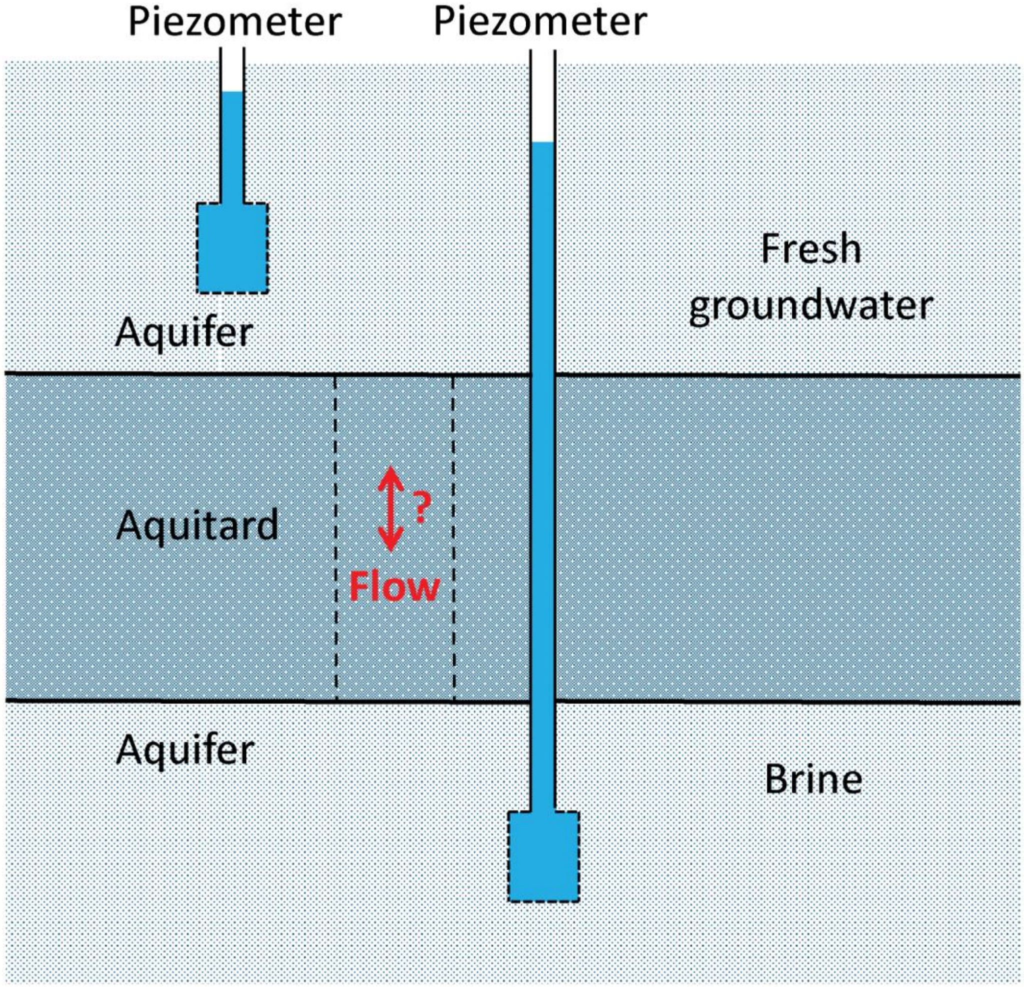

Darcy's Law in Variable Density Groundwater Systems

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THE GROUNDWATER PROJECT

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Dedication

This book is dedicated to numerous professors and private-sector mentors who, over many years, guided me in the direction of practical quantitative hydrology.

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The Groundwater Project Foreword

The United Nations (UN) - Water Summit on Groundwater, held from 7 to 8 December 2022 at the UNESCO headquarters in Paris, France, concluded with a call for governments and other stakeholders to scale up their efforts to better manage groundwater. The intent of the call to action was to inform relevant discussions at the UN 2023 Water Conference held from 22 to 24 March 2023 at the UN headquarters in New York City. One of the required actions is *strengthening human and institutional capacity*, for which groundwater education is fundamental.

The 2024 World Water Day theme is *Water for Peace*, which focuses on the critical role water plays in the stability and prosperity of the world. The UN-Water website↗ states that *more than three billion people worldwide depend on water that crosses national borders*. There are 592 transboundary aquifers, yet most countries do not have an intergovernmental cooperation agreement in place for sharing and managing the aquifer. Moreover, while groundwater plays a key role in global stability and prosperity, it also makes up 99 percent of all liquid freshwater—accordingly, groundwater is at the heart of the freshwater crisis. *Groundwater is an invaluable resource*.

The Groundwater Project (GW-Project), a registered Canadian charity founded in 2018 is committed to advancement of groundwater education as a means to accelerate action related to our essential groundwater resources. We are committed to *making groundwater understandable* and, thus, enable *building the human capacity for sustainable development and management of groundwater.* To that end, the GW-Project creates and publishes high-quality books about *all-things-groundwater*, for all who want to learn about groundwater. Our books are unique. They synthesize knowledge, are rigorously peer reviewed and translated into many languages, and are free of charge. An important tenet of GW-Project books is a strong emphasis on visualization: Clear illustrations stimulate spatial and critical thinking. The GW-Project started publishing books in August 2020; by the end of 2023, we had published 44 original books and 58 translations. The books can be downloaded at gw-project.org↗.

The GW-Project embodies a new type of global educational endeavor made possible by the contributions of a dedicated international group of volunteer professionals from a broad range of disciplines. Academics, practitioners, and retirees contribute by writing and/or reviewing books aimed at diverse levels of readers including children, teenagers, undergraduate and graduate students, professionals in groundwater fields, and the general public. More than 1,000 dedicated volunteers from 70 countries and six continents are involved—and participation is growing. Revised editions of the books are published from time to time. Readers are invited to propose revisions.

We thank our sponsors for their ongoing financial support. Please consider donating to the GW-Project so we can continue to publish books free of charge.

The GW-Project Board of Directors, January 2024

Foreword

This is the first publication in a new series of educational products offered by The Groundwater Project. Described as mini-books, these products focus on very specific subjects in groundwater hydrology with an emphasis on performing practical and insightful calculations. The aim is to provide students and practitioners with useful quantitative tools for evaluating groundwater systems without the use of complex numerical models or proprietary software. These publications are formatted to be brief, concise, and well-focused. We encourage the groundwater community to submit ideas for mini-books on other subjects to augment this series.

This mini-book addresses an issue in groundwater hydrology that is unfamiliar to many groundwater practitioners: using Darcy's law to estimate groundwater flow direction and flux in systems with variable pore water density. Fortunately, in most natural groundwater systems the variation in groundwater density is not extreme and the familiar constant density (hydraulic head) form of Darcy's law is sufficiently accurate for practical application. However, in systems with variable groundwater density, the head form of Darcy's law can break down and result in incorrect flow directions and inaccurate fluxes.

Some examples of systems with extreme density conditions include—but are not limited to—seawater incursions, geothermal areas, deep oil field brines, playa lakes, liners below chemical storage ponds, and highly contaminated groundwater chemical plumes. In these types of systems, a different (pressure-based) form of Darcy's law may be required to obtain defensible results. While presented in the literature for decades, the pressure-based form of Darcy's law is unfamiliar to many practicing groundwater hydrologists even though it is the more universal and used in other industries such as petroleum engineering. This mini-book walks through the basic principles of Darcy's law including pressure, gravity, and the concept of hydraulic head. Illustrative problems are presented and solved to accentuate the difference between the two forms of Darcy's law.

John Cherry, The Groundwater Project Leader

Guelph, Ontario, Canada, February 2024

Preface

The value of numerical models is well recognized in the groundwater industry. However, I feel that many practitioners are too quick to jump to complex numerical models before a fundamental understanding of the groundwater system has been developed. In many situations, simple analytical solutions can be valuable in answering the questions being posed or troubleshooting numerical modeling results. I refer to these as scoping-level calculations that get close (or close enough) to answering the issue being evaluated.

Over 40 years, my coworkers and I have identified numerous errors in numerical models that were uncovered by scoping-level calculations done with a calculator (including more than a few of my own!). Such modeling errors can result from seemingly innocuous causes such as a slipped decimal point in an input parameter, an incorrect unit conversion, or misrepresentation of a boundary condition. While a scoping-level calculation may not necessarily substitute for a numerical model (particularly in the eyes of many government regulators), it has been shown time and time again that a well-posed analytical calculation provided a result that was fully consistent with an eventual model prediction and in some cases could have answered the question at hand without resorting to the time and expense of the subsequent modeling effort.

In groundwater hydrology, the value of analytical solutions is well known as a method for analyzing pumping tests. One advantage is that the solutions can be cast in terms of dimensionless parameters, and these can be assigned unique values through a curve-matching procedure. The process would be more tedious and uncertain if done by matching field data to graphical predictions of a numerical model.

Darcy's law is the most fundamental concept in groundwater hydrology and forms the basis of essentially all quantitative methods for evaluating groundwater flow and transport. I hope this mini-book provides insight into certain aspects of Darcy's law that are not readily apparent in many textbooks and course curricula. A greater understanding of Darcy's law, as it pertains to variable density groundwater, will hopefully allow students and practitioners to devise their own scoping-level calculations to evaluate questions of practical importance for these types of groundwater systems.

Fred Marinelli, February 2024

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I greatly appreciate the thorough and useful reviews and contributions to this book by the following individuals:

- Dr. David Rudolph, professor, University of Waterloo, Waterloo, Ontario, Canada
- Dr. Vincent Post, associate professor, Flinders University, Bedford Park, South Australia, Australia
- Dr. Neil Thompson, professor, University of Waterloo, Waterloo, Ontario, Canada

These reviews led to many changes in the original text and greatly improved this published version. I am grateful to Amanda Sills and the formatting team of The Groundwater Project for their oversight and copyediting of this book. I also thank Dr. Eileen Poeter (professor emeritus, Colorado School of Mines, Golden, Colorado, USA) for her review, editing, and encouragement in producing this book.

The source of material presented in figures and tables are acknowledged in the captions; however, figures and tables without a citation to source are original to this book.

1 Introduction

In most natural systems, the variation in groundwater density is sufficiently small to be safely ignored when performing routine calculations. However, there are situations, both natural and human-induced, where groundwater density must be considered to achieve accurate calculations of flow and transport. These include—but are not necessarily limited to—elevated groundwater temperatures in deep mines and geothermal areas, seawater intrusion into freshwater aquifers along coastlines, and groundwater chemical plumes with very high total dissolved solids (TDS).

Because these situations are regarded as non-routine, many hydrologists do not consider if the effects of groundwater density need to be incorporated into analytical calculations or numerical models. For this reason, the significance of variable groundwater density is not explicitly discussed in many introductory textbooks on groundwater hydrology. An excellent treatment of the subject is presented in Post and Simmons (2022), which provides transient multidimensional simulations to illustrate how variable groundwater density can impart significant and sometimes nonintuitive flow behaviors in natural systems.

In this mini-book, we focus on Darcy’s law for one-dimensional flow in a homogeneous porous medium. We first review the hydraulic head form of Darcy’s law that is familiar to most groundwater hydrologists and discuss the concept of head as a hydraulic potential. We then devise a thought experiment to show that the head form of Darcy’s law cannot work for a hypothetical system with variable-density groundwater. This leads us to the pressure-based form of Darcy’s law, which is more universal and can be used to estimate flux in systems with variable-density groundwater. Finally, through illustrative examples, we use the pressure-based form to evaluate horizontal and vertical fluxes in systems that are not amenable to analysis by the head form.

2 Head Form of Darcy's Law

Hydrologists are generally familiar with the following form of Darcy's law for one-dimensional flow in an isotropic saturated medium.

$$q(s') = -K \left. \frac{dH}{ds} \right|_{s'} \tag{1}$$

where (parameter dimensions are in dark green font with mass as M, length as L, time as T, temperature as Θ):

s = distance variable (L)

s' = specified distance coordinate (L)

q = specific discharge; volumetric flow rate per unit cross-sectional area of the medium normal to the flow direction (LT^{-1})

H = hydraulic head (L)

K = hydraulic conductivity (LT^{-1})

In Equation (1), the derivative (dH/ds) is the hydraulic gradient or rate of change in head with distance along the flow direction. The subscript at the end of the equation implies that the derivative is evaluated at the specified coordinate s'. The negative sign indicates that flow is in the direction of *decreasing* hydraulic head. For this form of Darcy's law to work in any direction, head must exist as a hydraulic potential as explained subsequently and discussed in Section 3. In this book, we express hydraulic head with a capital "H" when it satisfies the conditions for being a hydraulic potential, and as a lower case "h" when it is not a hydraulic potential.

