	.4717044	.4670206	.4623872	.4578032	.4532679
.0600	.4487805	.4443401	.4399462	.4355980	.4312949	.4270361	.4228211	.4186493	.4145201	.4104329
.0700	.4063873	.4023826	.3984185	.3944943	.3906096	.3867639	.3829568	.3791879	.3754566	.3717626
.0800	.3681054	.3644847	.3609000	.3573509	.3538371	.3503581	.3469137	.3435034	.3401268	.3367837
.0900	.3334736	.3301963	.3269513	.3237384	.3205572	.3174074	.3142886	.3112006	.3081431	.3051157
.1000	.3021181	.2991500	.2962112	.2933013	.2904201	.2875672	.2847424	.2819453	.2791758	.2764336
.1100	.2737183	.2710297	.2683675	.2657315	.2631215	.2605371	.2579781	.2554442	.2529353	.2504510
.1200	.2479911	.2455555	.2431437	.2407556	.2383910	.2360497	.2337313	.2314357	.2291627	.2269120
.1300	.2246834	.2224767	.2202917	.2181281	.2159858	.2138646	.2117642	.2096844	.2076250	.2055859
.1400	.2035668	.2015675	.1995879	.1976277	.1956868	.1937649	.1918619	.1899776	.1881118	.1862643
.1500	.1844350	.1826237	.1808301	.1790541	.1772956	.1755544	.1738303	.1721230	.1704326	.1687588
.1600	.1671014	.1654603	.1638353	.1622262	.1606330	.1590554	.1574933	.1559466	.1544150	.1528985
.1700	.1513968	.1499100	.1484377	.1469799	.1455364	.1441070	.1426918	.1412904	.1399028	.1385288
.1800	.1371683	.1358211	.1344872	.1331664	.1318586	.1305636	.1292813	.1280116	.1267544	.1255096
.1900	.1242769	.1230564	.1218478	.1206512	.1194662	.1182930	.1171312	.1159808	.1148418	.1137139
.2000	.1125971	.1114913	.1103963	.1093121	.1082386	.1071756	.1061230	.1050807	.1040487	.1030269
.2100	.1020150	.1010131	.1000211	.0990388	.0980661	.0971030	.0961493	.0952050	.0942700	.0933442
.2200	.0924275	.0915197	.0906209	.0897309	.0888497	.0879771	.0871130	.0862575	.0854103	.0845715
.2300	.0837409	.0829185	.0821042	.0812978	.0804994	.0797088	.0789260	.0781508	.0773833	.0766233
.2400	.0758708	.0751257	.0743879	.0736573	.0729339	.0722176	.0715084	.0708061	.0701107	.0694221
.2500	.0687403	.0680652	.0673967	.0667348	.0660794	.0654305	.0647879	.0641516	.0635215	.0628977

$\left(\frac{\alpha t}{L^2}\right)$	0	1	2	3	4	5	6	7	8	9
.2600	.0622800	.0616683	.0610627	.0604630	.0598692	.0592812	.0586990	.0581225	.0575517	.0569864
.2700	.0564268	.0558726	.0553239	.0547805	.0542425	.0537098	.0531823	.0526600	.0521428	.0516307
.2800	.0511237	.0506216	.0501244	.0496322	.0491447	.0486621	.0481842	.0477109	.0472424	.0467784
.2900	.0463190	.0458641	.0454136	.0449676	.0445260	.0440887	.0436557	.0432270	.0428024	.0423821
.3000	.0419658	.0415537	.0411456	.0407415	.0403414	.0399452	.0395529	.0391644	.0387798	.0383989
.3100	.0380218	.0376484	.0372786	.0369125	.0365500	.0361910	.0358356	.0354837	.0351352	.0347901
.3200	.0344484	.0341101	.0337751	.0334434	.0331150	.0327897	.0324677	.0321488	.0318331	.0315205
.3300	.0312109	.0309044	.0306009	.0303003	.0300028	.0297081	.0294163	.0291274	.0288414	.0285581
.3400	.0282776	.0279999	.0277249	.0274527	.0271830	.0269161	.0266517	.0263900	.0261308	.0258742
.3500	.0256201	.0253684	.0251193	.0248726	.0246283	.0243864	.0241469	.0239098	.0236750	.0234425
.3600	.0232122	.0229843	.0227585	.0225350	.0223137	.0220946	.0218776	.0216627	.0214500	.0212393
.3700	.0210307	.0208242	.0206196	.0204171	.0202166	.0200181	.0198215	.0196268	.0194340	.0192432
.3800	.0190542	.0188671	.0186818	.0184983	.0183166	.0181367	.0179586	.0177822	.0176076	.0174347
.3900	.0172634	.0170939	.0169260	.0167598	.0165952	.0164322	.0162708	.0161110	.0159528	.0157961
.4000	.0156410	.0154874	.0153353	.0151847	.0150355	.0148879	.0147417	.0145969	.0144535	.0143116
.4100	.0141710	.0140318	.0138940	.0137576	.0136225	.0134887	.0133562	.0132250	.0130952	.0129665
.4200	.0128392	.0127131	.0125883	.0124646	.0123422	.0122210	.0121010	.0119821	.0118644	.0117479
.4300	.0116325	.0115183	.0114052	.0112932	.0111823	.0110724	.0109637	.0108560	.0107494	.0106438
.4400	.0105393	.0104358	.0103333	.0102318	.0101313	.0100318	.0099333	.0098358	.0097392	.0096435
.4500	.0095488	.0094550	.0093622	.0092702	.0091792	.0090890	.0089998	.0089114	.0088238	.0087372
.4600	.0086514	.0085664	.0084823	.0083990	.0083165	.0082348	.0081539	.0080739	.0079946	.0079161
.4700	.0078383	.0077613	.0076851	.0076096	.0075349	.0074609	.0073876	.0073151	.0072432	.0071721
.4800	.0071016	.0070319	.0069628	.0068945	.0068267	.0067597	.0066933	.0066276	.0065625	.0064980
.4900	.0064342	.0063710	.0063085	.0062465	.0061852	.0061244	.0060643	.0060047	.0059457	.0058873
.5000	.0058295	.0057723	.0057156	.0056594	.0056039	.0055488	.0054943	.0054404	.0053869	.0053340
.5100	.0052817	.0052298	.0051784	.0051276	.0050772	.0050273	.0049780	.0049291	.0048807	.0048327
.5200	.0047853	.0047383	.0046917	.0046457	.0046000	.0045549	.0045101	.0044658	.0044220	.0043785
.5300	.0043355	.0042930	.0042508	.0042091	.0041677	.0041268	.0040863	.0040461	.0040064	.0039670
.5400	.0039281	.0038895	.0038513	.0038135	.0037760	.0037389	.0037022	.0036659	.0036299	.0035942
.5500	.0035589	.0035240	.0034893	.0034551	.0034211	.0033875	.0033543	.0033213	.0032887	.0032564
.5600	.0032244	.0031928	.0031614	.0031304	.0030996	.0030692	.0030390	.0030092	.0029796	.0029504
.5700	.0029214	.0028927	.0028643	.0028362	.0028083	.0027807	.0027534	.0027264	.0026996	.0026731
.5800	.0026468	.0026208	.0025951	.0025696	.0025444	.0025194	.0024947	.0024702	.0024459	.0024219
.5900	.0023981	.0023745	.0023512	.0023281	.0023053	.0022826	.0022602	.0022380	.0022160	.0021943
.6000	.0021727	.0021514	.0021302	.0021093	.0020886	.0020681	.0020478	.0020277	.0020078	.0019880

$\left(\frac{\alpha t}{L^2}\right)$	0	1	2	3	4	5	6	7	8	9
.6100	.0019685	.0019492	.0019300	.0019111	.0018923	.0018737	.0018553	.0018371	.0018191	.0018012
.6200	.0017835	.0017660	.0017486	.0017315	.0017145	.0016976	.0016810	.0016645	.0016481	.0016319
.6300	.0016159	.0016000	.0015843	.0015687	.0015533	.0015381	.0015230	.0015080	.0014932	.0014785
.6400	.0014640	.0014496	.0014354	.0014213	.0014074	.0013935	.0013798	.0013663	.0013529	.0013396
.6500	.0013264	.0013134	.0013005	.0012877	.0012751	.0012626	.0012502	.0012379	.0012257	.0012137
.6600	.0012018	.0011900	.0011783	.0011667	.0011553	.0011439	.0011327	.0011215	.0011105	.0010996
.6700	.0010888	.0010781	.0010675	.0010571	.0010467	.0010364	.0010262	.0010161	.0010062	.0009963
.6800	.0009865	.0009768	.0009672	.0009577	.0009483	.0009390	.0009298	.0009206	.0009116	.0009027
.6900	.0008938	.0008850	.0008763	.0008677	.0008592	.0008507	.0008424	.0008341	.0008259	.0008178
.7000	.0008098	.0008018	.0007940	.0007862	.0007784	.0007708	.0007632	.0007557	.0007483	.0007410
.7100	.0007337	.0007265	.0007193	.0007123	.0007053	.0006984	.0006915	.0006847	.0006780	.0006713
.7200	.0006647	.0006582	.0006517	.0006453	.0006390	.0006327	.0006265	.0006204	.0006143	.0006082
.7300	.0006023	.0005963	.0005905	.0005847	.0005789	.0005733	.0005676	.0005621	.0005565	.0005511
.7400	.0005457	.0005403	.0005350	.0005297	.0005245	.0005194	.0005143	.0005092	.0005042	.0004993
.7500	.0004944	.0004895	.0004847	.0004799	.0004752	.0004706	.0004659	.0004614	.0004568	.0004524
.7600	.0004479	.0004435	.0004392	.0004348	.0004306	.0004263	.0004222	.0004180	.0004139	.0004098
.7700	.0004058	.0004018	.0003979	.0003940	.0003901	.0003863	.0003825	.0003787	.0003750	.0003713
.7800	.0003677	.0003641	.0003605	.0003569	.0003534	.0003500	.0003465	.0003431	.0003398	.0003364
.7900	.0003331	.0003298	.0003266	.0003234	.0003202	.0003171	.0003140	.0003109	.0003078	.0003048
.8000	.0003018	.0002988	.0002959	.0002930	.0002901	.0002873	.0002845	.0002817	.0002789	.0002762
.8100	.0002734	.0002708	.0002681	.0002655	.0002629	.0002603	.0002577	.0002552	.0002527	.0002502
.8200	.0002477	.0002453	.0002429	.0002405	.0002382	.0002358	.0002335	.0002312	.0002289	.0002267
.8300	.0002245	.0002223	.0002201	.0002179	.0002158	.0002137	.0002116	.0002095	.0002074	.0002054
.8400	.0002034	.0002014	.0001994	.0001974	.0001955	.0001936	.0001917	.0001898	.0001879	.0001861
.8500	.0001843	.0001824	.0001807	.0001789	.0001771	.0001754	.0001737	.0001720	.0001703	.0001686
.8600	.0001669	.0001653	.0001637	.0001621	.0001605	.0001589	.0001573	.0001558	.0001543	.0001528
.8700	.0001513	.0001498	.0001483	.0001468	.0001454	.0001440	.0001426	.0001412	.0001398	.0001384
.8800	.0001370	.0001357	.0001344	.0001330	.0001317	.0001304	.0001292	.0001279	.0001266	.0001254
.8900	.0001242	.0001229	.0001217	.0001205	.0001194	.0001182	.0001170	.0001159	.0001147	.0001136
.9000	.0001125	.0001114	.0001103	.0001092	.0001081	.0001071	.0001060	.0001050	.0001039	.0001029
.9100	.0001019	.0001009	.0000999	.0000989	.0000980	.0000970	.0000961	.0000951	.0000942	.0000933
.9200	.0000923	.0000914	.0000905	.0000896	.0000888	.0000879	.0000870	.0000862	.0000853	.0000845
.9300	.0000837	.0000828	.0000820	.0000812	.0000804	.0000796	.0000788	.0000781	.0000773	.0000765
.9400	.0000758	.0000751	.0000743	.0000736	.0000729	.0000721	.0000714	.0000707	.0000700	.0000694
.9500	.0000687	.0000680	.0000673	.0000667	.0000660	.0000654	.0000647	.0000641	.0000635	.0000628
.9600	.0000622	.0000616	.0000610	.0000604	.0000598	.0000592	.0000586	.0000581	.0000575	.0000569
.9700	.0000564	.0000558	.0000553	.0000547	.0000542	.0000537	.0000531	.0000526	.0000521	.0000516
.9800	.0000511	.0000506	.0000501	.0000496	.0000491	.0000486	.0000481	.0000477	.0000472	.0000467
.9900	.0000463	.0000458	.0000454	.0000449	.0000445	.0000440	.0000436	.0000432	.0000428	.0000423
1.0000	.0000419									

Table 12

Values of:

$$R = 1 - \frac{1}{12}\left(\frac{L^2}{\alpha t}\right) + \frac{8}{\pi^4}\left(\frac{L^2}{\alpha t}\right) \sum_{n=1,3,S}^{n=\infty} \frac{e^{-n^2\pi^2\left(\frac{\alpha t}{L^2}\right)}}{n^4}$$

This function is useful for estimating return flows from irrigation or stream depletions due to distributed pumping. Examples of its use are given in Chapter 10. The factors described are well adapted to digital computer evaluations of return flows or pump depletions.

$\left(\frac{\alpha t}{L^2}\right)$	0	1	2	3	4	5	6	7	8	9
0.0000	0.0000000	.0475766	.0672835	.0824052	.0951533	.1063846	.1165385	.1258760	.1345671	.1427299
.0100	.1504506	.1577939	.1648103	.1715400	.1780155	.1842635	.1903066	.1961635	.2018506	.2073818
.0200	.2127692	.2180236	.2231542	.2281695	.2330769	.2378831	.2425941	.2472152	.2517515	.2562075
.0300	.2605872	.2648944	.2691325	.2733048	.2774142	.2814634	.2854549	.2893911	.2932741	.2971060
.0400	.3008888	.3046240	.3083135	.3119588	.3155613	.3191224	.3226435	.3261256	.3295701	.3329779
.0500	.3363501	.3396877	.3429916	.3462627	.3495017	.3527096	.3558869	.3590345	.3621530	.3652431
.0600	.3683054	.3713405	.3743489	.3773311	.3802878	.3832193	.3861262	.3890089	.3918679	.3947036
.0700	.3975163	.4003064	.4030744	.4058206	.4085454	.4112490	.4139318	.4165941	.4192363	.4218585
.0800	.4244611	.4270444	.4296087	.4321541	.4346809	.4371894	.4396799	.4421524	.4446074	.4470449
.0900	.4494652	.4518686	.4542551	.4566250	.4589785	.4613158	.4636370	.4659424	.4682321	.4705062
.1000	.4727650	.4750086	.4772372	.4794509	.4816498	.4838342	.4860041	.4881598	.4903013	.4924288
.1100	.4945424	.4966423	.4987285	.5008013	.5028607	.5049069	.5069400	.5089601	.5109673	.5129618
.1200	.5149436	.5169129	.5188698	.5208144	.5227467	.5246670	.5265753	.5284717	.5303563	.5322293
.1300	.5340906	.5359405	.5377789	.5396061	.5414220	.5432269	.5450207	.5468036	.5485757	.5503370
.1400	.5520876	.5538276	.5555572	.5572763	.5589851	.5606837	.5623721	.5640504	.5657187	.5673770
.1500	.5690255	.5706642	.5722933	.5739126	.5755225	.5771228	.5787138	.5802954	.5818677	.5834308
.1600	.5849848	.5865298	.5880657	.5895927	.5911109	.5926203	.5941209	.5956129	.5970963	.5985711
.1700	.6000375	.6014954	.6029450	.6043864	.6058194	.6072444	.6086612	.6100699	.6114707	.6128635
.1800	.6142484	.6156255	.6169949	.6183565	.6197105	.6210569	.6223957	.6237270	.6250509	.6263674
.1900	.6276766	.6289785	.6302731	.6315605	.6328409	.6341141	.6353803	.6366395	.6378918	.6391372
.2000	.6403757	.6416074	.6428324	.6440507	.6452624	.6464674	.6476659	.6488578	.6500433	.6512223
.2100	.6523950	.6535613	.6547213	.6558751	.6570226	.6581640	.6592992	.6604284	.6615515	.6626685
.2200	.6637797	.6648849	.6659842	.6670776	.6681653	.6692471	.6703233	.6713937	.6724585	.6735177
.2300	.6745713	.6756193	.6766619	.6776990	.6787306	.6797568	.6807777	.6817933	.6828035	.6838085
.2400	.6848083	.6858029	.6867923	.6877766	.6887558	.6897300	.6906991	.6916632	.6926224	.6935767
.2500	.6945261	.6954706	.6964103	.6973451	.6982752	.6992006	.7001213	.7010373	.7019486	.7028553
.2600	.7037575	.7046551	.7055481	.7064367	.7073208	.7082004	.7090756	.7099465	.7108129	.7116751
.2700	.7125329	.7133865	.7142358	.7150809	.7159218	.7167586	.7175912	.7184196	.7192440	.7200644
.2800	.7208806	.7216929	.7225012	.7233055	.7241059	.7249023	.7256949	.7264836	.7272685	.7280495
.2900	.7288268	.7296002	.7303700	.7311360	.7318983	.7326569	.7334119	.7341632	.7349109	.7356551
.3000	.7363956	.7371327	.7378662	.7385962	.7393227	.7400458	.7407654	.7414816	.7421944	.7429039
.3100	.7436099	.7443127	.7450121	.7457083	.7464011	.7470907	.7477771	.7484603	.7491403	.7498171
.3200	.7504907	.7511612	.7518286	.7524929	.7531541	.7538122	.7544673	.7551193	.7557684	.7564145
.3300	.7570576	.7576977	.7583349	.7589692	.7596005	.7602290	.7608547	.7614774	.7620974	.7627145
.3400	.7633288	.7639403	.7645491	.7651551	.7657584	.7663589	.7669568	.7675520	.7681445	.7687343
.3500	.7693215	.7699061	.7704880	.7710674	.7716442	.7722184	.7727901	.7733592	.7739258	.7744899
.3600	.7750515	.7756107	.7761674	.7767216	.7772734	.7778228	.7783697	.7789143	.7794565	.7799964
.3700	.7805338	.7810690	.7816018	.7821323	.7826605	.7831865	.7837101	.7842315	.7847507	.7852676
.3800	.7857823	.7862948	.7868050	.7873131	.7878191	.7883229	.7888245	.7893240	.7898214	.7903166
.3900	.7908098	.7913009	.7917899	.7922768	.7927617	.7932446	.7937254	.7942042	.7946810	.7951558
.4000	.7956286	.7960994	.7965683	.7970352	.7975002	.7979633	.7984244	.7988836	.7993410	.7997964
.4100	.8002500	.8007017	.8011515	.8015995	.8020457	.8024900	.8029325	.8033732	.8038122	.8042493
.4200	.8046846	.8051182	.8055501	.8059801	.8064085	.8068351	.8072600	.8076832	.8081046	.8085244
.4300	.8089425	.8093590	.8097737	.8101868	.8105983	.8110081	.8114163	.8118229	.8122279	.8126312
.4400	.8130330	.8134332	.8138318	.8142288	.8146243	.8150182	.8154106	.8158014	.8161907	.8165785
.4500	.8169648	.8173496	.8177329	.8181146	.8184950	.8188738	.8192512	.8196271	.8200015	.8203746
.4600	.8207462	.8211163	.8214851	.8218524	.8222183	.8225829	.8229460	.8233078	.8236681	.8240271
.4700	.8243848	.8247411	.8250960	.8254496	.8258019	.8261529	.8265025	.8268508	.8271978	.8275435
.4800	.8278879	.8282311	.8285729	.8289135	.8292528	.8295909	.8299277	.8302632	.8305975	.8309306
.4900	.8312624	.8315930	.8319225	.8322506	.8325776	.8329034	.8332280	.8335515	.8338737	.8341947
.5000	.8345146	.8348334	.8351509	.8354674	.8357827	.8360968	.8364098	.8367217	.8370324	.8373421

$\left(\frac{\alpha t}{L^2}\right)$	0	1	2	3	4	5	6	7	8	9
.5100	.8376506	.8379580	.8382644	.8385696	.8388737	.8391768	.8394788	.8397797	.8400795	.8403783
.5200	.8406760	.8409727	.8412683	.8415629	.8418564	.8421489	.8424404	.8427308	.8430203	.8433087
.5300	.8435961	.8438826	.8441680	.8444524	.8447359	.8450183	.8452998	.8455803	.8458598	.8461384
.5400	.8464160	.8466927	.8469684	.8472432	.8475170	.8477899	.8480619	.8483329	.8486030	.8488722
.5500	.8491405	.8494078	.8496743	.8499399	.8502045	.8504683	.8507312	.8509932	.8512543	.8515145
.5600	.8517739	.8520324	.8522900	.8525468	.8528027	.8530578	.8533120	.8535654	.8538179	.8540696
.5700	.8543205	.8545705	.8548197	.8550681	.8553157	.8555625	.8558084	.8560536	.8562979	.8565415
.5800	.8567842	.8570262	.8572674	.8575078	.8577474	.8579862	.8582243	.8584616	.8586981	.8589339
.5900	.8591689	.8594031	.8596366	.8598694	.8601014	.8603327	.8605632	.8607930	.8610221	.8612504
.6000	.8614780	.8617049	.8619311	.8621565	.8623813	.8626053	.8628286	.8630513	.8632732	.8634944
.6100	.8637149	.8639348	.8641540	.8643724	.8645902	.8648073	.8650238	.8652396	.8654547	.8656691
.6200	.8658829	.8660960	.8663084	.8665202	.8667314	.8669419	.8671517	.8673609	.8675695	.8677774
.6300	.8679847	.8681914	.8683975	.8686029	.8688077	.8690118	.8692154	.8694183	.8696206	.8698223
.6400	.8700234	.8702239	.8704238	.8706231	.8708218	.8710199	.8712175	.8714144	.8716107	.8718065
.6500	.8720016	.8721962	.8723902	.8725837	.8727765	.8729688	.8731606	.8733517	.8735423	.8737324
.6600	.8739219	.8741108	.8742992	.8744870	.8746743	.8748610	.8750472	.8752328	.8754179	.8756025
.6700	.8757865	.8759701	.8761530	.8763355	.8765174	.8766988	.8768797	.8770600	.8772398	.8774192
.6800	.8775980	.8777763	.8779541	.8781313	.8783081	.8784844	.8786602	.8788354	.8790102	.8791845
.6900	.8793583	.8795316	.8797044	.8798767	.8800486	.8802200	.8803908	.8805612	.8807312	.8809006
.7000	.8810696	.8812381	.8814061	.8815737	.8817408	.8819075	.8820737	.8822394	.8824046	.8825695
.7100	.8827338	.8828977	.8830612	.8832242	.8833867	.8835488	.8837105	.8838717	.8840325	.8841929
.7200	.8843528	.8845123	.8846713	.8848299	.8849881	.8851459	.8853032	.8854601	.8856166	.8857727
.7300	.8859283	.8860836	.8862384	.8863928	.8865468	.8867003	.8868535	.8870063	.8871586	.8873106
.7400	.8874621	.8876132	.8877640	.8879143	.8880643	.8882138	.8883630	.8885117	.8886601	.8888081
.7500	.8889557	.8891029	.8892497	.8893961	.8895422	.8896879	.8898332	.8899781	.8901226	.8902668
.7600	.8904106	.8905540	.8906971	.8908397	.8909821	.8911240	.8912656	.8914068	.8915477	.8916882
.7700	.8918283	.8919681	.8921075	.8922466	.8923853	.8925236	.8926616	.8927993	.8929366	.8930735
.7800	.8932102	.8933464	.8934823	.8936179	.8937532	.8938881	.8940226	.8941568	.8942907	.8944243
.7900	.8945575	.8946904	.8948229	.8949552	.8950870	.8952186	.8953498	.8954808	.8956113	.8957416
.8000	.8958716	.8960012	.8961305	.8962595	.8963881	.8965165	.8966445	.8967722	.8968997	.8970268
.8100	.8971535	.8972800	.8974062	.8975321	.8976576	.8977829	.8979078	.8980325	.8981568	.8982809
.8200	.8984046	.8985280	.8986512	.8987740	.8988966	.8990189	.8991408	.8992625	.8993839	.8995050
.8300	.8996258	.8997463	.8998665	.8999865	.9001062	.9002255	.9003446	.9004634	.9005820	.9007002
.8400	.9008182	.9009359	.9010533	.9011704	.9012873	.9014039	.9015202	.9016362	.9017520	.9018675
.8500	.9019827	.9020977	.9022124	.9023268	.9024410	.9025549	.9026685	.9027819	.9028950	.9030079
.8600	.9031204	.9032328	.9033448	.9034566	.9035682	.9036795	.9037905	.9039013	.9040119	.9041221
.8700	.9042322	.9043420	.9044515	.9045608	.9046698	.9047786	.9048871	.9049954	.9051034	.9052113
.8800	.9053188	.9054261	.9055332	.9056400	.9057466	.9058530	.9059591	.9060650	.9061706	.9062760
.8900	.9063812	.9064861	.9065908	.9066953	.9067995	.9069035	.9070073	.9071108	.9072141	.9073172
.9000	.9074201	.9075227	.9076251	.9077273	.9078292	.9079310	.9080325	.9081337	.9082348	.9083356
.9100	.9084363	.9085367	.9086368	.9087368	.9088365	.9089361	.9090354	.9091345	.9092333	.9093320
.9200	.9094305	.9095287	.9096267	.9097245	.9098221	.9099195	.9100167	.9101137	.9102105	.9103070
.9300	.9104034	.9104995	.9105955	.9106912	.9107867	.9108821	.9109772	.9110721	.9111668	.9112614
.9400	.9113557	.9114498	.9115437	.9116375	.9117310	.9118243	.9119174	.9120104	.9121031	.9121957
.9500	.9122880	.9123802	.9124722	.9125639	.9126555	.9127469	.9128381	.9129291	.9130199	.9131106
.9600	.9132010	.9132913	.9133813	.9134712	.9135609	.9136504	.9137398	.9138289	.9139179	.9140066
.9700	.9140952	.9141836	.9142719	.9143599	.9144478	.9145355	.9146230	.9147103	.9147975	.9148845
.9800	.9149713	.9150579	.9151443	.9152306	.9153167	.9154026	.9154884	.9155740	.9156594	.9157446
.9900	.9158297	.9159145	.9159993	.9160838	.9161682	.9162524	.9163364	.9164203	.9165040	.9165875
1.0000	.9166709									

Table 13

Values of the integral:

$$\int_x^{\infty} e^{-u^2 - \frac{m^2}{u^2}}\, du$$

This table was prepared by Simpson's rule integration. The Simpson's rule evaluation of the integral is approximate but an exact evaluation can be obtained in the form:

$$\int_x^{\infty} e^{-u^2 - \frac{m^2}{u^2}}\, du = \frac{\sqrt{\pi}}{4}\,[e^{2m} + e^{-2m}] - \frac{\sqrt{\pi}}{4}\left[e^{2m}\,\frac{2}{\sqrt{\pi}}\int_0^{(x + \frac{m}{x})} e^{-u^2} du + e^{-2m}\,\frac{2}{\sqrt{\pi}}\int_0^{(x - \frac{m}{x})} e^{-u^2} du\right]$$

The integral from zero to infinity is:

$$\int_0^{\infty} e^{-u^2 - \frac{m^2}{u^2}}\, du = \frac{\sqrt{\pi}}{2}\, e^{-2m}$$

The integral from zero to x is:

$$\int_0^{x} e^{-u^2 - \frac{m^2}{u^2}}\, du = \frac{\sqrt{\pi}}{4}\left[e^{2m}\,\frac{2}{\sqrt{\pi}}\int_0^{(x + \frac{m}{x})} e^{-u^2} du + e^{-2m}\,\frac{2}{\sqrt{\pi}}\int_0^{(x - \frac{m}{x})} e^{-u^2} du\right] - \frac{\sqrt{\pi}}{4}\,[e^{2m} - e^{-2m}]$$

A check of the tabulation can be obtained by comparing values computed from these integrals with the tabular values. Such comparisons are shown below:

m	$\frac{\sqrt{\pi}}{2} e^{-2m}$	Tabular Values
0.001	0.88446	0.88448
0.002	0.88269	0.88264
0.003	0.88093	0.88091
0.004	0.87917	0.87915
0.005	0.87741	0.87739
0.006	0.87566	0.87564
0.007	0.87391	0.87389
0.008	0.87216	0.87214
0.009	0.87042	0.87040
0.010	0.86868	0.86866
0.020	0.85148	0.85146
0.030	0.83462	0.83460
0.040	0.81809	0.81807
0.050	0.80189	0.80187
0.060	0.78601	0.78599
0.070	0.77045	0.77043
0.080	0.75519	0.75517
0.090	0.74024	0.74022
0.100	0.72558	0.72556
0.200	0.59406	0.59404
0.300	0.48637	0.48635
0.400	0.39821	0.39819
0.500	0.32602	0.32601
0.600	0.26693	0.26691
0.700	0.21854	0.21852
0.800	0.17893	0.17891
0.900	0.14649	0.14647
1.000	0.11994	0.11992

Since the Simpson's rule integration was made from $x = 3.0$ backward toward $x = 0$ a check between the integral and tabular values in the above comparison provides, for each value of m, a check of all tabular values between those limits.

Some spot checks for m = 0.001 are the following:

x	$\int_x^\infty e^{-u^2 - \frac{m^2}{u^2}}\,du$	Tabular Values
0.1	0.78655	0.78653
0.2	0.68886	0.68884
0.3	0.59499	0.59497
0.4	0.50657	0.50655
0.5	0.42495	0.42493
1.0	0.13940	0.13938
1.5	0.03004	0.03002
2.0	0.00415	0.00413

The approximate integration is seen to be within about two units in the fifth place. Values shown are rounded to the nearest figure in the fifth place.

Table 13. $\int_x^\infty e^{-u^2-\frac{m^2}{u^2}}\,du.$

M= .001

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.88448	.87611	.86616	.85618	.84621	.83623	.82626	.81631	.80637	.79644
.1	.78653	.77664	.76677	.75693	.74711	.73732	.72756	.71783	.70813	.69847
.2	.68884	.67925	.66970	.66020	.65073	.64132	.63195	.62263	.61335	.60413
.3	.59497	.58586	.57680	.56780	.55887	.54999	.54117	.53242	.52373	.51511
.4	.50655	.49807	.48965	.48130	.47303	.46482	.45669	.44864	.44066	.43275
.5	.42493	.41718	.40951	.40192	.39440	.38697	.37963	.37236	.36517	.35807
.6	.35105	.34412	.33727	.33050	.32382	.31722	.31071	.30429	.29795	.29169
.7	.28552	.27944	.27344	.26753	.26170	.25596	.25031	.24474	.23925	.23385
.8	.22854	.22331	.21816	.21310	.20812	.20322	.19841	.19367	.18902	.18445
.9	.17997	.17556	.17123	.16698	.16281	.15871	.15469	.15075	.14689	.14310
1.0	.13938	.13574	.13217	.12867	.12525	.12189	.11861	.11539	.11224	.10916
1.1	.10615	.10320	.10031	.09749	.09473	.09204	.08940	.08683	.08432	.08186
1.2	.07946	.07712	.07484	.07261	.07043	.06831	.06624	.06422	.06225	.06033
1.3	.05846	.05664	.05487	.05314	.05146	.04982	.04823	.04667	.04516	.04370
1.4	.04227	.04088	.03953	.03821	.03694	.03570	.03450	.03333	.03219	.03109
1.5	.03002	.02898	.02797	.02700	.02605	.02513	.02424	.02337	.02254	.02173
1.6	.02094	.02018	.01944	.01873	.01804	.01737	.01673	.01610	.01550	.01491
1.7	.01435	.01380	.01327	.01276	.01227	.01179	.01133	.01089	.01046	.01005
1.8	.00965	.00926	.00889	.00854	.00819	.00786	.00754	.00723	.00693	.00665
1.9	.00637	.00610	.00585	.00560	.00537	.00514	.00492	.00471	.00451	.00431
2.0	.00413	.00395	.00377	.00361	.00345	.00330	.00315	.00301	.00287	.00275
2.1	.00262	.00250	.00239	.00228	.00217	.00207	.00198	.00188	.00180	.00171
2.2	.00163	.00155	.00148	.00141	.00134	.00128	.00121	.00116	.00110	.00105
2.3	.00099	.00094	.00090	.00085	.00081	.00077	.00073	.00069	.00066	.00062
2.4	.00059	.00056	.00053	.00050	.00048	.00045	.00043	.00040	.00038	.00036
2.5	.00034	.00032	.00030	.00029	.00027	.00026	.00024	.00023	.00021	.00020
2.6	.00019	.00018	.00017	.00016	.00015	.00014	.00013	.00012	.00011	.00011
2.7	.00010	.00009	.00009	.00008	.00007	.00007	.00006	.00006	.00006	.00005
2.8	.00005	.00004	.00004	.00004	.00003	.00003	.00003	.00002	.00002	.00002
2.9	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00000	.00000	.00000
3.0	.00000									

1.

M= .002

χ	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.88264	.87582	.86602	.85609	.84614	.83618	.82622	.81627	.80633	.79641
.1	.78651	.77662	.76675	.75691	.74709	.73730	.72754	.71781	.70812	.69845
.2	.68883	.67924	.66969	.66019	.65073	.64131	.63194	.62262	.61335	.60413
.3	.59496	.58585	.57680	.56780	.55886	.54998	.54117	.53242	.52373	.51511
.4	.50655	.49806	.48965	.48130	.47302	.46482	.45669	.44863	.44065	.43275
.5	.42492	.41717	.40950	.40191	.39440	.38697	.37962	.37236	.36517	.35807
.6	.35105	.34412	.33727	.33050	.32382	.31722	.31071	.30428	.29794	.29169
.7	.28552	.27944	.27344	.26753	.26170	.25596	.25031	.24474	.23925	.23385
.8	.22854	.22331	.21816	.21310	.20812	.20322	.19841	.19367	.18902	.18445
.9	.17997	.17556	.17123	.16698	.16281	.15871	.15469	.15075	.14689	.14310
1.0	.13938	.13574	.13217	.12867	.12525	.12189	.11861	.11539	.11224	.10916
1.1	.10615	.10320	.10031	.09749	.09473	.09204	.08940	.08683	.08432	.08186
1.2	.07946	.07712	.07484	.07261	.07043	.06831	.06624	.06422	.06225	.06033
1.3	.05846	.05664	.05487	.05314	.05146	.04982	.04823	.04667	.04516	.04369
1.4	.04227	.04088	.03953	.03821	.03694	.03570	.03450	.03333	.03219	.03109
1.5	.03002	.02898	.02797	.02700	.02605	.02513	.02424	.02337	.02254	.02173
1.6	.02094	.02018	.01944	.01873	.01804	.01737	.01673	.01610	.01550	.01491
1.7	.01435	.01380	.01327	.01276	.01227	.01179	.01133	.01089	.01046	.01005
1.8	.00965	.00926	.00889	.00854	.00819	.00786	.00754	.00723	.00693	.00665
1.9	.00637	.00610	.00585	.00560	.00537	.00514	.00492	.00471	.00451	.00431
2.0	.00413	.00395	.00377	.00361	.00345	.00330	.00315	.00301	.00287	.00274
2.1	.00262	.00250	.00239	.00228	.00217	.00207	.00198	.00188	.00180	.00171
2.2	.00163	.00155	.00148	.00141	.00134	.00128	.00121	.00116	.00110	.00105
2.3	.00099	.00094	.00090	.00085	.00081	.00077	.00073	.00069	.00066	.00062
2.4	.00059	.00056	.00053	.00050	.00048	.00045	.00043	.00040	.00038	.00036
2.5	.00034	.00032	.00030	.00029	.00027	.00026	.00024	.00023	.00021	.00020
2.6	.00019	.00018	.00017	.00016	.00015	.00014	.00013	.00012	.00011	.00011
2.7	.00010	.00009	.00009	.00008	.00007	.00007	.00006	.00006	.00006	.00005
2.8	.00005	.00004	.00004	.00004	.00003	.00003	.00003	.00002	.00002	.00002
2.9	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00000	.00000	.00000
3.0	.0000C									

2.