The relationships associated with Equation (1) are shown diagrammatically in Figure 1.

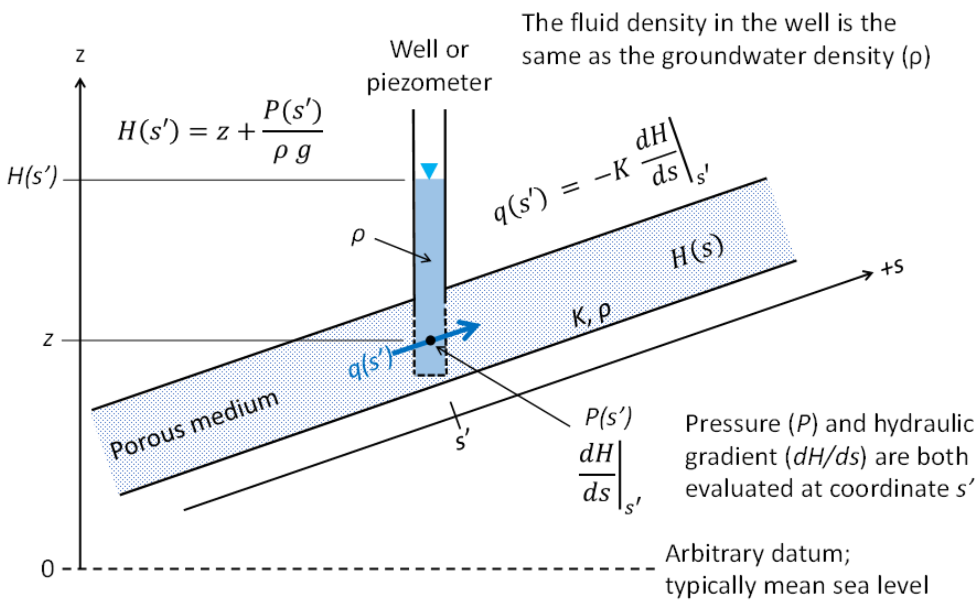


Figure 1 - One-dimensional head form of Darcy's law traditionally used in groundwater hydrology. The derivative dH/ds is the gradient of hydraulic head (H) evaluated at coordinate s'. Specific discharge (q) is positive in the $+s$ direction. The negative sign indicates that flow is in the direction of decreasing hydraulic head.

In groundwater textbooks (for example, Freeze & Cherry, 1979; Domenico & Schwartz, 1990), Equation (2) is often given for hydraulic head.

$$H = z + \frac{P}{\rho\, g} \tag{2}$$

where:

P = pore fluid gauge pressure ($ML^{-1}T^{-2}$), for example as $kg/m/s^2$, or Pascal which is abbreviated with Pa, or Newton/m^2

z = vertical height above an arbitrary datum at which pressure is measured (L)

ρ = groundwater (pore fluid) density (ML^{-3})

g = acceleration of gravity (MT^{-2}), which is 9.807 m/s^2

In Equation (2), gauge pressure is equal to absolute pressure minus the prevailing atmospheric pressure. Its value is 0 Pa when sensing the atmosphere and is the pressure typically measured by a pressure gauge or open-end manometer. In fluid mechanics textbooks, an additional term is added to the right side of Equation (2) to account for the kinetic energy (or momentum) associated with a moving fluid. However, in groundwater systems, the velocity tends to be very small, so this term is generally ignored in the groundwater discipline.

We refer to Equation (1) as the head form of Darcy's law. Hydraulic head (H) is typically viewed as the water level attained in a well or piezometer that is in hydraulic communication with the pore water in the geologic medium. The well operates as a manometer. Hydraulic head is expressed as a height above an arbitrary datum (typically, elevation above mean sea level).

Hydraulic conductivity (K) is a parameter that depends on properties of both the medium and the pore fluid:

$$K = \frac{k\,\rho\,g}{\mu} \tag{3}$$

where:

k = intrinsic permeability of the porous medium (L^2)

μ = groundwater dynamic viscosity at the prevailing system temperature ($ML^{-1}T^{-1}$)

Dynamic viscosity is sometimes described with a unit called *poise*, which can be defined as: 1 poise = 1 gram/cm/s = 0.1 kg/m/s. Pure water at 20 °C and atmospheric pressure has a dynamic viscosity of 0.001 kg/m/s, which is equal to 0.01 poise, described in many older textbooks as one centipoise.

Several conditions must be met for Equation (1) to be strictly valid. First is that hydraulic head (H) must exist as a hydraulic potential. Section 3 shows that for all practical purposes, this is true only for systems with uniform pore water density (ρ). Second, the same density value must be used in both Equation (2) and Equation (3). Referring to Figure 1, if hydraulic head is taken as the physical water level elevation in a monitoring well, the density of fluid in the well-water column must be the same as the groundwater density. As discussed in Section 4.1, water viscosity (μ) is sensitive to temperature. Thus, the value of viscosity in Equation (3) should reflect the prevailing temperature in the groundwater system under consideration.

As a practical matter, the above conditions are approximately met in most groundwater systems that hydrologists deal with. For this reason, issues regarding groundwater density and viscosity are commonly not considered by hydrologists when performing Darcy's law calculations. The parameter that usually controls the overall reliability of a groundwater calculation is hydraulic conductivity (K), which is always uncertain due to heterogeneity of geologic materials and difficulty associated with measuring it. As a consequence, attention is usually focused on hydraulic conductivity and its variability, with little or no consideration of pore fluid density and viscosity. Experience has shown that this strategy is appropriate for most applications of Darcy's law in typical groundwater systems. However, there are systems in which failing to account for the magnitude/variation of groundwater density and viscosity can lead to erroneous results. Some of these situations are addressed throughout this book.

At this point, we consider isotropic (nondirectional characteristics) porous media with regard to hydraulic conductivity and intrinsic permeability. In an isotropic medium, the flow is in the direction of the maximum hydraulic gradient, which is the direction with the *maximum* rate of decrease in hydraulic head with distance. If the medium has directional characteristics (i.e., is anisotropic), the expression of Darcy's law becomes more complex with respect to hydraulic head (H). In Sections 7 and 8, we will partially relax the condition of an isotropic medium when evaluating strictly horizontal and vertical flow.

Figure 2 shows a laboratory column filled with a porous medium that can rotate to different vertical orientations. With flexible tubing, the ends of the column are each connected to water reservoirs that also function as piezometers. If there are no hydraulic losses in the tubing, the flow rate through the column is given by Equation (4).

$$Q = q\,A = -K\,A\left(\frac{dH}{ds}\right) \approx -K\,A\left(\frac{H_2 - H_1}{L}\right) \tag{4}$$

where:

Q = flow rate, positive in the +s direction ($L^{-3}T^{-1}$)

A = Column cross-sectional area (L^2)

L = column length (L)

H_i = water level above the datum (head) in piezometer i (L)

The other parameters were previously defined. The column can rotate to any orientation in the vertical plane, which changes the flow direction. However, the flow magnitude (Q) remains constant as long as the head differential ($H_2 - H_1$) does not change. This behavior is a powerful property of the head form of Darcy's law; however, it occurs only when head (H) satisfies the conditions for being a hydraulic potential.

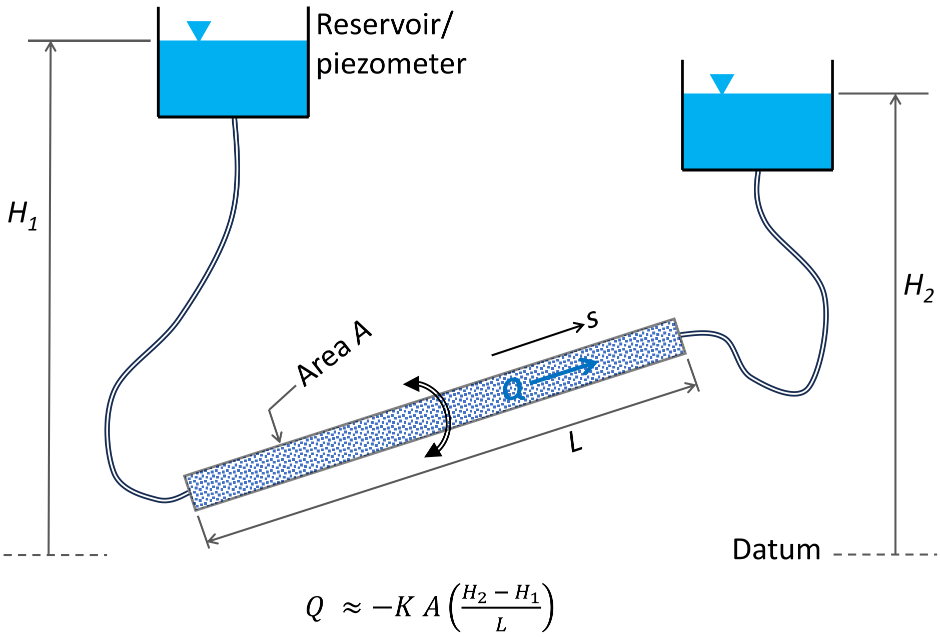


Figure 2 - Laboratory column with water reservoirs that also serve as piezometers. The column can rotate to any orientation, which changes the flow direction. If the head differential ($H_1 - H_2$) is fixed, the flow magnitude (Q) does not change regardless of the flow direction. This behavior occurs only if head (H) is a hydraulic potential.

3 Hydraulic Head as a Potential

In a groundwater flow system hydraulic head (*H*) must have the following properties to be a hydraulic potential quantity.

- It is a scalar quantity with no implied directionality.
- It varies continuously with no discontinuities or jumps.
- In going from one point to another, the change in H is the same regardless of the path taken.
- The direction of flow (or components of flow) is always in the direction of decreasing H.
- A point in space can have only one value of H.

To investigate this further, let us define what is meant by a *thought experiment*. One can define a thought experiment as a hypothetical situation in which a hypothesis, theory, or principle is laid out for the purpose of thinking through its consequences. A thought experiment does not have to be real, but it must entail characteristics that are consistent and relevant to the process or issue being investigated.

To evaluate the existence of hydraulic potentials in a variable-density system, consider the thought experiment for a thermally driven convection cell shown in Figure 3. In this experiment, externally controlled temperature variations affect the fluid density (and viscosity) in each leg of the cell.

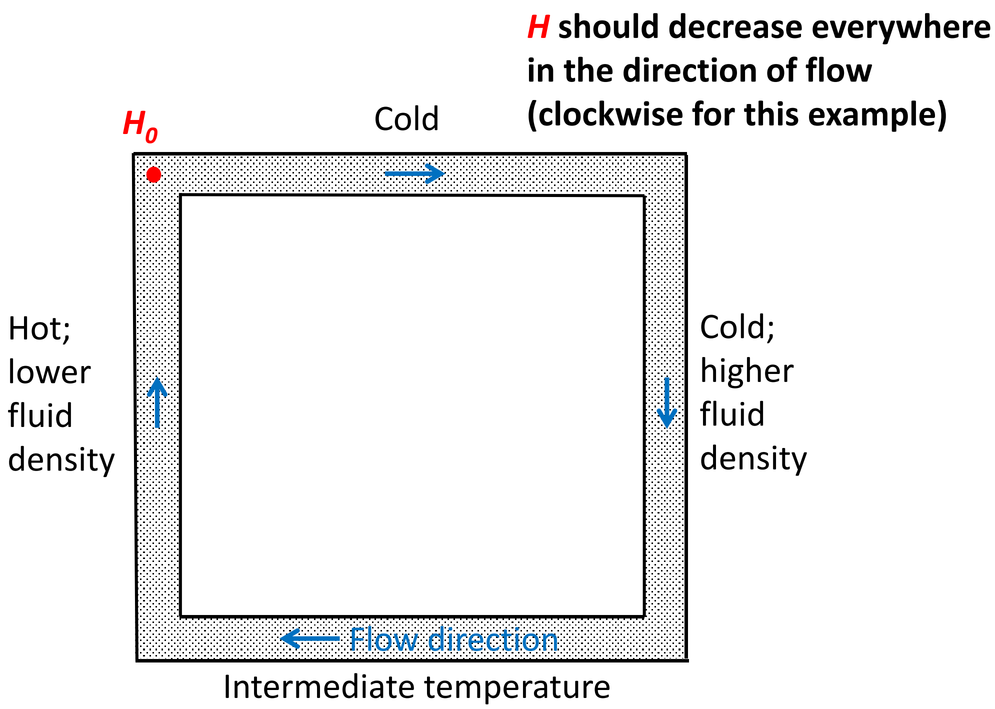


Figure 3 - Thermally driven convection cell. This figure illustrates a hypothetical closed loop consisting of a pipe filled with a saturated porous medium. The temperature in each leg of the cell can be externally controlled and maintained. In a laboratory setting, this might be accomplished using heating/cooling coils wrapped around each leg.

For this situation, experience indicates that density-driven flow will be induced in the pipe loop as shown. Our mission is to define a location-specific scaler quantity H that satisfies the rules for being a potential. H_0 is an arbitrary value assigned at the upper left corner of the pipe loop. To be a potential, H must decrease in the direction of flow *and* it must vary continuously with no jumps or discontinuities.

Do you see the problem? If we assign a starting value of H_0 at the upper left corner and follow the flow path, it is possible for H to decrease in the direction of flow. However, as we come full circle and return to the starting location, the value of H would have to jump from a lower value to the higher starting value (H_0). This would be a discontinuity, which cannot occur for a potential quantity. If H changes smoothly with no jumps, then somewhere in the system H would have to increase in the direction of flow, which also violates the rules for a potential.

In fact, for this variable-density system, one cannot invent *any* scalar quantity that can be used in the head form of Darcy's law to compute the correct flow direction at all locations. The head form of Darcy's law might work for certain portions of this system, but it doesn't work at all locations in the system. This is true no matter how you choose to define H. The bottom line is that the head form of Darcy's law cannot work for this variable-density system because no matter how one chooses to define head, it cannot exist as a hydraulic potential.

Using energy considerations, M. K. Hubbert (1940, 1956) developed a complete definition of hydraulic head as shown in Equation (5).

$$H(z', P') = \frac{\Phi}{g} = (z' - z^*) + \frac{1}{g} \oint_{P^*}^{P'} \frac{1}{\rho(P)} dP \tag{5}$$

where (parameter dimensions are in dark green font with mass as M, length as L, time as T, temperature as Θ):

Φ = potential; work required to transform a unit mass of the fluid from an arbitrary reference state to the current state at a location of interest ($L^{-2}T^{-2}$)

z^* = arbitrary elevation of the reference state datum (L)

P^* = arbitrary pressure for the reference state ($ML^{-1}T^{-2}$)

z' = elevation above the datum at the location of interest(L)

P' = pore fluid pressure at the location of interest ($ML^{-1}T^{-2}$)

The small circle in the integral indicates that it is *path independent*, which implies that the integral has the same value regardless of the path taken in going from the reference state to the location of interest. Hubbert (1940, 1956) showed that for H to exist as a potential, the integral must be path independent.

Since the reference state is arbitrary, we can assign $z^*= 0$ and $P^*= 0$. With these substitutions, Equation (5) simplifies to Equation (6).

$$H(z', P') = z' + \frac{1}{g} \oint_{0}^{P'} \frac{1}{\rho(P)} dP \tag{6}$$

For the integral to be path independent, density must be a *unique* function of pressure. That means there can be no two points in the system where the pore water has the same pressure but different fluid densities. Otherwise, the integral is indeterminant and H cannot exist as a potential quantity. This is illustrated in Figure 4.

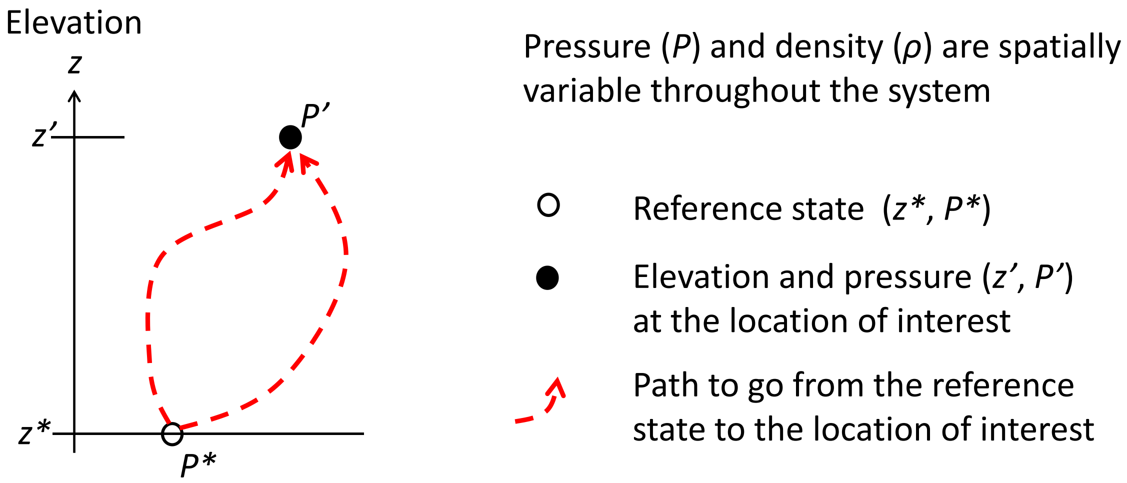


Figure 4 - Path from the reference state to the location of interest. In this system, pore water pressure and density both vary spatially. For hydraulic head to be a potential, the value of the integral in Equation (6) must be the same regardless of the path taken in going from P^* to P'.

If the water density is everywhere constant, ρ can come out of the integral in Equation (6) and the relationship simplifies to Equation (7), which is equivalent to the previously presented Equation (2):

$$H(z', P') = z' + \frac{P'}{\rho\, g} \tag{7}$$

However, this requires that the groundwater density (ρ) is everywhere constant. We conclude that, from a theoretical perspective, the head form of Darcy's law is strictly valid only in groundwater systems with spatially uniform fluid density.

For variable-density groundwater flow systems, previous investigators have proposed different ways of defining head at a location of interest. For example, h_f in Equation (8) is referred to as the *freshwater head*, while h_p in Equation (9) is referred to as *pointwater head*.

$$h_f(z', P') = z' + \frac{P'}{\rho_f\, g} \tag{8}$$

$$h_p(z', P') = z' + \frac{P'}{\rho' \, g} \tag{9}$$

where:

z'	=	elevation at a specified location of interest (L)
P'	=	pore water pressure at the location of interest ($ML^{-1}T^{-2}$)
h_f	=	freshwater head (L)
ρ_f	=	density of pure water at 20 °C (ML^{-3}), which is 998.2 kg/m^3
h_p	=	pointwater head (L)
ρ'	=	pore fluid density at the location of interest, that is, where P' is measured (ML^{-3})

At this point, we define these proposed heads with a lowercase h, as it has not been demonstrated that they are true hydraulic potentials that can work universally in the head form of Darcy's law. We evaluate the applicability of these modified representations of head throughout this book.

4 Pressure Form of Darcy's Law

Referring to Figure 5, there is a second form of Darcy's law that does not require use of a hydraulic potential. It is based on separate terms pertaining to gravity and pressure. This is the pressure form of Darcy's law shown in Equation (10).

$$q(s') = -\frac{k}{\mu}\left[\rho(s')\, g \sin(\theta) + \left.\frac{dP}{ds}\right|_{s'}\right] \tag{10}$$

where:

θ = direction of flow from horizontal; counter-clockwise positive [radians or degrees]

The other parameters were defined previously. In Equation 10 (and other subsequent equations), a combination of parentheses and brackets is used to better identify the computational grouping of terms.

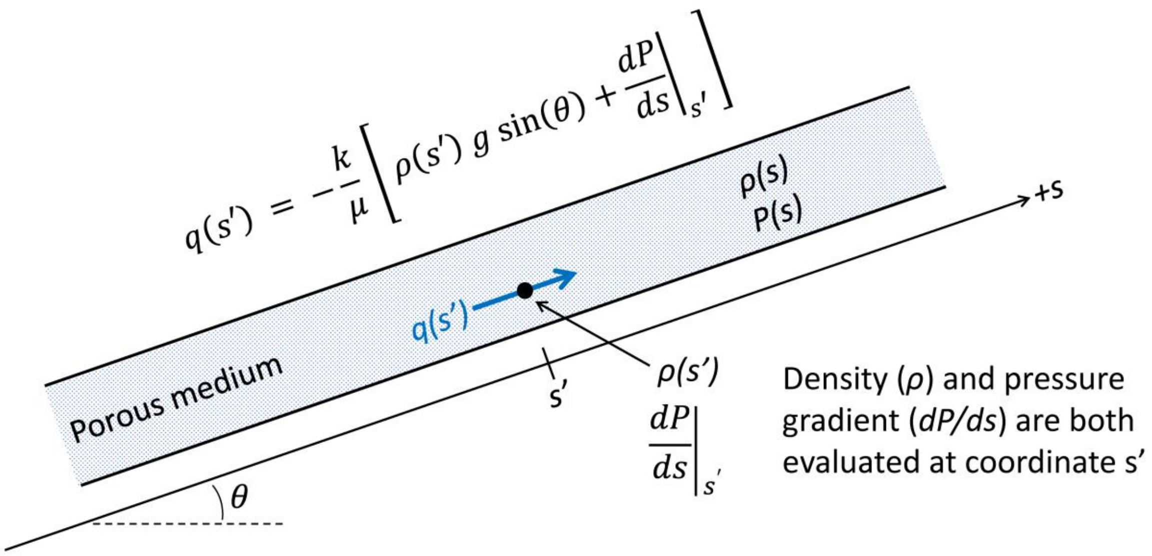


Figure 5 - The one-dimensional pressure form of Darcy's law. The derivative dP/ds is the gradient of pressure evaluated at coordinate s'. Because there are two terms within the brackets, flow is not necessarily in the direction of decreasing pressure. Specific discharge (q) is positive in the $+s$ direction.

The specific discharge at coordinate s', $q(s')$, is the flow rate per unit cross-sectional area of the medium perpendicular to the s direction, the same as for the head form of Darcy's law in Equation (1). The first term within the brackets is the s-direction component of the force per unit water volume due to gravity. The second term within the brackets is the s-direction component of the net force per unit water volume due to the pressure gradient (McWhorter & Sunada, 1977). Interestingly, when the s direction is horizontal (θ is either 0 or π radians), there is no gravity effect and flow is driven solely by the pressure gradient in the direction of decreasing pressure. It has been said the head form of Darcy's law sometimes works, but the pressure form of Darcy's law *always* works. This is the version of Darcy's law used in petroleum engineering.

For an isotropic medium with a given pressure gradient (dP/ds), this pressure form of Darcy's law depends on the flow direction as defined by the angle θ. For an anisotropic medium where intrinsic permeability (k) has directional properties, the pressure form of Darcy's law becomes more complex with regard to gravity and pressure.

4.1 Effect of Temperature

Let us first consider a low-salinity groundwater system with elevated temperatures, such as in a geothermal area. The density and viscosity of pure water change significantly with temperature. For example, when temperature increases from 20 to 80 °C:

- density of pure water decreases from 997.94 to 971.20 kg/m^3 (a decrease of 2.68 percent), and
- viscosity decreases from 0.0010 to 0.00036 kg/m/s (i.e., lower by a factor of 2.82).

These variations are illustrated in Figure 6. Over this temperature range, the relative change in viscosity is much greater than the change in density.