M= .003

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.88091	.87534	.86578	.85593	.84602	.83608	.82614	.81621	.80628	.79636
.1	.78646	.77658	.76672	.75688	.74707	.73728	.72752	.71779	.70810	.69844
.2	.68881	.67923	.66968	.66017	.65071	.64130	.63193	.62261	.61334	.60412
.3	.59495	.58584	.57679	.56779	.55885	.54998	.54116	.53241	.52372	.51510
.4	.50654	.49806	.48964	.48129	.47302	.46481	.45669	.44863	.44065	.43275
.5	.42492	.41717	.40950	.40191	.39440	.38697	.37962	.37235	.36517	.35807
.6	.35105	.34411	.33726	.33050	.32382	.31722	.31071	.30428	.29794	.29169
.7	.28552	.27944	.27344	.26753	.26170	.25596	.25030	.24474	.23925	.23385
.8	.22854	.22330	.21816	.21310	.20812	.20322	.19840	.19367	.18902	.18445
.9	.17996	.17556	.17123	.16698	.16280	.15871	.15469	.15075	.14689	.14310
1.0	.13938	.13574	.13217	.12867	.12525	.12189	.11861	.11539	.11224	.10916
1.1	.10615	.10320	.10031	.09749	.09473	.09204	.08940	.08683	.08432	.08186
1.2	.07946	.07712	.07484	.07261	.07043	.06831	.06624	.06422	.06225	.06033
1.3	.05846	.05664	.05487	.05314	.05146	.04982	.04823	.04667	.04516	.04369
1.4	.04227	.04088	.03953	.03821	.03694	.03570	.03450	.03333	.03219	.03109
1.5	.03002	.02898	.02797	.02700	.02605	.02513	.02424	.02337	.02254	.02173
1.6	.02094	.02018	.01944	.01873	.01804	.01737	.01673	.01610	.01550	.01491
1.7	.01435	.01380	.01327	.01276	.01227	.01179	.01133	.01089	.01046	.01005
1.8	.00965	.00926	.00889	.00854	.00819	.00786	.00754	.00723	.00693	.00665
1.9	.00637	.00610	.00585	.00560	.00537	.00514	.00492	.00471	.00451	.00431
2.0	.00413	.00395	.00377	.00361	.00345	.00330	.00315	.00301	.00287	.00274
2.1	.00262	.00250	.00239	.00228	.00217	.00207	.00198	.00188	.00180	.00171
2.2	.00163	.00155	.00148	.00141	.00134	.00128	.00121	.00116	.00110	.00105
2.3	.00099	.00094	.00090	.00085	.00081	.00077	.00073	.00069	.00066	.00062
2.4	.00059	.00056	.00053	.00050	.00048	.00045	.00043	.00040	.00038	.00036
2.5	.00034	.00032	.00030	.00029	.00027	.00026	.00024	.00023	.00021	.00020
2.6	.00019	.00018	.00017	.00016	.00015	.00014	.00013	.00012	.00011	.00011
2.7	.00010	.00009	.00009	.00008	.00007	.00007	.00006	.00006	.00006	.00005
2.8	.00005	.00004	.00004	.00004	.00003	.00003	.00003	.00002	.00002	.00002
2.9	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00000	.00000	.00000
3.0	.00000									

M= .004

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.87915	.87468	.86544	.85571	.84586	.83596	.82604	.81612	.80620	.79630
.1	.78641	.77653	.76667	.75684	.74703	.73724	.72749	.71776	.70807	.69841
.2	.68879	.67920	.66966	.66015	.65069	.64128	.63191	.62259	.61332	.60410
.3	.59494	.58583	.57678	.56778	.55884	.54997	.54115	.53240	.52371	.51509
.4	.50654	.49805	.48963	.48129	.47301	.46481	.45668	.44862	.44064	.43274
.5	.42491	.41717	.40950	.40191	.39440	.38697	.37962	.37235	.36517	.35806
.6	.35105	.34411	.33726	.33050	.32381	.31722	.31071	.30428	.29794	.29169
.7	.28552	.27943	.27344	.26752	.26170	.25596	.25030	.24473	.23925	.23385
.8	.22853	.22330	.21816	.21309	.20811	.20322	.19840	.19367	.18902	.18445
.9	.17996	.17556	.17123	.16698	.16280	.15871	.15469	.15075	.14689	.14310
1.0	.13938	.13574	.13217	.12867	.12525	.12189	.11861	.11539	.11224	.10916
1.1	.10614	.10320	.10031	.09749	.09473	.09204	.08940	.08683	.08432	.08186
1.2	.07946	.07712	.07484	.07261	.07043	.06831	.06624	.06422	.06225	.06033
1.3	.05846	.05664	.05487	.05314	.05146	.04982	.04822	.04667	.04516	.04369
1.4	.04227	.04088	.03953	.03821	.03694	.03570	.03450	.03333	.03219	.03109
1.5	.03002	.02898	.02797	.02700	.02605	.02513	.02424	.02337	.02254	.02173
1.6	.02094	.02018	.01944	.01873	.01804	.01737	.01673	.01610	.01550	.01491
1.7	.01435	.01380	.01327	.01276	.01227	.01179	.01133	.01089	.01046	.01005
1.8	.00965	.00926	.00889	.00854	.00819	.00786	.00754	.00723	.00693	.00665
1.9	.00637	.00610	.00585	.00560	.00537	.00514	.00492	.00471	.00451	.00431
2.0	.00413	.00395	.00377	.00361	.00345	.00330	.00315	.00301	.00287	.00274
2.1	.00262	.00250	.00239	.00228	.00217	.00207	.00198	.00188	.00180	.00171
2.2	.00163	.00155	.00148	.00141	.00134	.00128	.00121	.00116	.00110	.00105
2.3	.00099	.00094	.00090	.00085	.00081	.00077	.00073	.00069	.00066	.00062
2.4	.00059	.00056	.00053	.00050	.00048	.00045	.00043	.00040	.00038	.00036
2.5	.00034	.00032	.00030	.00029	.00027	.00026	.00024	.00023	.00021	.00020
2.6	.00019	.00018	.00017	.00016	.00015	.00014	.00013	.00012	.00011	.00011
2.7	.00010	.00009	.00009	.00008	.00007	.00007	.00006	.00006	.00006	.00005
2.8	.00005	.00004	.00004	.00004	.00003	.00003	.00003	.00002	.00002	.00002
2.9	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00000	.00000	.00000
3.0	.00000									

4.

Table 13. $\int_x^\infty e^{-u^2-\frac{m^2}{u^2}}\,du.$

M= .005

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.87739	.87385	.86502	.85543	.84565	.83579	.82591	.81601	.80611	.79621
.1	.78633	.77646	.76661	.75678	.74698	.73720	.72745	.71772	.70803	.69838
.2	.68876	.67917	.66963	.66013	.65067	.64126	.63189	.62257	.61330	.60409
.3	.59492	.58581	.57676	.56777	.55883	.54995	.54114	.53239	.52370	.51508
.4	.50653	.49804	.48962	.48128	.47300	.46480	.45667	.44862	.44064	.43273
.5	.42491	.41716	.40949	.40190	.39439	.38696	.37961	.37235	.36516	.35806
.6	.35104	.34411	.33726	.33049	.32381	.31721	.31070	.30428	.29794	.29168
.7	.28551	.27943	.27343	.26752	.26170	.25596	.25030	.24473	.23925	.23385
.8	.22853	.22330	.21816	.21309	.20811	.20322	.19840	.19367	.18902	.18445
.9	.17996	.17555	.17122	.16697	.16280	.15871	.15469	.15075	.14689	.14310
1.0	.13938	.13574	.13217	.12867	.12525	.12189	.11861	.11539	.11224	.10916
1.1	.10614	.10320	.10031	.09749	.09473	.09204	.08940	.08683	.08431	.08186
1.2	.07946	.07712	.07484	.07261	.07043	.06831	.06624	.06422	.06225	.06033
1.3	.05846	.05664	.05487	.05314	.05146	.04982	.04822	.04667	.04516	.04369
1.4	.04227	.04088	.03953	.03821	.03694	.03570	.03450	.03333	.03219	.03109
1.5	.03002	.02898	.02797	.02700	.02605	.02513	.02424	.02337	.02254	.02173
1.6	.02094	.02018	.01944	.01873	.01804	.01737	.01673	.01610	.01550	.01491
1.7	.01435	.01380	.01327	.01276	.01227	.01179	.01133	.01089	.01046	.01005
1.8	.00965	.00926	.00889	.00854	.00819	.00786	.00754	.00723	.00693	.00665
1.9	.00637	.00610	.00585	.00560	.00537	.00514	.00492	.00471	.00451	.00431
2.0	.00413	.00395	.00377	.00361	.00345	.00330	.00315	.00301	.00287	.00274
2.1	.00262	.00250	.00239	.00228	.00217	.00207	.00198	.00188	.00180	.00171
2.2	.00163	.00155	.00148	.00141	.00134	.00128	.00121	.00116	.00110	.00105
2.3	.00099	.00094	.00090	.00085	.00081	.00077	.00073	.00069	.00066	.00062
2.4	.00059	.00056	.00053	.00050	.00048	.00045	.00043	.00040	.00038	.00036
2.5	.00034	.00032	.00030	.00029	.00027	.00026	.00024	.00023	.00021	.00020
2.6	.00019	.00018	.00017	.00016	.00015	.00014	.00013	.00012	.00011	.00011
2.7	.00010	.00009	.00009	.00008	.00007	.00007	.00006	.00006	.00006	.00005
2.8	.00005	.00004	.00004	.00004	.00003	.00003	.00003	.00002	.00002	.00002
2.9	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00000	.00000	.00000
3.0	.00000									

5.

M= .006

χ	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.87564	.87287	.86450	.85509	.84539	.83559	.82574	.81587	.80599	.79611
.1	.78624	.77638	.76654	.75672	.74692	.73714	.72740	.71768	.70799	.69834
.2	.68872	.67914	.66960	.66010	.65064	.64123	.63187	.62255	.61328	.60407
.3	.59490	.58579	.57674	.56775	.55881	.54994	.54112	.53237	.52369	.51507
.4	.50651	.49803	.48961	.48127	.47299	.46479	.45666	.44861	.44063	.43273
.5	.42490	.41715	.40948	.40189	.39438	.38695	.37961	.37234	.36516	.35806
.6	.35104	.34410	.33725	.33049	.32381	.31721	.31070	.30427	.29793	.29168
.7	.28551	.27943	.27343	.26752	.26169	.25595	.25030	.24473	.23924	.23385
.8	.22853	.22330	.21815	.21309	.20811	.20321	.19840	.19367	.18902	.18445
.9	.17996	.17555	.17122	.16697	.16280	.15871	.15469	.15075	.14689	.14310
1.0	.13938	.13574	.13217	.12867	.12525	.12189	.11861	.11559	.11224	.10916
1.1	.10614	.10319	.10031	.09749	.09473	.09204	.08940	.08683	.08431	.08186
1.2	.07946	.07712	.07484	.07261	.07043	.06831	.06624	.06422	.06225	.06033
1.3	.05846	.05664	.05487	.05314	.05146	.04982	.04822	.04667	.04516	.04369
1.4	.04227	.04088	.03953	.03821	.03694	.03570	.03450	.03333	.03219	.03109
1.5	.03002	.02898	.02797	.02700	.02605	.02513	.02424	.02337	.02254	.02173
1.6	.02094	.02018	.01944	.01873	.01804	.01737	.01673	.01610	.01550	.01491
1.7	.01435	.01380	.01327	.01276	.01227	.01179	.01133	.01089	.01046	.01005
1.8	.00965	.00926	.00889	.00854	.00819	.00786	.00754	.00723	.00693	.00665
1.9	.00637	.00610	.00585	.00560	.00537	.00514	.00492	.00471	.00451	.00431
2.0	.00413	.00395	.00377	.00361	.00345	.00330	.00315	.00301	.00287	.00274
2.1	.00262	.00250	.00239	.00228	.00217	.00207	.00198	.00188	.00180	.00171
2.2	.00163	.00155	.00148	.00141	.00134	.00128	.00121	.00116	.00110	.00105
2.3	.00099	.00094	.00090	.00085	.00081	.00077	.00073	.00069	.00066	.00062
2.4	.00059	.00056	.00053	.00050	.00048	.00045	.00043	.00040	.00038	.00036
2.5	.00034	.00032	.00030	.00029	.00027	.00026	.00024	.00023	.00021	.00020
2.6	.00019	.00018	.00017	.00016	.00015	.00014	.00013	.00012	.00011	.00011
2.7	.00010	.00009	.00009	.00008	.00007	.00007	.00006	.00006	.00006	.00005
2.8	.00005	.00004	.00004	.00004	.00003	.00003	.00003	.00002	.00002	.00002
2.9	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00000	.00000	.00000
3.0	.00000									

Table 13 $\int_x^\infty e^{-u^2 - \frac{m^2}{u^2}} du.$

M= .007

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.87389	.87176	.86389	.85468	.84509	.83536	.82555	.81571	.80585	.79599
.1	.78613	.77629	.76645	.75664	.74685	.73708	.72734	.71762	.70794	.69829
.2	.68867	.67910	.66956	.66006	.65061	.64120	.63183	.62252	.61325	.60404
.3	.59488	.58577	.57672	.56773	.55879	.54992	.54111	.53236	.52367	.51505
.4	.50650	.49802	.48960	.48125	.47298	.46478	.45665	.44860	.44062	.43272
.5	.42489	.41714	.40948	.40189	.39438	.38695	.37960	.37233	.36515	.35805
.6	.35103	.34410	.33725	.33048	.32380	.31721	.31069	.30427	.29793	.29168
.7	.28551	.27942	.27343	.26752	.26169	.25595	.25030	.24473	.23924	.23384
.8	.22853	.22330	.21815	.21309	.20811	.20321	.19840	.19367	.18902	.18445
.9	.17996	.17555	.17122	.16697	.16280	.15871	.15469	.15075	.14688	.14309
1.0	.13938	.13574	.13217	.12867	.12524	.12189	.11860	.11539	.11224	.10916
1.1	.10614	.10319	.10031	.09749	.09473	.09204	.08940	.08683	.08431	.08186
1.2	.07946	.07712	.07483	.07260	.07043	.06831	.06624	.06422	.06225	.06033
1.3	.05846	.05664	.05487	.05314	.05146	.04982	.04822	.04667	.04516	.04369
1.4	.04227	.04088	.03953	.03821	.03694	.03570	.03450	.03333	.03219	.03109
1.5	.03002	.02898	.02797	.02700	.02605	.02513	.02424	.02337	.02254	.02173
1.6	.02094	.02018	.01944	.01873	.01804	.01737	.01673	.01610	.01550	.01491
1.7	.01435	.01380	.01327	.01276	.01227	.01179	.01133	.01089	.01046	.01005
1.8	.00965	.00926	.00889	.00854	.00819	.00786	.00754	.00723	.00693	.00665
1.9	.00637	.00610	.00585	.00560	.00537	.00514	.00492	.00471	.00451	.00431
2.0	.00413	.00395	.00377	.00361	.00345	.00330	.00315	.00301	.00287	.00274
2.1	.00262	.00250	.00239	.00228	.00217	.00207	.00198	.00188	.00180	.00171
2.2	.00163	.00155	.00148	.00141	.00134	.00128	.00121	.00116	.00110	.00105
2.3	.00099	.00094	.00090	.00085	.00081	.00077	.00073	.00069	.00066	.00062
2.4	.00059	.00056	.00053	.00050	.00048	.00045	.00043	.00040	.00038	.00036
2.5	.00034	.00032	.00030	.00029	.00027	.00026	.00024	.00023	.00021	.00020
2.6	.00019	.00018	.00017	.00016	.00015	.00014	.00013	.00012	.00011	.00011
2.7	.00010	.00009	.00009	.00008	.00007	.00007	.00006	.00006	.00006	.00005
2.8	.00005	.00004	.00004	.00004	.00003	.00003	.00003	.00002	.00002	.00002
2.9	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00000	.00000	.00000
3.0	.00000									

7.

M= .008

χ	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.87214	.87052	.86320	.85422	.84475	.83508	.82533	.81552	.80569	.79585
.1	.78601	.77617	.76635	.75655	.74676	.73700	.72727	.71756	.70788	.69823
.2	.68862	.67905	.66951	.66002	.65057	.64116	.63180	.62249	.61322	.60401
.3	.59485	.58575	.57670	.56770	.55877	.54990	.54109	.53234	.52365	.51503
.4	.50648	.49800	.48958	.48124	.47297	.46477	.45664	.44859	.44061	.43271
.5	.42488	.41713	.40947	.40188	.39437	.38694	.37959	.37233	.36514	.35804
.6	.35102	.34409	.33724	.33048	.32380	.31720	.31069	.30426	.29792	.29167
.7	.28550	.27942	.27342	.26751	.26169	.25595	.25029	.24472	.23924	.23384
.8	.22852	.22329	.21815	.21309	.20811	.20321	.19840	.19366	.18901	.18445
.9	.17996	.17555	.17122	.16697	.16280	.15870	.15469	.15075	.14688	.14309
1.0	.13938	.13574	.13217	.12867	.12524	.12189	.11860	.11539	.11224	.10916
1.1	.10614	.10319	.10031	.09749	.09473	.09204	.08940	.08683	.08431	.08186
1.2	.07946	.07712	.07483	.07260	.07043	.06831	.06624	.06422	.06225	.06033
1.3	.05846	.05664	.05487	.05314	.05146	.04982	.04822	.04667	.04516	.04369
1.4	.04227	.04088	.03953	.03821	.03694	.03570	.03450	.03333	.03219	.03109
1.5	.03002	.02898	.02797	.02700	.02605	.02513	.02424	.02337	.02254	.02173
1.6	.02094	.02018	.01944	.01873	.01804	.01737	.01673	.01610	.01550	.01491
1.7	.01435	.01380	.01327	.01276	.01227	.01179	.01133	.01089	.01046	.01005
1.8	.00965	.00926	.00889	.00854	.00819	.00786	.00754	.00723	.00693	.00665
1.9	.00637	.00610	.00585	.00560	.00537	.00514	.00492	.00471	.00451	.00431
2.0	.00413	.00395	.00377	.00361	.00345	.00330	.00315	.00301	.00287	.00274
2.1	.00262	.00250	.00239	.00228	.00217	.00207	.00198	.00188	.00180	.00171
2.2	.00163	.00155	.00148	.00141	.00134	.00128	.00121	.00116	.00110	.00105
2.3	.00099	.00094	.00090	.00085	.00081	.00077	.00073	.00069	.00066	.00062
2.4	.00059	.00056	.00053	.00050	.00048	.00045	.00043	.00040	.00038	.00036
2.5	.00034	.00032	.00030	.00029	.00027	.00026	.00024	.00023	.00021	.00020
2.6	.00019	.00018	.00017	.00016	.00015	.00014	.00013	.00012	.00011	.00011
2.7	.00010	.00009	.00009	.00008	.00007	.00007	.00006	.00006	.00006	.00005
2.8	.00005	.00004	.00004	.00004	.00003	.00003	.00003	.00002	.00002	.00002
2.9	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00000	.00000	.00000
3.0	.00000									

8

Table 13. $\int_x^\infty e^{-u^2 - \frac{m^2}{u^2}}\,du.$

M= .009

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.87040	.86919	.86243	.85370	.84436	.83478	.82507	.81531	.80550	.79569
.1	.78587	.77605	.76624	.75645	.74667	.73692	.72719	.71748	.70781	.69817
.2	.68856	.67899	.66946	.65997	.65052	.64112	.63176	.62245	.61319	.60398
.3	.59482	.58572	.57667	.56768	.55874	.54987	.54106	.53232	.52363	.51501
.4	.50646	.49798	.48957	.48122	.47295	.46475	.45662	.44857	.44059	.43269
.5	.42487	.41712	.40945	.40187	.39436	.38693	.37958	.37232	.36513	.35803
.6	.35102	.34408	.33723	.33047	.32379	.31719	.31068	.30426	.29792	.29167
.7	.28550	.27941	.27342	.26751	.26168	.25594	.25029	.24472	.23923	.23384
.8	.22852	.22329	.21814	.21308	.20810	.20321	.19839	.19366	.18901	.18444
.9	.17995	.17555	.17122	.16697	.16280	.15870	.15469	.15075	.14688	.14309
1.0	.13938	.13573	.13217	.12867	.12524	.12189	.11860	.11539	.11224	.10916
1.1	.10614	.10319	.10031	.09749	.09473	.09203	.08940	.08683	.08431	.08186
1.2	.07946	.07712	.07483	.07260	.07043	.06831	.06624	.06422	.06225	.06033
1.3	.05846	.05664	.05487	.05314	.05146	.04982	.04822	.04667	.04516	.04369
1.4	.04227	.04088	.03953	.03821	.03694	.03570	.03449	.03333	.03219	.03109
1.5	.03002	.02898	.02797	.02700	.02605	.02513	.02424	.02337	.02254	.02173
1.6	.02094	.02018	.01944	.01873	.01804	.01737	.01673	.01610	.01550	.01491
1.7	.01435	.01380	.01327	.01276	.01227	.01179	.01133	.01089	.01046	.01005
1.8	.00965	.00926	.00889	.00854	.00819	.00786	.00754	.00723	.00693	.00665
1.9	.00637	.00610	.00585	.00560	.00537	.00514	.00492	.00471	.00451	.00431
2.0	.00413	.00395	.00377	.00361	.00345	.00330	.00315	.00301	.00287	.00274
2.1	.00262	.00250	.00239	.00228	.00217	.00207	.00198	.00188	.00180	.00171
2.2	.00163	.00155	.00148	.00141	.00134	.00128	.00121	.00116	.00110	.00105
2.3	.00099	.00094	.00090	.00085	.00081	.00077	.00073	.00069	.00066	.00062
2.4	.00059	.00056	.00053	.00050	.00048	.00045	.00043	.00040	.00038	.00036
2.5	.00034	.00032	.00030	.00029	.00027	.00026	.00024	.00023	.00021	.00020
2.6	.00019	.00018	.00017	.00016	.00015	.00014	.00013	.00012	.00011	.00011
2.7	.00010	.00009	.00009	.00008	.00007	.00007	.00006	.00006	.00006	.00005
2.8	.00005	.00004	.00004	.00004	.00003	.00003	.00003	.00002	.00002	.00002
2.9	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00000	.00000	.00000
3.0	.00000									

9.

M= .010

χ	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.86866	.86777	.86158	.85312	.84393	.83443	.82479	.81507	.80530	.79551
.1	.78571	.77591	.76611	.75633	.74657	.73682	.72710	.71740	.70774	.69810
.2	.68850	.67893	.66941	.65992	.65047	.64107	.63172	.62241	.61315	.60394
.3	.59478	.58568	.57664	.56765	.55872	.54985	.54104	.53229	.52361	.51499
.4	.50644	.49796	.48955	.48120	.47293	.46473	.45661	.44856	.44058	.43268
.5	.42486	.41711	.40944	.40185	.39435	.38692	.37957	.37231	.36512	.35802
.6	.35101	.34407	.33723	.33046	.32378	.31719	.31068	.30425	.29791	.29166
.7	.28549	.27941	.27341	.26750	.26168	.25594	.25028	.24471	.23923	.23383
.8	.22852	.22329	.21814	.21308	.20810	.20320	.19839	.19366	.18901	.18444
.9	.17995	.17554	.17122	.16697	.16279	.15870	.15468	.15074	.14688	.14309
1.0	.13937	.13573	.13216	.12867	.12524	.12189	.11860	.11538	.11224	.10915
1.1	.10614	.10319	.10031	.09749	.09473	.09203	.08940	.08683	.08431	.08186
1.2	.07946	.07712	.07483	.07260	.07043	.06831	.06624	.06422	.06225	.06033
1.3	.05846	.05664	.05487	.05314	.05146	.04982	.04822	.04667	.04516	.04369
1.4	.04226	.04088	.03953	.03821	.03694	.03570	.03449	.03333	.03219	.03109
1.5	.03002	.02898	.02797	.02700	.02605	.02513	.02424	.02337	.02254	.02173
1.6	.02094	.02018	.01944	.01873	.01804	.01737	.01673	.01610	.01550	.01491
1.7	.01435	.01380	.01327	.01276	.01227	.01179	.01133	.01089	.01046	.01005
1.8	.00965	.00926	.00889	.00854	.00819	.00786	.00754	.00723	.00693	.00665
1.9	.00637	.00610	.00585	.00560	.00537	.00514	.00492	.00471	.00451	.00431
2.0	.00413	.00395	.00377	.00361	.00345	.00330	.00315	.00301	.00287	.00274
2.1	.00262	.00250	.00239	.00228	.00217	.00207	.00198	.00188	.00180	.00171
2.2	.00163	.00155	.00148	.00141	.00134	.00128	.00121	.00116	.00110	.00105
2.3	.00099	.00094	.00090	.00085	.00081	.00077	.00073	.00069	.00066	.00062
2.4	.00059	.00056	.00053	.00050	.00048	.00045	.00043	.00040	.00038	.00036
2.5	.00034	.00032	.00030	.00029	.00027	.00026	.00024	.00023	.00021	.00020
2.6	.00019	.00018	.00017	.00016	.00015	.00014	.00013	.00012	.00011	.00011
2.7	.00010	.00009	.00009	.00008	.00007	.00007	.00006	.00006	.00006	.00005
2.8	.00005	.00004	.00004	.00004	.00003	.00003	.00003	.00002	.00002	.00002
2.9	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00000	.00000	.00000
3.0	.00000									

Table 13. $\int_{x}^{\infty} e^{-u^2 - \frac{m^2}{u^2}} du$

M= .020

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.85146	.85144	.84968	.84448	.83732	.82914	.82042	.81136	.80211	.79271
.1	.78323	.77370	.76412	.75453	.74492	.73531	.72571	.71612	.70655	.69700
.2	.68747	.67798	.66851	.65908	.64969	.64033	.63102	.62175	.61253	.60335
.3	.59423	.58516	.57614	.56717	.55827	.54942	.54063	.53190	.52324	.51464
.4	.50611	.49764	.48924	.48091	.47266	.46447	.45635	.44831	.44035	.43246
.5	.42464	.41691	.40925	.40167	.39417	.38675	.37941	.37215	.36497	.35788
.6	.35087	.34394	.33710	.33034	.32366	.31707	.31057	.30415	.29781	.29156
.7	.28540	.27932	.27333	.26742	.26160	.25586	.25021	.24465	.23916	.23377
.8	.22846	.22323	.21809	.21303	.20805	.20315	.19834	.19361	.18897	.18440
.9	.17991	.17551	.17118	.16693	.16276	.15867	.15465	.15071	.14685	.14306
1.0	.13935	.13571	.13214	.12864	.12522	.12186	.11858	.11536	.11222	.10914
1.1	.10612	.10317	.10029	.09747	.09471	.09202	.08939	.08681	.08430	.08184
1.2	.07945	.07711	.07482	.07259	.07042	.06830	.06623	.06421	.06224	.06032
1.3	.05845	.05663	.05486	.05313	.05145	.04981	.04822	.04667	.04516	.04369
1.4	.04226	.04087	.03952	.03821	.03693	.03569	.03449	.03332	.03219	.03108
1.5	.03001	.02898	.02797	.02699	.02604	.02513	.02423	.02337	.02253	.02172
1.6	.02094	.02018	.01944	.01873	.01804	.01737	.01672	.01610	.01549	.01491
1.7	.01434	.01380	.01327	.01276	.01227	.01179	.01133	.01089	.01046	.01005
1.8	.00965	.00926	.00889	.00853	.00819	.00786	.00754	.00723	.00693	.00664
1.9	.00637	.00610	.00585	.00560	.00537	.00514	.00492	.00471	.00451	.00431
2.0	.00413	.00395	.00377	.00361	.00345	.00330	.00315	.00301	.00287	.00274
2.1	.00262	.00250	.00239	.00228	.00217	.00207	.00198	.00188	.00180	.00171
2.2	.00163	.00155	.00148	.00141	.00134	.00128	.00121	.00116	.00110	.00105
2.3	.00099	.00094	.00090	.00085	.00081	.00077	.00073	.00069	.00066	.00062
2.4	.00059	.00056	.00053	.00050	.00048	.00045	.00043	.00040	.00038	.00036
2.5	.00034	.00032	.00030	.00029	.00027	.00026	.00024	.00023	.00021	.00020
2.6	.00019	.00018	.00017	.00016	.00015	.00014	.00013	.00012	.00011	.00011
2.7	.00010	.00009	.00009	.00008	.00007	.00007	.00006	.00006	.00006	.00005
2.8	.00005	.00004	.00004	.00004	.00003	.00003	.00003	.00002	.00002	.00002
2.9	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00000	.00000	.00000
3.0	.00000									

11:

M= .030

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.83460	.83460	.83429	.83193	.82717	.82080	.81341	.80537	.79690	.78814
.1	.77917	.77006	.76084	.75155	.74220	.73282	.72342	.71401	.70459	.69518
.2	.68577	.67639	.66702	.65769	.64838	.63910	.62986	.62065	.61149	.60237
.3	.59330	.58428	.57531	.56638	.55752	.54871	.53995	.53126	.52263	.51406
.4	.50555	.49711	.48874	.48043	.47219	.46403	.45593	.44791	.43996	.43209
.5	.42429	.41657	.40892	.40136	.39387	.38646	.37914	.37189	.36472	.35764
.6	.35064	.34372	.33689	.33014	.32347	.31689	.31039	.30398	.29765	.29141
.7	.28525	.27918	.27319	.26729	.26147	.25574	.25009	.24453	.23906	.23366
.8	.22836	.22313	.21799	.21294	.20796	.20307	.19826	.19354	.18889	.18433
.9	.17984	.17544	.17112	.16687	.16270	.15861	.15460	.15066	.14680	.14302
1.0	.13930	.13566	.13210	.12860	.12518	.12183	.11855	.11533	.11218	.10911
1.1	.10609	.10315	.10026	.09744	.09469	.09200	.08936	.08679	.08428	.08182
1.2	.07943	.07709	.07480	.07258	.07040	.06828	.06621	.06419	.06223	.06031
1.3	.05844	.05662	.05485	.05312	.05144	.04980	.04821	.04666	.04515	.04368
1.4	.04225	.04086	.03951	.03820	.03693	.03569	.03448	.03332	.03218	.03108
1.5	.03001	.02897	.02796	.02699	.02604	.02512	.02423	.02337	.02253	.02172
1.6	.02094	.02017	.01944	.01873	.01804	.01737	.01672	.01610	.01549	.01491
1.7	.01434	.01380	.01327	.01276	.01227	.01179	.01133	.01089	.01046	.01005
1.8	.00965	.00926	.00889	.00853	.00819	.00786	.00754	.00723	.00693	.00664
1.9	.00637	.00610	.00585	.00560	.00537	.00514	.00492	.00471	.00451	.00431
2.0	.00413	.00395	.00377	.00361	.00345	.00330	.00315	.00301	.00287	.00274
2.1	.00262	.00250	.00239	.00228	.00217	.00207	.00198	.00188	.00180	.00171
2.2	.00163	.00155	.00148	.00141	.00134	.00128	.00121	.00116	.00110	.00104
2.3	.00099	.00094	.00090	.00085	.00081	.00077	.00073	.00069	.00066	.00062
2.4	.00059	.00056	.00053	.00050	.00048	.00045	.00043	.00040	.00038	.00036
2.5	.00034	.00032	.00030	.00029	.00027	.00026	.00024	.00023	.00021	.00020
2.6	.00019	.00018	.00017	.00016	.00015	.00014	.00013	.00012	.00011	.00011
2.7	.00010	.00009	.00009	.00008	.00007	.00007	.00006	.00006	.00006	.00005
2.8	.00005	.00004	.00004	.00004	.00003	.00003	.00003	.00002	.00002	.00002
2.9	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00000	.00000	.00000
3.0	.00000									

Table 13. $\int_{x}^{\infty} e^{-u^2 - \frac{m^2}{u^2}} du.$

M= .040

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.81807	.81807	.81804	.81721	.81451	.81000	.80415	.79734	.78987	.78191
.1	.77362	.76507	.75632	.74744	.73845	.72937	.72024	.71106	.70186	.69264
.2	.68341	.67418	.66496	.65574	.64655	.63738	.62824	.61913	.61005	.60101
.3	.59201	.58306	.57415	.56528	.55647	.54771	.53901	.53036	.52177	.51324
.4	.50477	.49637	.48803	.47975	.47155	.46341	.45534	.44735	.43942	.43157
.5	.42380	.41610	.40847	.40092	.39346	.38607	.37875	.37152	.36437	.35730
.6	.35032	.34341	.33659	.32985	.32320	.31663	.31014	.30374	.29742	.29118
.7	.28504	.27897	.27299	.26710	.26129	.25557	.24993	.24437	.23890	.23352
.8	.22821	.22300	.21786	.21281	.20784	.20296	.19815	.19343	.18879	.18423
.9	.17975	.17535	.17103	.16679	.16262	.15854	.15453	.15059	.14673	.14295
1.0	.13924	.13560	.13204	.12855	.12513	.12178	.11850	.11528	.11214	.10906
1.1	.10605	.10311	.10023	.09741	.09465	.09196	.08933	.08676	.08425	.08180
1.2	.07940	.07706	.07478	.07255	.07038	.06826	.06619	.06417	.06221	.06029
1.3	.05842	.05660	.05483	.05311	.05142	.04979	.04819	.04664	.04514	.04367
1.4	.04224	.04085	.03950	.03819	.03692	.03568	.03448	.03331	.03217	.03107
1.5	.03000	.02897	.02796	.02698	.02603	.02512	.02423	.02336	.02253	.02172
1.6	.02093	.02017	.01943	.01872	.01803	.01736	.01672	.01609	.01549	.01490
1.7	.01434	.01379	.01327	.01276	.01226	.01179	.01133	.01088	.01046	.01004
1.8	.00964	.00926	.00889	.00853	.00819	.00786	.00753	.00723	.00693	.00664
1.9	.00637	.00610	.00585	.00560	.00536	.00514	.00492	.00471	.00451	.00431
2.0	.00412	.00395	.00377	.00361	.00345	.00330	.00315	.00301	.00287	.00274
2.1	.00262	.00250	.00239	.00228	.00217	.00207	.00198	.00188	.00180	.00171
2.2	.00163	.00155	.00148	.00141	.00134	.00128	.00121	.00116	.00110	.00104
2.3	.00099	.00094	.00090	.00085	.00081	.00077	.00073	.00069	.00066	.00062
2.4	.00059	.00056	.00053	.00050	.00048	.00045	.00043	.00040	.00038	.00036
2.5	.00034	.00032	.00030	.00029	.00027	.00026	.00024	.00023	.00021	.00020
2.6	.00019	.00018	.00017	.00016	.00015	.00014	.00013	.00012	.00011	.00011
2.7	.00010	.00009	.00009	.00008	.00007	.00007	.00006	.00006	.00006	.00005
2.8	.00005	.00004	.00004	.00004	.00003	.00003	.00003	.00002	.00002	.00002
2.9	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00000	.00000	.00000
3.0	.00000									

Table 13.

M= .050

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.80187	.80187	.80187	.80164	.80032	.79743	.79308	.78758	.78121	.77419
.1	.76669	.75881	.75064	.74225	.73369	.72500	.71620	.70733	.69839	.68941
.2	.68039	.67136	.66231	.65327	.64422	.63519	.62617	.61718	.60821	.59927
.3	.59036	.58149	.57266	.56387	.55513	.54644	.53780	.52921	.52067	.51219
.4	.50377	.49541	.48712	.47888	.47072	.46262	.45458	.44662	.43873	.43091
.5	.42316	.41549	.40789	.40037	.39292	.38555	.37826	.37105	.36392	.35687
.6	.34990	.34302	.33621	.32949	.32285	.31629	.30982	.30343	.29712	.29090
.7	.28476	.27871	.27274	.26686	.26106	.25534	.24971	.24417	.23871	.23333
.8	.22803	.22282	.21770	.21265	.20769	.20281	.19801	.19330	.18866	.18411
.9	.17963	.17524	.17092	.16668	.16252	.15844	.15443	.15050	.14665	.14287
1.0	.13916	.13553	.13197	.12848	.12506	.12171	.11843	.11522	.11208	.10901
1.1	.10600	.10306	.10018	.09736	.09461	.09192	.08929	.08672	.08421	.08176
1.2	.07937	.07703	.07475	.07252	.07035	.06823	.06616	.06415	.06218	.06027
1.3	.05840	.05658	.05481	.05309	.05141	.04977	.04818	.04663	.04512	.04365
1.4	.04223	.04084	.03949	.03818	.03691	.03567	.03447	.03330	.03216	.03106
1.5	.02999	.02896	.02795	.02697	.02603	.02511	.02422	.02336	.02252	.02171
1.6	.02093	.02017	.01943	.01872	.01803	.01736	.01671	.01609	.01548	.01490
1.7	.01434	.01379	.01326	.01275	.01226	.01178	.01133	.01088	.01045	.01004
1.8	.00964	.00926	.00889	.00853	.00819	.00785	.00753	.00722	.00693	.00664
1.9	.00637	.00610	.00585	.00560	.00536	.00514	.00492	.00471	.00450	.00431
2.0	.00412	.00394	.00377	.00361	.00345	.00329	.00315	.00301	.00287	.00274
2.1	.00262	.00250	.00239	.00228	.00217	.00207	.00198	.00188	.00180	.00171
2.2	.00163	.00155	.00148	.00141	.00134	.00128	.00121	.00116	.00110	.00104
2.3	.00099	.00094	.00090	.00085	.00081	.00077	.00073	.00069	.00066	.00062
2.4	.00059	.00056	.00053	.00050	.00048	.00045	.00043	.00040	.00038	.00036
2.5	.00034	.00032	.00030	.00029	.00027	.00026	.00024	.00023	.00021	.00020
2.6	.00019	.00018	.00017	.00016	.00015	.00014	.00013	.00012	.00011	.00011
2.7	.00010	.00009	.00009	.00008	.00007	.00007	.00006	.00006	.00006	.00005
2.8	.00005	.00004	.00004	.00004	.00003	.00003	.00003	.00002	.00002	.00002
2.9	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00000	.00000	.00000
3.0	.00000									

14.