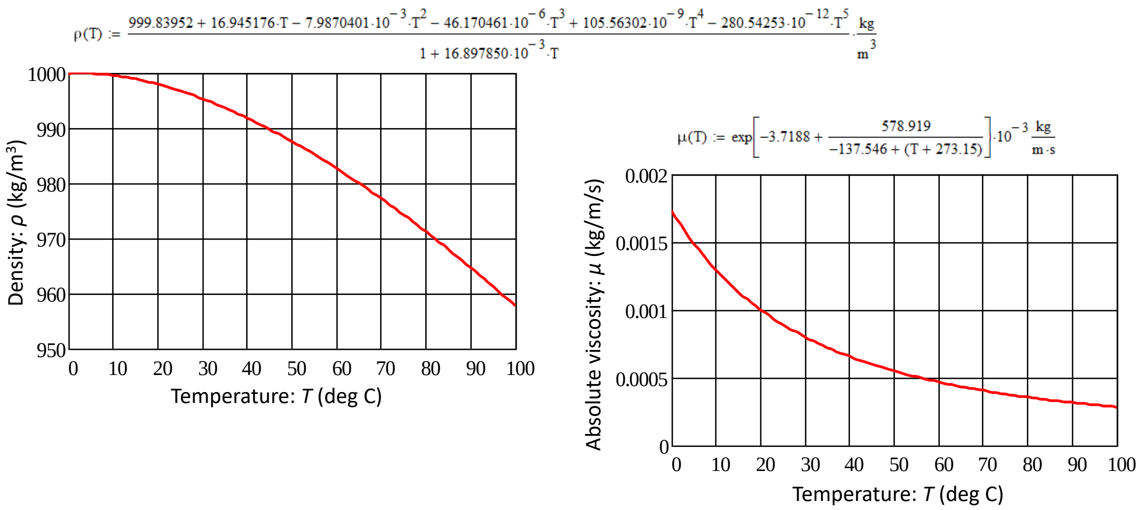


Figure 6 - The effect of temperature on the density and dynamic viscosity of pure water at atmospheric pressure (i.e., zero gauge-pressure). *T* is temperature in °C. Above each graph is the empirical equation used to construct the curve. The empirical equation for water density is provided in Kell (1975). The empirical equation for viscosity is commonly referred to as the Vogel-Fulcher-Tamman equation or VFT equation, which is presented in CRC (2023).

If the temperature distribution along a one-dimensional flow path is known or estimated, the pressure form of Darcy's law can be modified to:

$$q(s') = -\frac{k}{\mu[T(s')]}\left[\rho[T(s')]\, g\, sin(\theta) + \left.\frac{dP}{ds}\right|_{s'}\right] \tag{11}$$

where (parameter dimensions are in dark green font with mass as M, length as L, time as T, temperature as Θ):

T = temperature (Θ) in °C for Celsius or °F for Fahrenheit

$T(s')$ = temperature (Θ) at coordinate s′ as shown in Figure 7 (°C or °F)

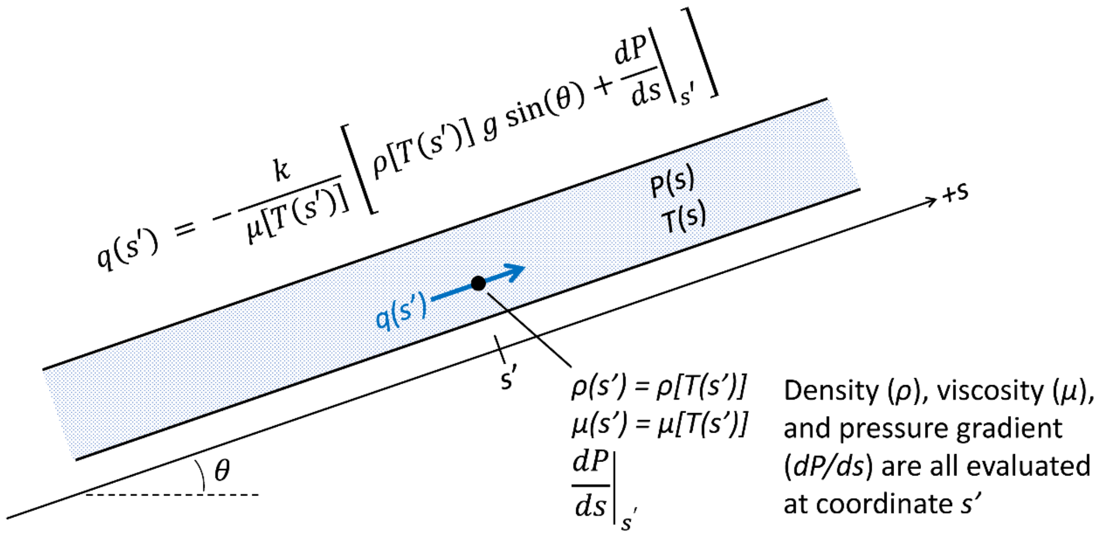


Figure 7 - The pressure form of Darcy's law for variable temperature. $T(s)$ is the temperature distribution along the s direction. Groundwater density and dynamic viscosity are both functions of temperature as shown in Figure 6.

For a groundwater flow system with variable temperature, viscosity $\mu[T(s)]$ and density $\rho[T(s)]$ can only have positive values. Inspection of Equation (11) shows that variations in viscosity can change the magnitude of specific discharge (q), but not the flow direction, and in variable temperature systems the effect on the magnitude of q can be very significant. In contrast, variations in density have the potential to change both the magnitude of q *and* the flow direction depending on whether the density change causes the summation within the brackets to change from positive to negative or *vice versa*.

4.2 Effect of Salinity

Now consider a groundwater system at atmospheric pressure (zero gauge-pressure) and a uniform temperature of 20 °C but with significant variations in salinity. The effect of salinity on groundwater density is complex and solute-specific. In general, the density of water increases with salinity, but there are some dissolved compounds for which the density decreases with increasing concentration in certain concentration ranges. Figure 8 shows the effect of a sodium chloride (NaCl) solution on water density. As can be seen, the effect is significant at high concentrations. In real systems, such high concentrations could occur at industrial/mining facilities, evapo-concentrated surface water bodies, deep connate groundwater brines associated with oil and gas development, and high-concentration groundwater contaminant plumes.

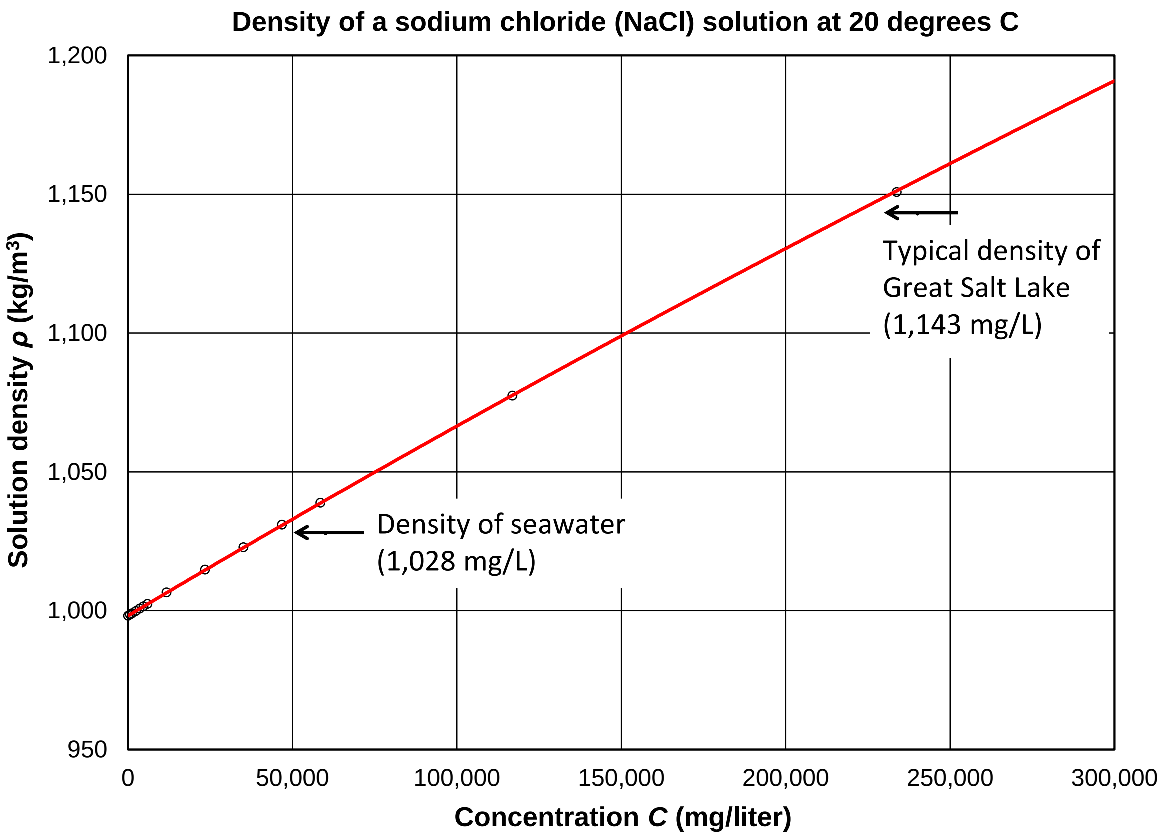


Figure 8 - The effect of dissolved sodium chloride (NaCl) on water density. The system is at atmospheric pressure (zero gauge-pressure) and has a temperature of 20 °C. Relationships between water density and the concentrations of various dissolved solutes can be evaluated using the Drefahl (2023) online calculator.

If the concentration distribution along a one-dimensional flow path is known or estimated, the pressure form of Darcy's law can be modified as shown in Equation (12).

$$q(s') = -\frac{k}{\mu}\left[\rho[C(s')]\, g \sin(\theta) + \left.\frac{dP}{ds}\right|_{s'}\right] \tag{12}$$

where (parameter dimensions are in dark green font with mass as M, length as L, time as T, temperature as Θ):

C = solute concentration (ML^{-3})

$C(s)$ = concentration (ML^{-3}) along the s flow direction as shown in Figure 9

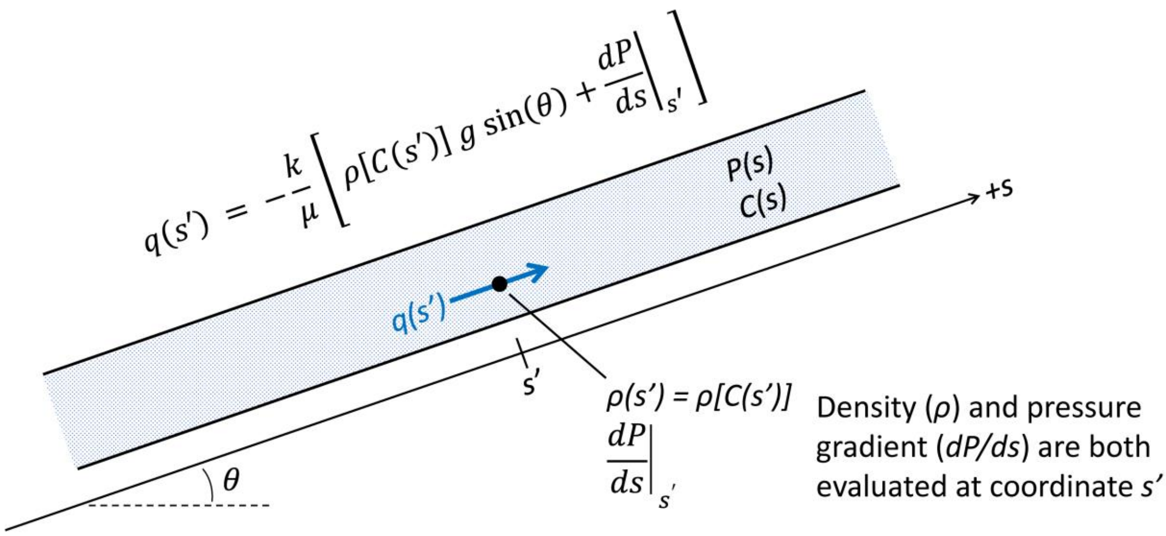


Figure 9 - The pressure form of Darcy's law for variable solute concentration. *C*(*s*) is the solute concentration distribution along the *s* direction. For sodium chloride (NaCl), the groundwater density as a function of concentration is shown in Figure 8.