Table 13. $\int_x^\infty e^{-u^2 - \frac{m^2}{u^2}} du.$

M= .060

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.78599	.78599	.78599	.78594	.78538	.78369	.78066	.77642	.77119	.76516
.1	.75852	.75139	.74388	.73606	.72800	.71975	.71135	.70282	.69420	.68550
.2	.67675	.66795	.65911	.65026	.64140	.63253	.62366	.61481	.60597	.59715
.3	.58835	.57959	.57085	.56216	.55350	.54489	.53632	.52780	.51933	.51092
.4	.50256	.49425	.48601	.47783	.46971	.46165	.45366	.44574	.43789	.43010
.5	.42239	.41475	.40718	.39969	.39227	.38493	.37767	.37048	.36338	.35635
.6	.34940	.34253	.33575	.32904	.32242	.31588	.30942	.30305	.29676	.29055
.7	.28443	.27839	.27243	.26656	.26078	.25507	.24945	.24392	.23846	.23310
.8	.22781	.22261	.21749	.21245	.20750	.20263	.19784	.19313	.18850	.18395
.9	.17948	.17509	.17078	.16655	.16240	.15832	.15432	.15039	.14654	.14277
1.0	.13906	.13543	.13188	.12839	.12498	.12163	.11836	.11515	.11201	.10894
1.1	.10593	.10299	.10012	.09731	.09456	.09187	.08924	.08667	.08417	.08172
1.2	.07932	.07699	.07471	.07248	.07031	.06820	.06613	.06412	.06215	.06024
1.3	.05837	.05656	.05479	.05306	.05138	.04975	.04816	.04661	.04510	.04363
1.4	.04221	.04082	.03947	.03816	.03689	.03565	.03445	.03328	.03215	.03105
1.5	.02998	.02895	.02794	.02696	.02602	.02510	.02421	.02335	.02251	.02170
1.6	.02092	.02016	.01942	.01871	.01802	.01735	.01671	.01608	.01548	.01490
1.7	.01433	.01379	.01326	.01275	.01226	.01178	.01132	.01088	.01045	.01004
1.8	.00964	.00926	.00889	.00853	.00818	.00785	.00753	.00722	.00693	.00664
1.9	.00636	.00610	.00584	.00560	.00536	.00513	.00492	.00471	.00450	.00431
2.0	.00412	.00394	.00377	.00361	.00345	.00329	.00315	.00301	.00287	.00274
2.1	.00262	.00250	.00239	.00228	.00217	.00207	.00198	.00188	.00180	.00171
2.2	.00163	.00155	.00148	.00141	.00134	.00128	.00121	.00115	.00110	.00104
2.3	.00099	.00094	.00090	.00085	.00081	.00077	.00073	.00069	.00066	.00062
2.4	.00059	.00056	.00053	.00050	.00048	.00045	.00043	.00040	.00038	.00036
2.5	.00034	.00032	.00030	.00029	.00027	.00026	.00024	.00023	.00021	.00020
2.6	.00019	.00018	.00017	.00016	.00015	.00014	.00013	.00012	.00011	.00011
2.7	.00010	.00009	.00009	.00008	.00007	.00007	.00006	.00006	.00006	.00005
2.8	.00005	.00004	.00004	.00004	.00003	.00003	.00003	.00002	.00002	.00002
2.9	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00000	.00000	.00000
3.0	.00000									

M= .070

χ	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.77043	.77043	.77043	.77042	.77021	.76931	.76733	.76422	.76006	.75503
.1	.74928	.74294	.73613	.72894	.72144	.71368	.70572	.69759	.68933	.68096
.2	.67250	.66396	.65538	.64675	.63809	.62941	.62072	.61203	.60334	.59466
.3	.58599	.57735	.56873	.56014	.55158	.54306	.53458	.52615	.51776	.50941
.4	.50112	.49289	.48471	.47658	.46852	.46051	.45257	.44470	.43689	.42915
.5	.42148	.41388	.40635	.39889	.39151	.38420	.37696	.36981	.36273	.35573
.6	.34881	.34196	.33520	.32852	.32192	.31540	.30896	.30261	.29633	.29014
.7	.28404	.27801	.27207	.26622	.26044	.25475	.24914	.24362	.23818	.23282
.8	.22755	.22236	.21725	.21222	.20728	.20242	.19763	.19293	.18831	.18377
.9	.17931	.17493	.17062	.16640	.16225	.15818	.15418	.15026	.14642	.14264
1.0	.13895	.13532	.13177	.12829	.12488	.12154	.11827	.11506	.11193	.10886
1.1	.10586	.10292	.10005	.09724	.09449	.09181	.08918	.08662	.08411	.08166
1.2	.07927	.07694	.07466	.07244	.07027	.06816	.06609	.06408	.06212	.06021
1.3	.05834	.05653	.05476	.05303	.05136	.04972	.04813	.04658	.04508	.04361
1.4	.04219	.04080	.03946	.03815	.03687	.03564	.03444	.03327	.03214	.03104
1.5	.02997	.02893	.02793	.02695	.02601	.02509	.02420	.02334	.02250	.02169
1.6	.02091	.02015	.01942	.01870	.01801	.01735	.01670	.01608	.01547	.01489
1.7	.01433	.01378	.01325	.01274	.01225	.01178	.01132	.01088	.01045	.01003
1.8	.00964	.00925	.00888	.00853	.00818	.00785	.00753	.00722	.00692	.00664
1.9	.00636	.00610	.00584	.00560	.00536	.00513	.00491	.00470	.00450	.00431
2.0	.00412	.00394	.00377	.00360	.00345	.00329	.00315	.00301	.00287	.00274
2.1	.00262	.00250	.00239	.00228	.00217	.00207	.00198	.00188	.00179	.00171
2.2	.00163	.00155	.00148	.00141	.00134	.00128	.00121	.00115	.00110	.00104
2.3	.00099	.00094	.00090	.00085	.00081	.00077	.00073	.00069	.00066	.00062
2.4	.00059	.00056	.00053	.00050	.00048	.00045	.00043	.00040	.00038	.00036
2.5	.00034	.00032	.00030	.00029	.00027	.00026	.00024	.00023	.00021	.00020
2.6	.00019	.00018	.00017	.00016	.00015	.00014	.00013	.00012	.00011	.00011
2.7	.00010	.00009	.00009	.00008	.00007	.00007	.00006	.00006	.00006	.00005
2.8	.00005	.00004	.00004	.00004	.00003	.00003	.00003	.00002	.00002	.00002
2.9	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00000	.00000	.00000
3.0	.00000									

Table 13. $\int_x^\infty e^{-u^2-\frac{m^2}{u^2}}\,du$.

M= .080

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.75517	.75517	.75517	.75517	.75510	.75466	.75345	.75126	.74808	.74399
.1	.73912	.73359	.72751	.72098	.71407	.70685	.69937	.69168	.68381	.67580
.2	.66766	.65943	.65112	.64274	.63431	.62585	.61736	.60885	.60033	.59181
.3	.58329	.57479	.56630	.55783	.54938	.54097	.53259	.52425	.51595	.50769
.4	.49948	.49132	.48321	.47515	.46715	.45921	.45132	.44350	.43575	.42805
.5	.42043	.41287	.40539	.39797	.39062	.38335	.37615	.36903	.36198	.35501
.6	.34812	.34131	.33457	.32791	.32134	.31484	.30843	.30209	.29584	.28967
.7	.28358	.27758	.27165	.26581	.26006	.25438	.24879	.24328	.23785	.23251
.8	.22725	.22207	.21697	.21196	.20702	.20217	.19740	.19271	.18809	.18356
.9	.17911	.17473	.17044	.16622	.16208	.15801	.15402	.15011	.14627	.14251
1.0	.13881	.13520	.13165	.12817	.12477	.12143	.11816	.11497	.11183	.10877
1.1	.10577	.10284	.09997	.09716	.09442	.09173	.08911	.08655	.08405	.08160
1.2	.07922	.07689	.07461	.07239	.07022	.06811	.06605	.06404	.06208	.06017
1.3	.05830	.05649	.05472	.05300	.05132	.04969	.04810	.04656	.04505	.04359
1.4	.04216	.04078	.03943	.03813	.03685	.03562	.03442	.03325	.03212	.03102
1.5	.02995	.02892	.02791	.02694	.02599	.02508	.02419	.02333	.02249	.02168
1.6	.02090	.02014	.01941	.01869	.01801	.01734	.01669	.01607	.01547	.01488
1.7	.01432	.01378	.01325	.01274	.01225	.01177	.01131	.01087	.01044	.01003
1.8	.00963	.00925	.00888	.00852	.00818	.00785	.00753	.00722	.00692	.00664
1.9	.00636	.00610	.00584	.00559	.00536	.00513	.00491	.00470	.00450	.00431
2.0	.00412	.00394	.00377	.00360	.00344	.00329	.00315	.00301	.00287	.00274
2.1	.00262	.00250	.00238	.00228	.00217	.00207	.00197	.00188	.00179	.00171
2.2	.00163	.00155	.00148	.00141	.00134	.00128	.00121	.00115	.00110	.00104
2.3	.00099	.00094	.00090	.00085	.00081	.00077	.00073	.00069	.00066	.00062
2.4	.00059	.00056	.00053	.00050	.00048	.00045	.00043	.00040	.00038	.00036
2.5	.00034	.00032	.00030	.00029	.00027	.00026	.00024	.00023	.00021	.00020
2.6	.00019	.00018	.00017	.00016	.00015	.00014	.00013	.00012	.00011	.00011
2.7	.00010	.00009	.00009	.00008	.00007	.00007	.00006	.00006	.00006	.00005
2.8	.00005	.00004	.00004	.00004	.00003	.00003	.00003	.00002	.00002	.00002
2.9	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00000	.00000	.00000
3.0	.00000									

M= .090

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.74022	.74022	.74022	.74022	.74020	.74000	.73930	.73784	.73548	.73225
.1	.72821	.72347	.71813	.71227	.70598	.69932	.69235	.68513	.67768	.67006
.2	.66228	.65437	.64636	.63826	.63009	.62186	.61359	.60528	.59695	.58861
.3	.58026	.57190	.56356	.55523	.54691	.53862	.53035	.52211	.51391	.50575
.4	.49762	.48955	.48152	.47353	.46561	.45773	.44991	.44215	.43445	.42682
.5	.41925	.41174	.40430	.39693	.38963	.38240	.37524	.36815	.36114	.35421
.6	.34735	.34056	.33386	.32723	.32068	.31421	.30782	.30151	.29529	.28914
.7	.28307	.27709	.27118	.26536	.25962	.25396	.24839	.24290	.23748	.23216
.8	.22691	.22174	.21666	.21165	.20673	.20189	.19713	.19245	.18785	.18333
.9	.17888	.17452	.17023	.16602	.16189	.15783	.15385	.14994	.14611	.14235
1.0	.13866	.13505	.13151	.12804	.12464	.12131	.11805	.11485	.11173	.10867
1.1	.10567	.10274	.09988	.09707	.09433	.09165	.08904	.08648	.08398	.08154
1.2	.07915	.07682	.07455	.07233	.07017	.06806	.06600	.06399	.06203	.06012
1.3	.05826	.05645	.05468	.05296	.05129	.04966	.04807	.04653	.04502	.04356
1.4	.04214	.04075	.03941	.03810	.03683	.03560	.03440	.03323	.03210	.03100
1.5	.02994	.02890	.02790	.02692	.02598	.02506	.02418	.02331	.02248	.02167
1.6	.02089	.02013	.01940	.01869	.01800	.01733	.01669	.01606	.01546	.01488
1.7	.01431	.01377	.01324	.01273	.01224	.01177	.01131	.01087	.01044	.01003
1.8	.00963	.00925	.00888	.00852	.00817	.00784	.00752	.00722	.00692	.00663
1.9	.00636	.00609	.00584	.00559	.00536	.00513	.00491	.00470	.00450	.00431
2.0	.00412	.00394	.00377	.00360	.00344	.00329	.00314	.00300	.00287	.00274
2.1	.00262	.00250	.00238	.00227	.00217	.00207	.00197	.00188	.00179	.00171
2.2	.00163	.00155	.00148	.00141	.00134	.00127	.00121	.00115	.00110	.00104
2.3	.00099	.00094	.00090	.00085	.00081	.00077	.00073	.00069	.00066	.00062
2.4	.00059	.00056	.00053	.00050	.00048	.00045	.00043	.00040	.00038	.00036
2.5	.00034	.00032	.00030	.00029	.00027	.00026	.00024	.00023	.00021	.00020
2.6	.00019	.00018	.00017	.00016	.00015	.00014	.00013	.00012	.00011	.00011
2.7	.00010	.00009	.00009	.00008	.00007	.00007	.00006	.00006	.00006	.00005
2.8	.00005	.00004	.00004	.00004	.00003	.00003	.00003	.00002	.00002	.00002
2.9	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00000	.00000	.00000
3.0	.00000									

Table 13. $\int_x^\infty e^{-u^2 - \frac{m^2}{u^2}}\,du.$

M= .100

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.72556	.72556	.72556	.72556	.72556	.72548	.72510	.72416	.72247	.71999
.1	.71672	.71273	.70810	.70291	.69724	.69116	.68472	.67798	.67099	.66377
.2	.65637	.64882	.64113	.63333	.62543	.61746	.60943	.60134	.59322	.58507
.3	.57690	.56871	.56053	.55234	.54417	.53600	.52786	.51974	.51165	.50359
.4	.49556	.48758	.47964	.47174	.46389	.45609	.44834	.44065	.43302	.42544
.5	.41793	.41048	.40309	.39577	.38852	.38133	.37422	.36718	.36021	.35331
.6	.34649	.33974	.33307	.32647	.31995	.31351	.30715	.30087	.29467	.28854
.7	.28250	.27654	.27066	.26486	.25914	.25350	.24794	.24247	.23707	.23176
.8	.22653	.22138	.21631	.21132	.20641	.20158	.19683	.19216	.18757	.18306
.9	.17863	.17427	.17000	.16579	.16167	.15762	.15365	.14975	.14593	.14217
1.0	.13850	.13489	.13135	.12789	.12450	.12117	.11792	.11473	.11161	.10855
1.1	.10556	.10263	.09977	.09697	.09424	.09156	.08895	.08639	.08390	.08146
1.2	.07908	.07675	.07448	.07227	.07011	.06800	.06594	.06394	.06198	.06007
1.3	.05822	.05640	.05464	.05292	.05125	.04962	.04803	.04649	.04499	.04353
1.4	.04211	.04072	.03938	.03807	.03681	.03557	.03437	.03321	.03208	.03098
1.5	.02992	.02888	.02788	.02691	.02596	.02505	.02416	.02330	.02247	.02166
1.6	.02088	.02012	.01939	.01868	.01799	.01732	.01668	.01605	.01545	.01487
1.7	.01431	.01376	.01324	.01273	.01224	.01176	.01130	.01086	.01043	.01002
1.8	.00962	.00924	.00887	.00851	.00817	.00784	.00752	.00721	.00692	.00663
1.9	.00636	.00609	.00584	.00559	.00535	.00513	.00491	.00470	.00450	.00430
2.0	.00412	.00394	.00377	.00360	.00344	.00329	.00314	.00300	.00287	.00274
2.1	.00262	.00250	.00238	.00227	.00217	.00207	.00197	.00188	.00179	.00171
2.2	.00163	.00155	.00148	.00141	.00134	.00127	.00121	.00115	.00110	.00104
2.3	.00099	.00094	.00090	.00085	.00081	.00077	.00073	.00069	.00066	.00062
2.4	.00059	.00056	.00053	.00050	.00048	.00045	.00043	.00040	.00038	.00036
2.5	.00034	.00032	.00030	.00029	.00027	.00026	.00024	.00023	.00021	.00020
2.6	.00019	.00018	.00017	.00016	.00015	.00014	.00013	.00012	.00011	.00011
2.7	.00010	.00009	.00009	.00008	.00007	.00007	.00006	.00006	.00006	.00005
2.8	.00005	.00004	.00004	.00004	.00003	.00003	.00003	.00002	.00002	.00002
2.9	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00000	.00000	.00000
3.0	.00000									

Table 13.

M= .200

χ	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.59404	.59404	.59404	.59404	.59404	.59404	.59404	.59404	.59403	.59398
.1	.59386	.59360	.59312	.59235	.59126	.58980	.58795	.58571	.58308	.58008
.2	.57672	.57302	.56900	.56469	.56010	.55527	.55020	.54493	.53947	.53383
.3	.52805	.52212	.51607	.50991	.50365	.49731	.49089	.48441	.47787	.47129
.4	.46467	.45802	.45134	.44465	.43796	.43125	.42455	.41786	.41117	.40450
.5	.39786	.39123	.38464	.37807	.37154	.36505	.35859	.35218	.34582	.33950
.6	.33323	.32701	.32085	.31474	.30869	.30270	.29677	.29090	.28509	.27934
.7	.27367	.26805	.26251	.25703	.25162	.24628	.24100	.23580	.23067	.22561
.8	.22062	.21570	.21086	.20608	.20138	.19675	.19219	.18771	.18330	.17895
.9	.17468	.17048	.16636	.16230	.15832	.15440	.15056	.14678	.14308	.13944
1.0	.13587	.13237	.12894	.12557	.12227	.11903	.11587	.11276	.10972	.10674
1.1	.10382	.10097	.09818	.09544	.09277	.09016	.08760	.08510	.08266	.08027
1.2	.07794	.07566	.07344	.07127	.06915	.06708	.06506	.06309	.06117	.05930
1.3	.05747	.05570	.05396	.05227	.05063	.04903	.04746	.04595	.04447	.04303
1.4	.04163	.04027	.03895	.03766	.03641	.03519	.03401	.03286	.03175	.03067
1.5	.02961	.02859	.02760	.02664	.02571	.02481	.02393	.02308	.02226	.02146
1.6	.02069	.01994	.01921	.01851	.01783	.01717	.01653	.01592	.01532	.01475
1.7	.01419	.01365	.01313	.01263	.01214	.01167	.01122	.01078	.01035	.00995
1.8	.00955	.00917	.00881	.00845	.00811	.00778	.00747	.00716	.00687	.00658
1.9	.00631	.00605	.00580	.00555	.00532	.00509	.00488	.00467	.00447	.00428
2.0	.00409	.00391	.00374	.00358	.00342	.00327	.00313	.00299	.00285	.00272
2.1	.00260	.00248	.00237	.00226	.00216	.00206	.00196	.00187	.00178	.00170
2.2	.00162	.00154	.00147	.00140	.00133	.00127	.00121	.00115	.00109	.00104
2.3	.00099	.00094	.00089	.00085	.00080	.00076	.00072	.00069	.00065	.00062
2.4	.00059	.00056	.00053	.00050	.00047	.00045	.00042	.00040	.00038	.00036
2.5	.00034	.00032	.00030	.00029	.00027	.00025	.00024	.00023	.00021	.00020
2.6	.00019	.00018	.00017	.00016	.00015	.00014	.00013	.00012	.00011	.00011
2.7	.00010	.00009	.00009	.00008	.00007	.00007	.00006	.00006	.00005	.00005
2.8	.00005	.00004	.00004	.00004	.00003	.00003	.00003	.00002	.00002	.00002
2.9	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00000	.00000	.00000
3.0	.00000									

20.

Table 13. $\int_{x}^{\infty} e^{-u^2 - \frac{m^2}{u^2}}\, du.$

M= .300

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.48635	.48635	.48635	.48635	.48635	.48635	.48635	.48635	.48635	.48635
.1	.48635	.48635	.48634	.48631	.48623	.48610	.48587	.48551	.48499	.48430
.2	.48339	.48226	.48090	.47929	.47744	.47534	.47299	.47040	.46758	.46454
.3	.46128	.45782	.45416	.45033	.44632	.44215	.43783	.43338	.42880	.42410
.4	.41930	.41439	.40940	.40433	.39919	.39398	.38872	.38341	.37805	.37266
.5	.36724	.36179	.35633	.35086	.34537	.33988	.33440	.32891	.32344	.31798
.6	.31254	.30712	.30172	.29634	.29100	.28569	.28041	.27516	.26996	.26480
.7	.25968	.25460	.24957	.24459	.23966	.23478	.22995	.22517	.22045	.21578
.8	.21117	.20662	.20213	.19769	.19331	.18900	.18474	.18055	.17641	.17234
.9	.16833	.16438	.16049	.15666	.15290	.14920	.14556	.14198	.13846	.13501
.0	.13162	.12829	.12501	.12180	.11865	.11556	.11253	.10956	.10665	.10380
.1	.10100	.09826	.09558	.09295	.09038	.08786	.08540	.08299	.08063	.07833
.2	.07608	.07388	.07173	.06963	.06758	.06558	.06362	.06171	.05985	.05803
.3	.05626	.05453	.05285	.05121	.04961	.04805	.04653	.04505	.04361	.04221
.4	.04085	.03952	.03823	.03697	.03575	.03457	.03341	.03229	.03120	.03014
.5	.02912	.02812	.02715	.02621	.02530	.02441	.02355	.02272	.02191	.02113
.6	.02037	.01964	.01893	.01824	.01757	.01692	.01630	.01569	.01511	.01454
.7	.01399	.01346	.01295	.01246	.01198	.01152	.01107	.01064	.01022	.00982
.8	.00943	.00906	.00870	.00835	.00801	.00769	.00738	.00708	.00679	.00651
.9	.00624	.00598	.00573	.00549	.00526	.00504	.00482	.00462	.00442	.00423
2.0	.00405	.00387	.00370	.00354	.00339	.00324	.00309	.00296	.00282	.00270
2.1	.00258	.00246	.00235	.00224	.00214	.00204	.00194	.00185	.00177	.00168
2.2	.00161	.00153	.00146	.00139	.00132	.00126	.00120	.00114	.00108	.00103
2.3	.00098	.00093	.00088	.00084	.00080	.00076	.00072	.00068	.00065	.00061
2.4	.00058	.00055	.00052	.00050	.00047	.00044	.00042	.00040	.00038	.00036
2.5	.00034	.00032	.00030	.00028	.00027	.00025	.00024	.00022	.00021	.00020
2.6	.00019	.00018	.00017	.00016	.00015	.00014	.00013	.00012	.00011	.00011
2.7	.00010	.00009	.00009	.00008	.00007	.00007	.00006	.00006	.00005	.00005
2.8	.00005	.00004	.00004	.00004	.00003	.00003	.00003	.00002	.00002	.00002
2.9	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00000	.00000	.00000
3.0	.00000									

Table 13.

M= .400

χ	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.39819	.39819	.39819	.39819	.39819	.39819	.39819	.39819	.39819	.39819
.1	.39819	.39819	.39819	.39819	.39819	.39818	.39817	.39814	.39809	.39800
.2	.39785	.39764	.39734	.39694	.39641	.39576	.39496	.39400	.39288	.39160
.3	.39014	.38851	.38670	.38472	.38258	.38026	.37778	.37515	.37237	.36944
.4	.36637	.36317	.35985	.35640	.35285	.34919	.34544	.34160	.33767	.33367
.5	.32959	.32546	.32126	.31701	.31272	.30838	.30401	.29961	.29518	.29073
.6	.28627	.28179	.27730	.27281	.26831	.26382	.25934	.25486	.25040	.24595
.7	.24152	.23712	.23273	.22837	.22404	.21973	.21546	.21123	.20702	.20286
.8	.19873	.19465	.19060	.18660	.18264	.17873	.17486	.17104	.16726	.16354
.9	.15986	.15624	.15266	.14914	.14566	.14224	.13887	.13555	.13228	.12907
1.0	.12591	.12280	.11974	.11674	.11379	.11089	.10805	.10525	.10251	.09982
1.1	.09718	.09460	.09206	.08958	.08714	.08475	.08242	.08013	.07789	.07570
1.2	.07356	.07146	.06941	.06740	.06545	.06353	.06166	.05983	.05805	.05631
1.3	.05461	.05295	.05134	.04976	.04822	.04672	.04526	.04383	.04245	.04110
1.4	.03978	.03850	.03725	.03604	.03486	.03371	.03260	.03151	.03046	.02943
1.5	.02843	.02747	.02653	.02562	.02473	.02387	.02304	.02223	.02144	.02068
1.6	.01994	.01923	.01854	.01786	.01721	.01658	.01598	.01539	.01481	.01426
1.7	.01373	.01321	.01271	.01223	.01176	.01131	.01087	.01045	.01004	.00965
1.8	.00927	.00890	.00855	.00821	.00788	.00756	.00726	.00696	.00668	.00640
1.9	.00614	.00589	.00564	.00541	.00518	.00496	.00475	.00455	.00436	.00417
2.0	.00399	.00382	.00365	.00349	.00334	.00319	.00305	.00292	.00279	.00266
2.1	.00254	.00243	.00232	.00221	.00211	.00201	.00192	.00183	.00175	.00166
2.2	.00159	.00151	.00144	.00137	.00130	.00124	.00118	.00112	.00107	.00102
2.3	.00097	.00092	.00087	.00083	.00079	.00075	.00071	.00068	.00064	.00061
2.4	.00058	.00055	.00052	.00049	.00046	.00044	.00042	.00039	.00037	.00035
2.5	.00033	.00031	.00030	.00028	.00027	.00025	.00024	.00022	.00021	.00020
2.6	.00019	.00017	.00016	.00015	.00014	.00014	.00013	.00012	.00011	.00010
2.7	.00010	.00009	.00008	.00008	.00007	.00007	.00006	.00006	.00005	.00005
2.8	.00005	.00004	.00004	.00004	.00003	.00003	.00003	.00002	.00002	.00002
2.9	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00000	.00000	.00000
3.0	.00000									

Table 13. $\int_x^\infty e^{-u^2-\frac{m^2}{u^2}}\,du.$

M= .500

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.32601	.32601	.32601	.32601	.32601	.32601	.32601	.32601	.32601	.32601
.1	.32601	.32601	.32601	.32601	.32601	.32601	.32601	.32600	.32600	.32599
.2	.32598	.32596	.32591	.32584	.32574	.32559	.32539	.32513	.32479	.32436
.3	.32384	.32322	.32249	.32165	.32069	.31960	.31839	.31705	.31558	.31398
.4	.31226	.31041	.30844	.30635	.30414	.30182	.29939	.29685	.29422	.29149
.5	.28867	.28576	.28277	.27971	.27657	.27337	.27011	.26679	.26341	.25999
.6	.25653	.25303	.24949	.24592	.24233	.23871	.23507	.23142	.22776	.22409
.7	.22041	.21673	.21306	.20938	.20571	.20206	.19841	.19478	.19116	.18756
.8	.18398	.18042	.17689	.17339	.16991	.16646	.16304	.15965	.15629	.15297
.9	.14969	.14644	.14323	.14005	.13692	.13383	.13077	.12776	.12479	.12186
1.0	.11897	.11613	.11333	.11057	.10786	.10519	.10257	.09998	.09745	.09496
1.1	.09251	.09011	.08775	.08543	.08316	.08093	.07875	.07661	.07451	.07246
1.2	.07044	.06847	.06654	.06466	.06281	.06100	.05924	.05751	.05582	.05417
1.3	.05256	.05099	.04946	.04796	.04649	.04507	.04368	.04232	.04100	.03971
1.4	.03845	.03723	.03604	.03487	.03374	.03265	.03158	.03053	.02952	.02854
1.5	.02758	.02665	.02575	.02487	.02402	.02319	.02239	.02161	.02085	.02012
1.6	.01940	.01871	.01804	.01740	.01677	.01616	.01557	.01500	.01444	.01391
1.7	.01339	.01289	.01240	.01193	.01148	.01104	.01062	.01021	.00981	.00943
1.8	.00906	.00871	.00836	.00803	.00771	.00740	.00710	.00682	.00654	.00627
1.9	.00602	.00577	.00553	.00530	.00508	.00487	.00466	.00446	.00427	.00409
2.0	.00392	.00375	.00358	.00343	.00328	.00314	.00300	.00286	.00274	.00261
2.1	.00250	.00239	.00228	.00217	.00207	.00198	.00189	.00180	.00172	.00164
2.2	.00156	.00149	.00142	.00135	.00128	.00122	.00116	.00111	.00105	.00100
2.3	.00095	.00091	.00086	.00082	.00078	.00074	.00070	.00067	.00063	.00060
2.4	.00057	.00054	.00051	.00048	.00046	.00043	.00041	.00039	.00037	.00035
2.5	.00033	.00031	.00029	.00028	.00026	.00025	.00023	.00022	.00021	.00019
2.6	.00018	.00017	.00016	.00015	.00014	.00013	.00013	.00012	.00011	.00010
2.7	.00010	.00009	.00008	.00008	.00007	.00007	.00006	.00006	.00005	.00005
2.8	.00005	.00004	.00004	.00003	.00003	.00003	.00003	.00002	.00002	.00002
2.9	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00000	.00000	.00000
3.0	.00000									

M= .600

χ	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.26691	.26691	.26691	.26691	.26691	.26691	.26691	.26691	.26691	.26691
.1	.26691	.26691	.26691	.26691	.26691	.26691	.26691	.26691	.26691	.26691
.2	.26691	.26690	.26690	.26689	.26688	.26685	.26682	.26676	.26668	.26657
.3	.26643	.26624	.26599	.26570	.26533	.26490	.26440	.26381	.26314	.26238
.4	.26153	.26058	.25954	.25840	.25717	.25584	.25441	.25288	.25126	.24955
.5	.24775	.24586	.24389	.24183	.23970	.23749	.23520	.23285	.23043	.22795
.6	.22541	.22282	.22018	.21748	.21475	.21197	.20916	.20631	.20343	.20053
.7	.19760	.19466	.19169	.18871	.18572	.18272	.17971	.17670	.17369	.17068
.8	.16767	.16467	.16167	.15869	.15572	.15276	.14982	.14690	.14399	.14110
.9	.13824	.13540	.13258	.12979	.12703	.12429	.12159	.11891	.11626	.11365
1.0	.11107	.10852	.10600	.10352	.10107	.09866	.09628	.09394	.09163	.08936
1.1	.08713	.08493	.08277	.08065	.07856	.07652	.07451	.07253	.07059	.06869
1.2	.06683	.06500	.06321	.06146	.05974	.05806	.05641	.05480	.05322	.05168
1.3	.05017	.04870	.04726	.04585	.04447	.04313	.04182	.04054	.03929	.03808
1.4	.03689	.03573	.03460	.03350	.03243	.03139	.03037	.02939	.02842	.02749
1.5	.02658	.02569	.02483	.02399	.02318	.02239	.02162	.02087	.02015	.01945
1.6	.01877	.01810	.01746	.01684	.01624	.01565	.01509	.01454	.01400	.01349
1.7	.01299	.01251	.01204	.01159	.01115	.01073	.01032	.00992	.00954	.00917
1.8	.00882	.00847	.00814	.00782	.00751	.00721	.00692	.00664	.00637	.00612
1.9	.00587	.00563	.00539	.00517	.00496	.00475	.00455	.00436	.00417	.00400
2.0	.00383	.00366	.00350	.00335	.00321	.00307	.00293	.00280	.00268	.00256
2.1	.00245	.00234	.00223	.00213	.00203	.00194	.00185	.00177	.00168	.00161
2.2	.00153	.00146	.00139	.00132	.00126	.00120	.00114	.00109	.00103	.00098
2.3	.00094	.00089	.00085	.00080	.00076	.00073	.00069	.00065	.00062	.00059
2.4	.00056	.00053	.00050	.00048	.00045	.00043	.00040	.00038	.00036	.00034
2.5	.00032	.00031	.00029	.00027	.00026	.00024	.00023	.00022	.00020	.00019
2.6	.00018	.00017	.00016	.00015	.00014	.00013	.00012	.00012	.00011	.00010
2.7	.00009	.00009	.00008	.00008	.00007	.00007	.00006	.00006	.00005	.00005
2.8	.00004	.00004	.00004	.00003	.00003	.00003	.00003	.00002	.00002	.00002
2.9	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00000	.00000	.00000
3.0	.00000									

Table 13. $\int_x^\infty e^{-u^2-\frac{m^2}{u^2}}\,du.$

M= .700

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.21852	.21852	.21852	.21852	.21852	.21852	.21852	.21852	.21852	.21852
.1	.21852	.21852	.21852	.21852	.21852	.21852	.21852	.21852	.21852	.21852
.2	.21852	.21852	.21852	.21852	.21852	.21852	.21851	.21850	.21849	.21847
.3	.21843	.21839	.21832	.21823	.21812	.21798	.21779	.21757	.21731	.21699
.4	.21662	.21619	.21570	.21515	.21453	.21384	.21307	.21224	.21133	.21034
.5	.20929	.20815	.20694	.20566	.20430	.20288	.20138	.19981	.19818	.19648
.6	.19473	.19291	.19103	.18910	.18712	.18509	.18301	.18089	.17873	.17652
.7	.17429	.17202	.16972	.16739	.16504	.16267	.16027	.15786	.15543	.15300
.8	.15055	.14809	.14563	.14317	.14070	.13824	.13577	.13332	.13086	.12842
.9	.12598	.12356	.12115	.11875	.11637	.11401	.11166	.10933	.10702	.10473
1.0	.10247	.10023	.09801	.09582	.09365	.09151	.08939	.08730	.08524	.08321
1.1	.08121	.07923	.07729	.07537	.07349	.07163	.06981	.06801	.06625	.06452
1.2	.06282	.06115	.05951	.05790	.05632	.05477	.05326	.05177	.05031	.04889
1.3	.04749	.04613	.04479	.04348	.04220	.04096	.03973	.03854	.03738	.03624
1.4	.03513	.03404	.03299	.03196	.03095	.02997	.02902	.02808	.02718	.02630
1.5	.02544	.02460	.02379	.02299	.02223	.02148	.02075	.02004	.01936	.01869
1.6	.01804	.01741	.01680	.01621	.01563	.01508	.01454	.01401	.01350	.01301
1.7	.01254	.01207	.01163	.01119	.01077	.01037	.00998	.00960	.00923	.00888
1.8	.00854	.00820	.00788	.00758	.00728	.00699	.00671	.00644	.00619	.00594
1.9	.00570	.00546	.00524	.00502	.00482	.00462	.00442	.00424	.00406	.00389
2.0	.00372	.00356	.00341	.00326	.00312	.00299	.00286	.00273	.00261	.00250
2.1	.00238	.00228	.00218	.00208	.00198	.00189	.00181	.00172	.00164	.00157
2.2	.00150	.00143	.00136	.00129	.00123	.00117	.00112	.00106	.00101	.00096
2.3	.00092	.00087	.00083	.00079	.00075	.00071	.00068	.00064	.00061	.00058
2.4	.00055	.00052	.00049	.00047	.00044	.00042	.00040	.00038	.00036	.00034
2.5	.00032	.00030	.00028	.00027	.00025	.00024	.00023	.00021	.00020	.00019
2.6	.00018	.00017	.00016	.00015	.00014	.00013	.00012	.00011	.00011	.00010
2.7	.00009	.00009	.00008	.00008	.00007	.00007	.00006	.00006	.00005	.00005
2.8	.00004	.00004	.00004	.00003	.00003	.00003	.00003	.00002	.00002	.00002
2.9	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00000	.00000	.00000
3.0	.00000									

Table 13.

M= .800

χ	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.17891	.17891	.17891	.17891	.17891	.17891	.17891	.17891	.17891	.17891
.1	.17891	.17891	.17891	.17891	.17891	.17891	.17891	.17891	.17891	.17891
.2	.17891	.17891	.17891	.17891	.17891	.17891	.17891	.17891	.17890	.17890
.3	.17889	.17888	.17887	.17885	.17882	.17878	.17872	.17865	.17856	.17844
.4	.17830	.17813	.17793	.17768	.17740	.17708	.17671	.17629	.17582	.17530
.5	.17473	.17410	.17341	.17267	.17187	.17100	.17008	.16911	.16807	.16697
.6	.16582	.16462	.16335	.16204	.16067	.15926	.15779	.15628	.15472	.15313
.7	.15149	.14981	.14809	.14634	.14456	.14275	.14091	.13904	.13715	.13524
.8	.13331	.13136	.12940	.12742	.12543	.12344	.12143	.11942	.11740	.11539
.9	.11337	.11135	.10933	.10732	.10531	.10332	.10132	.09934	.09737	.09541
1.0	.09346	.09153	.08961	.08771	.08583	.08396	.08211	.08028	.07847	.07668
1.1	.07492	.07317	.07145	.06974	.06807	.06641	.06478	.06318	.06159	.06004
1.2	.05851	.05700	.05552	.05406	.05263	.05123	.04985	.04850	.04717	.04587
1.3	.04459	.04334	.04211	.04091	.03974	.03859	.03746	.03636	.03529	.03423
1.4	.03321	.03220	.03122	.03026	.02933	.02842	.02753	.02666	.02581	.02499
1.5	.02419	.02340	.02264	.02190	.02118	.02047	.01979	.01912	.01848	.01785
1.6	.01724	.01664	.01607	.01551	.01497	.01444	.01393	.01343	.01295	.01248
1.7	.01203	.01159	.01117	.01075	.01036	.00997	.00960	.00924	.00889	.00855
1.8	.00822	.00791	.00760	.00730	.00702	.00674	.00648	.00622	.00597	.00573
1.9	.00550	.00528	.00507	.00486	.00466	.00447	.00428	.00410	.00393	.00377
2.0	.00361	.00346	.00331	.00317	.00303	.00290	.00277	.00265	.00254	.00242
2.1	.00232	.00221	.00212	.00202	.00193	.00184	.00176	.00168	.00160	.00153
2.2	.00146	.00139	.00132	.00126	.00120	.00114	.00109	.00104	.00099	.00094
2.3	.00089	.00085	.00081	.00077	.00073	.00069	.00066	.00063	.00059	.00056
2.4	.00054	.00051	.00048	.00046	.00043	.00041	.00039	.00037	.00035	.00033
2.5	.00031	.00029	.00028	.00026	.00025	.00023	.00022	.00021	.00020	.00018
2.6	.00017	.00016	.00015	.00014	.00014	.00013	.00012	.00011	.00010	.00010
2.7	.00009	.00009	.00008	.00007	.00007	.00006	.00006	.00006	.00005	.00005
2.8	.00004	.00004	.00004	.00003	.00003	.00003	.00002	.00002	.00002	.00002
2.9	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00000	.00000	.00000
3.0	.00000									

Table 13. $\int_x^\infty e^{-u^2-\frac{m^2}{u^2}}\,du.$

M= .900

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.14647	.14647	.14647	.14647	.14647	.14647	.14647	.14647	.14647	.14647
.1	.14647	.14647	.14647	.14647	.14647	.14647	.14647	.14647	.14647	.14647
.2	.14647	.14647	.14647	.14647	.14647	.14647	.14647	.14647	.14647	.14647
.3	.14647	.14647	.14647	.14646	.14646	.14645	.14643	.14641	.14639	.14635
.4	.14630	.14624	.14616	.14607	.14596	.14582	.14566	.14547	.14524	.14499
.5	.14471	.14438	.14402	.14362	.14317	.14269	.14216	.14158	.14096	.14030
.6	.13959	.13883	.13802	.13717	.13628	.13533	.13435	.13332	.13225	.13113
.7	.12998	.12879	.12756	.12629	.12499	.12366	.12229	.12090	.11947	.11802
.8	.11655	.11505	.11353	.11199	.11043	.10886	.10727	.10566	.10405	.10242
.9	.10079	.09915	.09751	.09586	.09421	.09255	.09090	.08925	.08760	.08596
1.0	.08432	.08268	.08106	.07944	.07783	.07623	.07465	.07307	.07151	.06996
1.1	.06843	.06691	.06540	.06392	.06245	.06099	.05956	.05814	.05674	.05536
1.2	.05400	.05266	.05134	.05004	.04877	.04751	.04627	.04505	.04386	.04268
1.3	.04153	.04040	.03929	.03820	.03713	.03608	.03506	.03405	.03307	.03210
1.4	.03116	.03024	.02934	.02846	.02760	.02676	.02594	.02514	.02435	.02359
1.5	.02284	.02212	.02141	.02072	.02005	.01939	.01876	.01814	.01753	.01695
1.6	.01637	.01582	.01528	.01475	.01424	.01375	.01327	.01280	.01235	.01191
1.7	.01148	.01107	.01067	.01028	.00990	.00954	.00918	.00884	.00851	.00819
1.8	.00788	.00758	.00729	.00701	.00674	.00648	.00622	.00598	.00574	.00551
1.9	.00529	.00508	.00488	.00468	.00449	.00431	.00413	.00396	.00379	.00364
2.0	.00348	.00334	.00319	.00306	.00293	.00280	.00268	.00256	.00245	.00235
2.1	.00224	.00214	.00205	.00196	.00187	.00178	.00170	.00163	.00155	.00148
2.2	.00141	.00135	.00128	.00122	.00117	.00111	.00106	.00101	.00096	.00091
2.3	.00087	.00083	.00079	.00075	.00071	.00068	.00064	.00061	.00058	.00055
2.4	.00052	.00049	.00047	.00044	.00042	.00040	.00038	.00036	.00034	.00032
2.5	.00030	.00029	.00027	.00026	.00024	.00023	.00022	.00020	.00019	.00018
2.6	.00017	.00016	.00015	.00014	.00013	.00012	.00012	.00011	.00010	.00010
2.7	.00009	.00008	.00008	.00007	.00007	.00006	.00006	.00005	.00005	.00005
2.8	.00004	.00004	.00004	.00003	.00003	.00003	.00002	.00002	.00002	.00002
2.9	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00000	.00000	.00000
3.0	.00000									

M=1.000

χ	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.11992	.11992	.11992	.11992	.11992	.11992	.11992	.11992	.11992	.11992
.1	.11992	.11992	.11992	.11992	.11992	.11992	.11992	.11992	.11992	.11992
.2	.11992	.11992	.11992	.11992	.11992	.11992	.11992	.11992	.11992	.11992
.3	.11992	.11992	.11992	.11992	.11992	.11992	.11991	.11991	.11990	.11989
.4	.11988	.11986	.11983	.11980	.11976	.11970	.11964	.11956	.11946	.11935
.5	.11922	.11907	.11889	.11869	.11846	.11820	.11792	.11760	.11725	.11687
.6	.11645	.11600	.11551	.11499	.11443	.11383	.11320	.11253	.11183	.11108
.7	.11031	.10949	.10864	.10776	.10685	.10590	.10492	.10391	.10287	.10181
.8	.10072	.09960	.09846	.09729	.09611	.09490	.09367	.09243	.09117	.08990
.9	.08861	.08731	.08600	.08468	.08335	.08201	.08067	.07932	.07797	.07662
1.0	.07527	.07391	.07256	.07121	.06986	.06852	.06718	.06585	.06453	.06321
1.1	.06190	.06060	.05931	.05803	.05676	.05550	.05426	.05303	.05181	.05060
1.2	.04941	.04824	.04708	.04593	.04480	.04369	.04259	.04151	.04045	.03940
1.3	.03837	.03736	.03636	.03539	.03443	.03348	.03256	.03165	.03076	.02989
1.4	.02904	.02820	.02738	.02658	.02579	.02502	.02427	.02354	.02282	.02212
1.5	.02144	.02077	.02012	.01948	.01886	.01826	.01767	.01710	.01654	.01599
1.6	.01546	.01495	.01444	.01395	.01348	.01302	.01257	.01213	.01171	.01130
1.7	.01090	.01051	.01014	.00977	.00942	.00908	.00874	.00842	.00811	.00781
1.8	.00752	.00723	.00696	.00669	.00644	.00619	.00595	.00572	.00550	.00528
1.9	.00507	.00487	.00467	.00449	.00431	.00413	.00396	.00380	.00364	.00349
2.0	.00335	.00321	.00307	.00294	.00282	.00270	.00258	.00247	.00236	.00226
2.1	.00216	.00207	.00198	.00189	.00180	.00172	.00165	.00157	.00150	.00143
2.2	.00137	.00130	.00124	.00118	.00113	.00108	.00103	.00098	.00093	.00089
2.3	.00084	.00080	.00076	.00073	.00069	.00066	.00062	.00059	.00056	.00053
2.4	.00051	.00048	.00046	.00043	.00041	.00039	.00037	.00035	.00033	.00031
2.5	.00030	.00028	.00026	.00025	.00024	.00022	.00021	.00020	.00019	.00018
2.6	.00017	.00016	.00015	.00014	.00013	.00012	.00011	.00011	.00010	.00009
2.7	.00009	.00008	.00008	.00007	.00007	.00006	.00006	.00005	.00005	.00005
2.8	.00004	.00004	.00003	.00003	.00003	.00003	.00002	.00002	.00002	.00002
2.9	.00001	.00001	.00001	.00001	.00001	.00001	.00000	.00000	.00000	.00000
3.0	.00000									

Table 14

Values of:

$$\int_x^\infty \frac{e^{-u^2 - \frac{m^2}{u^2}} du}{u}$$

This table was prepared by Simpson's rule integration. Since this is an approximate numerical process it is desirable to have some means of evaluating the degree of approximation obtained.

When $(m^2/u^2) << 1$ the integral reduces approximately to

$$\int_x^\infty \frac{e^{-u^2} du}{u}$$

The substitution $v = u^2$ will reduce this to a form of the Exponential Integral which is a tabulated function. Then

$$\int_x^\infty \frac{e^{-u^2} du}{u} = \frac{1}{2} \int_{x^2}^\infty \frac{e^{-v} dv}{v} = \frac{1}{2} E_1(x^2)$$

When $m = 0.001$ $x = 1.0$ $(m^2/u^2) < 0.000001$ $e^{-\frac{m^2}{u^2}} > 0.999999$

$\frac{1}{2} E_1(x)^2 = 0.1096919$. The tabular value is 0.10969. The check is satisfactory when $m = 0.001$ $x = 0.1$ $(m^2/u^2) < 0.0001$ $e^{-\frac{m^2}{u^2}} >= 0.999900$

$\frac{1}{2} E_1(x^2) = 2.0189648$. The tabular value is 2.01891. The check is good to the fourth decimal place which is all that could be expected of the approximation.

Another check can be obtained from the integral:

$$\int_0^\infty \frac{e^{-u^2 - \frac{m^2}{u^2}} du}{u} = K_0\ (2m). \qquad \text{If } m > 0 .$$

A comparison of values computed by Simpson's rule with values of the above integral is shown below:

m	K_0 (2m)	Values computed by Simpson's rule
0.001	6.33055	6.42824
0.002	5.63742	5.61213
0.003	5.23198	5.23290
0.004	4.94434	4.94644
0.005	4.72124	4.72140
0.006	4.53898	4.53863
0.007	4.38489	4.38476
0.008	4.25143	4.25146
0.009	4.13373	4.13377
0.010	4.02846	4.02846
0.020	3.33654	3.33654
0.030	2.93288	2.93287
0.040	2.64749	2.64748
0.050	2.42707	2.42706
0.060	2.24786	2.24785
0.070	2.09717	2.09716
0.080	1.96742	1.96741
0.090	1.85371	1.85371
0.100	1.75270	1.75270

Agreement is observed for the $m = 0.010$ and larger values of m. In this range the table can be accepted since the integration was carried backward from $x = 3.0$. The Simpson's rule procedure makes every value dependent upon the preceding one and a check of the value for $x = 0$ checks the entire table for that value of m.

Discrepancies are noted in the range $m = 0.001$ to $m = 0.009$. In this range the integrand drops sharply toward zero as x approaches zero and the shape of the curve may here be ill adapted to evaluation by the Simpson's rule procedure. The checks obtained for $x = 1.0$ and $x = 0.1$, when m = 0.001, lend support to this presumption. To evaluate the integral in the area near zero make use of the relations

$$e^{-u^2} = 1 - u^2 + \frac{u^4}{2!} - \frac{u^6}{3!} + \ldots$$

and

$$\int_x^\infty \frac{e^{-u^2 - \frac{m^2}{u^2}}du}{u} = \int_0^\infty \frac{e^{-u^2 - \frac{m^2}{u^2}}du}{u} - \int_0^x \frac{e^{-u^2 - \frac{m^2}{u^2}}du}{u}$$

Then the last integral above can be expressed approximately as

$$\int_0^x \frac{e^{-u^2 - \frac{m^2}{u^2}}du}{u} \doteq \int_0^x \frac{e^{-\frac{m^2}{u^2}}du}{u} - \int_0^x \frac{u^2\, e^{-\frac{m^2}{u^2}}du}{u} + \frac{1}{2}\int_0^x \frac{u^4\, e^{-\frac{m^2}{u^2}}du}{u} - \ldots$$

Let $\frac{m}{u} = v$ $\quad u = \frac{m}{v}$ $\quad du = \frac{-mdv}{v^2}$ $\quad \frac{1}{u} = \frac{v}{m}$. Then, by substitution

$$\int_0^x \frac{e^{-\frac{m^2}{u^2}}du}{u} = \int_{(\frac{m}{x})}^\infty \frac{e^{-v^2}dv}{v}$$

By similar substitutions and integration by parts the remaining two integrals can be evaluated and the integral approximated as

$$\int_0^x \frac{e^{-u^2 - \frac{m^2}{u^2}}du}{u} \doteq \left(- \frac{x^2}{2} + \frac{x^4}{8} - \frac{m^2x^2}{8}\right) e^{-(\frac{m^2}{x^2})} + \left(1 + m^2 + \frac{m^4}{4}\right) \int_{(\frac{m}{x})}^\infty \frac{e^{-v^2}dv}{v}$$

(Approximate if $m \ll 1$ $\quad x \ll 1$)

As noted previously, the integral which appears here can be expressed in terms of the exponential integral as

$$\int_{(\frac{m}{x})}^{\infty} \frac{e^{-v^2} dv}{v} = \frac{1}{2} \int_{(\frac{m^2}{x})}^{\infty} \frac{e^{-u}\, du}{u} = -\frac{1}{2}\, \mathrm{Ei}(-\frac{m^2}{x^2}) = \frac{1}{2}\, E_1(\frac{m^2}{x^2}) \; .$$

The results of computations made by this approximation are

m	x = 0.01	x = 0.02
0.001	4.31163	3.62237
0.002	4.29683	

Since the approximation values for m = .001 x = 0.02 and for m = 0.002 x = 0.01 agree well with the tabular values the presumption is confirmed. The tabular values affected by this difficulty have been replaced by computed values.

Table 14. $\int_x^{\infty} \frac{e^{-u^2 - \frac{m^2}{u^2}}}{u} du$

M= .001

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	6.33055	4.31163	3.62236	3.21784	2.93075	2.70817	2.52646	2.37299	2.24023	2.12331
.1	2.01891	1.92465	1.83879	1.75999	1.68723	1.61967	1.55667	1.49767	1.44224	1.38999
.2	1.34061	1.29383	1.24942	1.20716	1.16689	1.12844	1.09169	1.05651	1.02279	.99044
.3	.95936	.92948	.90074	.87305	.84636	.82063	.79579	.77181	.74864	.72625
.4	.70459	.68363	.66334	.64370	.62467	.60624	.58837	.57105	.55424	.53795
.5	.52213	.50679	.49189	.47743	.46340	.44976	.43652	.42366	.41116	.39902
.6	.38722	.37576	.36462	.35379	.34327	.33304	.32310	.31344	.30405	.29491
.7	.28604	.27741	.26902	.26087	.25294	.24523	.23774	.23046	.22338	.21651
.8	.20982	.20332	.19701	.19087	.18491	.17911	.17348	.16801	.16269	.15753
.9	.15251	.14764	.14291	.13832	.13386	.12952	.12532	.12123	.11727	.11342
1.0	.10969	.10606	.10254	.09913	.09582	.09261	.08950	.08648	.08355	.08071
1.1	.07795	.07529	.07270	.07019	.06776	.06541	.06313	.06092	.05878	.05670
1.2	.05470	.05276	.05087	.04905	.04729	.04559	.04394	.04234	.04080	.03931
1.3	.03786	.03647	.03512	.03381	.03255	.03134	.03016	.02902	.02792	.02686
1.4	.02584	.02485	.02390	.02297	.02209	.02123	.02040	.01960	.01883	.01809
1.5	.01737	.01668	.01602	.01538	.01476	.01417	.01359	.01304	.01251	.01200
1.6	.01151	.01103	.01058	.01014	.00972	.00931	.00892	.00854	.00818	.00783
1.7	.00750	.00718	.00687	.00658	.00629	.00602	.00576	.00551	.00527	.00503
1.8	.00481	.00460	.00439	.00420	.00401	.00383	.00366	.00349	.00333	.00318
1.9	.00304	.00290	.00276	.00264	.00251	.00240	.00228	.00218	.00208	.00198
2.0	.00188	.00179	.00171	.00163	.00155	.00147	.00140	.00133	.00127	.00121
2.1	.00115	.00109	.00104	.00099	.00094	.00089	.00085	.00080	.00076	.00072
2.2	.00069	.00065	.00062	.00059	.00056	.00053	.00050	.00047	.00045	.00043
2.3	.00040	.00038	.00036	.00034	.00032	.00031	.00029	.00027	.00026	.00024
2.4	.00023	.00022	.00021	.00019	.00018	.00017	.00016	.00015	.00015	.00014
2.5	.00013	.00012	.00011	.00011	.00010	.00010	.00009	.00008	.00008	.00007
2.6	.00007	.00007	.00006	.00006	.00005	.00005	.00005	.00004	.00004	.00004
2.7	.00004	.00003	.00003	.00003	.00003	.00002	.00002	.00002	.00002	.00002
2.8	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.9	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

Table 14.

M= .002

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	5.63742	4.29683	3.61864	3.21619	2.92983	2.70758	2.52605	2.37270	2.24001	2.12314
.1	2.01877	1.92454	1.83869	1.75991	1.68716	1.61961	1.55662	1.49763	1.44220	1.38996
.2	1.34058	1.29380	1.24939	1.20714	1.16686	1.12842	1.09167	1.05649	1.02278	.99042
.3	.95935	.92947	.90072	.87304	.84636	.82062	.79579	.77181	.74864	.72624
.4	.70458	.68362	.66334	.64370	.62467	.60623	.58837	.57104	.55424	.53794
.5	.52213	.50679	.49189	.47743	.46339	.44976	.43652	.42365	.41116	.39902
.6	.38722	.37576	.36462	.35379	.34327	.33304	.32310	.31344	.30404	.29491
.7	.28604	.27741	.26902	.26086	.25294	.24523	.23774	.23046	.22338	.21650
.8	.20982	.20332	.19701	.19087	.18490	.17911	.17348	.16801	.16269	.15753
.9	.15251	.14764	.14291	.13832	.13385	.12952	.12532	.12123	.11727	.11342
1.0	.10969	.10606	.10254	.09913	.09582	.09261	.08950	.08648	.08355	.08071
1.1	.07795	.07529	.07270	.07019	.06776	.06541	.06313	.06092	.05878	.05670
1.2	.05470	.05276	.05087	.04905	.04729	.04559	.04394	.04234	.04080	.03931
1.3	.03786	.03647	.03512	.03381	.03255	.03134	.03016	.02902	.02792	.02686
1.4	.02584	.02485	.02390	.02297	.02209	.02123	.02040	.01960	.01883	.01809
1.5	.01737	.01668	.01602	.01538	.01476	.01417	.01359	.01304	.01251	.01200
1.6	.01151	.01103	.01058	.01014	.00972	.00931	.00892	.00854	.00818	.00783
1.7	.00750	.00718	.00687	.00658	.00629	.00602	.00576	.00551	.00527	.00503
1.8	.00481	.00460	.00439	.00420	.00401	.00383	.00366	.00349	.00333	.00318
1.9	.00304	.00290	.00276	.00264	.00251	.00240	.00228	.00218	.00208	.00198
2.0	.00188	.00179	.00171	.00163	.00155	.00147	.00140	.00133	.00127	.00121
2.1	.00115	.00109	.00104	.00099	.00094	.00089	.00085	.00080	.00076	.00072
2.2	.00069	.00065	.00062	.00059	.00056	.00053	.00050	.00047	.00045	.00043
2.3	.00040	.00038	.00036	.00034	.00032	.00031	.00029	.00027	.00026	.00024
2.4	.00023	.00022	.00021	.00019	.00018	.00017	.00016	.00015	.00015	.00014
2.5	.00013	.00012	.00011	.00011	.00010	.00010	.00009	.00008	.00008	.00007
2.6	.00007	.00007	.00006	.00006	.00005	.00005	.00005	.00004	.00004	.00004
2.7	.00004	.00003	.00003	.00003	.00003	.00002	.00002	.00002	.00002	.00002
2.8	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.9	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

2.

Table 14. $\int_x^\infty \frac{e^{-u^2 - \frac{m^2}{u^2}}\,du}{u}$

M= .003

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	5.23198	4.27264	3.61246	3.21344	2.92828	2.70660	2.52537	2.37220	2.23963	2.12284
.1	2.01853	1.92434	1.83853	1.75978	1.68704	1.61951	1.55653	1.49755	1.44213	1.38990
.2	1.34053	1.29376	1.24935	1.20710	1.16683	1.12839	1.09164	1.05647	1.02275	.99040
.3	.95933	.92945	.90071	.87302	.84634	.82061	.79577	.77179	.74862	.72623
.4	.70457	.68361	.66333	.64369	.62466	.60623	.58836	.57104	.55423	.53794
.5	.52213	.50678	.49189	.47743	.46339	.44976	.43651	.42365	.41116	.39902
.6	.38722	.37576	.36462	.35379	.34327	.33304	.32310	.31344	.30404	.29491
.7	.28604	.27741	.26902	.26086	.25294	.24523	.23774	.23046	.22338	.21650
.8	.20982	.20332	.19700	.19087	.18490	.17911	.17348	.16801	.16269	.15753
.9	.15251	.14764	.14291	.13832	.13385	.12952	.12532	.12123	.11727	.11342
1.0	.10969	.10606	.10254	.09913	.09582	.09261	.08950	.08648	.08355	.08071
1.1	.07795	.07528	.07270	.07019	.06776	.06541	.06313	.06092	.05878	.05670
1.2	.05470	.05276	.05087	.04905	.04729	.04559	.04394	.04234	.04080	.03931
1.3	.03786	.03647	.03512	.03381	.03255	.03133	.03016	.02902	.02792	.02686
1.4	.02584	.02485	.02390	.02297	.02209	.02123	.02040	.01960	.01883	.01809
1.5	.01737	.01668	.01602	.01538	.01476	.01417	.01359	.01304	.01251	.01200
1.6	.01151	.01103	.01058	.01014	.00972	.00931	.00892	.00854	.00818	.00783
1.7	.00750	.00718	.00687	.00658	.00629	.00602	.00576	.00551	.00527	.00503
1.8	.00481	.00460	.00439	.00420	.00401	.00383	.00366	.00349	.00333	.00318
1.9	.00304	.00290	.00276	.00264	.00251	.00240	.00228	.00218	.00208	.00198
2.0	.00188	.00179	.00171	.00163	.00155	.00147	.00140	.00133	.00127	.00121
2.1	.00115	.00109	.00104	.00099	.00094	.00089	.00085	.00080	.00076	.00072
2.2	.00069	.00065	.00062	.00059	.00056	.00053	.00050	.00047	.00045	.00043
2.3	.00040	.00038	.00036	.00034	.00032	.00031	.00029	.00027	.00026	.00024
2.4	.00023	.00022	.00021	.00019	.00018	.00017	.00016	.00015	.00015	.00014
2.5	.00013	.00012	.00011	.00011	.00010	.00010	.00009	.00008	.00008	.00007
2.6	.00007	.00007	.00006	.00006	.00005	.00005	.00005	.00004	.00004	.00004
2.7	.00004	.00003	.00003	.00003	.00003	.00002	.00002	.00002	.00002	.00002
2.8	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.9	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

3.

M= .004

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	4.94434	4.23977	3.60387	3.20960	2.92613	2.70522	2.52442	2.37151	2.23910	2.12243
.1	2.01820	1.92407	1.83831	1.75958	1.68688	1.61937	1.55641	1.49744	1.44204	1.38981
.2	1.34045	1.29369	1.24929	1.20704	1.16678	1.12835	1.09160	1.05643	1.02272	.99037
.3	.95930	.92943	.90068	.87300	.84632	.82059	.79576	.77178	.74861	.72621
.4	.70456	.68360	.66332	.64368	.62465	.60622	.58835	.57103	.55423	.53793
.5	.52212	.50677	.49188	.47742	.46338	.44975	.43651	.42365	.41115	.39901
.6	.38722	.37575	.36461	.35379	.34327	.33304	.32310	.31343	.30404	.29491
.7	.28603	.27740	.26902	.26086	.25293	.24523	.23774	.23046	.22338	.21650
.8	.20982	.20332	.19700	.19087	.18490	.17911	.17348	.16801	.16269	.15753
.9	.15251	.14764	.14291	.13832	.13385	.12952	.12531	.12123	.11727	.11342
1.0	.10968	.10606	.10254	.09913	.09582	.09261	.08950	.08648	.08355	.08071
1.1	.07795	.07528	.07270	.07019	.06776	.06541	.06313	.06092	.05878	.05670
1.2	.05470	.05276	.05087	.04905	.04729	.04559	.04394	.04234	.04080	.03931
1.3	.03786	.03647	.03512	.03381	.03255	.03133	.03016	.02902	.02792	.02686
1.4	.02584	.02485	.02390	.02297	.02209	.02123	.02040	.01960	.01883	.01809
1.5	.01737	.01668	.01602	.01538	.01476	.01417	.01359	.01304	.01251	.01200
1.6	.01151	.01103	.01058	.01014	.00972	.00931	.00892	.00854	.00818	.00783
1.7	.00750	.00718	.00687	.00658	.00629	.00602	.00576	.00551	.00527	.00503
1.8	.00481	.00460	.00439	.00420	.00401	.00383	.00366	.00349	.00333	.00318
1.9	.00304	.00290	.00276	.00264	.00251	.00240	.00228	.00218	.00208	.00198
2.0	.00188	.00179	.00171	.00163	.00155	.00147	.00140	.00133	.00127	.00121
2.1	.00115	.00109	.00104	.00099	.00094	.00089	.00085	.00080	.00076	.00072
2.2	.00069	.00065	.00062	.00059	.00056	.00053	.00050	.00047	.00045	.00043
2.3	.00040	.00038	.00036	.00034	.00032	.00031	.00029	.00027	.00026	.00024
2.4	.00023	.00022	.00021	.00019	.00018	.00017	.00016	.00015	.00015	.00014
2.5	.00013	.00012	.00011	.00011	.00010	.00010	.00009	.00008	.00008	.00007
2.6	.00007	.00007	.00006	.00006	.00005	.00005	.00005	.00004	.00004	.00004
2.7	.00004	.00003	.00003	.00003	.00003	.00002	.00002	.00002	.00002	.00002
2.8	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.9	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

4.

Table 14. $\int_x^\infty \frac{e^{-u^2 - \frac{m^2}{u^2}}}{u} du$

M= .005

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	4.72124	4.19912	3.59294	3.20469	2.92337	2.70346	2.52320	2.37062	2.23843	2.12190
.1	2.01777	1.92372	1.83801	1.75934	1.68667	1.61919	1.55625	1.49731	1.44192	1.38971
.2	1.34036	1.29360	1.24921	1.20697	1.16672	1.12829	1.09155	1.05638	1.02267	.99033
.3	.95926	.92939	.90065	.87297	.84629	.82056	.79573	.77175	.74859	.72620
.4	.70454	.68358	.66330	.64366	.62464	.60620	.58834	.57102	.55422	.53792
.5	.52211	.50677	.49187	.47741	.46338	.44974	.43650	.42364	.41115	.39901
.6	.38721	.37575	.36461	.35378	.34326	.33303	.32309	.31343	.30404	.29491
.7	.28603	.27740	.26901	.26086	.25293	.24523	.23774	.23046	.22338	.21650
.8	.20981	.20332	.19700	.19086	.18490	.17911	.17348	.16801	.16269	.15753
.9	.15251	.14764	.14291	.13831	.13385	.12952	.12531	.12123	.11727	.11342
1.0	.10968	.10606	.10254	.09913	.09582	.09261	.08950	.08648	.08355	.08071
1.1	.07795	.07528	.07270	.07019	.06776	.06541	.06313	.06092	.05878	.05670
1.2	.05470	.05275	.05087	.04905	.04729	.04559	.04394	.04234	.04080	.03931
1.3	.03786	.03647	.03512	.03381	.03255	.03133	.03016	.02902	.02792	.02686
1.4	.02584	.02485	.02390	.02297	.02209	.02123	.02040	.01960	.01883	.01809
1.5	.01737	.01668	.01602	.01538	.01476	.01417	.01359	.01304	.01251	.01200
1.6	.01151	.01103	.01058	.01014	.00971	.00931	.00892	.00854	.00818	.00783
1.7	.00750	.00718	.00687	.00658	.00629	.00602	.00576	.00551	.00527	.00503
1.8	.00481	.00460	.00439	.00420	.00401	.00383	.00366	.00349	.00333	.00318
1.9	.00304	.00290	.00276	.00264	.00251	.00240	.00228	.00218	.00208	.00198
2.0	.00188	.00179	.00171	.00163	.00155	.00147	.00140	.00133	.00127	.00121
2.1	.00115	.00109	.00104	.00099	.00094	.00089	.00085	.00080	.00076	.00072
2.2	.00069	.00065	.00062	.00059	.00056	.00053	.00050	.00047	.00045	.00043
2.3	.00040	.00038	.00036	.00034	.00032	.00031	.00029	.00027	.00026	.00024
2.4	.00023	.00022	.00021	.00019	.00018	.00017	.00016	.00015	.00015	.00014
2.5	.00013	.00012	.00011	.00011	.00010	.00010	.00009	.00008	.00008	.00007
2.6	.00007	.00007	.00006	.00006	.00005	.00005	.00005	.00004	.00004	.00004
2.7	.00004	.00003	.00003	.00003	.00003	.00002	.00002	.00002	.00002	.00002
2.8	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.9	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

5.

M= .006

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	4.53898	4.15176	3.57975	3.19873	2.92000	2.70131	2.52172	2.36953	2.23760	2.12125
.1	2.01725	1.92329	1.83766	1.75904	1.68641	1.61897	1.55606	1.49714	1.44177	1.38957
.2	1.34024	1.29350	1.24911	1.20689	1.16664	1.12822	1.09149	1.05632	1.02262	.99028
.3	.95922	.92935	.90061	.87293	.84626	.82053	.79570	.77173	.74856	.72617
.4	.70452	.68356	.66328	.64364	.62462	.60619	.58832	.57100	.55420	.53791
.5	.52210	.50675	.49186	.47740	.46337	.44973	.43649	.42363	.41114	.39900
.6	.38720	.37574	.36460	.35378	.34326	.33303	.32309	.31343	.30403	.29490
.7	.28603	.27740	.26901	.26086	.25293	.24522	.23773	.23045	.22338	.21650
.8	.20981	.20331	.19700	.19086	.18490	.17910	.17347	.16800	.16269	.15753
.9	.15251	.14764	.14291	.13831	.13385	.12952	.12531	.12123	.11727	.11342
1.0	.10968	.10606	.10254	.09913	.09582	.09261	.08950	.08648	.08355	.08071
1.1	.07795	.07528	.07270	.07019	.06776	.06541	.06313	.06092	.05878	.05670
1.2	.05470	.05275	.05087	.04905	.04729	.04559	.04394	.04234	.04080	.03931
1.3	.03786	.03647	.03512	.03381	.03255	.03133	.03016	.02902	.02792	.02686
1.4	.02584	.02485	.02390	.02297	.02209	.02123	.02040	.01960	.01883	.01809
1.5	.01737	.01668	.01602	.01538	.01476	.01417	.01359	.01304	.01251	.01200
1.6	.01151	.01103	.01058	.01014	.00971	.00931	.00892	.00854	.00818	.00783
1.7	.00750	.00718	.00687	.00658	.00629	.00602	.00576	.00551	.00527	.00503
1.8	.00481	.00460	.00439	.00420	.00401	.00383	.00366	.00349	.00333	.00318
1.9	.00304	.00290	.00276	.00264	.00251	.00240	.00228	.00218	.00208	.00198
2.0	.00188	.00179	.00171	.00163	.00155	.00147	.00140	.00133	.00127	.00121
2.1	.00115	.00109	.00104	.00099	.00094	.00089	.00085	.00080	.00076	.00072
2.2	.00069	.00065	.00062	.00059	.00056	.00053	.00050	.00047	.00045	.00043
2.3	.00040	.00038	.00036	.00034	.00032	.00031	.00029	.00027	.00026	.00024
2.4	.00023	.00022	.00021	.00019	.00018	.00017	.00016	.00015	.00015	.00014
2.5	.00013	.00012	.00011	.00011	.00010	.00010	.00009	.00008	.00008	.00007
2.6	.00007	.00007	.00006	.00006	.00005	.00005	.00005	.00004	.00004	.00004
2.7	.00004	.00003	.00003	.00003	.00003	.00002	.00002	.00002	.00002	.00002
2.8	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.9	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

Table 14. $\int_x^\infty \frac{e^{-u^2 - \frac{m^2}{u^2}}}{u} du$

M= .007

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	4.38489	4.09886	3.56439	3.19172	2.91603	2.69877	2.51996	2.36825	2.23662	2.12048
.1	2.01663	1.92279	1.83724	1.75868	1.68611	1.61871	1.55583	1.49694	1.44159	1.38942
.2	1.34010	1.29337	1.24900	1.20679	1.16655	1.12814	1.09141	1.05625	1.02256	.99022
.3	.95916	.92930	.90057	.87289	.84622	.82050	.79567	.77170	.74853	.72614
.4	.70449	.68354	.66326	.64362	.62460	.60617	.58831	.57099	.55419	.53789
.5	.52208	.50674	.49185	.47739	.46336	.44972	.43648	.42362	.41113	.39899
.6	.38720	.37573	.36460	.35377	.34325	.33302	.32308	.31342	.30403	.29490
.7	.28602	.27739	.26901	.26085	.25293	.24522	.23773	.23045	.22337	.21650
.8	.20981	.20331	.19700	.19086	.18490	.17910	.17347	.16800	.16269	.15752
.9	.15251	.14764	.14291	.13831	.13385	.12952	.12531	.12123	.11726	.11342
1.0	.10968	.10606	.10254	.09913	.09582	.09261	.08949	.08647	.08355	.08071
1.1	.07795	.07528	.07270	.07019	.06776	.06541	.06312	.06092	.05878	.05670
1.2	.05470	.05275	.05087	.04905	.04729	.04559	.04394	.04234	.04080	.03931
1.3	.03786	.03647	.03512	.03381	.03255	.03133	.03016	.02902	.02792	.02686
1.4	.02584	.02485	.02390	.02297	.02209	.02123	.02040	.01960	.01883	.01809
1.5	.01737	.01668	.01602	.01538	.01476	.01417	.01359	.01304	.01251	.01200
1.6	.01151	.01103	.01058	.01014	.00971	.00931	.00892	.00854	.00818	.00783
1.7	.00750	.00718	.00687	.00658	.00629	.00602	.00576	.00551	.00527	.00503
1.8	.00481	.00460	.00439	.00420	.00401	.00383	.00366	.00349	.00333	.00318
1.9	.00304	.00290	.00276	.00264	.00251	.00240	.00228	.00218	.00208	.00198
2.0	.00188	.00179	.00171	.00163	.00155	.00147	.00140	.00133	.00127	.00121
2.1	.00115	.00109	.00104	.00099	.00094	.00089	.00085	.00080	.00076	.00072
2.2	.00069	.00065	.00062	.00059	.00056	.00053	.00050	.00047	.00045	.00043
2.3	.00040	.00038	.00036	.00034	.00032	.00031	.00029	.00027	.00026	.00024
2.4	.00023	.00022	.00021	.00019	.00018	.00017	.00016	.00015	.00015	.00014
2.5	.00013	.00012	.00011	.00011	.00010	.00010	.00009	.00008	.00008	.00007
2.6	.00007	.00007	.00006	.00006	.00005	.00005	.00005	.00004	.00004	.00004
2.7	.00004	.00003	.00003	.00003	.00003	.00002	.00002	.00002	.00002	.00002
2.8	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.9	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

7.

M= .008

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	4.25143	4.04161	3.54696	3.18370	2.91148	2.69585	2.51794	2.36677	2.23550	2.11960
.1	2.01592	1.92221	1.83675	1.75827	1.68576	1.61841	1.55557	1.49671	1.44139	1.38924
.2	1.33994	1.29323	1.24887	1.20667	1.16644	1.12804	1.09132	1.05618	1.02248	.99016
.3	.95910	.92925	.90051	.87284	.84617	.82045	.79563	.77166	.74850	.72611
.4	.70446	.68351	.66323	.64360	.62458	.60615	.58829	.57097	.55417	.53788
.5	.52207	.50673	.49184	.47738	.46334	.44971	.43647	.42361	.41112	.39898
.6	.38719	.37573	.36459	.35376	.34324	.33302	.32308	.31341	.30402	.29489
.7	.28602	.27739	.26900	.26085	.25292	.24522	.23773	.23045	.22337	.21649
.8	.20981	.20331	.19699	.19086	.18489	.17910	.17347	.16800	.16269	.15752
.9	.15251	.14764	.14290	.13831	.13385	.12952	.12531	.12123	.11726	.11342
1.0	.10968	.10606	.10254	.09913	.09582	.09261	.08949	.08647	.08354	.08070
1.1	.07795	.07528	.07270	.07019	.06776	.06540	.06312	.06091	.05878	.05670
1.2	.05470	.05275	.05087	.04905	.04729	.04559	.04394	.04234	.04080	.03931
1.3	.03786	.03647	.03512	.03381	.03255	.03133	.03016	.02902	.02792	.02686
1.4	.02584	.02485	.02390	.02297	.02209	.02123	.02040	.01960	.01883	.01809
1.5	.01737	.01668	.01602	.01538	.01476	.01417	.01359	.01304	.01251	.01200
1.6	.01151	.01103	.01058	.01014	.00971	.00931	.00892	.00854	.00818	.00783
1.7	.00750	.00718	.00687	.00658	.00629	.00602	.00576	.00551	.00527	.00503
1.8	.00481	.00460	.00439	.00420	.00401	.00383	.00366	.00349	.00333	.00318
1.9	.00304	.00290	.00276	.00264	.00251	.00240	.00228	.00218	.00208	.00198
2.0	.00188	.00179	.00171	.00163	.00155	.00147	.00140	.00133	.00127	.00121
2.1	.00115	,00109	.00104	.00099	.00094	.00089	.00085	.00080	.00076	.00072
2.2	.00069	.00065	.00062	.00059	.00056	.00053	.00050	.00047	.00045	.00043
2.3	.00040	.00038	.00036	.00034	.00032	.00031	.00029	.00027	.00026	.00024
2.4	.00023	.00022	.00021	.00019	.00018	.00017	.00016	.00015	.00015	.00014
2.5	.00013	.00012	.00011	.00011	.00010	.00010	.00009	.00008	.00008	.00007
2.6	.00007	.00007	.00006	.00006	.00005	.00005	.00005	.00004	.00004	.00004
2.7	.00004	.00003	.00003	.00003	.00003	.00002	.00002	.00002	.00002	.00002
2.8	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.9	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

Table 14. $\int_x^{\infty} \frac{e^{-u^2 - \frac{m^2}{u^2}}}{u}\,du$

M= .009

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	4.13373	3.98121	3.52759	3.17469	2.90634	2.69256	2.51565	2.36509	2.23422	2.11860
.1	2.01512	1.92155	1.83620	1.75781	1.68536	1.61807	1.55527	1.49645	1.44116	1.38904
.2	1.33976	1.29307	1.24873	1.20654	1.16632	1.12793	1.09123	1.05608	1.02240	.99008
.3	.95903	.92918	.90045	.87279	.84612	.82041	.79559	.77162	.74846	.72608
.4	.70443	.68348	.66321	.64357	.62455	.60613	.58826	.57095	.55415	.53786
.5	.52205	.50671	.49182	.47737	.46333	.44970	.43646	.42360	.41111	.39897
.6	.38718	.37572	.36458	.35376	.34324	.33301	.32307	.31341	.30402	.29489
.7	.28601	.27738	.26900	.26084	.25292	.24521	.23772	.23044	.22337	.21649
.8	.20980	.20331	.19699	.19086	.18489	.17910	.17347	.16800	.16268	.15752
.9	.15251	.14763	.14290	.13831	.13385	.12951	.12531	.12123	.11726	.11341
1.0	.10968	.10606	.10254	.09913	.09582	.09261	.08949	.08647	.08354	.08070
1.1	.07795	.07528	.07269	.07019	.06776	.06540	.06312	.06091	.05877	.05670
1.2	.05470	.05275	.05087	.04905	.04729	.04559	.04394	.04234	.04080	.03930
1.3	.03786	.03647	.03512	.03381	.03255	.03133	.03016	.02902	.02792	.02686
1.4	.02584	.02485	.02390	.02297	.02209	.02123	.02040	.01960	.01883	.01809
1.5	.01737	.01668	.01602	.01538	.01476	.01417	.01359	.01304	.01251	.01200
1.6	.01151	.01103	.01058	.01014	.00971	.00931	.00892	.00854	.00818	.00783
1.7	.00750	.00718	.00687	.00658	.00629	.00602	.00576	.00551	.00527	.00503
1.8	.00481	.00460	.00439	.00420	.00401	.00383	.00366	.00349	.00333	.00318
1.9	.00304	.00290	.00276	.00264	.00251	.00240	.00228	.00218	.00208	.00198
2.0	.00188	.00179	.00171	.00163	.00155	.00147	.00140	.00133	.00127	.00121
2.1	.00115	.00109	.00104	.00099	.00094	.00089	.00085	.00080	.00076	.00072
2.2	.00069	.00065	.00062	.00059	.00056	.00053	.00050	.00047	.00045	.00043
2.3	.00040	.00038	.00036	.00034	.00032	.00031	.00029	.00027	.00026	.00024
2.4	.00023	.00022	.00021	.00019	.00018	.00017	.00016	.00015	.00015	.00014
2.5	.00013	.00012	.00011	.00011	.00010	.00010	.00009	.00008	.00008	.00007
2.6	.00007	.00007	.00006	.00006	.00005	.00005	.00005	.00004	.00004	.00004
2.7	.00004	.00003	.00003	.00003	.00003	.00002	.00002	.00002	.00002	.00002
2.8	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.9	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

9.