4.3 Effect of Pressure

Water is a slightly compressible fluid. In going from 1 to 200 atm of pressure, the density of pure water at 20 °C increases from 998 to only 1,007 kg/m^3; 200 atm of pressure is what would occur at the bottom of a static water column about 2,000 m high. For this reason, the effect of pressure on groundwater density is usually ignored in groundwater calculations. However, it could be important when evaluating very deep flow systems.

5 Solution of the Convection Cell Thought Experiment

Now let's return to the thermally driven convection cell. For this thought experiment, we consider that the temperature in each leg of the pipe loop can be externally controlled using heating or cooling coils. Each leg of length L is maintained at a uniform temperature along its *entire* length. In the real world, if two legs have different temperatures, there would need to be a gradient (or transition) of temperature at the corner where the legs meet. Our thought experiment assumes that the temperature changes abruptly at each corner of the cell, which would not happen physically. However, for the issues being evaluated by this thought experiment, this departure from reality does not affect the conclusions reached.

The pipe loop is a closed system, that is, there is no addition of water or chemical mass from outside this system. The only thing that affects water density and viscosity is the externally imposed temperature in each leg. For steady-state flow, these conditions dictate that the mass flux of water in each leg must be the same. The mass flux (j) is equal to the product of specific discharge times the fluid density $(\rho\, q)$. Accordingly, we can multiply both sides of Equation (11) by density (ρ) to obtain the Mass Flux Equation (13).

$$j(s') = -\frac{k\,\rho[T(s')]}{\mu[T(s')]}\left[\rho[T(s')]\,g\sin(\theta) + \left.\frac{dP}{ds}\right|_{s'}\right] \tag{13}$$

where (parameter dimensions are in dark green font with mass as M, length as L, time as T, temperature as Θ):

j = mass flux ($MT^{-1}L^{-2}$)

The setup for mathematical solution of the convection cell problem is shown in Figure 10. The cell contains four legs in a vertical plane, each of length L and with the same cross-sectional area. For our thought experiment, the mass flux (j) is the same everywhere within the convection cell, and each leg has the temperature T_i where i is the leg number 1, 2, 3, or 4.

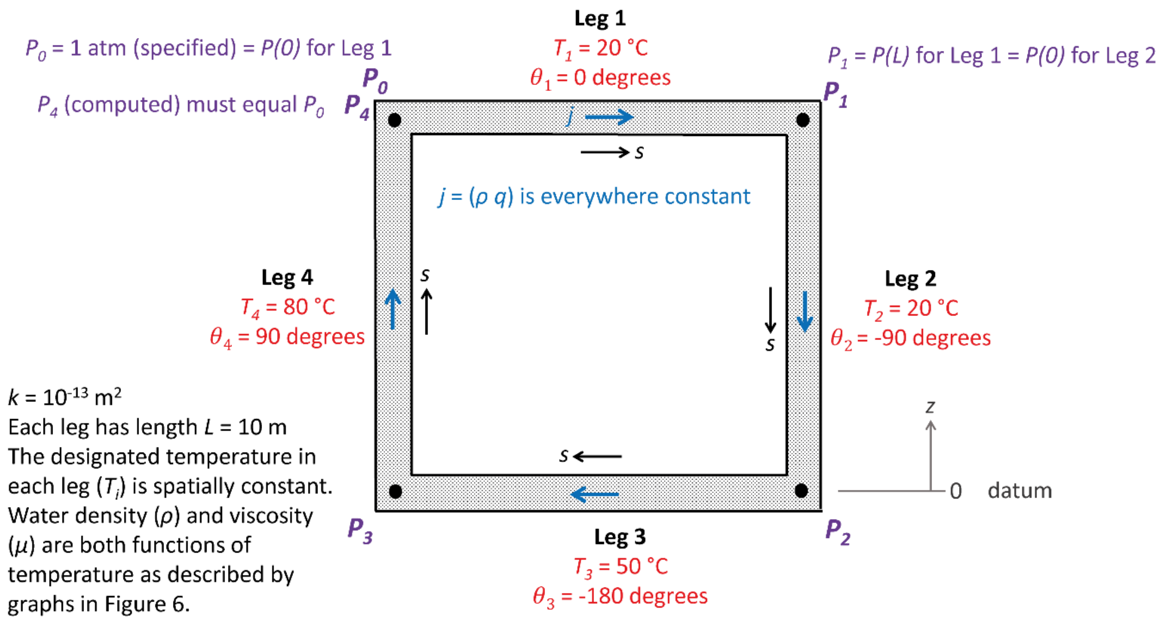


Figure 10 - Setup for solution of the convection cell thought experiment. P_0 is an arbitrary fluid pressure assigned to the upper left corner of the cell. Each leg is assigned and index number $i = 1, 2, 3, 4$ as shown. Mass flux (*j*) is the same in each leg of the cell. P_1, P_2, P_3, and P_4 are computed pressures at the corners. P_0 and P_4 are the same point and therefore must have the same pressure.

For leg i of the cell, the mass flux equation shown in Equation (14) applies (modified from Equation (13).

$$j = -\frac{k\,\rho[T_i]}{\mu[T_i]}\left[\rho[T_i]\,g\,\sin(\theta_i) + \left.\frac{dP}{ds}\right|_i\right] \tag{14}$$

where:

T_i = uniform temperature along the length of leg i (Θ)

Referring to Figure 10, the mass flux (j) and temperature (T) are specified to be uniform, but different, within each leg, so the derivative of pressure (dP/ds) within a particular leg must also be uniform. The pressure gradient in leg i is therefore computed by Equation (15).

$$\left.\frac{dP}{ds}\right|_i = \frac{P_i(L) - P_i(0)}{L} \tag{15}$$

where:

$P_i(0)$ = water pressure at the upstream end of leg i ($ML^{-1}T^{-2}$)

$P_i(L)$ = water pressure at the downstream end of leg i ($ML^{-1}T^{-2}$)

L = length of the convection cell leg (L)

Substituting Equation (15) for the derivative in Equation (14) and solving for $P_i(L)$ leads to Equation (16) which applies to the conditions illustrated in Figure 11.

$$P_i(L) = P_i(0) - \frac{j\, L\, \mu[T_i]}{k\, \rho[T_i]} - L\, \rho[T_i]\, g \sin(\theta_i) \tag{16}$$

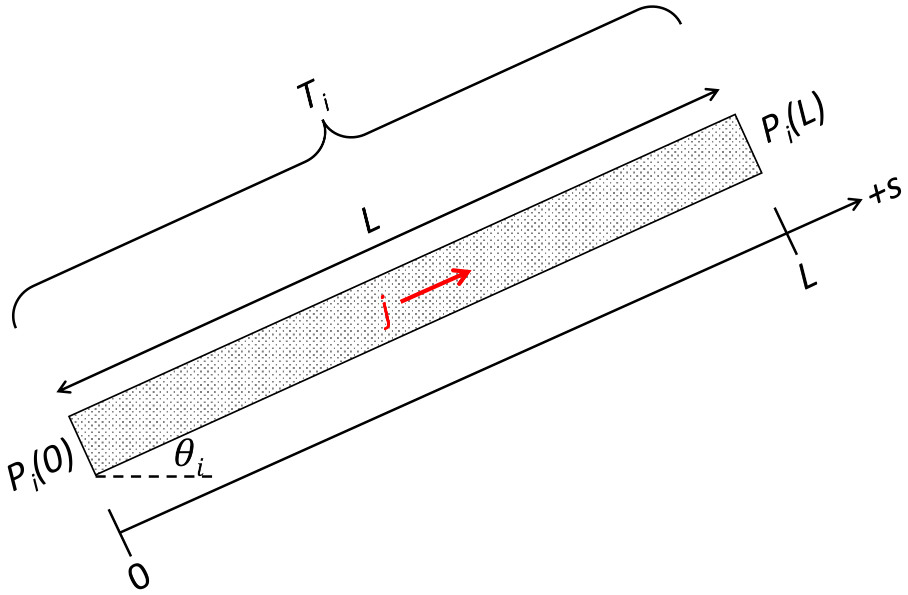


Figure 11 - Pressure at each end of a convection cell leg. The entire leg has constant mass flux (*j*) and uniform temperature T_i.

For our convection cell, this formulation leads to a set of coupled equations shown in Equations (17) through (21).

Leg 1

$$P_1 = P_0 - \frac{j\, L\, \mu[20\,^{\circ}C]}{k\, \rho[20\,^{\circ}C]} - L\, \rho[20\,^{\circ}C]\, g \sin(0\ deg) \tag{17}$$

Leg 2

$$P_2 = P_1 - \frac{j\, L\, \mu[20\,^{\circ}C]}{k\, \rho[20\,^{\circ}C]} - L\, \rho[20\,^{\circ}C]\, g \sin(-90\ deg) \tag{18}$$

Leg 3

$$P_3 = P_2 - \frac{j\, L\, \mu[50\,^{\circ}C]}{k\, \rho[50\,^{\circ}C]} - L\, \rho[50\,^{\circ}C]\, g \sin(-180\ deg) \tag{19}$$

Leg 4

$$P_4 = P_3 - \frac{j\, L\, \mu[80\,^{\circ}C]}{k\, \rho[80\,^{\circ}C]} - L\, \rho[80\,^{\circ}C]\, g \sin(90\ deg) \tag{20}$$

$$P_4 = P_0 \tag{21}$$

To find a solution, Equations (17) through (20) are evaluated sequentially using an arbitrary P_0 in Equation (17) and a trial value for mass flux (j). Then, the computed P_4 from Equation (20) is compared to the input P_0. If they differ, a new mass flux value is tested. This process is repeated in a trial-and-error manner until a value of j is found that results in Equation (21) being satisfied. When this is achieved, the associated j value is the uniform mass flux within the system being considered. In fluid mechanics, this iterative solution technique is known as the Hardy-Cross method. Armed with Equations (17) through (21), and the empirical relationships in Figure 6, we can now solve the convection cell problem shown in Figure 10 to obtain the system mass flux and the pressures at each corner of the convection cell. The calculations of pressure at each corner of the call are summarized in Table 1. With the solution obtained, we wish to further investigate the applicability of proposed alternate forms of hydraulic head within the system. Because the pressure gradient in each leg is linear, the pressure at the midpoint of a leg is equal to the average of the pressures at each end of the leg. Thus, at the midpoint of leg i, the freshwater head is given by Equation (22) and the pointwater head is given by Equation (23).

$$h_{fmi} = z_{mi} + \frac{P_{i-1} + P_i}{2\ g\ \rho_f} \tag{22}$$

$$h_{pmi} = z_{mi} + \frac{P_{i-1} + P_i}{2\ g\ \rho(T_i)} \tag{23}$$

where:

h_{fmi}	=	freshwater head at the midpoint of convection cell leg i (L)
z_{mi}	=	midpoint elevation of convection cell leg i (L)
h_{pmi}	=	pointwater head at the midpoint of convection cell leg i (L)

Calculations of freshwater head and pointwater head are summarized in Table 1, and the results are shown graphically in Figure 12.

Table 1 - Calculations for the convection cell thought experiment. To provide detailed documentation, these calculations are performed using MathCad® software. However, the equations could also be evaluated using a calculator or spreadsheet. The MathCad® output contains a semicolon before the equal sign and a dot to indicate multiplication of parameters and units (MathCad® treats units as parameters).