Table 14.

M= .010

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	4.02846	3.91876	3.50641	3.16471	2.90063	2.68889	2.51310	2.36323	2.23280	2.11748
.1	2.01422	1.92081	1.83559	1.75729	1.68492	1.61768	1.55494	1.49616	1.44091	1.38881
.2	1.33956	1.29289	1.24857	1.20639	1.16619	1.12781	1.09111	1.05598	1.02231	.99000
.3	.95895	.92911	.90039	.87273	.84607	.82035	.79554	.77157	.74842	.72604
.4	.70439	.68345	.66317	.64354	.62453	.60610	.58824	.57092	.55413	.53784
.5	.52203	.50669	.49180	.47735	.46331	.44968	.43645	.42359	.41110	.39896
.6	.38717	.37571	.36457	.35375	.34323	.33300	.32306	.31340	.30401	.29488
.7	.28600	.27738	.26899	.26084	.25291	.24521	.23772	.23044	.22336	.21648
.8	.20980	.20330	.19699	.19085	.18489	.17909	.17346	.16799	.16268	.15752
.9	.15250	.14763	.14290	.13831	.13385	.12951	.12531	.12122	.11726	.11341
1.0	.10968	.10605	.10254	.09913	.09582	.09261	.08949	.08647	.08354	.08070
1.1	.07795	.07528	.07269	.07019	.06776	.06540	.06312	.06091	.05877	.05670
1.2	.05470	.05275	.05087	.04905	.04729	.04559	.04394	.04234	.04080	.03930
1.3	.03786	.03647	.03512	.03381	.03255	.03133	.03016	.02902	.02792	.02686
1.4	.02584	.02485	.02389	.02297	.02209	.02123	.02040	.01960	.01883	.01809
1.5	.01737	.01668	.01602	.01538	.01476	.01417	.01359	.01304	.01251	.01200
1.6	.01151	.01103	.01058	.01014	.00971	.00931	.00892	.00854	.00818	.00783
1.7	.00750	.00718	.00687	.00658	.00629	.00602	.00576	.00551	.00527	.00503
1.8	.00481	.00460	.00439	.00420	.00401	.00383	.00366	.00349	.00333	.00318
1.9	.00304	.00290	.00276	.00264	.00251	.00240	.00228	.00218	.00208	.00198
2.0	.00188	.00179	.00171	.00163	.00155	.00147	.00140	.00133	.00127	.00121
2.1	.00115	.00109	.00104	.00099	.00094	.00089	.00085	.00080	.00076	.00072
2.2	.00069	.00065	.00062	.00059	.00056	.00053	.00050	.00047	.00045	.00043
2.3	.00040	.00038	.00036	.00034	.00032	.00031	.00029	.00027	.00026	.00024
2.4	.00023	.00022	.00021	.00019	.00018	.00017	.00016	.00015	.00015	.00014
2.5	.00013	.00012	.00011	.00011	.00010	.00010	.00009	.00008	.00008	.00007
2.6	.00007	.00007	.00006	.00006	.00005	.00005	.00005	.00004	.00004	.00004
2.7	.00004	.00003	.00003	.00003	.00003	.00002	.00002	.00002	.00002	.00002
2.8	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.9	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

Table 14. $\int_x^\infty \frac{e^{-u^2 - \frac{m^2}{u^2}}}{u}\,du$

M= .020

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	3.33654	3.33465	3.22687	3.02006	2.81481	2.63272	2.47374	2.33424	2.21063	2.10003
.1	2.00016	1.90926	1.82596	1.74916	1.67797	1.61169	1.54972	1.49159	1.43687	1.38523
.2	1.33637	1.29003	1.24599	1.20407	1.16409	1.12590	1.08937	1.05439	1.02085	.98865
.3	.95772	.92797	.89934	.87175	.84517	.81952	.79476	.77085	.74775	.72541
.4	.70380	.68290	.66266	.64306	.62407	.60568	.58784	.57055	.55378	.53751
.5	.52172	.50640	.49153	.47709	.46307	.44945	.43623	.42338	.41090	.39878
.6	.38699	.37554	.36441	.35360	.34309	.33287	.32294	.31328	.30390	.29477
.7	.28590	.27728	.26890	.26075	.25283	.24513	.23764	.23037	.22329	.21642
.8	.20974	.20324	.19693	.19080	.18484	.17905	.17342	.16795	.16264	.15748
.9	.15247	.14760	.14287	.13828	.13382	.12948	.12528	.12120	.11724	.11339
1.0	.10966	.10603	.10252	.09911	.09580	.09259	.08948	.08646	.08353	.08069
1.1	.07794	.07527	.07268	.07018	.06775	.06539	.06311	.06090	.05876	.05669
1.2	.05469	.05274	.05086	.04904	.04728	.04558	.04393	.04233	.04079	.03930
1.3	.03786	.03646	.03511	.03381	.03255	.03133	.03015	.02902	.02792	.02686
1.4	.02583	.02485	.02389	.02297	.02208	.02123	.02040	.01960	.01883	.01809
1.5	.01737	.01668	.01602	.01538	.01476	.01416	.01359	.01304	.01251	.01200
1.6	.01150	.01103	.01057	.01014	.00971	.00931	.00892	.00854	.00818	.00783
1.7	.00750	.00718	.00687	.00658	.00629	.00602	.00576	.00551	.00527	.00503
1.8	.00481	.00460	.00439	.00420	.00401	.00383	.00366	.00349	.00333	.00318
1.9	.00304	.00290	.00276	.00264	.00251	.00240	.00228	.00218	.00207	.00198
2.0	.00188	.00179	.00171	.00163	.00155	.00147	.00140	.00133	.00127	.00121
2.1	.00115	.00109	.00104	.00099	.00094	.00089	.00085	.00080	.00076	.00072
2.2	.00069	.00065	.00062	.00059	.00056	.00053	.00050	.00047	.00045	.00043
2.3	.00040	.00038	.00036	.00034	.00032	.00031	.00029	.00027	.00026	.00024
2.4	.00023	.00022	.00021	.00019	.00018	.00017	.00016	.00015	.00015	.00014
2.5	.00013	.00012	.00011	.00011	.00010	.00010	.00009	.00008	.00008	.00007
2.6	.00007	.00007	.00006	.00006	.00005	.00005	.00005	.00004	.00004	.00004
2.7	.00004	.00003	.00003	.00003	.00003	.00002	.00002	.00002	.00002	.00002
2.8	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.9	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

11.

Table 14.

M= .030

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	2.93287	2.93287	2.91550	2.82325	2.68787	2.54617	2.41166	2.28785	2.17483	2.07166
.1	1.97720	1.89035	1.81014	1.73576	1.66651	1.60179	1.54110	1.48402	1.43020	1.37931
.2	1.33108	1.28529	1.24173	1.20022	1.16060	1.12272	1.08648	1.05174	1.01842	.98643
.3	.95567	.92608	.89759	.87014	.84367	.81813	.79347	.76965	.74663	.72436
.4	.70283	.68198	.66181	.64226	.62332	.60497	.58718	.56993	.55319	.53696
.5	.52120	.50591	.49107	.47666	.46266	.44907	.43586	.42304	.41058	.39847
.6	.38670	.37527	.36415	.35335	.34285	.33265	.32273	.31308	.30371	.29459
.7	.28573	.27712	.26875	.26061	.25269	.24500	.23752	.23025	.22318	.21631
.8	.20964	.20315	.19684	.19071	.18476	.17897	.17335	.16788	.16257	.15742
.9	.15241	.14754	.14281	.13822	.13377	.12944	.12523	.12115	.11719	.11335
1.0	.10962	.10600	.10248	.09908	.09577	.09256	.08945	.08643	.08350	.08067
1.1	.07791	.07525	.07266	.07016	.06773	.06538	.06310	.06089	.05875	.05668
1.2	.05467	.05273	.05085	.04903	.04727	.04557	.04392	.04233	.04078	.03929
1.3	.03785	.03645	.03510	.03380	.03254	.03132	.03015	.02901	.02791	.02685
1.4	.02583	.02484	.02389	.02297	.02208	.02122	.02039	.01960	.01883	.01808
1.5	.01737	.01668	.01602	.01537	.01476	.01416	.01359	.01304	.01251	.01200
1.6	.01150	.01103	.01057	.01013	.00971	.00931	.00892	.00854	.00818	.00783
1.7	.00750	.00718	.00687	.00658	.00629	.00602	.00576	.00551	.00526	.00503
1.8	.00481	.00460	.00439	.00420	.00401	.00383	.00366	.00349	.00333	.00318
1.9	.00304	.00290	.00276	.00264	.00251	.00240	.00228	.00218	.00207	.00198
2.0	.00188	.00179	.00171	.00163	.00155	.00147	.00140	.00133	.00127	.00121
2.1	.00115	.00109	.00104	.00099	.00094	.00089	.00085	.00080	.00076	.00072
2.2	.00069	.00065	.00062	.00059	.00056	.00053	.00050	.00047	.00045	.00043
2.3	.00040	.00038	.00036	.00034	.00032	.00031	.00029	.00027	.00026	.00024
2.4	.00023	.00022	.00021	.00019	.00018	.00017	.00016	.00015	.00015	.00014
2.5	.00013	.00012	.00011	.00011	.00010	.00010	.00009	.00008	.00008	.00007
2.6	.00007	.00007	.00006	.00006	.00005	.00005	.00005	.00004	.00004	.00004
2.7	.00004	.00003	.00003	.00003	.00003	.00002	.00002	.00002	.00002	.00002
2.8	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.9	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

12.

Table 14. $\int_x^\infty \frac{e^{-u^2-\frac{m^2}{u^2}}du}{u}$

M= .040

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	2.64748	2.64748	2.64559	2.61411	2.53791	2.43798	2.33149	2.22672	2.12700	2.03340
.1	1.94601	1.86452	1.78847	1.71735	1.65071	1.58812	1.52917	1.47355	1.42094	1.37108
.2	1.32374	1.27871	1.23580	1.19486	1.15573	1.11830	1.08244	1.04806	1.01504	.98332
.3	.95281	.92344	.89515	.86788	.84158	.81618	.79166	.76797	.74506	.72290
.4	.70146	.68071	.66061	.64114	.62228	.60399	.58626	.56906	.55238	.53619
.5	.52048	.50523	.49043	.47605	.46209	.44853	.43536	.42256	.41012	.39804
.6	.38630	.37488	.36379	.35301	.34253	.33234	.32243	.31280	.30344	.29434
.7	.28550	.27689	.26853	.26040	.25250	.24482	.23735	.23008	.22303	.21617
.8	.20950	.20301	.19671	.19059	.18464	.17886	.17324	.16778	.16248	.15733
.9	.15232	.14746	.14274	.13815	.13370	.12937	.12517	.12109	.11714	.11330
1.0	.10957	.10595	.10244	.09903	.09573	.09252	.08941	.08639	.08347	.08063
1.1	.07788	.07522	.07263	.07013	.06770	.06535	.06307	.06087	.05873	.05666
1.2	.05465	.05271	.05083	.04902	.04726	.04555	.04391	.04231	.04077	.03928
1.3	.03784	.03644	.03509	.03379	.03253	.03131	.03014	.02900	.02791	.02685
1.4	.02582	.02483	.02388	.02296	.02207	.02122	.02039	.01959	.01882	.01808
1.5	.01737	.01668	.01601	.01537	.01475	.01416	.01359	.01304	.01250	.01199
1.6	.01150	.01103	.01057	.01013	.00971	.00930	.00891	.00854	.00818	.00783
1.7	.00750	.00718	.00687	.00657	.00629	.00602	.00576	.00550	.00526	.00503
1.8	.00481	.00460	.00439	.00420	.00401	.00383	.00366	.00349	.00333	.00318
1.9	.00304	.00290	.00276	.00263	.00251	.00240	.00228	.00218	.00207	.00198
2.0	.00188	.00179	.00171	.00163	.00155	.00147	.00140	.00133	.00127	.00121
2.1	.00115	.00109	.00104	.00099	.00094	.00089	.00085	.00080	.00076	.00072
2.2	.00069	.00065	.00062	.00059	.00056	.00053	.00050	.00047	.00045	.00043
2.3	.00040	.00038	.00036	.00034	.00032	.00031	.00029	.00027	.00026	.00024
2.4	.00023	.00022	.00021	.00019	.00018	.00017	.00016	.00015	.00015	.00014
2.5	.00013	.00012	.00011	.00011	.00010	.00010	.00009	.00008	.00008	.00007
2.6	.00007	.00007	.00006	.00006	.00005	.00005	.00005	.00004	.00004	.00004
2.7	.00004	.00003	.00003	.00003	.00003	.00002	.00002	.00002	.00002	.00002
2.8	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.9	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

M= .050

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	2.42706	2.42706	2.42693	2.41840	2.38152	2.31756	2.23862	2.15406	2.06915	1.98656
.1	1.90750	1.83242	1.76138	1.69426	1.63083	1.57086	1.51409	1.46028	1.40920	1.36063
.2	1.31440	1.27032	1.22824	1.18802	1.14953	1.11266	1.07729	1.04334	1.01072	.97935
.3	.94915	.92007	.89203	.86499	.83890	.81370	.78935	.76581	.74305	.72103
.4	.69971	.67907	.65908	.63971	.62093	.60273	.58507	.56795	.55133	.53521
.5	.51955	.50436	.48960	.47528	.46136	.44784	.43470	.42194	.40954	.39749
.6	.38577	.37439	.36332	.35256	.34211	.33194	.32206	.31245	.30311	.29402
.7	.28519	.27661	.26826	.26014	.25225	.24458	.23712	.22987	.22283	.21597
.8	.20931	.20284	.19655	.19044	.18449	.17872	.17311	.16765	.16236	.15721
.9	.15221	.14735	.14264	.13805	.13361	.12928	.12509	.12102	.11706	.11323
1.0	.10950	.10589	.10238	.09897	.09567	.09247	.08936	.08635	.08342	.08059
1.1	.07784	.07518	.07260	.07009	.06767	.06532	.06304	.06084	.05870	.05663
1.2	.05463	.05269	.05081	.04900	.04724	.04553	.04389	.04229	.04075	.03926
1.3	.03782	.03643	.03508	.03378	.03252	.03130	.03013	.02899	.02790	.02684
1.4	.02581	.02483	.02387	.02295	.02207	.02121	.02038	.01958	.01882	.01807
1.5	.01736	.01667	.01601	.01537	.01475	.01416	.01358	.01303	.01250	.01199
1.6	.01150	.01102	.01057	.01013	.00971	.00930	.00891	.00854	.00818	.00783
1.7	.00750	.00718	.00687	.00657	.00629	.00602	.00575	.00550	.00526	.00503
1.8	.00481	.00460	.00439	.00420	.00401	.00383	.00366	.00349	.00333	.00318
1.9	.00303	.00290	.00276	.00263	.00251	.00240	.00228	.00218	.00207	.00198
2.0	.00188	.00179	.00171	.00163	.00155	.00147	.00140	.00133	.00127	.00121
2.1	.00115	.00109	.00104	.00099	.00094	.00089	.00084	.00080	.00076	.00072
2.2	.00069	.00065	.00062	.00059	.00056	.00053	.00050	.00047	.00045	.00043
2.3	.00040	.00038	.00036	.00034	.00032	.00031	.00029	.00027	.00026	.00024
2.4	.00023	.00022	.00021	.00019	.00018	.00017	.00016	.00015	.00015	.00014
2.5	.00013	.00012	.00011	.00011	.00010	.00010	.00009	.00008	.00008	.00007
2.6	.00007	.00007	.00006	.00006	.00005	.00005	.00005	.00004	.00004	.00004
2.7	.00004	.00003	.00003	.00003	.00003	.00002	.00002	.00002	.00002	.00002
2.8	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.9	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

Table 14. $\int_x^\infty \frac{e^{-u^2 - \frac{m^2}{u^2}}}{u}\,du$

M= .060

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	2.24785	2.24785	2.24784	2.24596	2.23049	2.19325	2.13843	2.07332	2.00355	1.93267
.1	1.86271	1.79478	1.72942	1.66687	1.60717	1.55026	1.49604	1.44436	1.39508	1.34805
.2	1.30314	1.26020	1.21911	1.17975	1.14202	1.10582	1.07105	1.03763	1.00548	.97453
.3	.94471	.91597	.88824	.86148	.83564	.81067	.78654	.76319	.74061	.71875
.4	.69758	.67708	.65722	.63796	.61929	.60119	.58363	.56659	.55006	.53401
.5	.51842	.50329	.48860	.47433	.46046	.44699	.43391	.42119	.40883	.39681
.6	.38514	.37379	.36275	.35203	.34160	.33146	.32160	.31201	.30269	.29363
.7	.28482	.27625	.26792	.25983	.25195	.24429	.23685	.22962	.22258	.21574
.8	.20909	.20263	.19635	.19025	.18431	.17855	.17294	.16750	.16221	.15707
.9	.15208	.14723	.14251	.13794	.13350	.12918	.12499	.12092	.11697	.11314
1.0	.10942	.10581	.10230	.09890	.09560	.09240	.08930	.08629	.08337	.08054
1.1	.07779	.07513	.07255	.07005	.06763	.06528	.06301	.06080	.05867	.05660
1.2	.05460	.05266	.05078	.04897	.04721	.04551	.04386	.04227	.04073	.03924
1.3	.03780	.03641	.03506	.03376	.03250	.03129	.03011	.02898	.02788	.02682
1.4	.02580	.02482	.02386	.02294	.02206	.02120	.02037	.01958	.01881	.01807
1.5	.01735	.01666	.01600	.01536	.01474	.01415	.01358	.01303	.01250	.01199
1.6	.01149	.01102	.01056	.01013	.00970	.00930	.00891	.00853	.00817	.00783
1.7	.00749	.00717	.00687	.00657	.00629	.00601	.00575	.00550	.00526	.00503
1.8	.00481	.00459	.00439	.00419	.00401	.00383	.00365	.00349	.00333	.00318
1.9	.00303	.00289	.00276	.00263	.00251	.00239	.00228	.00218	.00207	.00198
2.0	.00188	.00179	.00171	.00163	.00155	.00147	.00140	.00133	.00127	.00121
2.1	.00115	.00109	.00104	.00099	.00094	.00089	.00084	.00080	.00076	.00072
2.2	.00069	.00065	.00062	.00059	.00056	.00053	.00050	.00047	.00045	.00043
2.3	.00040	.00038	.00036	.00034	.00032	.00031	.00029	.00027	.00026	.00024
2.4	.00023	.00022	.00021	.00019	.00018	.00017	.00016	.00015	.00015	.00014
2.5	.00013	.00012	.00011	.00011	.00010	.00010	.00009	.00008	.00008	.00007
2.6	.00007	.00007	.00006	.00006	.00005	.00005	.00005	.00004	.00004	.00004
2.7	.00004	.00003	.00003	.00003	.00003	.00002	.00002	.00002	.00002	.00002
2.8	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.9	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

15.

Table 14.

M= .070

χ	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	2.09716	2.09716	2.09716	2.09682	2.09114	2.07137	2.03567	1.98783	1.93250	1.87332
.1	1.81277	1.75242	1.69319	1.63565	1.58006	1.52657	1.47521	1.42594	1.37871	1.33344
.2	1.29004	1.24841	1.20846	1.17010	1.13325	1.09782	1.06374	1.03093	.99933	.96887
.3	.93950	.91115	.88379	.85736	.83181	.80712	.78323	.76011	.73774	.71607
.4	.69508	.67474	.65502	.63591	.61737	.59938	.58193	.56499	.54855	.53259
.5	.51709	.50204	.48742	.47321	.45941	.44600	.43297	.42030	.40799	.39602
.6	.38439	.37308	.36208	.35139	.34099	.33088	.32106	.31150	.30221	.29317
.7	.28438	.27584	.26753	.25945	.25159	.24396	.23653	.22931	.22229	.21547
.8	.20883	.20238	.19611	.19002	.18410	.17834	.17275	.16732	.16203	.15690
.9	.15192	.14708	.14237	.13780	.13337	.12906	.12487	.12081	.11687	.11304
1.0	.10932	.10572	.10222	.09882	.09552	.09233	.08923	.08622	.08330	.08048
1.1	.07773	.07507	.07250	.07000	.06758	.06523	.06296	.06076	.05863	.05656
1.2	.05456	.05263	.05075	.04894	.04718	.04548	.04384	.04225	.04071	.03922
1.3	.03778	.03639	.03504	.03374	.03249	.03127	.03010	.02896	.02787	.02681
1.4	.02579	.02480	.02385	.02293	.02205	.02119	.02036	.01957	.01880	.01806
1.5	.01735	.01666	.01599	.01535	.01474	.01414	.01357	.01302	.01249	.01198
1.6	.01149	.01102	.01056	.01012	.00970	.00930	.00891	.00853	.00817	.00782
1.7	.00749	.00717	.00686	.00657	.00628	.00601	.00575	.00550	.00526	.00503
1.8	.00481	.00459	.00439	.00419	.00401	.00383	.00365	.00349	.00333	.00318
1.9	.00303	.00289	.00276	.00263	.00251	.00239	.00228	.00218	.00207	.00198
2.0	.00188	.00179	.00171	.00162	.00155	.00147	.00140	.00133	.00127	.00121
2.1	.00115	.00109	.00104	.00099	.00094	.00089	.00084	.00080	.00076	.00072
2.2	.00069	.00065	.00062	.00059	.00056	.00053	.00050	.00047	.00045	.00042
2.3	.00040	.00038	.00036	.00034	.00032	.00031	.00029	.00027	.00026	.00024
2.4	.00023	.00022	.00021	.00019	.00018	.00017	.00016	.00015	.00015	.00014
2.5	.00013	.00012	.00011	.00011	.00010	.00010	.00009	.00008	.00008	.00007
2.6	.00007	.00007	.00006	.00006	.00005	.00005	.00005	.00004	.00004	.00004
2.7	.00004	.00003	.00003	.00003	.00003	.00002	.00002	.00002	.00002	.00002
2.8	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.9	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

Table 14. $\int_x^\infty \frac{e^{-u^2 - \frac{m^2}{u^2}}\,du}{u}$

M= .080

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	1.96741	1.96741	1.96741	1.96736	1.96553	1.95592	1.93411	1.90056	1.85819	1.81013
.1	1.75888	1.70621	1.65335	1.60108	1.54990	1.50010	1.45185	1.40522	1.36025	1.31693
.2	1.27520	1.23504	1.19636	1.15913	1.12327	1.08871	1.05541	1.02329	.99231	.96241
.3	.93354	.90565	.87869	.85264	.82743	.80304	.77944	.75658	.73444	.71299
.4	.69220	.67205	.65250	.63354	.61515	.59730	.57998	.56316	.54683	.53097
.5	.51556	.50060	.48606	.47193	.45820	.44486	.43189	.41928	.40702	.39511
.6	.38352	.37226	.36131	.35066	.34030	.33023	.32043	.31091	.30165	.29264
.7	.28388	.27536	.26707	.25902	.25119	.24357	.23616	.22896	.22196	.21515
.8	.20853	.20210	.19584	.18976	.18385	.17811	.17253	.16710	.16183	.15671
.9	.15174	.14690	.14221	.13765	.13322	.12892	.12474	.12068	.11674	.11292
1.0	.10921	.10561	.10212	.09872	.09543	.09224	.08915	.08614	.08323	.08040
1.1	.07767	.07501	.07244	.06994	.06752	.06518	.06291	.06071	.05858	.05652
1.2	.05452	.05259	.05072	.04890	.04715	.04545	.04381	.04222	.04068	.03919
1.3	.03776	.03637	.03502	.03372	.03247	.03125	.03008	.02895	.02785	.02680
1.4	.02577	.02479	.02384	.02292	.02203	.02118	.02035	.01956	.01879	.01805
1.5	.01734	.01665	.01599	.01535	.01473	.01414	.01357	.01302	.01249	.01197
1.6	.01148	.01101	.01056	.01012	.00970	.00929	.00890	.00853	.00817	.00782
1.7	.00749	.00717	.00686	.00657	.00628	.00601	.00575	.00550	.00526	.00503
1.8	.00480	.00459	.00439	.00419	.00400	.00382	.00365	.00349	.00333	.00318
1.9	.00303	.00289	.00276	.00263	.00251	.00239	.00228	.00217	.00207	.00197
2.0	.00188	.00179	.00171	.00162	.00155	.00147	.00140	.00133	.00127	.00121
2.1	.00115	.00109	.00104	.00098	.00094	.00089	.00084	.00080	.00076	.00072
2.2	.00069	.00065	.00062	.00059	.00056	.00053	.00050	.00047	.00045	.00042
2.3	.00040	.00038	.00036	.00034	.00032	.00031	.00029	.00027	.00026	.00024
2.4	.00023	.00022	.00021	.00019	.00018	.00017	.00016	.00015	.00015	.00014
2.5	.00013	.00012	.00011	.00011	.00010	.00010	.00009	.00008	.00008	.00007
2.6	.00007	.00007	.00006	.00006	.00005	.00005	.00005	.00004	.00004	.00004
2.7	.00004	.00003	.00003	.00003	.00003	.00002	.00002	.00002	.00002	.00002
2.8	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.9	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

Table 14.

M= .090

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	1.85371	1.85371	1.85371	1.85370	1.85317	1.84890	1.83638	1.81391	1.78259	1.74461
.1	1.70217	1.65705	1.61057	1.56370	1.51708	1.47115	1.42620	1.38241	1.33987	1.29864
.2	1.25875	1.22017	1.18290	1.14689	1.11212	1.07853	1.04608	1.01474	.98445	.95516
.3	.92685	.89947	.87297	.84733	.82250	.79846	.77517	.75261	.73073	.70953
.4	.68896	.66902	.64966	.63088	.61265	.59496	.57777	.56109	.54488	.52913
.5	.51383	.49897	.48452	.47048	.45683	.44356	.43067	.41813	.40593	.39408
.6	.38255	.37133	.36043	.34983	.33951	.32948	.31973	.31024	.30101	.29204
.7	.28331	.27482	.26656	.25853	.25072	.24313	.23574	.22856	.22158	.21479
.8	.20819	.20177	.19553	.18947	.18357	.17784	.17228	.16686	.16160	.15649
.9	.15153	.14671	.14202	.13747	.13305	.12875	.12458	.12054	.11661	.11279
1.0	.10909	.10549	.10200	.09862	.09533	.09214	.08905	.08605	.08314	.08032
1.1	.07759	.07494	.07237	.06988	.06746	.06512	.06285	.06066	.05853	.05647
1.2	.05448	.05254	.05067	.04886	.04711	.04541	.04377	.04219	.04065	.03916
1.3	.03773	.03634	.03500	.03370	.03244	.03123	.03006	.02893	.02783	.02678
1.4	.02576	.02477	.02382	.02290	.02202	.02116	.02034	.01955	.01878	.01804
1.5	.01733	.01664	.01598	.01534	.01472	.01413	.01356	.01301	.01248	.01197
1.6	.01148	.01100	.01055	.01011	.00969	.00929	.00890	.00852	.00816	.00782
1.7	.00748	.00716	.00686	.00656	.00628	.00601	.00575	.00550	.00525	.00502
1.8	.00480	.00459	.00439	.00419	.00400	.00382	.00365	.00349	.00333	.00318
1.9	.00303	.00289	.00276	.00263	.00251	.00239	.00228	.00217	.00207	.00197
2.0	.00188	.00179	.00171	.00162	.00155	.00147	.00140	.00133	.00127	.00121
2.1	.00115	.00109	.00104	.00098	.00094	.00089	.00084	.00080	.00076	.00072
2.2	.00069	.00065	.00062	.00059	.00056	.00053	.00050	.00047	.00045	.00042
2.3	.00040	.00038	.00036	.00034	.00032	.00031	.00029	.00027	.00026	.00024
2.4	.00023	.00022	.00021	.00019	.00018	.00017	.00016	.00015	.00015	.00014
2.5	.00013	.00012	.00011	.00011	.00010	.00010	.00009	.00008	.00008	.00007
2.6	.00007	.00007	.00006	.00006	.00005	.00005	.00005	.00004	.00004	.00004
2.7	.00004	.00003	.00003	.00003	.00003	.00002	.00002	.00002	.00002	.00002
2.8	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.9	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

Table 14. $\int_x^\infty \frac{e^{-u^2 - \frac{m^2}{u^2}}\,du}{u}$

M= .100

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	1.75270	1.75270	1.75270	1.75270	1.75256	1.75081	1.74405	1.72967	1.70732	1.67812
.1	1.64375	1.60577	1.56553	1.52403	1.48203	1.44008	1.39855	1.35771	1.31773	1.27873
.2	1.24078	1.20391	1.16814	1.13347	1.09987	1.06733	1.03582	1.00531	.97577	.94716
.3	.91946	.89263	.86664	.84145	.81704	.79338	.77044	.74820	.72662	.70568
.4	.68537	.66565	.64651	.62792	.60988	.59235	.57532	.55878	.54271	.52709
.5	.51191	.49715	.48281	.46887	.45531	.44213	.42931	.41684	.40472	.39293
.6	.38146	.37031	.35946	.34890	.33864	.32865	.31894	.30949	.30030	.29137
.7	.28267	.27421	.26599	.25799	.25020	.24264	.23528	.22812	.22116	.21439
.8	.20781	.20141	.19519	.18914	.18326	.17755	.17199	.16660	.16135	.15625
.9	.15130	.14649	.14181	.13727	.13286	.12857	.12441	.12037	.11645	.11264
1.0	.10895	.10536	.10187	.09850	.09522	.09203	.08895	.08595	.08305	.08023
1.1	.07750	.07486	.07229	.06980	.06739	.06505	.06279	.06060	.05847	.05642
1.2	.05442	.05249	.05063	.04882	.04707	.04537	.04373	.04215	.04061	.03913
1.3	.03770	.03631	.03497	.03367	.03242	.03121	.03003	.02890	.02781	.02676
1.4	.02574	.02475	.02380	.02289	.02200	.02115	.02033	.01953	.01877	.01803
1.5	.01731	.01663	.01597	.01533	.01471	.01412	.01355	.01300	.01247	.01196
1.6	.01147	.01100	.01054	.01011	.00969	.00928	.00889	.00852	.00816	.00781
1.7	.00748	.00716	.00685	.00656	.00628	.00600	.00574	.00549	.00525	.00502
1.8	.00480	.00459	.00438	.00419	.00400	.00382	.00365	.00348	.00333	.00317
1.9	.00303	.00289	.00276	.00263	.00251	.00239	.00228	.00217	.00207	.00197
2.0	.00188	.00179	.00170	.00162	.00155	.00147	.00140	.00133	.00127	.00120
2.1	.00115	.00109	.00104	.00098	.00094	.00089	.00084	.00080	.00076	.00072
2.2	.00069	.00065	.00062	.00059	.00056	.00053	.00050	.00047	.00045	.00042
2.3	.00040	.00038	.00036	.00034	.00032	.00031	.00029	.00027	.00026	.00024
2.4	.00023	.00022	.00021	.00019	.00018	.00017	.00016	.00015	.00015	.00014
2.5	.00013	.00012	.00011	.00011	.00010	.00010	.00009	.00008	.00008	.00007
2.6	.00007	.00007	.00006	.00006	.00005	.00005	.00005	.00004	.00004	.00004
2.7	.00004	.00003	.00003	.00003	.00003	.00002	.00002	.00002	.00002	.00002
2.8	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.9	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

M= .200

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	1.11452	1.11452	1.11452	1.11452	1.11452	1.11452	1.11452	1.11451	1.11439	1.11391
.1	1.11265	1.11013	1.10595	1.09986	1.09176	1.08169	1.06978	1.05622	1.04122	1.02499
.2	1.00776	.98971	.97102	.95186	.93234	.91261	.89275	.87285	.85299	.83322
.3	.81360	.79416	.77496	.75600	.73732	.71893	.70085	.68309	.66566	.64856
.4	.63180	.61538	.59930	.58356	.56816	.55309	.53837	.52397	.50990	.49615
.5	.48272	.46960	.45679	.44429	.43208	.42016	.40853	.39718	.38611	.37531
.6	.36478	.35450	.34448	.33471	.32518	.31589	.30683	.29800	.28940	.28101
.7	.27284	.26488	.25712	.24957	.24220	.23503	.22805	.22125	.21463	.20818
.8	.20191	.19580	.18985	.18407	.17844	.17296	.16763	.16244	.15739	.15249
.9	.14772	.14308	.13857	.13418	.12992	.12578	.12175	.11784	.11404	.11035
1.0	.10676	.10328	.09989	.09661	.09342	.09032	.08732	.08440	.08157	.07883
1.1	.07617	.07358	.07108	.06865	.06629	.06401	.06180	.05965	.05757	.05556
1.2	.05361	.05172	.04989	.04811	.04640	.04474	.04313	.04157	.04007	.03861
1.3	.03720	.03584	.03452	.03324	.03201	.03082	.02967	.02856	.02748	.02644
1.4	.02544	.02447	.02353	.02263	.02176	.02092	.02011	.01932	.01857	.01784
1.5	.01713	.01646	.01580	.01517	.01457	.01398	.01342	.01288	.01235	.01185
1.6	.01136	.01090	.01045	.01002	.00960	.00920	.00881	.00844	.00809	.00775
1.7	.00742	.00710	.00680	.00651	.00623	.00596	.00570	.00545	.00521	.00498
1.8	.00476	.00455	.00435	.00416	.00397	.00379	.00362	.00346	.00330	.00315
1.9	.00301	.00287	.00274	.00261	.00249	.00238	.00226	.00216	.00206	.00196
2.0	.00187	.00178	.00169	.00161	.00154	.00146	.00139	.00132	.00126	.00120
2.1	.00114	.00108	.00103	.00098	.00093	.00088	.00084	.00080	.00076	.00072
2.2	.00068	.00065	.00061	.00058	.00055	.00052	.00050	.00047	.00045	.00042
2.3	.00040	.00038	.00036	.00034	.00032	.00030	.00029	.00027	.00026	.00024
2.4	.00023	.00022	.00020	.00019	.00018	.00017	.00016	.00015	.00014	.00014
2.5	.00013	.00012	.00011	.00011	.00010	.00009	.00009	.00008	.00008	.00007
2.6	.00007	.00006	.00006	.00006	.00005	.00005	.00005	.00004	.00004	.00004
2.7	.00004	.00003	.00003	.00003	.00003	.00002	.00002	.00002	.00002	.00002
2.8	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.9	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

Table 14. $\displaystyle\int_x^\infty \frac{e^{-u^2 - \frac{m^2}{u^2}}}{u}\,du$

M= .300

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.77752	.77752	.77752	.77752	.77752	.77752	.77752	.77752	.77752	.77752
.1	.77751	.77748	.77738	.77713	.77660	.77566	.77417	.77200	.76907	.76530
.2	.76067	.75518	.74884	.74170	.73381	.72523	.71603	.70627	.69601	.68533
.3	.67429	.66294	.65133	.63953	.62756	.61548	.60333	.59113	.57891	.56671
.4	.55454	.54244	.53041	.51848	.50665	.49495	.48338	.47196	.46069	.44957
.5	.43862	.42784	.41723	.40680	.39655	.38648	.37659	.36689	.35737	.34804
.6	.33889	.32993	.32115	.31255	.30413	.29589	.28783	.27995	.27224	.26470
.7	.25733	.25013	.24310	.23623	.22952	.22297	.21657	.21033	.20423	.19829
.8	.19249	.18684	.18132	.17594	.17070	.16559	.16062	.15577	.15104	.14644
.9	.14195	.13759	.13334	.12920	.12518	.12126	.11745	.11374	.11014	.10663
1.0	.10322	.09991	.09668	.09355	.09051	.08755	.08468	.08189	.07918	.07655
1.1	.07399	.07151	.06911	.06677	.06451	.06231	.06018	.05811	.05610	.05416
1.2	.05228	.05045	.04868	.04697	.04531	.04370	.04214	.04063	.03917	.03776
1.3	.03639	.03506	.03378	.03255	.03135	.03019	.02907	.02798	.02694	.02593
1.4	.02495	.02400	.02309	.02221	.02136	.02054	.01974	.01898	.01824	.01753
1.5	.01684	.01618	.01554	.01492	.01433	.01375	.01320	.01267	.01216	.01166
1.6	.01119	.01073	.01029	.00987	.00946	.00906	.00869	.00832	.00797	.00764
1.7	.00731	.00700	.00670	.00642	.00614	.00588	.00562	.00538	.00514	.00492
1.8	.00470	.00450	.00430	.00411	.00392	.00375	.00358	.00342	.00326	.00312
1.9	.00297	.00284	.00271	.00258	.00246	.00235	.00224	.00214	.00204	.00194
2.0	.00185	.00176	.00168	.00160	.00152	.00145	.00138	.00131	.00125	.00119
2.1	.00113	.00107	.00102	.00097	.00092	.00088	.00083	.00079	.00075	.00071
2.2	.00068	.00064	.00061	.00058	.00055	.00052	.00049	.00047	.00044	.00042
2.3	.00040	.00038	.00036	.00034	.00032	.00030	.00029	.00027	.00026	.00024
2.4	.00023	.00022	.00020	.00019	.00018	.00017	.00016	.00015	.00014	.00014
2.5	.00013	.00012	.00011	.00011	.00010	.00009	.00009	.00008	.00008	.00007
2.6	.00007	.00006	.00006	.00006	.00005	.00005	.00005	.00004	.00004	.00004
2.7	.00004	.00003	.00003	.00003	.00003	.00002	.00002	.00002	.00002	.00002
2.8	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.9	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