$k := 1 \cdot 10^{-13} \cdot m^2$ Intrinsic permeability of porous medium

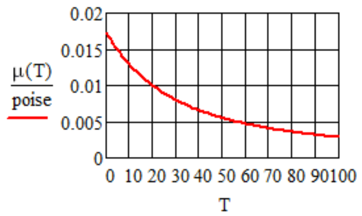
$L := 10 \cdot m$ Length of each leg of a square convection cell

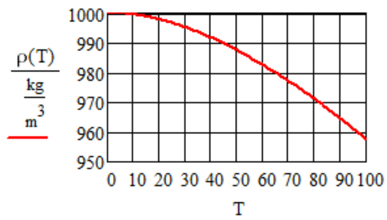
Water Density vs Temperature (C)

$$\rho(T) := \frac{999.83952 + 16.945176 \cdot T - 7.9870401 \cdot 10^{-3} \cdot T^2 - 46.170461 \cdot 10^{-6} \cdot T^3 + 105.56302 \cdot 10^{-9} \cdot T^4 - 280.54253 \cdot 10^{-12} \cdot T^5}{1 + 16.897850 \cdot 10^{-3} \cdot T} \cdot \frac{kg}{m^3}$$

$$\mu(T) := \exp\left[-3.7188 + \frac{578.919}{-137.546 + (T + 273.15)}\right] \cdot 10^{-2} poise$$

Water Viscosity vs Temperature (C) $T := 0, 1 .. 100$





$j := 282.4 \cdot \frac{kg}{yr \cdot m^2}$ Trial-and-error system mass flux value to get $P_4 = P_0$

P_i = pore water pressure and corner i

h_{fi} = fresh water head at midpoint of leg i

q_i = specific discharge in leg i

P_{mi} = pore water pressure at mid-point of leg i

h_{pi} = point water head at midpoint of leg i

$P_0 := 1.000 \cdot atm$ Arbitrary pore water pressure at upper left corner of the cell

Leg 1

$$P_1 := P_0 - \frac{j \cdot L \cdot \mu(20)}{k \cdot \rho(20)} - L \cdot \rho(20) \cdot g \cdot \sin\left(\frac{0 \cdot \pi}{180}\right) \qquad P_1 = 0.99114 \cdot atm$$

$$q_1 := \frac{j}{\rho(20)} \qquad q_1 = 0.2830 \cdot \frac{m}{yr}$$

$$h_{f1} := L + \frac{P_0 + P_1}{2 \cdot g \cdot \rho(20)} \qquad h_{f1} = 20.308 m$$

$$h_{p1} := L + \frac{P_0 + P_1}{2 \cdot g \cdot \rho(20)} \qquad h_{p1} = 20.308 m$$

Leg 2

$$P_2 := P_1 - \frac{j \cdot L \cdot \mu(20)}{k \cdot \rho(20)} - L \cdot \rho(20) \cdot g \cdot \sin\left(\frac{-90 \cdot \pi}{180}\right) \qquad P_2 = 1.94811 \cdot atm$$

$$q_2 := \frac{j}{\rho(20)} \qquad q_2 = 0.2830 \cdot \frac{m}{yr}$$

$$h_{f2} := \frac{L}{2} + \frac{P_1 + P_2}{2 \cdot g \cdot \rho(20)} \qquad h_{f2} = 20.216 m$$

$$h_{p2} := \frac{L}{2} + \frac{P_1 + P_2}{2 \cdot g \cdot \rho(20)} \qquad h_{p2} = 20.216 m$$

Leg 3

$$P_3 := P_2 - \frac{j \cdot L \cdot \mu(50)}{k \cdot \rho(50)} - L \cdot \rho(50) \cdot g \cdot \sin\left(\frac{-180 \cdot \pi}{180}\right) \qquad P_3 = 1.94320 \cdot atm$$

$$q_3 := \frac{j}{\rho(50)} \qquad q_3 = 0.2860 \cdot \frac{m}{yr}$$

$$h_{f3} := 0 + \frac{P_2 + P_3}{2 \cdot g \cdot \rho(20)} \qquad h_{f3} = 20.145 m$$

$$h_{p3} := 0 + \frac{P_2 + P_3}{2g \cdot \rho(50)} \qquad h_{p3} = 20.356 m$$

Leg 4

$$P_4 := P_3 - \frac{j \cdot L \cdot \mu(80)}{k \cdot \rho(80)} - L \cdot \rho(80) \cdot g \cdot \sin\left(\frac{90 \cdot \pi}{180}\right) \qquad P_4 = 1.00000 \cdot atm$$

$$q_4 := \frac{j}{\rho(80)} \qquad q_4 = 0.2908 \cdot \frac{m}{yr}$$

$$h_{f4} := \frac{L}{2} + \frac{P_3 + P_4}{2 \cdot g \cdot \rho(20)} \qquad h_{f4} = 20.236 m$$

$$h_{p4} := \frac{L}{2} + \frac{P_3 + P_4}{2 \cdot g \cdot \rho(80)} \qquad h_{p4} = 20.656 m$$

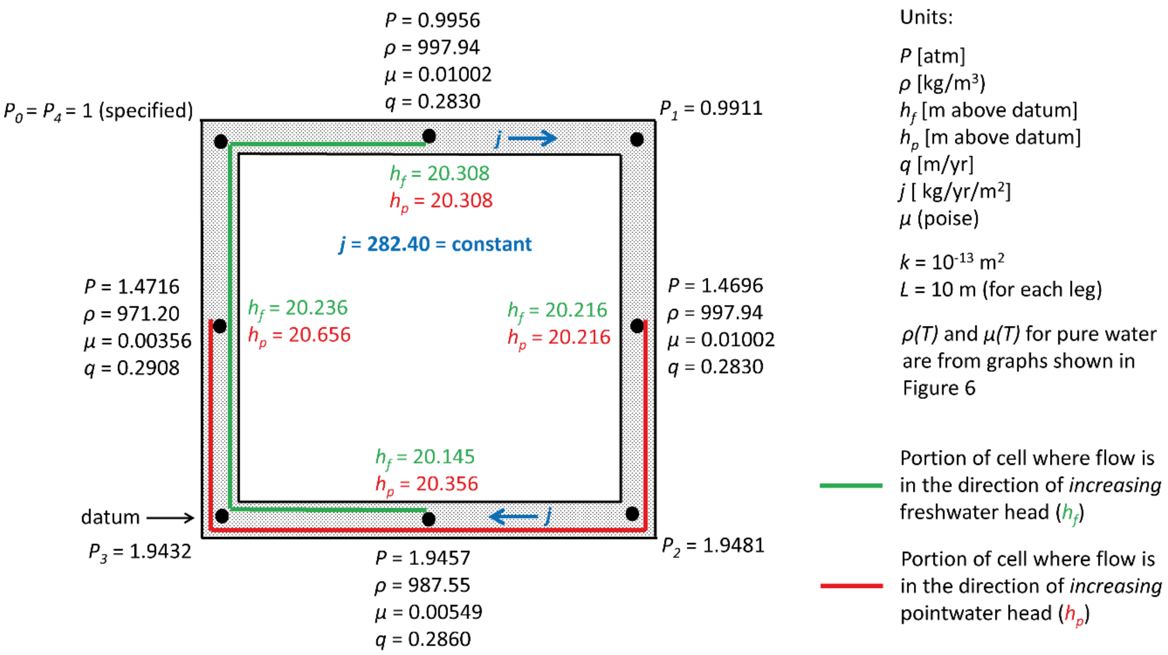


Figure 12 - Results of the convection cell thought experiment. In portions of the cell, the fluid flow is in the direction of increasing freshwater head (h_f) and/or increasing pointwater head (h_p), indicating that these parameters are not true hydraulic potentials.

For this thought experiment, the mathematical results show that freshwater head (h_f) and pointwater head (h_p) are not hydraulic potentials because in some portions of the cell the flow is in the direction of increasing values. As such, it is somewhat misleading to describe these parameters as heads and interpret them as alternate representations of the parameter H used in the head form of Darcy's law. Blindly replacing H with h_f or h_p in Equation (1) would not provide the true flow directions or magnitudes at all locations within the cell. For this variable-density flow system, the hydraulics can only be solved using the pressure form of Darcy's law. The bottom line is that the hydraulic potential H is a different "animal" from h_f or h_p because the latter are not true hydraulic potentials. Further, the downward flow in the right leg of the cell in Figure 12 is in the direction of increasing pressure, so pressure alone is not a true potential for describing flow.

In Figure 12, an interesting observation is that specific discharge increases slightly where the cell attains higher temperature. One might ask: Why does the specific discharge change if there are no sources or sinks that add or subtract flow to/from the system? For this closed flow system, conservation of mass implies that the mass flux ($j = \rho q$) is everywhere constant. If the fluid density ρ decreases (at higher temperature), the specific discharge q must increase. This can also be explained by thermal expansion of the fluid at higher temperature.

6 Application of Darcy's Law to Groundwater Systems

Having shown that the head form of Darcy's law did not work for our convection cell thought experiment, there is some good news for groundwater hydrologists. In most natural groundwater systems, the difference in groundwater density is sufficiently small that its variation can be ignored, and calculations can proceed using a uniform (average) density. When this assumption is justified, the hydraulic head form of Darcy's law can be used, and the errors imposed are not large enough to be of practical concern. This explains why the hydraulic head form of Darcy's law is embedded in the thinking of the groundwater discipline: It usually works! However, there are situations where one may need to use the pressure form of Darcy's law to obtain more representative results. Examples include the following:

- in a freshwater aquifer, the horizontal migration of a groundwater chemical plume with very high TDS;
- vertical groundwater flow in an aquitard that is sandwiched between two aquifers, one with relatively fresh water and one with brine, a situation known to occur adjacent to Great Salt Lake in Utah, USA;
- deep groundwater flow in a geothermal area;
- vertical upward flow of lower TDS groundwater toward shallow higher TDS groundwater associated with a playa lake;
- groundwater flow passing over the top of a salt dome;
- along coastlines where a freshwater aquifer is in hydraulic contact with an aquifer containing ocean water; and
- where high-concentration chemical reagents are injected into groundwater for in situ treatment.

A practical approach for groundwater professionals can be to perform scoping-level, one-dimensional flow solutions using both the head and pressure forms of Darcy's law and compare the results. If the results are very similar, one can usually proceed with the head form using a uniform (average) fluid density and have a reasonable expectation that the results will be sufficiently accurate for practical application. If results of the two forms of Darcy's law are significantly different, deferral to the pressure form may be required to obtain defensible results. For example, if both equations give the same flow direction but a magnitude difference less than, say, 10 percent, one might conclude that the uncertainty of the results is small compared to the uncertainty in the value of k (or K), and either method is adequate for practical application. However, if the two methods give different flow directions, the pressure form of Darcy's law should take precedent.

The relative effect of temperature (T) on water viscosity (μ) is always greater than its effect on water density (ρ). At face value, one might conclude that the temperature effect on viscosity could swamp the effect on density. However, the system hydraulics may be more complicated than this simple interpretation. The flux equation presented previously for a system with zero solute concentration—but with elevated/variable temperature as shown in Equation (11)—is reproduced here as Equation (24).

$$q(s') = -\frac{k}{\mu\,[T(s')]}\left[\rho[T(s')]\;g\,\sin(\theta) + \left.\frac{dP}{ds}\right|_{s'}\right] \qquad (24)$$

This equation shows that the magnitude of flux (q) is inversely proportional to viscosity, and this effect can be significant. However, a change in viscosity alone does *not* affect the flow direction. The direction of flow is controlled by the term $[\rho[T(s)]\;g\,\sin(\theta) + \;dP/ds|_{s'}\;]$. If this term is negative, flow is in the $+s$ direction; if positive, flow is in the $-s$ direction. Therefore, in a variable temperature system, one cannot necessarily neglect the magnitude/variation in groundwater density even though the relative variation in viscosity is much greater.

The flux equation presented as Equation (12) for a system at standard temperature, but with an elevated/variable solute concentration, is reproduced here as Equation (25).

$$q(s') = -\frac{k}{\mu}\left[\;\rho[C(s')]\;g\,\sin(\theta) + \left.\frac{dP}{ds}\right|_{s'}\;\right] \qquad (25)$$

Except for extreme cases of certain organic compounds (such as biodegradable slurries), we can usually ignore the effect of concentration on water viscosity. Elevated/variable concentration can significantly change the groundwater density, and this can affect both flow direction and magnitude as expressed by the bracketed term.

7 Horizontal Flow Calculations

Let's consider some practical situations. The first is essentially horizontal flow in an aquifer where the groundwater density varies laterally but not vertically. If the principal directions of hydraulic conductivity are horizontal and vertical, we can relax the isotropic assumption and consider horizontal hydraulic conductivity (K_h) or horizontal intrinsic permeability (k_h) in calculations of horizontal flow.

For horizontal flow, the direction angle (θ) is zero and the pressure form of Darcy's law shown by Equation (10) simplifies to Equation (26) where the coordinate direction x is used to indicate that the flow direction is horizontal.

$$q_h(x') = -\frac{k_h}{\mu}\frac{dP}{dx}\bigg|_{x'} \tag{26}$$

where (parameter dimensions are in dark green font with mass as M, length as L, time as T, temperature as Θ):

q_h = horizontal specific discharge (LT^{-1})

k_h = horizontal intrinsic permeability (L^2)

x' = specified horizontal coordinate (L)

Horizontal flow is controlled only by the gradient of pressure. Conveniently, the density of the groundwater in the aquifer and its spatial variation $\rho(s)$ falls out of the flow equation.

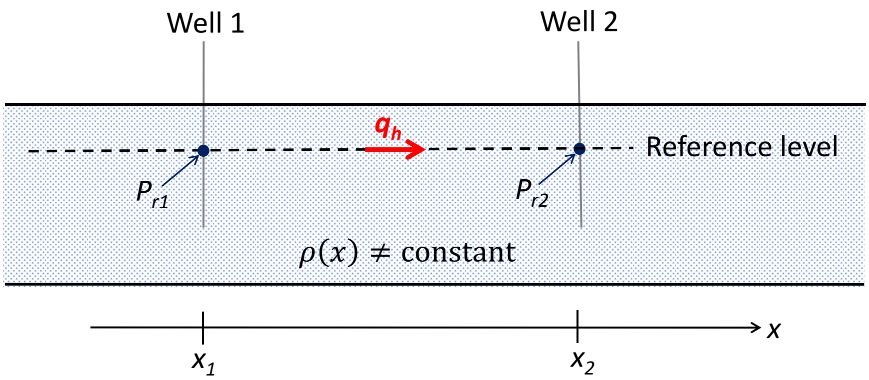
Let's evaluate the horizontal component of flow in an aquifer between two monitoring wells. To proceed, we must select an arbitrary horizontal reference level at elevation z_r and estimate the fluid pressure (P_r) at each well at that elevation as shown in Figure 13. Replacing the derivative in Equation (26) with a finite difference approximation, an estimate of the horizontal specific discharge is given by Equation (27) where an approximation sign is used because a linear hydraulic gradient is assumed between x_1 and x_2.

$$q_h \approx -\frac{k_h}{\mu}\frac{(P_{r2} - P_{r1})}{(x_2 - x_1)} \tag{27}$$

where:

P_{ri} = pressure at the reference elevation at the location of well i ($ML^{-1}T^{-2}$)

x_i = distance coordinate at the location of well i (L)



$$q_h \approx -\frac{k_h}{\mu}\frac{(P_{r2}-P_{r1})}{(x_2-x_1)}$$

Figure 13 - Estimation of horizontal specific discharge in an aquifer with variable-density groundwater. The magnitude/variation of groundwater density between the wells is not needed to perform the calculation. The equation for q_h has an approximation sign because a linear pressure gradient is assumed to exist between the wells.

7.1 Estimation of Pressure at the Reference Elevation

Equation (27) is easy to solve. However, a practical challenge is estimating the fluid pressures P_{r1} and P_{r2} at the *chosen* reference elevation (z_r). As shown in Figure 14, we typically install a groundwater monitoring well and measure the physical water-level elevation in the well or fluid pressure at the elevation of a pressure transducer installed somewhere within the water column.

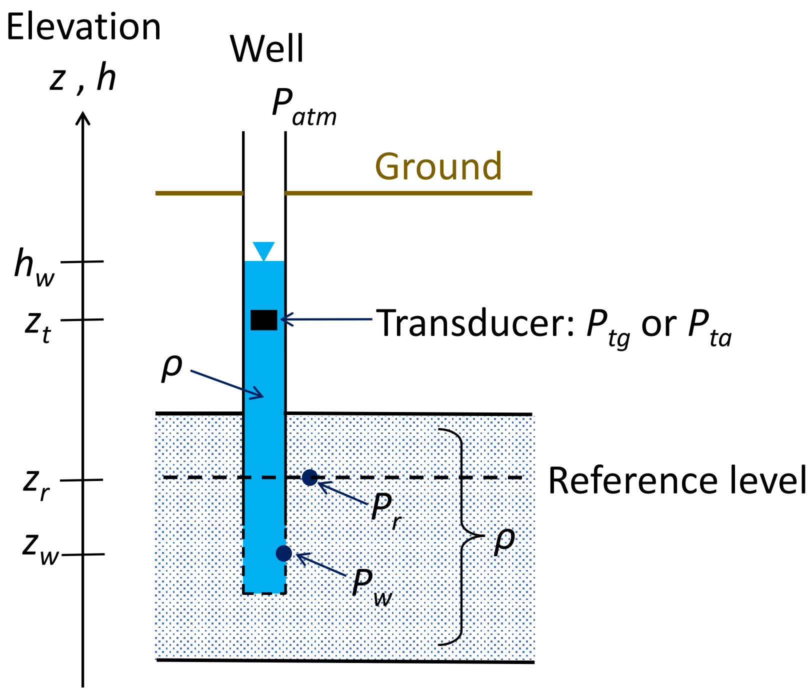


Figure 14 - Typical groundwater monitoring well installation. Parameters shown on the figure are described in the text.

The parameters represented in Figure 14 are as follows. Parameter dimensions are in dark green font with mass as M, length as L, time as T, temperature as Θ.

z_w = midpoint elevation of well completion zone (L)

z_r = reference level elevation (L)

z_t = pressure transducer elevation (L)

h_w = physical water-level elevation in a well (L)

ρ = groundwater density in the vicinity of the well (ML^{-3})

P_w = fluid pressure at the midpoint of a well completion zone ($ML^{-1}T^{-2}$)

P_r = groundwater pressure at the reference level ($ML^{-1}T^{-2}$)

P_{atm} = prevailing atmospheric pressure ($ML^{-1}T^{-2}$)

P_{tg} = gauge pressure measured by a vented submersible transducer ($ML^{-1}T^{-2}$)

P_{ta} = absolute pressure measured by an unvented submersible transducer ($ML^{-1}T^{-2}$)

We assume that the monitoring well is periodically purged, so the density of the water column in the casing is the same as groundwater in the aquifer (ρ). Using well measurements, we compute the groundwater pressure at the reference level (P_r) at the well location making the following assumptions.

- The vertical pressure distribution in the aquifer is hydrostatic (no upward or downward flow).
- The vertical pressure distribution in the well-water column is hydrostatic.
- P_w is the fluid pressure in the well-completion zone *and* in the adjacent geologic formation at elevation z_w.
- The groundwater density (ρ) is vertically uniform at the well location.
- The average density of the well-water column is the same as the groundwater density (ρ).

These assumptions are reasonable for many groundwater situations. The assumptions say the vertical pressure distribution inside the well is the same as the vertical pressure distribution in the adjacent aquifer.

Hydraulic conditions in a monitoring well or piezometer can be evaluated by measuring any of the following.

- The distance from a surveyed measuring point (typically the top of the well casing) down to the physical water level in the well. This is usually done using an electric depth-to-water tape that indicates when its bottom probe contacts the water surface. Subtracting this distance from the elevation of the measuring point gives the elevation of the water surface (h_w).

- Measurement of gauge pressure (P_{tg}) using a vented pressure transducer that does not require a barometric correction. For this type of instrument, the readout pressure is zero when sensing the atmosphere regardless of the prevailing barometric pressure.
- Measurement of absolute pressure (P_{ta}) by a submersible pressure transducer positioned at a known elevation in the water column. Referred to as an unvented pressure transducer, when this instrument senses the atmosphere at sea level, its readout pressure is approximately 100,300 pascals (Pa) or 14.7 pounds per square inch (psi). To convert absolute pressure to gauge pressure requires a correction using the prevailing barometric pressure, which is typically measured using an external barometer at ground surface or accessing barometric records from a nearby weather station.

Other measurement devices are available, but these are the ones most used.

We can now set up equations to compute P_r for three types of field measurements, and these are presented below in Equations (28), (29), and (30).

1. If the physical water-level elevation (h_w) is measured, then:

$$P_r = \rho\, g(h_w - z_r) \tag{28}$$

2. If a vented transducer measures true gauge pressure (P_{tg}):

$$P_r = P_{tg} + \rho\, g(z_t - z_r) \tag{29}$$

3. If an unvented transducer measures absolute pressure (P_{ta}):

$$P_r = P_{ta} - P_{atm} + \rho\, g(z_t - z_r) \tag{30}$$

P_{atm} in Equation (30) is the *prevailing* atmospheric pressure at the time of transducer measurement, not standard atmospheric pressure. In subsequent analyses, we compute P_r at the location of two monitoring wells. When doing so, certain parameters in Equations (28), (29), and (30) will be subscripted with "1" or "2" to identify the well being evaluated.

In general practice, the water density of the formation and in the well-water column can be directly measured in the lab using a grab sample taken during purging. However, if a pressure transducer is positioned below the physical water surface in the well, the average water column density *above* the transducer can be back-calculated using Equation (31) if the transducer is vented or Equation (32) if the transducer is unvented.

$$\rho = \frac{P_{tg}}{g(h_w - z_t)} \tag{31}$$

or

$$\rho = \frac{P_{ta} - P_{atm}}{g(h_w - z_t)} \tag{32}$$

If a transducer is positioned *at* the chosen reference level ($z_t = z_r$), the last term in Equation (29) and Equation (30) is zero and P_r can be determined directly without knowing the groundwater or well column water density.

To minimize inaccuracies that may occur when the above assumptions are not strictly correct, it is best to select the reference level z_r to pass through the midpoint of the well completion zone. To evaluate flow between two wells having different completion elevations, a good approach is to select z_r to have an elevation about midway between the well completion midpoints.

7.2 Use of Freshwater Head for Horizontal Flow

For horizontal flow problems, hydrologists tend to be more comfortable with fluid levels and hydraulic conductivity as opposed to pressure and intrinsic permeability. With algebraic manipulation, we can rewrite the Darcy equation for 1-D horizontal flow as Equation (33).

$$q_h = -\left(\frac{k_h \rho_f g}{\mu_{st}}\right)\left(\frac{\mu_{st}}{\mu}\right)\frac{d}{dx}\left[z_r + \frac{P}{\rho_f g}\right] \tag{33}$$

where (parameter dimensions are in dark green font with mass as M, length as L, time as T, temperature as Θ):

μ_{st} = standard dynamic viscosity of pure water at 20 °C ($ML^{-1}T^{-1}$) is 0.001 kg/m/s

The other parameters were defined previously.

We define the first term on the right-hand side of Equation (33) as shown in Equation (34).

$$K_{sth} = \frac{k_h\, \rho_f\, g}{\mu_{st}} \tag{34}$$

where:

K_{sth} = standard horizontal hydraulic conductivity for pure water at 20 °C (LT^{-1})

K_{sth} is the hydraulic conductivity that one would expect to measure in a laboratory permeameter using pure water at 20 °C. The bracketed term in Equation (33) is the freshwater head associated with the reference elevation (z_r) as shown in Equation (35).

$$h_f = z_r + \frac{P_r}{\rho_f\, g} \tag{35}$$

In Equation (33) we added the reference elevation (z_r) in the bracketed term, which is permissible because z_r is a constant and $dz_r/dx = 0$. Further, h_f depends on the chosen elevation of the reference level z_r. Substituting Equations (34) and (35) into Equation (33) gives an equivalent form of Darcy's law for one-dimensional horizontal flow expressed in terms of standard horizontal hydraulic conductivity (K_{sth}) and the freshwater head (h_f) associated with the reference level (z_r), as shown in Equation (36).

$$q_h = -K_{sth}\left(\frac{\mu_{st}}{\mu}\right)\frac{dh_f}{dx} \tag{36}$$

Equation (36) looks quite similar to the head form of Darcy's law. However, for a *variable-density system*, this equation is *not* the head form for the following reasons.

- It applies only to horizontal flow, while Darcy's law should work for any flow direction.
- h_f is not a hydraulic potential, as was demonstrated by the convection cell thought experiment.

h_f can be visualized as the water level achieved in a manometer filled with freshwater of density ρ_f that senses the fluid pressure at a point located at the reference elevation. h_f is simply a surrogate for describing fluid pressure and how pressure varies laterally at the reference level. Equation (36) is the pressure form of Darcy's law for horizontal flow using parameters that are more familiar to hydrologists.