M= .400

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.56534	.56534	.56534	.56534	.56534	.56534	.56534	.56534	.56534	.56534
.1	.56534	.56534	.56534	.56534	.56533	.56529	.56521	.56504	.56474	.56425
.2	.56351	.56247	.56108	.55928	.55706	.55439	.55125	.54765	.54358	.53907
.3	.53413	.52878	.52305	.51697	.51055	.50385	.49687	.48966	.48223	.47462
.4	.46686	.45896	.45095	.44285	.43468	.42646	.41821	.40995	.40168	.39343
.5	.38520	.37701	.36886	.36077	.35274	.34479	.33691	.32912	.32142	.31381
.6	.30630	.29890	.29160	.28442	.27734	.27038	.26353	.25680	.25019	.24370
.7	.23732	.23107	.22494	.21892	.21303	.20725	.20159	.19605	.19063	.18533
.8	.18014	.17506	.17010	.16525	.16050	.15587	.15135	.14693	.14262	.13841
.9	.13430	.13030	.12639	.12258	.11886	.11524	.11171	.10827	.10492	.10166
1.0	.09848	.09539	.09238	.08945	.08660	.08382	.08112	.07850	.07595	.07347
1.1	.07106	.06872	.06645	.06424	.06209	.06001	.05798	.05602	.05412	.05227
1.2	.05047	.04873	.04704	.04541	.04382	.04228	.04079	.03935	.03795	.03660
1.3	.03528	.03401	.03278	.03159	.03044	.02933	.02825	.02720	.02620	.02522
1.4	.02428	.02336	.02248	.02163	.02081	.02002	.01925	.01851	.01779	.01710
1.5	.01644	.01579	.01517	.01457	.01400	.01344	.01290	.01239	.01189	.01141
1.6	.01095	.01050	.01007	.00966	.00926	.00888	.00851	.00816	.00782	.00749
1.7	.00717	.00687	.00658	.00630	.00603	.00577	.00552	.00528	.00505	.00483
1.8	.00462	.00442	.00422	.00404	.00386	.00368	.00352	.00336	.00321	.00306
1.9	.00293	.00279	.00266	.00254	.00243	.00231	.00221	.00210	.00200	.00191
2.0	.00182	.00173	.00165	.00157	.00150	.00143	.00136	.00129	.00123	.00117
2.1	.00111	.00106	.00101	.00096	.00091	.00086	.00082	.00078	.00074	.00070
2.2	.00067	.00063	.00060	.00057	.00054	.00051	.00049	.00046	.00044	.00041
2.3	.00039	.00037	.00035	.00033	.00032	.00030	.00028	.00027	.00025	.00024
2.4	.00023	.00021	.00020	.00019	.00018	.00017	.00016	.00015	.00014	.00013
2.5	.00013	.00012	.00011	.00011	.00010	.00009	.00009	.00008	.00008	.00007
2.6	.00007	.00006	.00006	.00006	.00005	.00005	.00005	.00004	.00004	.00004
2.7	.00003	.00003	.00003	.00003	.00003	.00002	.00002	.00002	.00002	.00002
2.8	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.9	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

Table 14. $\displaystyle\int_x^{\infty} \frac{e^{-u^2-\frac{m^2}{u^2}}\,du}{u}$

M= .500

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.42102	.42102	.42102	.42102	.42102	.42102	.42102	.42102	.42102	.42102
.1	.42102	.42102	.42102	.42102	.42102	.42102	.42102	.42101	.42099	.42096
.2	.42089	.42076	.42056	.42026	.41982	.41923	.41844	.41744	.41620	.41471
.3	.41295	.41092	.40861	.40601	.40313	.39998	.39657	.39290	.38898	.38484
.4	.38047	.37591	.37116	.36624	.36116	.35595	.35061	.34516	.33961	.33398
.5	.32828	.32252	.31672	.31088	.30502	.29915	.29327	.28739	.28152	.27568
.6	.26986	.26407	.25832	.25261	.24695	.24134	.23579	.23030	.22487	.21951
.7	.21422	.20900	.20386	.19879	.19380	.18889	.18406	.17931	.17464	.17006
.8	.16556	.16114	.15680	.15255	.14839	.14430	.14030	.13639	.13255	.12880
.9	.12513	.12154	.11803	.11460	.11125	.10797	.10478	.10165	.09861	.09563
1.0	.09273	.08990	.08714	.08445	.08183	.07928	.07679	.07437	.07201	.06971
1.1	.06748	.06530	.06319	.06113	.05913	.05718	.05529	.05345	.05167	.04993
1.2	.04825	.04661	.04503	.04348	.04199	.04054	.03913	.03777	.03644	.03516
1.3	.03392	.03271	.03154	.03041	.02932	.02826	.02723	.02623	.02527	.02434
1.4	.02344	.02257	.02173	.02091	.02013	.01937	.01863	.01792	.01723	.01657
1.5	.01593	.01531	.01472	.01414	.01359	.01305	.01253	.01204	.01155	.01109
1.6	.01065	.01021	.00980	.00940	.00902	.00865	.00829	.00795	.00762	.00730
1.7	.00699	.00670	.00642	.00614	.00588	.00563	.00539	.00516	.00494	.00472
1.8	.00452	.00432	.00413	.00395	.00377	.00361	.00344	.00329	.00314	.00300
1.9	.00287	.00274	.00261	.00249	.00238	.00227	.00216	.00206	.00197	.00187
2.0	.00179	.00170	.00162	.00154	.00147	.00140	.00133	.00127	.00121	.00115
2.1	.00109	.00104	.00099	.00094	.00089	.00085	.00081	.00077	.00073	.00069
2.2	.00066	.00062	.00059	.00056	.00053	.00051	.00048	.00045	.00043	.00041
2.3	.00039	.00037	.00035	.00033	.00031	.00029	.00028	.00026	.00025	.00024
2.4	.00022	.00021	.00020	.00019	.00018	.00017	.00016	.00015	.00014	.00013
2.5	.00012	.00012	.00011	.00010	.00010	.00009	.00009	.00008	.00008	.00007
2.6	.00007	.00006	.00006	.00006	.00005	.00005	.00005	.00004	.00004	.00004
2.7	.00003	.00003	.00003	.00003	.00003	.00002	.00002	.00002	.00002	.00002
2.8	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.9	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

M= .600

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.31850	.31850	.31850	.31850	.31850	.31850	.31850	.31850	.31850	.31850
.1	.31850	.31850	.31850	.31850	.31850	.31850	.31850	.31850	.31850	.31850
.2	.31850	.31849	.31847	.31843	.31837	.31828	.31813	.31792	.31763	.31725
.3	.31675	.31613	.31536	.31445	.31337	.31212	.31069	.30908	.30729	.30532
.4	.30316	.30083	.29832	.29564	.29280	.28981	.28667	.28339	.27999	.27646
.5	.27282	.26908	.26525	.26134	.25734	.25329	.24917	.24501	.24080	.23656
.6	.23230	.22801	.22371	.21940	.21509	.21079	.20649	.20221	.19795	.19371
.7	.18950	.18532	.18117	.17706	.17299	.16896	.16498	.16104	.15715	.15332
.8	.14954	.14581	.14214	.13852	.13496	.13146	.12802	.12464	.12132	.11806
.9	.11486	.11172	.10864	.10562	.10267	.09977	.09694	.09416	.09145	.08880
1.0	.08620	.08366	.08118	.07876	.07640	.07409	.07183	.06963	.06749	.06540
1.1	.06336	.06137	.05943	.05755	.05571	.05392	.05218	.05048	.04884	.04723
1.2	.04567	.04416	.04268	.04125	.03986	.03851	.03720	.03592	.03468	.03348
1.3	.03232	.03119	.03009	.02903	.02800	.02700	.02604	.02510	.02419	.02331
1.4	.02246	.02164	.02084	.02007	.01932	.01860	.01790	.01723	.01658	.01594
1.5	.01534	.01475	.01418	.01363	.01310	.01259	.01209	.01162	.01116	.01072
1.6	.01029	.00988	.00948	.00910	.00873	.00837	.00803	.00770	.00738	.00708
1.7	.00678	.00650	.00623	.00596	.00571	.00547	.00524	.00501	.00480	.00459
1.8	.00439	.00420	.00402	.00384	.00367	.00351	.00335	.00321	.00306	.00293
1.9	.00279	.00267	.00255	.00243	.00232	.00221	.00211	.00201	.00192	.00183
2.0	.00175	.00166	.00158	.00151	.00144	.00137	.00130	.00124	.00118	.00112
2.1	.00107	.00102	.00097	.00092	.00088	.00083	.00079	.00075	.00071	.00068
2.2	.00064	.00061	.00058	.00055	.00052	.00050	.00047	.00045	.00042	.00040
2.3	.00038	.00036	.00034	.00032	.00031	.00029	.00027	.00026	.00024	.00023
2.4	.00022	.00021	.00020	.00018	.00017	.00016	.00015	.00015	.00014	.00013
2.5	.00012	.00012	.00011	.00010	.00010	.00009	.00009	.00008	.00008	.00007
2.6	.00007	.00006	.00006	.00005	.00005	.00005	.00004	.00004	.00004	.00004
2.7	.00003	.00003	.00003	.00003	.00003	.00002	.00002	.00002	.00002	.00002
2.8	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.9	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

Table 14. $\int_x^\infty \frac{e^{-u^2 - \frac{m^2}{u^2}}\,du}{u}$

M= .700

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.24365	.24365	.24365	.24365	.24365	.24365	.24365	.24365	.24365	.24365
.1	.24365	.24365	.24365	.24365	.24365	.24365	.24365	.24365	.24365	.24365
.2	.24365	.24365	.24365	.24364	.24364	.24363	.24361	.24358	.24352	.24345
.3	.24333	.24318	.24297	.24270	.24237	.24195	.24144	.24083	.24012	.23930
.4	.23836	.23730	.23612	.23482	.23339	.23184	.23017	.22837	.22645	.22442
.5	.22228	.22004	.21769	.21525	.21271	.21009	.20740	.20463	.20179	.19889
.6	.19593	.19293	.18988	.18679	.18367	.18052	.17735	.17416	.17095	.16774
.7	.16452	.16130	.15808	.15487	.15167	.14849	.14532	.14216	.13903	.13593
.8	.13285	.12980	.12678	.12379	.12084	.11792	.11504	.11220	.10940	.10664
.9	.10391	.10124	.09860	.09601	.09346	.09096	.08850	.08609	.08372	.08140
1.0	.07912	.07689	.07471	.07257	.07047	.06842	.06642	.06446	.06254	.06067
1.1	.05884	.05705	.05530	.05360	.05194	.05032	.04874	.04720	.04570	.04424
1.2	.04282	.04143	.04008	.03877	.03749	.03625	.03504	.03386	.03272	.03161
1.3	.03053	.02949	.02847	.02748	.02653	.02560	.02470	.02382	.02298	.02215
1.4	.02136	.02059	.01984	.01912	.01842	.01774	.01708	.01645	.01583	.01524
1.5	.01466	.01411	.01357	.01305	.01255	.01206	.01160	.01114	.01071	.01029
1.6	.00988	.00949	.00911	.00875	.00840	.00806	.00773	.00741	.00711	.00682
1.7	.00654	.00627	.00601	.00576	.00551	.00528	.00506	.00484	.00464	.00444
1.8	.00425	.00407	.00389	.00372	.00356	.00340	.00325	.00311	.00297	.00284
1.9	.00271	.00259	.00247	.00236	.00225	.00215	.00205	.00196	.00187	.00178
2.0	.00170	.00162	.00154	.00147	.00140	.00133	.00127	.00121	.00115	.00110
2.1	.00104	.00099	.00094	.00090	.00085	.00081	.00077	.00073	.00070	.00066
2.2	.00063	.00060	.00057	.00054	.00051	.00048	.00046	.00044	.00041	.00039
2.3	.00037	.00035	.00033	.00032	.00030	.00028	.00027	.00025	.00024	.00023
2.4	.00021	.00020	.00019	.00018	.00017	.00016	.00015	.00014	.00014	.00013
2.5	.00012	.00011	.00011	.00010	.00009	.00009	.00008	.00008	.00007	.00007
2.6	.00007	.00006	.00006	.00005	.00005	.00005	.00004	.00004	.00004	.00004
2.7	.00003	.00003	.00003	.00003	.00002	.00002	.00002	.00002	.00002	.00002
2.8	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.9	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

Table 14.

M= .800

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.18795	.18795	.18795	.18795	.18795	.18795	.18795	.18795	.18795	.18795
.1	.18795	.18795	.18795	.18795	.18795	.18795	.18795	.18795	.18795	.18795
.2	.18795	.18795	.18795	.18795	.18795	.18795	.18795	.18794	.18793	.18792
.3	.18790	.18787	.18783	.18776	.18767	.18755	.18740	.18720	.18696	.18666
.4	.18630	.18587	.18538	.18481	.18417	.18344	.18263	.18173	.18074	.17967
.5	.17851	.17726	.17593	.17451	.17301	.17143	.16977	.16804	.16624	.16437
.6	.16243	.16044	.15839	.15628	.15413	.15194	.14970	.14743	.14512	.14279
.7	.14043	.13805	.13565	.13324	.13081	.12838	.12594	.12350	.12106	.11863
.8	.11620	.11378	.11137	.10898	.10659	.10423	.10188	.09956	.09725	.09497
.9	.09272	.09049	.08828	.08611	.08396	.08185	.07976	.07771	.07569	.07370
1.0	.07174	.06982	.06793	.06607	.06425	.06247	.06071	.05900	.05731	.05566
1.1	.05405	.05247	.05092	.04941	.04793	.04649	.04508	.04370	.04235	.04104
1.2	.03976	.03851	.03729	.03610	.03494	.03381	.03271	.03164	.03060	.02959
1.3	.02860	.02764	.02671	.02581	.02493	.02407	.02324	.02244	.02165	.02089
1.4	.02016	.01944	.01875	.01808	.01742	.01679	.01618	.01559	.01502	.01446
1.5	.01392	.01340	.01290	.01241	.01194	.01149	.01105	.01062	.01021	.00982
1.6	.00943	.00906	.00871	.00836	.00803	.00771	.00740	.00710	.00681	.00654
1.7	.00627	.00601	.00577	.00553	.00530	.00508	.00486	.00466	.00446	.00427
1.8	.00409	.00391	.00375	.00358	.00343	.00328	.00314	.00300	.00287	.00274
1.9	.00262	.00250	.00239	.00228	.00218	.00208	.00199	.00189	.00181	.00172
2.0	.00164	.00157	.00150	.00143	.00136	.00129	.00123	.00117	.00112	.00106
2.1	.00101	.00096	.00092	.00087	.00083	.00079	.00075	.00071	.00068	.00064
2.2	.00061	.00058	.00055	.00052	.00050	.00047	.00045	.00043	.00040	.00038
2.3	.00036	.00034	.00033	.00031	.00029	.00028	.00026	.00025	.00023	.00022
2.4	.00021	.00020	.00019	.00018	.00017	.00016	.00015	.00014	.00013	.00012
2.5	.00012	.00011	.00010	.00010	.00009	.00009	.00008	.00008	.00007	.00007
2.6	.00006	.00006	.00006	.00005	.00005	.00005	.00004	.00004	.00004	.00004
2.7	.00003	.00003	.00003	.00003	.00002	.00002	.00002	.00002	.00002	.00002
2.8	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.9	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

Table 14. $\int_x^{\infty} \frac{e^{-u^2 \frac{m^2}{u^2}} du}{u}$

M= .900

χ	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.14593	.14593	.14593	.14593	.14593	.14593	.14593	.14593	.14593	.14593
.1	.14593	.14593	.14593	.14593	.14593	.14593	.14593	.14593	.14593	.14593
.2	.14593	.14593	.14593	.14593	.14593	.14593	.14593	.14593	.14592	.14592
.3	.14592	.14591	.14591	.14589	.14587	.14585	.14581	.14575	.14568	.14558
.4	.14546	.14531	.14513	.14491	.14464	.14433	.14398	.14357	.14310	.14258
.5	.14200	.14136	.14066	.13989	.13907	.13817	.13722	.13620	.13512	.13398
.6	.13279	.13153	.13023	.12886	.12745	.12599	.12449	.12294	.12135	.11973
.7	.11807	.11638	.11466	.11291	.11114	.10935	.10754	.10572	.10388	.10204
.8	.10018	.09832	.09645	.09459	.09272	.09086	.08900	.08714	.08530	.08346
.9	.08164	.07983	.07803	.07625	.07448	.07273	.07100	.06929	.06760	.06593
1.0	.06428	.06266	.06105	.05948	.05792	.05639	.05489	.05341	.05196	.05053
1.1	.04913	.04775	.04640	.04508	.04379	.04252	.04128	.04006	.03887	.03770
1.2	.03657	.03545	.03437	.03331	.03227	.03126	.03027	.02931	.02837	.02746
1.3	.02657	.02570	.02486	.02404	.02324	.02246	.02170	.02096	.02025	.01955
1.4	.01888	.01822	.01759	.01697	.01637	.01579	.01522	.01467	.01414	.01363
1.5	.01313	.01265	.01218	.01173	.01129	.01087	.01046	.01006	.00968	.00931
1.6	.00895	.00860	.00827	.00795	.00763	.00733	.00704	.00676	.00649	.00623
1.7	.00598	.00574	.00550	.00528	.00506	.00485	.00465	.00446	.00427	.00409
1.8	.00392	.00375	.00359	.00344	.00329	.00315	.00301	.00288	.00275	.00263
1.9	.00252	.00241	.00230	.00220	.00210	.00200	.00191	.00183	.00174	.00166
2.0	.00159	.00151	.00144	.00138	.00131	.00125	.00119	.00114	.00108	.00103
2.1	.00098	.00093	.00089	.00085	.00080	.00077	.00073	.00069	.00066	.00063
2.2	.00059	.00056	.00054	.00051	.00048	.00046	.00044	.00041	.00039	.00037
2.3	.00035	.00033	.00032	.00030	.00028	.00027	.00025	.00024	.00023	.00022
2.4	.00020	.00019	.00018	.00017	.00016	.00015	.00015	.00014	.00013	.00012
2.5	.00011	.00011	.00010	.00010	.00009	.00009	.00008	.00008	.00007	.00007
2.6	.00006	.00006	.00005	.00005	.00005	.00005	.00004	.00004	.00004	.00003
2.7	.00003	.00003	.00003	.00003	.00002	.00002	.00002	.00002	.00002	.00002
2.8	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.9	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

Table 14.

M=1.000

χ	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.11389	.11389	.11389	.11389	.11389	.11389	.11389	.11389	.11389	.11389
.1	.11389	.11389	.11389	.11389	.11389	.11389	.11389	.11389	.11389	.11389
.2	.11389	.11389	.11389	.11389	.11389	.11389	.11389	.11389	.11389	.11389
.3	.11389	.11389	.11389	.11388	.11388	.11387	.11387	.11385	.11383	.11381
.4	.11377	.11372	.11366	.11358	.11349	.11337	.11323	.11306	.11286	.11263
.5	.11236	.11205	.11171	.11133	.11090	.11043	.10991	.10935	.10875	.10809
.6	.10739	.10665	.10586	.10502	.10414	.10321	.10225	.10124	.10019	.09911
.7	.09799	.09684	.09565	.09443	.09319	.09192	.09062	.08930	.08796	.08661
.8	.08523	.08384	.08244	.08103	.07961	.07818	.07675	.07531	.07387	.07243
.9	.07099	.06955	.06812	.06669	.06527	.06386	.06245	.06106	.05967	.05830
1.0	.05694	.05559	.05426	.05295	.05164	.05036	.04909	.04784	.04661	.04539
1.1	.04420	.04302	.04186	.04073	.03961	.03851	.03743	.03637	.03534	.03432
1.2	.03332	.03235	.03139	.03046	.02954	.02865	.02778	.02692	.02609	.02527
1.3	.02448	.02370	.02295	.02221	.02149	.02079	.02010	.01944	.01879	.01816
1.4	.01755	.01696	.01638	.01581	.01527	.01473	.01422	.01372	.01323	.01276
1.5	.01230	.01186	.01143	.01101	.01061	.01022	.00984	.00947	.00912	.00877
1.6	.00844	.00812	.00781	.00751	.00722	.00694	.00667	.00640	.00615	.00591
1.7	.00567	.00544	.00522	.00501	.00481	.00461	.00442	.00424	.00407	.00390
1.8	.00373	.00358	.00343	.00328	.00314	.00301	.00288	.00275	.00263	.00252
1.9	.00241	.00230	.00220	.00210	.00201	.00192	.00184	.00175	.00167	.00160
2.0	.00152	.00145	.00139	.00132	.00126	.00120	.00115	.00109	.00104	.00099
2.1	.00095	.00090	.00086	.00082	.00078	.00074	.00070	.00067	.00064	.00060
2.2	.00057	.00055	.00052	.00049	.00047	.00044	.00042	.00040	.00038	.00036
2.3	.00034	.00032	.00031	.00029	.00028	.00026	.00025	.00023	.00022	.00021
2.4	.00020	.00019	.00018	.00017	.00016	.00015	.00014	.00013	.00013	.00012
2.5	.00011	.00011	.00010	.00009	.00009	.00008	.00008	.00007	.00007	.00006
2.6	.00006	.00006	.00005	.00005	.00005	.00004	.00004	.00004	.00004	.00003
2.7	.00003	.00003	.00003	.00003	.00002	.00002	.00002	.00002	.00002	.00002
2.8	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.9	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

Table 15

Values of:

$$\int_x^{\infty} \frac{e^{-u^2 - \frac{m^2}{u^2}}}{u^2} du$$

This table was prepared by Simpson's rule integration. It is to be expected that the difficulties met in the preparation of Table 14 would be present here also and that some of the values near zero would have to be replaced. The integral from zero to infinity is:

$$\int_0^{\infty} \frac{e^{-u^2 - \frac{m^2}{u^2}}}{u^2} du = \frac{\sqrt{\pi}\, e^{-2m}}{2m} \qquad \text{If} \quad m > 0 \quad .$$

A comparison of some computed values and values computed by Simpson's rule is shown below:

m	$\int_0^{\infty} \frac{e^{-u^2 - \frac{m^2}{u^2}}}{u^2} du$	Values computed by Simpson's rule
0.001	884.45624	1015.64397
0.002	441.34455	425.84105
0.003	293.64183	292.38882
0.004	219.79135	220.65825
0.005	175.48176	175.69769
0.01	86.86785	86.87255
0.02	42.57387	42.57386
0.03	27.82057	27.82057
0.04	20.45226	20.45226
0.05	16.03782	16.03782
0.1	7.25581	7.25581
0.2	2.97028	2.97028
0.3	1.62124	1.62124
0.4	0.99552	0.99552
0.5	0.65205	0.65205

From this comparison it can be concluded that the table can be accepted for the m values from 0.02 to 1.00 inclusive and that corrections will be needed for some tabular values in the range of m between 0.001 and 0.01 inclusive.

To obtain these corrections make use of the expression

$$e^{-u^2} = 1 - u^2 + \frac{u^4}{2!} - \frac{u^6}{3!} + \ldots$$

And write the approximation

$$\int_0^x \frac{e^{-u^2 - \frac{m^2}{u^2}} du}{u^2} \doteq \int_0^x \frac{e^{-\frac{m^2}{u^2}} du}{u^2} - \int_0^x \frac{u^2 e^{-\frac{m^2}{u^2}} du}{u^2} + \frac{1}{2}\int_0^x \frac{u^4 e^{-\frac{m^2}{u^2}} du}{u^2} - \ldots$$

For these purposes this approximation will only be used for values of x which are small compared to unity. Let

$$\frac{m}{u} = v \qquad u = \frac{m}{v} \qquad du = -\frac{mdv}{v^2}$$

Then, by substitution:

$$\int_0^x \frac{e^{-\frac{m^2}{u^2}} du}{u^2} = \frac{\sqrt{\pi}}{2m} \frac{2}{\sqrt{\pi}} \int_{(\frac{m}{x})}^{\infty} e^{-u^2} du \quad .$$

$$\int_0^x \frac{u^2 e^{-\frac{m^2}{u^2}} du}{u^2} = m \int_{(\frac{m}{x})}^{\infty} \frac{e^{-v^2} dv}{v^2}$$

$$\frac{1}{2}\int_0^x \frac{u^4 e^{-\frac{m^2}{u^2}} du}{u^2} = \frac{m^3}{2} \int_{(\frac{m}{x})}^{\infty} \frac{e^{-v^2} dv}{v^4}$$

When v is small compared to unity this last integral will always be less than

$$\frac{m^3}{2}\int_{(\frac{m}{x})}^{\infty} \frac{dv}{v^4} = -\frac{m^3}{6v^3}\Bigg]_{(\frac{m}{x})}^{\infty} = \frac{x^3}{6}$$

Since this integral must be smaller than the two preceding ones it appears that the approximation will be valid to five places of decimals out to $x = .03$. To obtain a value for the integral from x to infinity, as in the table, these integral evaluations must be subtracted, algebraically, from the value of the integral from 0 to ∞. The second integral can be evaluated by use of Table 9.

As an example, compute the tabular value for $m = 0.001$ and $x = 0.02$ $(m/x) = 0.05$. Then

$$\int_0^{\infty} \frac{e^{-u^2 - \frac{m^2}{u^2}}\, du}{u^2} = \qquad 884.45624$$

$$-\frac{\sqrt{\pi}}{2m}\frac{2}{\sqrt{\pi}}\int_{0.05}^{\infty} e^{-u^2}\, du = -(886.22693)\ (1 - 0.056371978)$$

$$= \qquad -836.26856$$

$$+\, m\int_{0.05}^{\infty} \frac{e^{-u^2}\, du}{u^2} = +\ (0.001)\ (18.27753) = +\ 0.01828$$

$$-\frac{0.02^3}{6} = \qquad 0.00000$$

Sum 48.20596

The tabular value is 48.20600

Replacement values have been computed by this procedure until the replacement and tabular values agree. The replacement values have been incorporated into the tables. Since the integration was made by working from

$x = 3$ back toward $x = 0$ an agreement between a replacement value and a tabular value at some x value constitutes a check of all tabular values between that point and $x = 3$.

Table 15. $\int_x^\infty \frac{e^{-u^2 - \frac{m^2}{u^2}}}{u^2}\,du$.

M= .001

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	884.45624	97.90531	48.20596	31.57857	23.26235	18.27488	14.95265	12.58224	10.80682	9.42809
.1	8.32705	7.42799	6.68040	6.04934	5.50983	5.04356	4.63679	4.27902	3.96208	3.67952
.2	3.42618	3.19789	2.99121	2.80334	2.63191	2.47495	2.33079	2.19800	2.07536	1.96181
.3	1.85645	1.75847	1.66718	1.58198	1.50231	1.42771	1.35774	1.29203	1.23023	1.17205
.4	1.11721	1.06545	1.01657	.97035	.92660	.88517	.84589	.80863	.77326	.73965
.5	.70771	.67732	.64839	.62085	.59461	.56959	.54573	.52296	.50123	.48047
.6	.46064	.44170	.42358	.40626	.38969	.37383	.35865	.34412	.33020	.31687
.7	.30410	.29186	.28012	.26887	.25809	.24774	.23782	.22831	.21917	.21041
.8	.20200	.19393	.18618	.17874	.17160	.16474	.15815	.15183	.14575	.13992
.9	.13431	.12893	.12376	.11879	.11402	.10943	.10503	.10080	.09673	.09283
1.0	.08907	.08547	.08200	.07867	.07547	.07240	.06945	.06661	.06389	.06127
1.1	.05876	.05634	.05402	.05179	.04965	.04759	.04562	.04372	.04190	.04015
1.2	.03847	.03686	.03531	.03383	.03240	.03103	.02972	.02846	.02724	.02608
1.3	.02497	.02390	.02287	.02189	.02094	.02004	.01917	.01834	.01754	.01677
1.4	.01604	.01534	.01466	.01401	.01339	.01280	.01223	.01169	.01117	.01067
1.5	.01019	.00973	.00929	.00887	.00847	.00808	.00771	.00736	.00702	.00670
1.6	.00639	.00610	.00581	.00554	.00529	.00504	.00480	.00458	.00436	.00416
1.7	.00396	.00377	.00359	.00342	.00326	.00310	.00295	.00281	.00267	.00254
1.8	.00242	.00230	.00219	.00208	.00198	.00188	.00179	.00170	.00161	.00153
1.9	.00146	.00138	.00131	.00125	.00118	.00112	.00107	.00101	.00096	.00091
2.0	.00086	.00082	.00078	.00074	.00070	.00066	.00063	.00059	.00056	.00053
2.1	.00051	.00048	.00045	.00043	.00041	.00038	.00036	.00034	.00032	.00031
2.2	.00029	.00027	.00026	.00025	.00023	.00022	.00021	.00019	.00018	.00017
2.3	.00016	.00015	.00015	.00014	.00013	.00012	.00012	.00011	.00010	.00010
2.4	.00009	.00009	.00008	.00008	.00007	.00007	.00006	.00006	.00006	.00005
2.5	.00005	.00005	.00004	.00004	.00004	.00004	.00003	.00003	.00003	.00003
2.6	.00003	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00001	.00001
2.7	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.8	.00001	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.9	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

Table 15.

M= .002

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	441.34455	96.92045	48.08157	31.54169	23.24681	18.26694	14.94807	12.57937	10.80490	9.42675
.1	8.32608	7.42726	6.67985	6.04891	5.50948	5.04328	4.63656	4.27883	3.96192	3.67939
.2	3.42607	3.19779	2.99113	2.80327	2.63185	2.47490	2.33074	2.19796	2.07532	1.96178
.3	1.85642	1.75844	1.66716	1.58196	1.50229	1.42769	1.35772	1.29201	1.23022	1.17204
.4	1.11719	1.06544	1.01656	.97034	.92659	.88516	.84589	.80863	.77325	.73965
.5	.70770	.67731	.64839	.62085	.59460	.56958	.54572	.52296	.50122	.48047
.6	.46064	.44169	.42358	.40626	.38968	.37383	.35865	.34412	.33020	.31687
.7	.30409	.29185	.28012	.26887	.25809	.24774	.23782	.22830	.21917	.21041
.8	.20200	.19393	.18618	.17874	.17160	.16474	.15815	.15183	.14575	.13992
.9	.13431	.12893	.12376	.11879	.11402	.10943	.10503	.10080	.09673	.09282
1.0	.08907	.08547	.08200	.07867	.07547	.07240	.06945	.06661	.06389	.06127
1.1	.05876	.05634	.05402	.05179	.04965	.04759	.04562	.04372	.04190	.04015
1.2	.03847	.03686	.03531	.03383	.03240	.03103	.02972	.02846	.02724	.02608
1.3	.02497	.02390	.02287	.02189	.02094	.02004	.01917	.01834	.01754	.01677
1.4	.01604	.01534	.01466	.01401	.01339	.01280	.01223	.01169	.01117	.01067
1.5	.01019	.00973	.00929	.00887	.00847	.00808	.00771	.00736	.00702	.00670
1.6	.00639	.00610	.00581	.00554	.00529	.00504	.00480	.00458	.00436	.00416
1.7	.00396	.00377	.00359	.00342	.00326	.00310	.00295	.00281	.00267	.00254
1.8	.00242	.00230	.00219	.00208	.00198	.00188	.00179	.00170	.00161	.00153
1.9	.00146	.00138	.00131	.00125	.00118	.00112	.00107	.00101	.00096	.00091
2.0	.00086	.00082	.00078	.00074	.00070	.00066	.00063	.00059	.00056	.00053
2.1	.00051	.00048	.00045	.00043	.00041	.00038	.00036	.00034	.00032	.00031
2.2	.00029	.00027	.00026	.00025	.00023	.00022	.00021	.00019	.00018	.00017
2.3	.00016	.00015	.00015	.00014	.00013	.00012	.00012	.00011	.00010	.00010
2.4	.00009	.00009	.00008	.00008	.00007	.00007	.00006	.00006	.00006	.00005
2.5	.00005	.00005	.00004	.00004	.00004	.00004	.00003	.00003	.00003	.00003
2.6	.00003	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00001	.00001
2.7	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.8	.00001	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.9	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

Table 15. $\int_x^\infty \frac{e^{-u^2 - \frac{m^2}{u^2}}}{u^2}\,du.$

M= .003

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	293.64183	95.31772	47.87550	31.48039	23.22095	18.25372	14.94044	12.57458	10.80170	9.42451
.1	8.32446	7.42605	6.67892	6.04818	5.50891	5.04281	4.63618	4.27852	3.96166	3.67917
.2	3.42588	3.19763	2.99099	2.80315	2.63174	2.47481	2.33066	2.19789	2.07526	1.96172
.3	1.85637	1.75840	1.66712	1.58192	1.50226	1.42766	1.35769	1.29199	1.23020	1.17202
.4	1.11718	1.06543	1.01654	.97032	.92658	.88515	.84588	.80862	.77324	.73964
.5	.70769	.67731	.64838	.62084	.59460	.56958	.54572	.52295	.50122	.48046
.6	.46064	.44169	.42358	.40625	.38968	.37382	.35865	.34411	.33020	.31687
.7	.30409	.29185	.28012	.26887	.25809	.24774	.23782	.22830	.21917	.21041
.8	.20200	.19393	.18618	.17874	.17159	.16474	.15815	.15183	.14575	.13992
.9	.13431	.12893	.12376	.11879	.11402	.10943	.10503	.10080	.09673	.09282
1.0	.08907	.08547	.08200	.07867	.07547	.07240	.06945	.06661	.06389	.06127
1.1	.05876	.05634	.05402	.05179	.04965	.04759	.04562	.04372	.04190	.04015
1.2	.03847	.03686	.03531	.03383	.03240	.03103	.02972	.02846	.02724	.02608
1.3	.02497	.02390	.02287	.02189	.02094	.02004	.01917	.01834	.01754	.01677
1.4	.01604	.01534	.01466	.01401	.01339	.01280	.01223	.01169	.01117	.01067
1.5	.01019	.00973	.00929	.00887	.00847	.00808	.00771	.00736	.00702	.00670
1.6	.00639	.00610	.00581	.00554	.00529	.00504	.00480	.00458	.00436	.00416
1.7	.00396	.00377	.00359	.00342	.00326	.00310	.00295	.00281	.00267	.00254
1.8	.00242	.00230	.00219	.00208	.00198	.00188	.00179	.00170	.00161	.00153
1.9	.00146	.00138	.00131	.00125	.00118	.00112	.00107	.00101	.00096	.00091
2.0	.00086	.00082	.00078	.00074	.00070	.00066	.00063	.00059	.00056	.00053
2.1	.00051	.00048	.00045	.00043	.00041	.00038	.00036	.00034	.00032	.00031
2.2	.00029	.00027	.00026	.00025	.00023	.00022	.00021	.00019	.00018	.00017
2.3	.00016	.00015	.00015	.00014	.00013	.00012	.00012	.00011	.00010	.00010
2.4	.00009	.00009	.00008	.00008	.00007	.00007	.00006	.00006	.00006	.00005
2.5	.00005	.00005	.00004	.00004	.00004	.00004	.00003	.00003	.00003	.00003
2.6	.00003	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00001	.00001
2.7	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.8	.00001	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.9	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

3.

M= .004

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	219.79135	93.15230	47.58958	31.39491	23.18483	18.23524	14.92976	12.56788	10.79723	9.42138
.1	8.32219	7.42435	6.67762	6.04717	5.50810	5.04216	4.63565	4.27807	3.96129	3.67886
.2	3.42562	3.19740	2.99080	2.80298	2.63160	2.47468	2.33055	2.19779	2.07517	1.96164
.3	1.85630	1.75834	1.66706	1.58187	1.50221	1.42762	1.35766	1.29195	1.23016	1.17199
.4	1.11715	1.06540	1.01652	.97030	.92656	.88513	.84586	.80860	.77323	.73963
.5	.70768	.67729	.64837	.62083	.59459	.56957	.54571	.52294	.50121	.48046
.6	.46063	.44169	.42357	.40625	.38968	.37382	.35864	.34411	.33019	.31686
.7	.30409	.29185	.28012	.26887	.25808	.24774	.23782	.22830	.21917	.21041
.8	.20200	.19392	.18617	.17874	.17159	.16474	.15815	.15182	.14575	.13992
.9	.13431	.12893	.12376	.11879	.11402	.10943	.10503	.10080	.09673	.09282
1.0	.08907	.08546	.08200	.07867	.07547	.07240	.06945	.06661	.06389	.06127
1.1	.05875	.05634	.05402	.05179	.04965	.04759	.04562	.04372	.04190	.04015
1.2	.03847	.03686	.03531	.03383	.03240	.03103	.02972	.02846	.02724	.02608
1.3	.02497	.02390	.02287	.02189	.02094	.02004	.01917	.01834	.01754	.01677
1.4	.01604	.01533	.01466	.01401	.01339	.01280	.01223	.01169	.01117	.01067
1.5	.01019	.00973	.00929	.00887	.00847	.00808	.00771	.00736	.00702	.00670
1.6	.00639	.00610	.00581	.00554	.00529	.00504	.00480	.00458	.00436	.00416
1.7	.00396	.00377	.00359	.00342	.00326	.00310	.00295	.00281	.00267	.00254
1.8	.00242	.00230	.00219	.00208	.00198	.00188	.00179	.00170	.00161	.00153
1.9	.00146	.00138	.00131	.00125	.00118	.00112	.00107	.00101	.00096	.00091
2.0	.00086	.00082	.00078	.00074	.00070	.00066	.00063	.00059	.00056	.00053
2.1	.00051	.00048	.00045	.00043	.00041	.00038	.00036	.00034	.00032	.00031
2.2	.00029	.00027	.00026	.00025	.00023	.00022	.00021	.00019	.00018	.00017
2.3	.00016	.00015	.00015	.00014	.00013	.00012	.00012	.00011	.00010	.00010
2.4	.00009	.00009	.00008	.00008	.00007	.00007	.00006	.00006	.00006	.00005
2.5	.00005	.00005	.00004	.00004	.00004	.00004	.00003	.00003	.00003	.00003
2.6	.00003	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00001	.00001
2.7	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.8	.00001	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.9	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

Table 15. $\int_{x}^{\infty} \frac{e^{-u^2 - \frac{m^2}{u^2}}}{u^2} du$.

M= .005

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	175.48176	90.49612	47.22633	31.28559	23.13853	18.21153	14.91606	12.55927	10.79149	9.41736
.1	8.31927	7.42217	6.67595	6.04586	5.50706	5.04132	4.63496	4.27751	3.96082	3.67846
.2	3.42528	3.19711	2.99055	2.80276	2.63141	2.47451	2.33040	2.19766	2.07505	1.96154
.3	1.85621	1.75825	1.66699	1.58180	1.50215	1.42756	1.35761	1.29191	1.23012	1.17195
.4	1.11712	1.06537	1.01649	.97028	.92654	.88511	.84584	.80858	.77321	.73961
.5	.70767	.67728	.64836	.62082	.59458	.56956	.54570	.52294	.50120	.48045
.6	.46063	.44168	.42357	.40624	.38967	.37382	.35864	.34411	.33019	.31686
.7	.30409	.29185	.28011	.26887	.25808	.24774	.23782	.22830	.21917	.21040
.8	.20199	.19392	.18617	.17873	.17159	.16473	.15815	.15182	.14575	.13991
.9	.13431	.12893	.12376	.11879	.11402	.10943	.10503	.10080	.09673	.09282
1.0	.08907	.08546	.08200	.07867	.07547	.07240	.06945	.06661	.06389	.06127
1.1	.05875	.05634	.05402	.05179	.04965	.04759	.04562	.04372	.04190	.04015
1.2	.03847	.03686	.03531	.03383	.03240	.03103	.02972	.02845	.02724	.02608
1.3	.02497	.02390	.02287	.02189	.02094	.02004	.01917	.01834	.01754	.01677
1.4	.01604	.01533	.01466	.01401	.01339	.01280	.01223	.01169	.01117	.01067
1.5	.01019	.00973	.00929	.00887	.00847	.00808	.00771	.00736	.00702	.00670
1.6	.00639	.00610	.00581	.00554	.00529	.00504	.00480	.00458	.00436	.00416
1.7	.00396	.00377	.00359	.00342	.00326	.00310	.00295	.00281	.00267	.00254
1.8	.00242	.00230	.00219	.00208	.00198	.00188	.00179	.00170	.00161	.00153
1.9	.00146	.00138	.00131	.00125	.00118	.00112	.00107	.00101	.00096	.00091
2.0	.00086	.00082	.00078	.00074	.00070	.00066	.00063	.00059	.00056	.00053
2.1	.00051	.00048	.00045	.00043	.00041	.00038	.00036	.00034	.00032	.00031
2.2	.00029	.00027	.00026	.00025	.00023	.00022	.00021	.00019	.00018	.00017
2.3	.00016	.00015	.00015	.00014	.00013	.00012	.00012	.00011	.00010	.00010
2.4	.00009	.00009	.00008	.00008	.00007	.00007	.00006	.00006	.00006	.00005
2.5	.00005	.00005	.00004	.00004	.00004	.00004	.00003	.00003	.00003	.00003
2.6	.00003	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00001	.00001
2.7	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.8	.00001	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.9	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

5.

M= .006

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	145.94262	87.43316	46.78892	31.15287	23.08215	18.18261	14.89934	12.54877	10.78447	9.41245
.1	8.31571	7.41951	6.67391	6.04427	5.50579	5.04030	4.63412	4.27681	3.96024	3.67797
.2	3.42486	3.19676	2.99024	2.80250	2.63118	2.47431	2.33023	2.19750	2.07492	1.96142
.3	1.85610	1.75816	1.66690	1.58172	1.50208	1.42750	1.35755	1.29185	1.23008	1.17191
.4	1.11708	1.06534	1.01646	.97025	.92651	.88509	.84582	.80856	.77319	.73959
.5	.70765	.67727	.64835	.62080	.59456	.56955	.54569	.52293	.50119	.48044
.6	.46062	.44167	.42356	.40624	.38967	.37381	.35863	.34410	.33018	.31685
.7	.30408	.29184	.28011	.26886	.25808	.24773	.23781	.22830	.21916	.21040
.8	.20199	.19392	.18617	.17873	.17159	.16473	.15815	.15182	.14575	.13991
.9	.13431	.12893	.12376	.11879	.11402	.10943	.10503	.10080	.09673	.09282
1.0	.08907	.08546	.08200	.07867	.07547	.07240	.06945	.06661	.06389	.06127
1.1	.05875	.05634	.05402	.05179	.04965	.04759	.04562	.04372	.04190	.04015
1.2	.03847	.03686	.03531	.03383	.03240	.03103	.02972	.02845	.02724	.02608
1.3	.02497	.02390	.02287	.02189	.02094	.02004	.01917	.01834	.01754	.01677
1.4	.01604	.01533	.01466	.01401	.01339	.01280	.01223	.01169	.01117	.01067
1.5	.01019	.00973	.00929	.00887	.00847	.00808	.00771	.00736	.00702	.00670
1.6	.00639	.00610	.00581	.00554	.00529	.00504	.00480	.00458	.00436	.00416
1.7	.00396	.00377	.00359	.00342	.00326	.00310	.00295	.00281	.00267	.00254
1.8	.00242	.00230	.00219	.00208	.00198	.00188	.00179	.00170	.00161	.00153
1.9	.00146	.00138	.00131	.00125	.00118	.00112	.00107	.00101	.00096	.00091
2.0	.00086	.00082	.00078	.00074	.00070	.00066	.00063	.00059	.00056	.00053
2.1	.00051	.00048	.00045	.00043	.00041	.00038	.00036	.00034	.00032	.00031
2.2	.00029	.00027	.00026	.00025	.00023	.00022	.00021	.00019	.00018	.00017
2.3	.00016	.00015	.00015	.00014	.00013	.00012	.00012	.00011	.00010	.00010
2.4	.00009	.00009	.00008	.00008	.00007	.00007	.00006	.00006	.00006	.00005
2.5	.00005	.00005	.00004	.00004	.00004	.00004	.00003	.00003	.00003	.00003
2.6	.00003	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00001	.00001
2.7	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.8	.00001	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.9	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

6.

Table 15. $\int_x^\infty \frac{e^{-u^2 - \frac{m^2}{u^2}} du}{u^2}$

M= .007

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	124.84374	84.05426	46.28113	30.99725	23.01582	18.14854	14.87962	12.53637	10.77619	9.40666
.1	8.31150	7.41637	6.67150	6.04238	5.50429	5.03909	4.63313	4.27599	3.95955	3.67739
.2	3.42437	3.19634	2.98988	2.80218	2.63090	2.47407	2.33002	2.19732	2.07475	1.96127
.3	1.85597	1.75804	1.66680	1.58163	1.50200	1.42742	1.35748	1.29179	1.23002	1.17185
.4	1.11703	1.06529	1.01642	.97021	.92648	.88505	.84579	.80854	.77317	.73957
.5	.70763	.67725	.64833	.62079	.59455	.56953	.54568	.52291	.50118	.48043
.6	.46061	.44166	.42355	.40623	.38966	.37380	.35863	.34409	.33018	.31685
.7	.30408	.29184	.28011	.26886	.25807	.24773	.23781	.22829	.21916	.21040
.8	.20199	.19392	.18617	.17873	.17159	.16473	.15814	.15182	.14575	.13991
.9	.13431	.12893	.12375	.11879	.11402	.10943	.10503	.10079	.09673	.09282
1.0	.08907	.08546	.08200	.07867	.07547	.07240	.06945	.06661	.06389	.06127
1.1	.05875	.05634	.05402	.05179	.04965	.04759	.04562	.04372	.04190	.04015
1.2	.03847	.03686	.03531	.03383	.03240	.03103	.02972	.02845	.02724	.02608
1.3	.02497	.02390	.02287	.02189	.02094	.02004	.01917	.01834	.01754	.01677
1.4	.01604	.01533	.01466	.01401	.01339	.01280	.01223	.01169	.01117	.01067
1.5	.01019	.00973	.00929	.00887	.00847	.00808	.00771	.00736	.00702	.00670
1.6	.00639	.00610	.00581	.00554	.00529	.00504	.00480	.00458	.00436	.00416
1.7	.00396	.00377	.00359	.00342	.00326	.00310	.00295	.00281	.00267	.00254
1.8	.00242	.00230	.00219	.00208	.00198	.00188	.00179	.00170	.00161	.00153
1.9	.00146	.00138	.00131	.00125	.00118	.00112	.00107	.00101	.00096	.00091
2.0	.00086	.00082	.00078	.00074	.00070	.00066	.00063	.00059	.00056	.00053
2.1	.00051	.00048	.00045	.00043	.00041	.00038	.00036	.00034	.00032	.00031
2.2	.00029	.00027	.00026	.00025	.00023	.00022	.00021	.00019	.00018	.00017
2.3	.00016	.00015	.00015	.00014	.00013	.00012	.00012	.00011	.00010	.00010
2.4	.00009	.00009	.00008	.00008	.00007	.00007	.00006	.00006	.00006	.00005
2.5	.00005	.00005	.00004	.00004	.00004	.00004	.00003	.00003	.00003	.00003
2.6	.00003	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00001	.00001
2.7	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.8	.00001	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.9	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

Table 15.

M= .008

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	109.02002	80.45200	45.70721	30.81936	22.93968	18.10936	14.85692	12.52209	10.76664	9.39998
.1	8.30665	7.41274	6.66872	6.04021	5.50256	5.03769	4.63199	4.27505	3.95876	3.67673
.2	3.42381	3.19585	2.98946	2.80182	2.63059	2.47380	2.32977	2.19710	2.07456	1.96110
.3	1.85581	1.75790	1.66668	1.58152	1.50190	1.42733	1.35740	1.29172	1.22995	1.17179
.4	1.11697	1.06524	1.01637	.97017	.92644	.88502	.84575	.80850	.77314	.73954
.5	.70761	.67722	.64831	.62077	.59453	.56952	.54566	.52290	.50117	.48042
.6	.46060	.44165	.42354	.40622	.38965	.37379	.35862	.34409	.33017	.31684
.7	.30407	.29183	.28010	.26885	.25807	.24773	.23781	.22829	.21916	.21040
.8	.20199	.19391	.18617	.17873	.17159	.16473	.15814	.15182	.14574	.13991
.9	.13431	.12892	.12375	.11879	.11401	.10943	.10502	.10079	.09673	.09282
1.0	.08907	.08546	.08200	.07867	.07547	.07240	.06945	.06661	.06389	.06127
1.1	.05875	.05634	.05402	.05179	.04965	.04759	.04562	.04372	.04190	.04015
1.2	.03847	.03686	.03531	.03383	.03240	.03103	.02972	.02845	.02724	.02608
1.3	.02497	.02390	.02287	.02189	.02094	.02004	.01917	.01834	.01754	.01677
1.4	.01604	.01533	.01466	.01401	.01339	.01280	.01223	.01169	.01117	.01067
1.5	.01019	.00973	.00929	.00887	.00847	.00808	.00771	.00736	.00702	.00670
1.6	.00639	.00610	.00581	.00554	.00529	.00504	.00480	.00458	.00436	.00416
1.7	.00396	.00377	.00359	.00342	.00326	.00310	.00295	.00281	.00267	.00254
1.8	.00242	.00230	.00219	.00208	.00198	.00188	.00179	.00170	.00161	.00153
1.9	.00146	.00138	.00131	.00125	.00118	.00112	.00107	.00101	.00096	.00091
2.0	.00086	.00082	.00078	.00074	.00070	.00066	.00063	.00059	.00056	.00053
2.1	.00051	.00048	.00045	.00043	.00041	.00038	.00036	.00034	.00032	.00031
2.2	.00029	.00027	.00026	.00025	.00023	.00022	.00021	.00019	.00018	.00017
2.3	.00016	.00015	.00015	.00014	.00013	.00012	.00012	.00011	.00010	.00010
2.4	.00009	.00009	.00008	.00008	.00007	.00007	.00006	.00006	.00006	.00005
2.5	.00005	.00005	.00004	.00004	.00004	.00004	.00003	.00003	.00003	.00003
2.6	.00003	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00001	.00001
2.7	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.8	.00001	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.9	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

Table 15. $\int_x^\infty \frac{e^{-u^2-\frac{m^2}{u^2}}}{u^2}\,du$

M= .009

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	96.71306	76.71589	45.07190	30.61987	22.85390	18.06512	14.83127	12.50593	10.75584	9.39241
.1	8.30116	7.40863	6.66558	6.03775	5.50061	5.03611	4.63070	4.27398	3.95787	3.67597
.2	3.42317	3.19530	2.98899	2.80141	2.63023	2.47348	2.32950	2.19686	2.07435	1.96091
.3	1.85564	1.75775	1.66654	1.58140	1.50179	1.42723	1.35731	1.29164	1.22988	1.17173
.4	1.11691	1.06518	1.01632	.97012	.92639	.88498	.84572	.80847	.77311	.73951
.5	.70758	.67720	.64828	.62075	.59451	.56950	.54564	.52288	.50115	.48041
.6	.46058	.44164	.42353	.40621	.38964	.37378	.35861	.34408	.33016	.31684
.7	.30406	.29183	.28009	.26885	.25806	.24772	.23780	.22829	.21915	.21039
.8	.20198	.19391	.18616	.17872	.17158	.16473	.15814	.15182	.14574	.13991
.9	.13430	.12892	.12375	.11878	.11401	.10943	.10502	.10079	.09673	.09282
1.0	.08907	.08546	.08200	.07867	.07547	.07240	.06945	.06661	.06388	.06127
1.1	.05875	.05634	.05402	.05179	.04965	.04759	.04562	.04372	.04190	.04015
1.2	.03847	.03686	.03531	.03383	.03240	.03103	.02972	.02845	.02724	.02608
1.3	.02497	.02390	.02287	.02189	.02094	.02004	.01917	.01834	.01754	.01677
1.4	.01604	.01533	.01466	.01401	.01339	.01280	.01223	.01169	.01117	.01067
1.5	.01019	.00973	.00929	.00887	.00847	.00808	.00771	.00736	.00702	.00670
1.6	.00639	.00610	.00581	.00554	.00529	.00504	.00480	.00458	.00436	.00416
1.7	.00396	.00377	.00359	.00342	.00326	.00310	.00295	.00281	.00267	.00254
1.8	.00242	.00230	.00219	.00208	.00198	.00188	.00179	.00170	.00161	.00153
1.9	.00146	.00138	.00131	.00125	.00118	.00112	.00107	.00101	.00096	.00091
2.0	.00086	.00082	.00078	.00074	.00070	.00066	.00063	.00059	.00056	.00053
2.1	.00051	.00048	.00045	.00043	.00041	.00038	.00036	.00034	.00032	.00031
2.2	.00029	.00027	.00026	.00025	.00023	.00022	.00021	.00019	.00018	.00017
2.3	.00016	.00015	.00015	.00014	.00013	.00012	.00012	.00011	.00010	.00010
2.4	.00009	.00009	.00008	.00008	.00007	.00007	.00006	.00006	.00006	.00005
2.5	.00005	.00005	.00004	.00004	.00004	.00004	.00003	.00003	.00003	.00003
2.6	.00003	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00001	.00001
2.7	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.8	.00001	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.9	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

9.

M= .010

χ	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	86.86785	72.92846	44.38033	30.39955	22.75868	18.01590	14.80268	12.48792	10.74379	9.38397
.1	8.29503	7.40405	6.66206	6.03500	5.49842	5.03434	4.62925	4.27278	3.95687	3.67513
.2	3.42245	3.19469	2.98846	2.80095	2.62983	2.47313	2.32919	2.19659	2.07411	1.96070
.3	1.85545	1.75758	1.66639	1.58126	1.50166	1.42712	1.35721	1.29154	1.22979	1.17165
.4	1.11684	1.06512	1.01626	.97006	.92634	.88493	.84568	.80843	.77307	.73948
.5	.70755	.67717	.64826	.62072	.59449	.56948	.54562	.52286	.50114	.48039
.6	.46057	.44163	.42352	.40620	.38963	.37377	.35860	.34407	.33016	.31683
.7	.30406	.29182	.28009	.26884	.25806	.24772	.23780	.22828	.21915	.21039
.8	.20198	.19391	.18616	.17872	.17158	.16472	.15814	.15181	.14574	.13991
.9	.13430	.12892	.12375	.11878	.11401	.10943	.10502	.10079	.09672	.09262
1.0	.08907	.08546	.08200	.07867	.07547	.07240	.06944	.06661	.06388	.06127
1.1	.05875	.05634	.05402	.05179	.04965	.04759	.04562	.04372	.04190	.04015
1.2	.03847	.03686	.03531	.03383	.03240	.03103	.02972	.02845	.02724	.02608
1.3	.02497	.02390	.02287	.02189	.02094	.02004	.01917	.01834	.01754	.01677
1.4	.01604	.01533	.01466	.01401	.01339	.01280	.01223	.01169	.01117	.01067
1.5	.01019	.00973	.00929	.00887	.00847	.00808	.00771	.00736	.00702	.00670
1.6	.00639	.00610	.00581	.00554	.00529	.00504	.00480	.00458	.00436	.00416
1.7	.00396	.00377	.00359	.00342	.00326	.00310	.00295	.00281	.00267	.00254
1.8	.00242	.00230	.00219	.00208	.00198	.00188	.00179	.00170	.00161	.00153
1.9	.00146	.00138	.00131	.00125	.00118	.00112	.00107	.00101	.00096	.00091
2.0	.00086	.00082	.00078	.00074	.00070	.00066	.00063	.00059	.00056	.00053
2.1	.00051	.00048	.00045	.00043	.00041	.00038	.00036	.00034	.00032	.00031
2.2	.00029	.00027	.00026	.00025	.00023	.00022	.00021	.00019	.00018	.00017
2.3	.00016	.00015	.00015	.00014	.00013	.00012	.00012	.00011	.00010	.00010
2.4	.00009	.00009	.00008	.00008	.00007	.00007	.00006	.00006	.00006	.00005
2.5	.00005	.00005	.00004	.00004	.00004	.00004	.00003	.00003	.00003	.00003
2.6	.00003	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00001	.00001
2.7	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.8	.00001	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.9	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

10

Table 15

$$\int_x^\infty \frac{e^{-u^2-\frac{m^2}{u^2}}\,du}{u^2}.$$

M= .020

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	42.57386	42.36671	35.60550	27.25893	21.34072	17.26750	14.36302	12.20899	10.55635	9.25227
.1	8.19915	7.33221	6.60694	5.99185	5.46405	5.00657	4.60652	4.25397	3.94114	3.66186
.2	3.41117	3.18503	2.98014	2.79374	2.62355	2.46763	2.32435	2.19231	2.07032	1.95733
.3	1.85244	1.75489	1.66397	1.57908	1.49970	1.42534	1.35560	1.29008	1.22846	1.17044
.4	1.11573	1.06411	1.01533	.96921	.92556	.88421	.84501	.80782	.77251	.73896
.5	.70707	.67672	.64784	.62034	.59413	.56915	.54532	.52258	.50087	.48014
.6	.46034	.44141	.42331	.40601	.38945	.37361	.35845	.34393	.33002	.31670
.7	.30394	.29171	.27998	.26875	.25797	.24763	.23772	.22820	.21908	.21032
.8	.20192	.19385	.18610	.17867	.17153	.16468	.15809	.15177	.14570	.13987
.9	.13427	.12889	.12372	.11875	.11398	.10940	.10500	.10077	.09670	.09280
1.0	.08905	.08544	.08198	.07865	.07545	.07238	.06943	.06660	.06387	.06126
1.1	.05874	.05633	.05401	.05178	.04964	.04758	.04561	.04371	.04189	.04014
1.2	.03847	.03685	.03531	.03382	.03239	.03103	.02971	.02845	.02724	.02608
1.3	.02496	.02389	.02287	.02188	.02094	.02004	.01917	.01833	.01754	.01677
1.4	.01604	.01533	.01466	.01401	.01339	.01280	.01223	.01169	.01116	.01066
1.5	.01019	.00973	.00929	.00887	.00847	.00808	.00771	.00736	.00702	.00670
1.6	.00639	.00610	.00581	.00554	.00529	.00504	.00480	.00458	.00436	.00416
1.7	.00396	.00377	.00359	.00342	.00326	.00310	.00295	.00281	.00267	.00254
1.8	.00242	.00230	.00219	.00208	.00198	.00188	.00179	.00170	.00161	.00153
1.9	.00146	.00138	.00131	.00125	.00118	.00112	.00107	.00101	.00096	.00091
2.0	.00086	.00082	.00078	.00074	.00070	.00066	.00063	.00059	.00056	.00053
2.1	.00051	.00048	.00045	.00043	.00041	.00038	.00036	.00034	.00032	.00031
2.2	.00029	.00027	.00026	.00025	.00023	.00022	.00021	.00019	.00018	.00017
2.3	.00016	.00015	.00015	.00014	.00013	.00012	.00012	.00011	.00010	.00010
2.4	.00009	.00009	.00008	.00008	.00007	.00007	.00006	.00006	.00006	.00005
2.5	.00005	.00005	.00004	.00004	.00004	.00004	.00003	.00003	.00003	.00003
2.6	.00003	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00001	.00001
2.7	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.8	.00001	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.9	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

11.

M= .030

χ	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	27.82057	27.81990	26.81959	23.17648	19.29527	16.13193	13.67691	11.76617	10.25540	9.03917
.1	8.04316	7.21485	6.51660	5.92095	5.40749	4.96078	4.56899	4.22287	3.91511	3.63989
.2	3.39248	3.16902	2.96634	2.78177	2.61312	2.45850	2.31631	2.18521	2.06402	1.95173
.3	1.84745	1.75041	1.65995	1.57547	1.49643	1.42239	1.35292	1.28765	1.22625	1.16842
.4	1.11389	1.06242	1.01379	.96780	.92426	.88302	.84391	.80681	.77157	.73810
.5	.70627	.67598	.64716	.61970	.59354	.56860	.54481	.52210	.50043	.47973
.6	.45995	.44105	.42298	.40570	.38916	.37334	.35819	.34369	.32980	.31649
.7	.30374	.29152	.27981	.26858	.25782	.24749	.23758	.22808	.21896	.21021
.8	.20181	.19375	.18601	.17858	.17145	.16460	.15802	.15170	.14564	.13981
.9	.13421	.12883	.12367	.11871	.11394	.10936	.10496	.10073	.09667	.09276
1.0	.08901	.08541	.08195	.07862	.07543	.07236	.06941	.06657	.06385	.06124
1.1	.05872	.05631	.05399	.05176	.04962	.04757	.04560	.04370	.04188	.04013
1.2	.03846	.03684	.03530	.03381	.03239	.03102	.02970	.02844	.02723	.02607
1.3	.02496	.02389	.02286	.02188	.02094	.02003	.01916	.01833	.01753	.01677
1.4	.01603	.01533	.01466	.01401	.01339	.01280	.01223	.01168	.01116	.01066
1.5	.01018	.00973	.00929	.00887	.00846	.00808	.00771	.00736	.00702	.00670
1.6	.00639	.00610	.00581	.00554	.00528	.00504	.00480	.00458	.00436	.00415
1.7	.00396	.00377	.00359	.00342	.00326	.00310	.00295	.00281	.00267	.00254
1.8	.00242	.00230	.00219	.00208	.00198	.00188	.00179	.00170	.00161	.00153
1.9	.00146	.00138	.00131	.00125	.00118	.00112	.00107	.00101	.00096	.00091
2.0	.00086	.00082	.00078	.00074	.00070	.00066	.00063	.00059	.00056	.00053
2.1	.00051	.00048	.00045	.00043	.00041	.00038	.00036	.00034	.00032	.00031
2.2	.00029	.00027	.00026	.00024	.00023	.00022	.00021	.00019	.00018	.00017
2.3	.00016	.00015	.00015	.00014	.00013	.00012	.00012	.00011	.00010	.00010
2.4	.00009	.00009	.00008	.00008	.00007	.00007	.00006	.00006	.00006	.00005
2.5	.00005	.00005	.00004	.00004	.00004	.00004	.00003	.00003	.00003	.00003
2.6	.00003	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00001	.00001
2.7	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.8	.00001	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.9	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

Table 15. $\int_x^\infty \frac{e^{-u^2-\frac{m^2}{u^2}}du}{u^2}$.

M= .040

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	20.45226	20.45226	20.34866	19.13826	16.97075	14.74641	12.80524	11.18935	9.85687	8.75373
.1	7.83248	7.05537	6.39327	5.82381	5.32975	4.89772	4.51720	4.17988	3.87910	3.60946
.2	3.36657	3.14680	2.94717	2.76514	2.59861	2.44578	2.30513	2.17533	2.05525	1.94392
.3	1.84048	1.74417	1.65434	1.57042	1.49188	1.41827	1.34918	1.28426	1.22316	1.16560
.4	1.11132	1.06007	1.01164	.96582	.92245	.88135	.84237	.80539	.77026	.73688
.5	.70515	.67495	.64620	.61881	.59271	.56783	.54409	.52144	.49981	.47915
.6	.45942	.44055	.42251	.40526	.38875	.37296	.35783	.34335	.32949	.31620
.7	.30347	.29127	.27957	.26836	.25760	.24729	.23740	.22790	.21880	.21005
.8	.20167	.19361	.18588	.17846	.17133	.16449	.15792	.15161	.14555	.13972
.9	.13413	.12876	.12360	.11864	.11388	.10930	.10490	.10068	.09662	.09272
1.0	.08897	.08537	.08191	.07859	.07539	.07233	.06938	.06655	.06382	.06121
1.1	.05870	.05629	.05397	.05174	.04960	.04755	.04558	.04368	.04187	.04012
1.2	.03844	.03683	.03528	.03380	.03237	.03101	.02969	.02843	.02722	.02606
1.3	.02495	.02388	.02286	.02187	.02093	.02002	.01916	.01832	.01753	.01676
1.4	.01603	.01533	.01465	.01401	.01339	.01279	.01223	.01168	.01116	.01066
1.5	.01018	.00972	.00928	.00886	.00846	.00808	.00771	.00736	.00702	.00670
1.6	.00639	.00609	.00581	.00554	.00528	.00504	.00480	.00458	.00436	.00415
1.7	.00396	.00377	.00359	.00342	.00326	.00310	.00295	.00281	.00267	.00254
1.8	.00242	.00230	.00219	.00208	.00198	.00188	.00179	.00170	.00161	.00153
1.9	.00146	.00138	.00131	.00125	.00118	.00112	.00107	.00101	.00096	.00091
2.0	.00086	.00082	.00078	.00074	.00070	.00066	.00063	.00059	.00056	.00053
2.1	.00051	.00048	.00045	.00043	.00041	.00038	.00036	.00034	.00032	.00031
2.2	.00029	.00027	.00026	.00024	.00023	.00022	.00021	.00019	.00018	.00017
2.3	.00016	.00015	.00015	.00014	.00013	.00012	.00012	.00011	.00010	.00010
2.4	.00009	.00009	.00008	.00008	.00007	.00007	.00006	.00006	.00006	.00005
2.5	.00005	.00005	.00004	.00004	.00004	.00004	.00003	.00003	.00003	.00003
2.6	.00003	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00001	.00001
2.7	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.8	.00001	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.9	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

M= .050

χ	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	16.03782	16.03782	16.03061	15.71153	14.67281	13.25422	11.81768	10.51460	9.38064	8.40753
.1	7.57419	6.85828	6.23991	5.70243	5.23225	4.81836	4.45187	4.12554	3.83348	3.57085
.2	3.33365	3.11855	2.92277	2.74395	2.58012	2.42957	2.29085	2.16270	2.04405	1.93394
.3	1.83156	1.73619	1.64717	1.56396	1.48605	1.41299	1.34440	1.27991	1.21920	1.16199
.4	1.10802	1.05705	1.00887	.96329	.92012	.87920	.84040	.80357	.76858	.73533
.5	.70371	.67362	.64496	.61767	.59165	.56684	.54318	.52059	.49902	.47841
.6	.45873	.43991	.42191	.40470	.38823	.37247	.35738	.34292	.32908	.31582
.7	.30312	.29094	.27926	.26807	.25733	.24703	.23716	.22768	.21858	.20986
.8	.20148	.19344	.18572	.17831	.17119	.16435	.15779	.15149	.14543	.13962
.9	.13403	.12866	.12351	.11855	.11380	.10922	.10483	.10061	.09655	.09266
1.0	.08891	.08532	.08186	.07854	.07535	.07228	.06934	.06651	.06379	.06118
1.1	.05867	.05626	.05394	.05172	.04958	.04753	.04556	.04366	.04185	.04010
1.2	.03842	.03681	.03527	.03378	.03236	.03099	.02968	.02842	.02721	.02605
1.3	.02494	.02387	.02285	.02186	.02092	.02002	.01915	.01832	.01752	.01676
1.4	.01602	.01532	.01465	.01400	.01338	.01279	.01222	.01168	.01116	.01066
1.5	.01018	.00972	.00928	.00886	.00846	.00808	.00771	.00736	.00702	.00670
1.6	.00639	.00609	.00581	.00554	.00528	.00504	.00480	.00457	.00436	.00415
1.7	.00396	.00377	.00359	.00342	.00325	.00310	.00295	.00281	.00267	.00254
1.8	.00242	.00230	.00219	.00208	.00198	.00188	.00179	.00170	.00161	.00153
1.9	.00146	.00138	.00131	.00125	.00118	.00112	.00107	.00101	.00096	.00091
2.0	.00086	.00082	.00078	.00074	.00070	.00066	.00063	.00059	.00056	.00053
2.1	.00050	.00048	.00045	.00043	.00041	.00038	.00036	.00034	.00032	.00031
2.2	.00029	.00027	.00026	.00024	.00023	.00022	.00021	.00019	.00018	.00017
2.3	.00016	.00015	.00015	.00014	.00013	.00012	.00012	.00011	.00010	.00010
2.4	.00009	.00009	.00008	.00008	.00007	.00007	.00006	.00006	.00006	.00005
2.5	.00005	.00005	.00004	.00004	.00004	.00004	.00003	.00003	.00003	.00003
2.6	.00003	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00001	.00001
2.7	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.8	.00001	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.9	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

Table 15. $\int_x^\infty \frac{e^{-u^2 - \frac{m^2}{u^2}} du}{u^2}$.

M= .060

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	13.10021	13.10021	13.09988	13.03117	12.60018	11.77781	10.78217	9.77991	8.84868	8.01379
.1	7.27654	6.62891	6.06007	5.55924	5.11668	4.72395	4.37389	4.06051	3.77878	3.52447
.2	3.29404	3.08450	2.89333	2.71836	2.55776	2.40995	2.27355	2.14740	2.03046	1.92184
.3	1.82075	1.72649	1.63846	1.55611	1.47896	1.40658	1.33858	1.27462	1.21439	1.15760
.4	1.10401	1.05338	1.00551	.96020	.91728	.87659	.83799	.80135	.76653	.73344
.5	.70196	.67199	.64346	.61627	.59036	.56564	.54206	.51955	.49805	.47751
.6	.45788	.43912	.42118	.40401	.38759	.37187	.35682	.34240	.32859	.31536
.7	.30269	.29053	.27888	.26771	.25700	.24672	.23686	.22740	.21832	.20961
.8	.20125	.19322	.18552	.17811	.17101	.16419	.15763	.15134	.14529	.13948
.9	.13390	.12855	.12340	.11845	.11370	.10913	.10474	.10053	.09648	.09258
1.0	.08884	.08525	.08180	.07848	.07529	.07223	.06929	.06646	.06374	.06113
1.1	.05863	.05622	.05391	.05168	.04955	.04750	.04553	.04364	.04182	.04008
1.2	.03840	.03679	.03525	.03377	.03234	.03098	.02967	.02841	.02720	.02604
1.3	.02493	.02386	.02284	.02185	.02091	.02001	.01914	.01831	.01751	.01675
1.4	.01602	.01531	.01464	.01399	.01338	.01278	.01222	.01167	.01115	.01065
1.5	.01017	.00972	.00928	.00886	.00846	.00807	.00770	.00735	.00702	.00669
1.6	.00638	.00609	.00581	.00554	.00528	.00503	.00480	.00457	.00436	.00415
1.7	.00396	.00377	.00359	.00342	.00325	.00310	.00295	.00281	.00267	.00254
1.8	.00242	.00230	.00219	.00208	.00198	.00188	.00179	.00170	.00161	.00153
1.9	.00146	.00138	.00131	.00125	.00118	.00112	.00107	.00101	.00096	.00091
2.0	.00086	.00082	.00078	.00074	.00070	.00066	.00063	.00059	.00056	.00053
2.1	.00050	.00048	.00045	.00043	.00041	.00038	.00036	.00034	.00032	.00031
2.2	.00029	.00027	.00026	.00024	.00023	.00022	.00021	.00019	.00018	.00017
2.3	.00016	.00015	.00015	.00014	.00013	.00012	.00012	.00011	.00010	.00010
2.4	.00009	.00009	.00008	.00008	.00007	.00007	.00006	.00006	.00006	.00005
2.5	.00005	.00005	.00004	.00004	.00004	.00004	.00003	.00003	.00003	.00003
2.6	.00003	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00001	.00001
2.7	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.8	.00001	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.9	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

M= .070

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	11.00641	11.00641	11.00640	10.99417	10.83788	10.40344	9.75663	9.02116	8.28311	7.58637
.1	6.94847	6.37315	5.85775	5.39703	4.98502	4.61589	4.28431	3.98557	3.71557	3.47075
.2	3.24808	3.04493	2.85907	2.68854	2.53168	2.38704	2.25335	2.12951	2.01457	1.90767
.3	1.80808	1.71513	1.62825	1.54690	1.47064	1.39905	1.33175	1.26841	1.20873	1.15244
.4	1.09929	1.04906	1.00155	.95656	.91394	.87352	.83516	.79874	.76412	.73121
.5	.69990	.67008	.64169	.61463	.58883	.56422	.54074	.51832	.49691	.47645
.6	.45689	.43820	.42031	.40321	.38683	.37116	.35616	.34178	.32802	.31482
.7	.30218	.29006	.27844	.26729	.25661	.24635	.23652	.22708	.21802	.20933
.8	.20098	.19297	.18528	.17789	.17080	.16399	.15744	.15116	.14513	.13933
.9	.13376	.12841	.12327	.11833	.11358	.10902	.10464	.10043	.09638	.09250
1.0	.08876	.08517	.08173	.07841	.07523	.07217	.06923	.06641	.06369	.06109
1.1	.05858	.05618	.05386	.05164	.04951	.04746	.04550	.04361	.04179	.04005
1.2	.03838	.03677	.03523	.03374	.03232	.03096	.02965	.02839	.02718	.02602
1.3	.02491	.02385	.02282	.02184	.02090	.02000	.01913	.01830	.01750	.01674
1.4	.01601	.01531	.01463	.01399	.01337	.01278	.01221	.01167	.01115	.01065
1.5	.01017	.00971	.00927	.00885	.00845	.00807	.00770	.00735	.00701	.00669
1.6	.00638	.00609	.00581	.00554	.00528	.00503	.00480	.00457	.00436	.00415
1.7	.00395	.00377	.00359	.00342	.00325	.00310	.00295	.00280	.00267	.00254
1.8	.00242	.00230	.00219	.00208	.00198	.00188	.00179	.00170	.00161	.00153
1.9	.00146	.00138	.00131	.00125	.00118	.00112	.00107	.00101	.00096	.00091
2.0	.00086	.00082	.00078	.00074	.00070	.00066	.00063	.00059	.00056	.00053
2.1	.00050	.00048	.00045	.00043	.00041	.00038	.00036	.00034	.00032	.00031
2.2	.00029	.00027	.00026	.00024	.00023	.00022	.00021	.00019	.00018	.00017
2.3	.00016	.00015	.00015	.00014	.00013	.00012	.00012	.00011	.00010	.00010
2.4	.00009	.00009	.00008	.00008	.00007	.00007	.00006	.00006	.00006	.00005
2.5	.00005	.00005	.00004	.00004	.00004	.00004	.00003	.00003	.00003	.00003
2.6	.00003	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00001	.00001
2.7	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.8	.00001	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.9	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

16.

M= .080

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	9.43991	9.43991	9.43991	9.43811	9.38816	9.17841	8.78420	8.26911	7.70448	7.13890
.1	6.59905	6.09717	5.63722	5.21879	4.83941	4.49576	4.18429	3.90160	3.64452	3.41022
.2	3.19617	3.00016	2.82024	2.65469	2.50204	2.36097	2.23033	2.10911	1.99643	1.89149
.3	1.79360	1.70214	1.61656	1.53636	1.46112	1.39042	1.32391	1.26128	1.20224	1.14652
.4	1.09388	1.04411	.99701	.95239	.91010	.86999	.83191	.79574	.76135	.72865
.5	.69752	.66789	.63965	.61274	.58708	.56259	.53922	.51691	.49559	.47522
.6	.45575	.43713	.41932	.40228	.38597	.37035	.35540	.34107	.32735	.31420
.7	.30159	.28951	.27792	.26681	.25615	.24593	.23612	.22670	.21767	.20899
.8	.20067	.19268	.18500	.17763	.17055	.16376	.15723	.15096	.14493	.13915
.9	.13359	.12825	.12311	.11818	.11345	.10889	.10452	.10032	.09628	.09240
1.0	.08867	.08508	.08164	.07833	.07515	.07210	.06916	.06634	.06363	.06103
1.1	.05853	.05613	.05382	.05160	.04947	.04742	.04546	.04357	.04176	.04002
1.2	.03835	.03674	.03520	.03372	.03230	.03093	.02963	.02837	.02716	.02601
1.3	.02490	.02383	.02281	.02183	.02089	.01998	.01912	.01829	.01749	.01673
1.4	.01600	.01530	.01462	.01398	.01336	.01277	.01220	.01166	.01114	.01064
1.5	.01016	.00971	.00927	.00885	.00845	.00806	.00770	.00735	.00701	.00669
1.6	.00638	.00608	.00580	.00553	.00528	.00503	.00479	.00457	.00435	.00415
1.7	.00395	.00376	.00359	.00341	.00325	.00309	.00295	.00280	.00267	.00254
1.8	.00242	.00230	.00218	.00208	.00198	.00188	.00179	.00170	.00161	.00153
1.9	.00146	.00138	.00131	.00125	.00118	.00112	.00107	.00101	.00096	.00091
2.0	.00086	.00082	.00078	.00074	.00070	.00066	.00063	.00059	.00056	.00053
2.1	.00050	.00048	.00045	.00043	.00041	.00038	.00036	.00034	.00032	.00031
2.2	.00029	.00027	.00026	.00024	.00023	.00022	.00021	.00019	.00018	.00017
2.3	.00016	.00015	.00015	.00014	.00013	.00012	.00012	.00011	.00010	.00010
2.4	.00009	.00009	.00008	.00008	.00007	.00007	.00006	.00006	.00006	.00005
2.5	.00005	.00005	.00004	.00004	.00004	.00004	.00003	.00003	.00003	.00003
2.6	.00003	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00001	.00001
2.7	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.8	.00001	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.9	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

M= .090

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	8.22488	8.22488	8.22488	8.22466	8.21049	8.11767	7.89203	7.54760	7.13059	6.68395
.1	6.23708	5.80714	5.40283	5.02765	4.68214	4.36524	4.07509	3.80954	3.56637	3.34344
.2	3.13876	2.95053	2.77711	2.61704	2.46902	2.33189	2.20463	2.08631	1.97613	1.87337
.3	1.77737	1.68757	1.60345	1.52453	1.45042	1.38072	1.31511	1.25327	1.19494	1.13985
.4	1.08778	1.03853	.99189	.94769	.90578	.86601	.82824	.79235	.75822	.72576
.5	.69485	.66541	.63736	.61061	.58510	.56075	.53751	.51531	.49411	.47384
.6	.45446	.43593	.41820	.40123	.38499	.36943	.35454	.34027	.32660	.31349
.7	.30093	.28889	.27734	.26627	.25564	.24545	.23567	.22628	.21727	.20862
.8	.20032	.19235	.18469	.17734	.17028	.16350	.15698	.15073	.14472	.13894
.9	.13340	.12806	.12294	.11802	.11329	.10875	.10438	.10019	.09616	.09228
1.0	.08856	.08498	.08155	.07824	.07507	.07202	.06909	.06627	.06357	.06097
1.1	.05847	.05607	.05376	.05155	.04942	.04738	.04542	.04353	.04172	.03998
1.2	.03831	.03671	.03517	.03369	.03227	.03091	.02960	.02835	.02714	.02599
1.3	.02488	.02381	.02279	.02181	.02087	.01997	.01910	.01828	.01748	.01672
1.4	.01599	.01529	.01461	.01397	.01335	.01276	.01220	.01165	.01113	.01063
1.5	.01016	.00970	.00926	.00884	.00844	.00806	.00769	.00734	.00701	.00668
1.6	.00638	.00608	.00580	.00553	.00527	.00503	.00479	.00457	.00435	.00415
1.7	.00395	.00376	.00358	.00341	.00325	.00309	.00294	.00280	.00267	.00254
1.8	.00241	.00230	.00218	.00208	.00197	.00188	.00178	.00170	.00161	.00153
1.9	.00145	.00138	.00131	.00125	.00118	.00112	.00107	.00101	.00096	.00091
2.0	.00086	.00082	.00078	.00074	.00070	.00066	.00063	.00059	.00056	.00053
2.1	.00050	.00048	.00045	.00043	.00041	.00038	.00036	.00034	.00032	.00031
2.2	.00029	.00027	.00026	.00024	.00023	.00022	.00021	.00019	.00018	.00017
2.3	.00016	.00015	.00015	.00014	.00013	.00012	.00012	.00011	.00010	.00010
2.4	.00009	.00009	.00008	.00008	.00007	.00007	.00006	.00006	.00006	.00005
2.5	.00005	.00005	.00004	.00004	.00004	.00004	.00003	.00003	.00003	.00003
2.6	.00003	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00001	.00001
2.7	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.8	.00001	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.9	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

18.

Table 15. $\int_x^\infty \frac{e^{-u^2 - \frac{m^2}{u^2}}}{u^2}\,du.$

M= .100

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	7.25581	7.25581	7.25581	7.25579	7.25221	7.21444	7.09301	6.87302	6.57563	6.23248
.1	5.87066	5.50896	5.15888	4.82677	4.51554	4.22607	3.95803	3.71041	3.48189	3.27101
.2	3.07633	2.89643	2.73000	2.57583	2.43282	2.29997	2.17637	2.06122	1.95377	1.85339
.3	1.75947	1.67148	1.58895	1.51145	1.43857	1.36998	1.30535	1.24440	1.18684	1.13246
.4	1.08102	1.03233	.98620	.94247	.90098	.86159	.82417	.78859	.75475	.72254
.5	.69187	.66265	.63480	.60824	.58289	.55870	.53561	.51354	.49245	.47230
.6	.45302	.43459	.41694	.40006	.38389	.36841	.35358	.33937	.32576	.31271
.7	.30020	.28820	.27670	.26566	.25507	.24491	.23516	.22581	.21683	.20820
.8	.19993	.19198	.18434	.17701	.16997	.16321	.15671	.15047	.14448	.13872
.9	.13318	.12786	.12275	.11784	.11312	.10859	.10423	.10005	.09602	.09216
1.0	.08844	.08487	.08144	.07814	.07497	.07193	.06900	.06619	.06349	.06090
1.1	.05840	.05601	.05370	.05149	.04937	.04733	.04537	.04349	.04168	.03994
1.2	.03827	.03667	.03513	.03366	.03224	.03088	.02957	.02832	.02712	.02596
1.3	.02486	.02379	.02277	.02179	.02085	.01995	.01909	.01826	.01747	.01671
1.4	.01597	.01527	.01460	.01396	.01334	.01275	.01219	.01164	.01113	.01063
1.5	.01015	.00969	.00926	.00884	.00844	.00806	.00769	.00734	.00700	.00668
1.6	.00637	.00608	.00580	.00553	.00527	.00502	.00479	.00456	.00435	.00414
1.7	.00395	.00376	.00358	.00341	.00325	.00309	.00294	.00280	.00267	.00254
1.8	.00241	.00230	.00218	.00208	.00197	.00188	.00178	.00170	.00161	.00153
1.9	.00145	.00138	.00131	.00125	.00118	.00112	.00107	.00101	.00096	.00091
2.0	.00086	.00082	.00078	.00074	.00070	.00066	.00063	.00059	.00056	.00053
2.1	.00050	.00048	.00045	.00043	.00040	.00038	.00036	.00034	.00032	.00031
2.2	.00029	.00027	.00026	.00024	.00023	.00022	.00021	.00019	.00018	.00017
2.3	.00016	.00015	.00015	.00014	.00013	.00012	.00011	.00011	.00010	.00010
2.4	.00009	.00009	.00008	.00008	.00007	.00007	.00006	.00006	.00006	.00005
2.5	.00005	.00005	.00004	.00004	.00004	.00004	.00003	.00003	.00003	.00003
2.6	.00003	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00001	.00001
2.7	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.8	.00001	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.9	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

M= .200

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	2.97028	2.97028	2.97028	2.97028	2.97028	2.97028	2.97027	2.97004	2.96848	2.96291
.1	2.94972	2.92582	2.88957	2.84091	2.78098	2.71158	2.63478	2.55261	2.46690	2.37921
.2	2.29083	2.20278	2.11586	2.03067	1.94763	1.86707	1.78918	1.71408	1.64184	1.57247
.3	1.50595	1.44223	1.3812[illegible]	1.32291	1.26714	1.21383	1.16290	1.11424	1.06775	1.02333
.4	.98089	.94034	.90159	.86456	.82915	.79530	.76292	.73196	.70233	.67398
.5	.64685	.62087	.59600	.57217	.54935	.52749	.50653	.48645	.46719	.44873
.6	.43102	.41403	.39774	.38210	.36709	.35269	.33886	.32559	.31284	.30060
.7	.28884	.27755	.26670	.25627	.24626	.23663	.22738	.21849	.20995	.20174
.8	.19384	.18625	.17896	.17194	.16520	.15871	.15248	.14648	.14072	.13517
.9	.12984	.12472	.11979	.11505	.11049	.10610	.10189	.09783	.09393	.09019
1.0	.08658	.08311	.07978	.07658	.07350	.07053	.06769	.06495	.06232	.05979
1.1	.05735	.05502	.05277	.05061	.04853	.04654	.04462	.04278	.04101	.03931
1.2	.03768	.03611	.03461	.03316	.03177	.03043	.02915	.02792	.02674	.02561
1.3	.02452	.02347	.02247	.02151	.02059	.01970	.01885	.01804	.01725	.01650
1.4	.01578	.01509	.01443	.01380	.01319	.01261	.01205	.01152	.01100	.01051
1.5	.01004	.00959	.00916	.00875	.00835	.00797	.00761	.00727	.00693	.00662
1.6	.00631	.00602	.00574	.00548	.00522	.00498	.00475	.00452	.00431	.00411
1.7	.00391	.00373	.00355	.00338	.00322	.00307	.00292	.00278	.00264	.00252
1.8	.00239	.00228	.00217	.00206	.00196	.00186	.00177	.00168	.00160	.00152
1.9	.00144	.00137	.00130	.00124	.00117	.00111	.00106	.00100	.00095	.00090
2.0	.00086	.00081	.00077	.00073	.00069	.00066	.00062	.00059	.00056	.00053
2.1	.00050	.00047	.00045	.00043	.00040	.00038	.00036	.00034	.00032	.00030
2.2	.00029	.00027	.00026	.00024	.00023	.00022	.00021	.00019	.00018	.00017
2.3	.00016	.00015	.00014	.00014	.00013	.00012	.00011	.00011	.00010	.00010
2.4	.00009	.00008	.00008	.00008	.00007	.00007	.00006	.00006	.00006	.00005
2.5	.00005	.00005	.00004	.00004	.00004	.00004	.00003	.00003	.00003	.00003
2.6	.00003	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00001	.00001
2.7	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.8	.00001	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.9	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

Table 15 $\int_x^\infty \frac{e^{-u^2 - \frac{m^2}{u^2}}\,du}{u^2}$.

M= .300

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	1.62124	1.62124	1.62124	1.62124	1.62124	1.62124	1.62124	1.62124	1.62124	1.62123
.1	1.62117	1.62090	1.62005	1.61803	1.61414	1.60768	1.59807	1.58495	1.56819	1.54784
.2	1.52412	1.49733	1.46787	1.43615	1.40259	1.36757	1.33148	1.29464	1.25736	1.21989
.3	1.18245	1.14523	1.10839	1.07206	1.03635	1.00133	.96709	.93365	.90108	.86938
.4	.83858	.80869	.77971	.75163	.72445	.69815	.67272	.64815	.62442	.60150
.5	.57937	.55802	.53742	.51755	.49839	.47991	.46210	.44493	.42837	.41242
.6	.39704	.38223	.36795	.35419	.34093	.32816	.31585	.30400	.29258	.28157
.7	.27097	.26076	.25092	.24144	.23231	.22352	.21504	.20688	.19902	.19145
.8	.18415	.17713	.17036	.16384	.15757	.15152	.14570	.14009	.13469	.12949
.9	.12448	.11966	.11501	.11054	.10624	.10209	.09810	.09426	.09056	.08700
1.0	.08357	.08027	.07710	.07404	.07110	.06827	.06555	.06293	.06041	.05798
1.1	.05565	.05341	.05125	.04917	.04718	.04526	.04341	.04164	.03993	.03829
1.2	.03671	.03520	.03374	.03234	.03100	.02970	.02846	.02727	.02612	.02502
1.3	.02397	.02295	.02198	.02104	.02015	.01929	.01846	.01766	.01690	.01617
1.4	.01547	.01480	.01415	.01354	.01294	.01237	.01183	.01131	.01081	.01033
1.5	.00987	.00943	.00900	.00860	.00821	.00784	.00749	.00715	.00682	.00651
1.6	.00621	.00593	.00565	.00539	.00514	.00490	.00468	.00446	.00425	.00405
1.7	.00386	.00368	.00350	.00334	.00318	.00303	.00288	.00274	.00261	.00248
1.8	.00236	.00225	.00214	.00203	.00193	.00184	.00175	.00166	.00158	.00150
1.9	.00143	.00136	.00129	.00122	.00116	.00110	.00105	.00099	.00094	.00089
2.0	.00085	.00080	.00076	.00072	.00069	.00065	.00062	.00058	.00055	.00052
2.1	.00050	.00047	.00045	.00042	.00040	.00038	.00036	.00034	.00032	.00030
2.2	.00029	.00027	.00026	.00024	.00023	.00022	.00020	.00019	.00018	.00017
2.3	.00016	.00015	.00014	.00014	.00013	.00012	.00011	.00011	.00010	.00009
2.4	.00009	.00008	.00008	.00007	.00007	.00007	.00006	.00006	.00005	.00005
2.5	.00005	.00005	.00004	.00004	.00004	.00004	.00003	.00003	.00003	.00003
2.6	.00003	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00001	.00001
2.7	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.8	.00001	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.9	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

M= .400

χ	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.99552	.99552	.99552	.99552	.99552	.99552	.99552	.99552	.99552	.99552
.1	.99552	.99552	.99551	.99549	.99540	.99516	.99464	.99362	.99191	.98927
.2	.98549	.98042	.97394	.96597	.95652	.94561	.93331	.91971	.90494	.88911
.3	.87237	.85483	.83664	.81792	.79878	.77934	.75969	.73992	.72012	.70036
.4	.68070	.66119	.64189	.62283	.60406	.58559	.56746	.54968	.53228	.51526
.5	.49864	.48241	.46659	.45118	.43617	.42158	.40739	.39359	.38020	.36720
.6	.35458	.34234	.33048	.31898	.30783	.29704	.28659	.27647	.26667	.25719
.7	.24802	.23915	.23057	.22227	.21425	.20650	.19901	.19177	.18477	.17801
.8	.17148	.16518	.15909	.15321	.14753	.14205	.13675	.13165	.12672	.12196
.9	.11737	.11295	.10867	.10455	.10058	.09675	.09305	.08949	.08605	.08274
1.0	.07955	.07647	.07350	.07064	.06789	.06523	.06268	.06021	.05784	.05556
1.1	.05336	.05124	.04920	.04723	.04534	.04352	.04177	.04009	.03846	.03690
1.2	.03540	.03396	.03257	.03123	.02995	.02871	.02753	.02638	.02529	.02423
1.3	.02322	.02225	.02131	.02041	.01955	.01872	.01792	.01716	.01643	.01572
1.4	.01505	.01440	.01377	.01318	.01260	.01205	.01153	.01102	.01054	.01007
1.5	.00963	.00920	.00879	.00840	.00802	.00766	.00732	.00699	.00667	.00637
1.6	.00608	.00580	.00553	.00528	.00504	.00480	.00458	.00437	.00416	.00397
1.7	.00378	.00360	.00343	.00327	.00312	.00297	.00283	.00269	.00256	.00244
1.8	.00232	.00221	.00210	.00200	.00190	.00181	.00172	.00163	.00155	.00148
1.9	.00140	.00133	.00127	.00120	.00114	.00109	.00103	.00098	.00093	.00088
2.0	.00084	.00079	.00075	.00071	.00068	.00064	.00061	.00058	.00055	.00052
2.1	.00049	.00046	.00044	.00042	.00039	.00037	.00035	.00033	.00032	.00030
2.2	.00028	.00027	.00025	.00024	.00023	.00021	.00020	.00019	.00018	.00017
2.3	.00016	.00015	.00014	.00013	.00013	.00012	.00011	.00011	.00010	.00009
2.4	.00009	.00008	.00008	.00007	.00007	.00007	.00006	.00006	.00005	.00005
2.5	.00005	.00004	.00004	.00004	.00004	.00003	.00003	.00003	.00003	.00003
2.6	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00001	.00001
2.7	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.8	.00001	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.9	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

Table 15. $\int_x^\infty \frac{e^{-u^2 \frac{m^2}{u^2}} du}{u^2}$.

M= .500

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.65205	.65205	.65205	.65205	.65205	.65205	.65205	.65205	.65205	.65205
.1	.65205	.65205	.65205	.65205	.65205	.65204	.65203	.65199	.65190	.65171
.2	.65135	.65075	.64982	.64847	.64662	.64418	.64109	.63732	.63282	.62759
.3	.62164	.61497	.60763	.59964	.59106	.58193	.57231	.56225	.55181	.54105
.4	.53000	.51874	.50729	.49571	.48405	.47232	.46059	.44887	.43719	.42558
.5	.41407	.40267	.39140	.38028	.36932	.35854	.34795	.33755	.32734	.31735
.6	.30756	.29799	.28864	.27951	.27059	.26190	.25343	.24517	.23713	.22931
.7	.22169	.21429	.20710	.20011	.19332	.18673	.18033	.17412	.16810	.16225
.8	.15659	.15110	.14579	.14063	.13564	.13081	.12613	.12160	.11722	.11298
.9	.10888	.10491	.10108	.09737	.09378	.09032	.08697	.08373	.08061	.07759
1.0	.07467	.07186	.06914	.06652	.06398	.06154	.05918	.05691	.05471	.05259
1.1	.05055	.04859	.04669	.04486	.04310	.04140	.03976	.03818	.03666	.03520
1.2	.03379	.03243	.03112	.02987	.02866	.02749	.02637	.02529	.02425	.02325
1.3	.02229	.02137	.02048	.01963	.01881	.01802	.01726	.01653	.01583	.01516
1.4	.01452	.01390	.01330	.01273	.01218	.01165	.01115	.01066	.01020	.00975
1.5	.00932	.00891	.00852	.00814	.00778	.00743	.00710	.00678	.00648	.00619
1.6	.00591	.00564	.00538	.00513	.00490	.00467	.00446	.00425	.00406	.00387
1.7	.00369	.00351	.00335	.00319	.00304	.00290	.00276	.00263	.00250	.00238
1.8	.00227	.00216	.00205	.00195	.00186	.00177	.00168	.00160	.00152	.00145
1.9	.00137	.00131	.00124	.00118	.00112	.00106	.00101	.00096	.00091	.00086
2.0	.00082	.00078	.00074	.00070	.00066	.00063	.00060	.00057	.00054	.00051
2.1	.00048	.00046	.00043	.00041	.00039	.00037	.00035	.00033	.00031	.00029
2.2	.00028	.00026	.00025	.00023	.00022	.00021	.00020	.00019	.00018	.00017
2.3	.00016	.00015	.00014	.00013	.00012	.00012	.00011	.00010	.00010	.00009
2.4	.00009	.00008	.00008	.00007	.00007	.00006	.00006	.00006	.00005	.00005
2.5	.00005	.00004	.00004	.00004	.00004	.00003	.00003	.00003	.00003	.00003
2.6	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00001	.00001
2.7	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.8	.00001	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.9	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

Table 15.

M= .600

χ	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.44488	.44488	.44488	.44488	.44488	.44488	.44488	.44488	.44488	.44488
.1	.44488	.44488	.44488	.44488	.44488	.44488	.44488	.44487	.44487	.44486
.2	.44484	.44480	.44471	.44456	.44430	.44391	.44334	.44255	.44150	.44015
.3	.43847	.43642	.43400	.43118	.42796	.42433	.42031	.41591	.41113	.40601
.4	.40055	.39479	.38875	.38245	.37592	.36920	.36230	.35525	.34808	.34081
.5	.33346	.32606	.31861	.31115	.30369	.29625	.28884	.28147	.27415	.26691
.6	.25973	.25265	.24566	.23877	.23198	.22531	.21875	.21231	.20600	.19981
.7	.19374	.18781	.18201	.17634	.17080	.16540	.16012	.15498	.14996	.14507
.8	.14032	.13569	.13118	.12680	.12253	.11839	.11437	.11046	.10666	.10298
.9	.09940	.09593	.09257	.08931	.08615	.08308	.08012	.07724	.07446	.07176
1.0	.06915	.06663	.06418	.06182	.05954	.05733	.05519	.05313	.05113	.04920
1.1	.04734	.04554	.04380	.04213	.04051	.03895	.03744	.03598	.03458	.03323
1.2	.03192	.03066	.02945	.02828	.02716	.02607	.02502	.02402	.02305	.02211
1.3	.02121	.02035	.01951	.01871	.01794	.01720	.01648	.01580	.01514	.01450
1.4	.01389	.01331	.01274	.01220	.01168	.01118	.01070	.01024	.00980	.00937
1.5	.00897	.00858	.00820	.00784	.00750	.00716	.00685	.00654	.00625	.00597
1.6	.00570	.00545	.00520	.00496	.00474	.00452	.00432	.00412	.00393	.00375
1.7	.00357	.00341	.00325	.00310	.00295	.00281	.00268	.00255	.00243	.00232
1.8	.00220	.00210	.00200	.00190	.00181	.00172	.00164	.00156	.00148	.00141
1.9	.00134	.00127	.00121	.00115	.00109	.00104	.00099	.00094	.00089	.00084
2.0	.00080	.00076	.00072	.00068	.00065	.00062	.00058	.00055	.00052	.00050
2.1	.00047	.00045	.00042	.00040	.00038	.00036	.00034	.00032	.00030	.00029
2.2	.00027	.00026	.00024	.00023	.00022	.00021	.00019	.00018	.00017	.00016
2.3	.00015	.00015	.00014	.00013	.00012	.00012	.00011	.00010	.00010	.00009
2.4	.00009	.00008	.00008	.00007	.00007	.00006	.00006	.00006	.00005	.00005
2.5	.00005	.00004	.00004	.00004	.00004	.00003	.00003	.00003	.00003	.00003
2.6	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00001	.00001	.00001
2.7	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.8	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.9	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

2A.

Table 15. $\int_{x}^{\infty} \frac{e^{-u^2} \frac{m^2}{u^2} du}{1^{12}}$.

M= .700

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.31220	.31220	.31220	.31220	.31220	.31220	.31220	.31220	.31220	.31220
.1	.31220	.31220	.31220	.31220	.31220	.31220	.31220	.31220	.31220	.31220
.2	.31220	.31220	.31219	.31218	.31215	.31211	.31203	.31191	.31172	.31145
.3	.31107	.31056	.30991	.30908	.30807	.30685	.30542	.30376	.30186	.29973
.4	.29736	.29475	.29191	.28884	.28556	.28207	.27839	.27453	.27050	.26631
.5	.26199	.25754	.25298	.24833	.24359	.23879	.23393	.22902	.22409	.21913
.6	.21416	.20920	.20424	.19930	.19438	.18950	.18465	.17986	.17511	.17042
.7	.16579	.16122	.15672	.15229	.14794	.14366	.13946	.13534	.13130	.12735
.8	.12347	.11969	.11598	.11236	.10882	.10537	.10200	.09871	.09551	.09239
.9	.08935	.08639	.08351	.08071	.07799	.07534	.07276	.07026	.06784	.06548
1.0	.06319	.06097	.05882	.05673	.05470	.05274	.05084	.04900	.04722	.04549
1.1	.04382	.04220	.04064	.03913	.03766	.03625	.03488	.03356	.03228	.03105
1.2	.02986	.02871	.02759	.02652	.02549	.02449	.02353	.02260	.02170	.02084
1.3	.02001	.01920	.01843	.01769	.01697	.01628	.01561	.01497	.01436	.01376
1.4	.01319	.01264	.01212	.01161	.01112	.01065	.01020	.00977	.00935	.00895
1.5	.00857	.00820	.00784	.00750	.00717	.00686	.00656	.00627	.00599	.00573
1.6	.00547	.00523	.00500	.00477	.00456	.00435	.00415	.00396	.00378	.00361
1.7	.00344	.00328	.00313	.00299	.00285	.00272	.00259	.00247	.00235	.00224
1.8	.00213	.00203	.00193	.00184	.00175	.00167	.00159	.00151	.00144	.00137
1.9	.00130	.00124	.00117	.00112	.00106	.00101	.00096	.00091	.00086	.00082
2.0	.00078	.00074	.00070	.00067	.00063	.00060	.00057	.00054	.00051	.00048
2.1	.00046	.00043	.00041	.00039	.00037	.00035	.00033	.00031	.00030	.00028
2.2	.00027	.00025	.00024	.00022	.00021	.00020	.00019	.00018	.00017	.00016
2.3	.00015	.00014	.00013	.00013	.00012	.00011	.00011	.00010	.00009	.00009
2.4	.00008	.00008	.00007	.00007	.00007	.00006	.00006	.00005	.00005	.00005
2.5	.00005	.00004	.00004	.00004	.00004	.00003	.00003	.00003	.00003	.00003
2.6	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00001	.00001	.00001
2.7	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.8	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.9	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

M= .800

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.22366	.22366	.22366	.22366	.22366	.22366	.22366	.22366	.22366	.22366
.1	.22366	.22366	.22366	.22366	.22366	.22366	.22366	.22366	.22366	.22366
.2	.22366	.22366	.22366	.22365	.22365	.22365	.22364	.22363	.22360	.22356
.3	.22349	.22339	.22324	.22304	.22278	.22243	.22200	.22146	.22081	.22003
.4	.21912	.21808	.21689	.21555	.21406	.21243	.21064	.20871	.20664	.20443
.5	.20209	.19962	.19703	.19433	.19152	.18862	.18563	.18257	.17943	.17623
.6	.17298	.16969	.16635	.16299	.15960	.15619	.15278	.14936	.14595	.14254
.7	.13914	.13577	.13241	.12908	.12579	.12252	.11929	.11610	.11296	.10986
.8	.10680	.10380	.10084	.09793	.09508	.09228	.08954	.08685	.08422	.08164
.9	.07912	.07666	.07425	.07190	.06960	.06736	.06518	.06305	.06098	.05896
1.0	.05699	.05508	.05322	.05141	.04965	.04794	.04628	.04466	.04310	.04158
1.1	.04010	.03867	.03729	.03594	.03464	.03338	.03216	.03097	.02983	.02872
1.2	.02765	.02661	.02560	.02463	.02370	.02279	.02192	.02107	.02025	.01946
1.3	.01870	.01797	.01726	.01658	.01592	.01528	.01467	.01408	.01351	.01296
1.4	.01243	.01192	.01143	.01096	.01051	.01007	.00965	.00925	.00886	.00848
1.5	.00812	.00778	.00745	.00713	.00682	.00653	.00624	.00597	.00571	.00546
1.6	.00522	.00499	.00477	.00456	.00435	.00416	.00397	.00379	.00362	.00346
1.7	.00330	.00315	.00300	.00287	.00273	.00261	.00249	.00237	.00226	.00215
1.8	.00205	.00195	.00186	.00177	.00169	.00161	.00153	.00146	.00139	.00132
1.9	.00125	.00119	.00113	.00108	.00103	.00097	.00093	.00088	.00084	.00079
2.0	.00075	.00072	.00068	.00065	.00061	.00058	.00055	.00052	.00050	.00047
2.1	.00045	.00042	.00040	.00038	.00036	.00034	.00032	.00031	.00029	.00027
2.2	.00026	.00024	.00023	.00022	.00021	.00020	.00018	.00017	.00017	.00016
2.3	.00015	.00014	.00013	.00012	.00012	.00011	.00010	.00010	.00009	.00009
2.4	.00008	.00008	.00007	.00007	.00006	.00006	.00006	.00005	.00005	.00005
2.5	.00004	.00004	.00004	.00004	.00003	.00003	.00003	.00003	.00003	.00003
2.6	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00001	.00001	.00001
2.7	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.8	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.9	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

Table 15.

$$\int_x^{\infty} \frac{e^{-u^2 - \frac{m^2}{u^2}}}{u^2}\,du$$

M= .900

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.16277	.16277	.16277	.16277	.16277	.16277	.16277	.16277	.16277	.16277
.1	.16277	.16277	.16277	.16277	.16277	.16277	.16277	.16277	.16277	.16277
.2	.16277	.16277	.16277	.16277	.16277	.16277	.16277	.16277	.16276	.16276
.3	.16275	.16273	.16270	.16266	.16261	.16252	.16241	.16226	.16206	.16182
.4	.16151	.16114	.16070	.16018	.15957	.15888	.15809	.15721	.15624	.15516
.5	.15399	.15272	.15136	.14990	.14835	.14672	.14500	.14320	.14132	.13938
.6	.13736	.13529	.13316	.13099	.12876	.12650	.12420	.12188	.11953	.11715
.7	.11477	.11237	.10996	.10755	.10515	.10274	.10035	.09796	.09559	.09324
.8	.09090	.08859	.08630	.08404	.08180	.07960	.07742	.07528	.07317	.07110
.9	.06906	.06706	.06509	.06317	.06128	.05943	.05761	.05584	.05411	.05241
1.0	.05076	.04914	.04756	.04602	.04452	.04306	.04163	.04024	.03889	.03757
1.1	.03629	.03505	.03384	.03267	.03152	.03042	.02934	.02830	.02728	.02630
1.2	.02535	.02443	.02353	.02267	.02183	.02101	.02023	.01947	.01873	.01802
1.3	.01733	.01667	.01603	.01541	.01481	.01423	.01367	.01313	.01261	.01211
1.4	.01162	.01116	.01071	.01027	.00986	.00945	.00906	.00869	.00833	.00799
1.5	.00765	.00733	.00702	.00673	.00644	.00617	.00590	.00565	.00541	.00517
1.6	.00495	.00473	.00453	.00433	.00414	.00395	.00378	.00361	.00345	.00329
1.7	.00314	.00300	.00287	.00274	.00261	.00249	.00238	.00227	.00216	.00206
1.8	.00196	.00187	.00178	.00170	.00162	.00154	.00147	.00140	.00133	.00127
1.9	.00121	.00115	.00109	.00104	.00099	.00094	.00089	.00085	.00081	.00077
2.0	.00073	.00069	.00066	.00062	.00059	.00056	.00053	.00051	.00048	.00045
2.1	.00043	.00041	.00039	.00037	.00035	.00033	.00031	.00030	.00028	.00027
2.2	.00025	.00024	.00022	.00021	.00020	.00019	.00018	.00017	.00016	.00015
2.3	.00014	.00014	.00013	.00012	.00011	.00011	.00010	.00010	.00009	.00008
2.4	.00008	.00008	.00007	.00007	.00006	.00006	.00006	.00005	.00005	.00005
2.5	.00004	.00004	.00004	.00004	.00003	.00003	.00003	.00003	.00003	.00002
2.6	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00001	.00001	.00001
2.7	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.8	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.9	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

M=1.000

χ	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.11994	.11994	.11994	.11994	.11994	.11994	.11994	.11994	.11994	.11994
.1	.11994	.11994	.11994	.11994	.11994	.11994	.11994	.11994	.11994	.11994
.2	.11994	.11994	.11994	.11994	.11994	.11994	.11994	.11994	.11994	.11994
.3	.11993	.11993	.11993	.11992	.11991	.11989	.11987	.11983	.11978	.11971
.4	.11962	.11951	.11936	.11918	.11895	.11869	.11838	.11801	.11759	.11711
.5	.11657	.11597	.11530	.11457	.11377	.11291	.11198	.11099	.10993	.10882
.6	.10764	.10641	.10512	.10378	.10239	.10096	.09949	.09797	.09642	.09484
.7	.09323	.09159	.08993	.08825	.08656	.08485	.08314	.08141	.07969	.07796
.8	.07623	.07450	.07279	.07107	.06937	.06768	.06600	.06434	.06270	.06107
.9	.05946	.05788	.05631	.05477	.05325	.05175	.05028	.04883	.04741	.04602
1.0	.04465	.04331	.04200	.04071	.03946	.03823	.03702	.03585	.03470	.03358
1.1	.03249	.03143	.03039	.02938	.02839	.02743	.02650	.02559	.02471	.02385
1.2	.02302	.02221	.02142	.02066	.01992	.01920	.01850	.01783	.01718	.01654
1.3	.01593	.01533	.01476	.01420	.01366	.01314	.01264	.01215	.01168	.01122
1.4	.01079	.01036	.00995	.00956	.00918	.00881	.00845	.00811	.00778	.00746
1.5	.00716	.00686	.00658	.00631	.00604	.00579	.00555	.00531	.00509	.00487
1.6	.00466	.00446	.00427	.00408	.00391	.00374	.00357	.00341	.00326	.00312
1.7	.00298	.00285	.00272	.00260	.00248	.00237	.00226	.00215	.00206	.00196
1.8	.00187	.00178	.00170	.00162	.00154	.00147	.00140	.00134	.00127	.00121
1.9	.00115	.00110	.00104	.00099	.00095	.00090	.00086	.00081	.00077	.00073
2.0	.00070	.00066	.00063	.00060	.00057	.00054	.00051	.00049	.00046	.00044
2.1	.00042	.00039	.00037	.00035	.00034	.00032	.00030	.00029	.00027	.00026
2.2	.00024	.00023	.00022	.00021	.00019	.00018	.00017	.00016	.00016	.00015
2.3	.00014	.00013	.00012	.00012	.00011	.00010	.00010	.00009	.00009	.00008
2.4	.00008	.00007	.00007	.00006	.00006	.00006	.00005	.00005	.00005	.00005
2.5	.00004	.00004	.00004	.00004	.00003	.00003	.00003	.00003	.00003	.00002
2.6	.00002	.00002	.00002	.00002	.00002	.00002	.00001	.00001	.00001	.00001
2.7	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2.8	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.9	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.0	.00000									

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Subject Index | Page

Analogs
Chapter 12 177

Artesian well
Chapter 4 37

Artesian well flow
Formula 4-7 41
Formula 4-11 43

Bank storage
Chapter 6 73

Boussinesq
Footnote 9
Development of Boussinesq 91

Canal seepage
Chapter 7 85

Darcy's law 7
206

Definition of terms 204

Depletion of a stream by an adjacent well
Formula 9-1 141

Determination of aquifer properties
From tests of a pumpted well 20
Impermeable outer boundary present 31
From Artesian well test data 44
From tests of an aquifer with a semi-permeable
confining bed 67

Digital computers
Ground water flow treatment by 184
Combination methods 197

Drawdowns due to pumping a well
Formula 3-1 19
Water table maintained at r = b 27
Impermeable outer boundary present 28
Due to an Artesian well
Formula 4-1 40
Formula 4-9 42
Well with a semi-permeable confining bed
Formula 5-2 58

Drift
Chapter 11 165

Subject index	Page
Dumm, Tapp and Moody	96
Dupuit	8,13,15,16,80,206
Forchheimer	13,15,16,80,206
Images	146
Ghyben-Herzberg principle	4
Impermeable outer boundary present	28
Index to tables	211
Intermittent operation	146,152
Laplace formulation	104
Line source	85
Local convergence	127
Maasland, M.	117
Maintained outer boundary	25
Notations	199
Parallel drains	89
Return flow	153
River valley idealization	137,162
Semi-permeable confining layer	57
Stream depletion	141
Ultimate steady state	
Confining layer present. Formula 5-6	61
Well	
In an infinitely extended aquifer	18
In a corner	151
In an aquifer of limited width	149,154
Intermittent operation	152
Werner	11,77,91

Figure index | Page

2-1	Volume element	7
2-2	Cross section	8
2-3	Parallel flow	10
2-4	Radial flow	12
3-1	$s/(Q/2\pi KD)$ vs $(r/\sqrt{4\alpha t})$	21
3-2	Plot of drawdowns s vs $(r/\sqrt{t})$	23
3-3	Master chart--impermeable outer boundary	32
3-4	Plot of drawdowns y vs t	33
4-1	Section showing an artesian stratum	37
4-2	Plot of $G(\sqrt{4\alpha t}/a)$	45
4-3	Observations	46
4-4	H vs $(\sqrt{4\alpha t}/a)$	54
5-1	Well drawing water from an aquifer overlain by a semi-permeable bed	57
5-2	$Y/(Q/2\pi KD)$ vs $(r/\sqrt{4\alpha t})$	60
5-3	Plot of data of table 5-1	68
6-1	Bank storage	74
6-2	Integral and derivative	75
6-3	Comparison of solutions for $H = d$	81
6-4	Comparison of solutions for $H = 2d$	82
7-1	Line source	85
8-1	Cross section normal to the line of drains	90
8-2	Comparison of solutions for $\frac{H}{d} = \frac{6}{7}$	114
8-3	Maasland's curve	120
8-4	Drainable depth at center of drain spacing	122
8-5	Drain	128
8-6	(h_c/H) vs $(\alpha t/L^2)$	132
8-7	(h_o/H) vs $(\alpha t/L^2)$	133
8-8	Parallel drain idealization	137
8-9	Idealization of a river valley	137
9-1	Stream depletion due to a well	143
10-1	Well in a corner	151
10-2	Pumping pattern	162
10-3	Cross section of a river valley	162
11-1	Positions	166
12-1	Transient type analog	179
12-2	View of back of panel showing capacitors and ground wires	179
12-3	Network arrangement	180
12-4	Steady state analog	183
12-5	Source-sink combination	194

~ About this Book ~

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