Figure 15 illustrates the relationship between the physical water level in a well and the hypothetical freshwater head associated with a reference level.

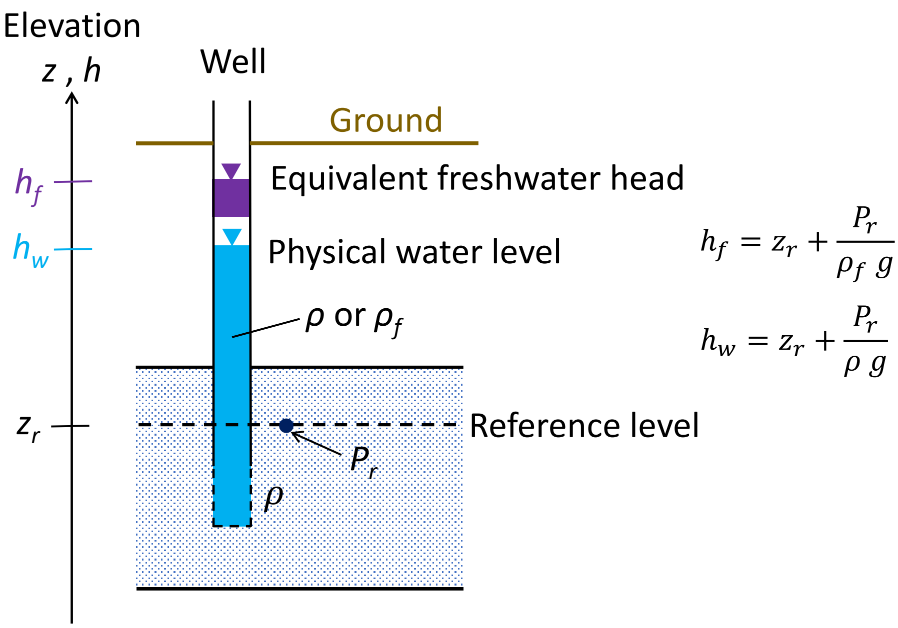


Figure 15 – Comparison of the physical water level in a well (h_w) with freshwater head (h_f). Freshwater head depends on the choice of the reference level elevation (z_r). When groundwater density (ρ) is greater than freshwater density ($\rho > \rho_f$), the freshwater head (h_f) is higher than the physical water level (h_w).

Using hydrostatics and the relationships shown in Figure 15, we note that the physical water level can be determined from the elevation and pressure at the reference level as expressed in Equation (37).

$$h_w = z_r + \frac{P_r}{\rho\, g} \tag{37}$$

When Equation (35) and Equation (37) are evaluated simultaneously and solved for h_f, we obtain Equation (38) for freshwater head (h_f) based on the measured physical water level in the well (h_w).

$$h_f = z_r + \frac{\rho}{\rho_f}(h_w - z_r) \tag{38}$$

Again, the computed freshwater head is not a fixed value, but depends on the chosen reference level elevation (z_r).

7.3 Horizontal Flow Example 1

As an illustrative example, Figure 16 shows the setup for a horizontal flow problem based on the measured physical water levels in two wells.

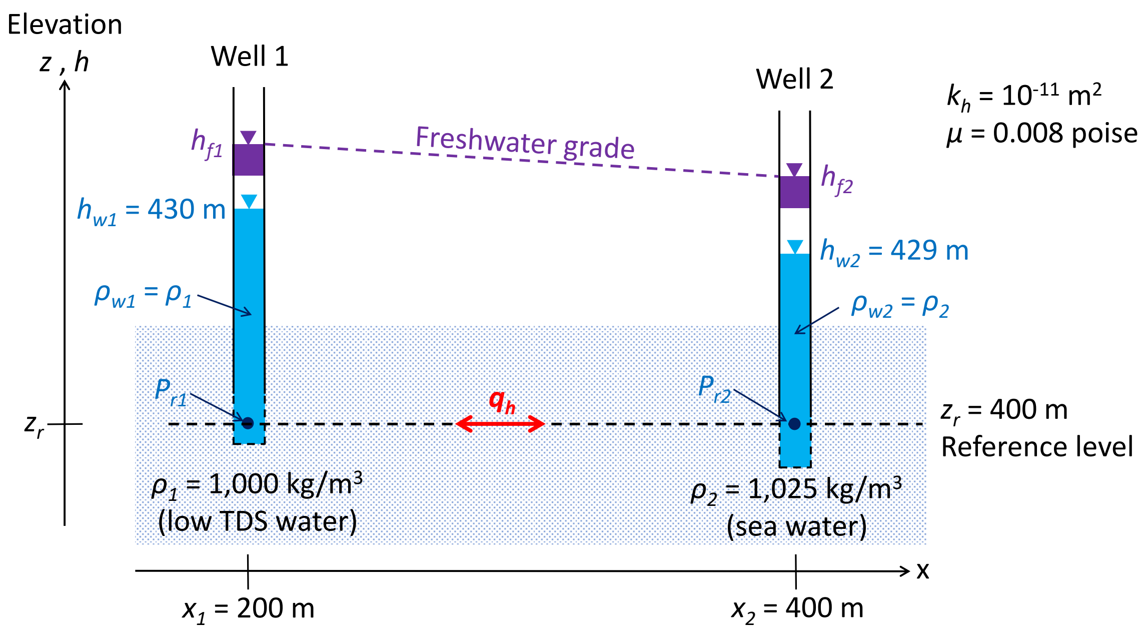


Figure 16 - Problem setup for horizontal flow in an aquifer with different groundwater density at the monitor well locations. The subscript 1 applies to parameters associated with Well 1 and subscript 2 applies to Well 2. The solution does not require knowledge of the groundwater density distribution between the wells.

The solution to the problem in Figure 16 is provided in Table 2. The computed specific discharge (5.32 m/yr) is the same regardless of whether the problem is solved using pressures (P_r) or freshwater heads (h_w). Knowledge of the groundwater density distribution *between* the two piezometers is not needed to solve this horizontal flow problem. Further, if a pressure transducer in each well is positioned *at* the reference elevation (z_r), the solution requires no knowledge of the groundwater density at each well location.

Table 2 - Mathematical solution to the horizontal flow problem shown in Figure 16. To provide good documentation, the results are provided in a MathCad® printout. However, the equations could also be evaluated using a calculator or spreadsheet. The MathCad® output contains a semicolon before the equal sign and a dot to indicate multiplication of parameters and units (MathCad® treats units as parameters).

Inputs

$k_h := 10^{-11} \cdot m^2$	Intrinsic permeability	
$\mu := 0.00800 \cdot poise$	Groundwater viscosity (at 30 degrees C)	$\mu = 0.00080 \frac{kg}{s \cdot m}$
$x_1 := 200 \cdot m \quad x_2 := 400 \cdot m$	Well coordinates along x axis	
$h_{w1} := 430 \cdot m$	Physical water level in Well 1	
$\rho_1 := 1000 \cdot \frac{kg}{m^3}$	Groundwater density at Well 1 (low TDS water)	
$h_{w2} := 429 \cdot m$	Physical water level in Well 2	
$\rho_2 := 1025 \cdot \frac{kg}{m^3}$	Groundwater density at Well 2 (sea water)	
$z_r := 400 \cdot m$	Reference level elevation	
$\rho_f := 998.2 \cdot \frac{kg}{m^3}$	Standard (freshwater) density (pure water at 20 degrees C)	
$\mu_{st} := 0.01 \cdot poise$	Standard water viscosity (at 20 degrees C)	$\mu_{st} = 0.00100 \cdot \frac{kg}{m \cdot s}$

Solution based on Pressures

$P_{r1} := \rho_1 \cdot g \cdot (h_{w1} - z_r)$	Pressure at reference level at Well 1	$P_{r1} = 2.942 \times 10^5 Pa$
$P_{r2} := \rho_2 \cdot g \cdot (h_{w2} - z_r)$	Pressure at reference level at Well 2	$P_{r2} = 2.915 \times 10^5 Pa$
$q_{hp} := -\left(\frac{k_h}{\mu}\right) \cdot \left(\frac{P_{r2} - P_{r1}}{x_2 - x_1}\right)$	Horizontal specific discharge based on pressure	$q_{hp} = 5.319 \cdot \frac{m}{yr}$

Solution based on Freshwater Heads

$K_{hf} := \frac{k_h \cdot \rho_f \cdot g}{\mu_{st}}$	Standard (freshwater) horizontal hydraulic conductivity	$K_{hf} = 9.789 \times 10^{-5} \cdot \frac{m}{sec}$
$h_{f1} := z_r + \frac{\rho_1}{\rho_f}(h_{w1} - z_r)$	Freshwater head at reference elevation at Well 1	$h_{f1} = 430.054 m$
$h_{f2} := z_r + \frac{\rho_2}{\rho_f}(h_{w2} - z_r)$	Freshwater head at reference elevation at Well 2	$h_{f2} = 429.779 m$
$q_{hf} := -K_{hf} \cdot \left(\frac{\mu_{st}}{\mu}\right) \cdot \left(\frac{h_{f2} - h_{f1}}{x_2 - x_1}\right)$	Horizontal specific discharge based on freshwater head	$q_{hf} = 5.319 \cdot \frac{m}{yr}$

Using the solution approach presented in Table 2, the sensitivity of specific discharge to variations in physical water-level elevation (h_w) and reference level elevation (z_r) are evaluated in Table 3.

Table 3 - Computed specific discharge (q_h) for different input values of physical water-level elevations (h_w) and the reference level elevation (z_r). Other (fixed) inputs are listed in Table 2.

	z_r(m)	h_{w1}(m)	h_{w2}(m)	q_h(m/yr)	Flow direction	Comments
Effect of different water levels in the wells	400	430	429	5.32	→	Base Case: **Figure** 16 and **Table 2**
	400	430	429.5	-4.59	←	Reversal of flow direction from base case
Effect of different reference level elevations used for analysis	410	430	429	10.15	→	Same flow direction as base case
	400	430	429	5.32	→	Base Case: **Figure** 16 and **Table 2**
	389	430	429	Zero		Computed specific discharge is zero
	380	430	429	-4.35	←	Reversal of flow direction from base case
	370	430	429	-9.19	←	Reversal of flow direction from base case

Physical water levels in wells do not always provide a reliable indication of flow direction. Consider the second line in Table 3, which indicates that the physical water level in Well 2 (h_{w2}) is 0.5 m lower than Well 1 (h_{w1}). At face value this might be interpreted to indicate flow to the right. However, the negative value of specific discharge shows that the true flow direction is to the left. For this example, the pressure at the reference level at Well 2 is greater than that of Well 1; however, the higher fluid density in Well 2 supports a shorter static water column in the well.

Results in Table 3 also show that the computed horizontal specific discharge (q_h) depends on the chosen reference elevation (z_r). Depending on the assigned value of z_r, the computed magnitude of q_h can vary significantly and the flow can change direction. To get a sense of the range in horizontal specific discharge that might occur in an aquifer, it may be advisable to compute q_h using several different values for the reference elevation. The uncertainty of the calculation increases as the elevation difference between the reference level and the well-completion zones becomes greater. This supports the suggestion at the end of Section 7.2 that it is best to select the reference level z_r to pass through the midpoint of the well completion zone, or if the wells have different completion elevations, to select z_r at an elevation about midway between the well completion midpoints.

The last two lines in Table 3 show that for a low z_r elevation, the computed flow direction is reversed from the base case. Well-completion depths and the reference level elevation must be judiciously chosen to be consistent with the hydrologic issue being evaluated while also striving for similar screen elevations.

7.4 Horizontal Flow Example 2

Next, we consider a horizontal flow thought experiment where groundwater density varies along the flow path as shown in Figure 17. As a first approximation, we assume the specific discharge (q_h) is laterally constant and the system temperature is 20 °C, which implies that $\mu = \mu_{st}$. For this uniform temperature problem, the variation in groundwater density would result from variations in solute concentration that, for example, might occur in a groundwater contaminant plume. The horizontal (segmented) density distribution for this problem should be viewed as a snapshot in time. In a field system, chemical transport would change the density distribution over time. The calculations used to evaluate this thought experiment are provided in Table 4 and the results are shown graphically in Figure 18.

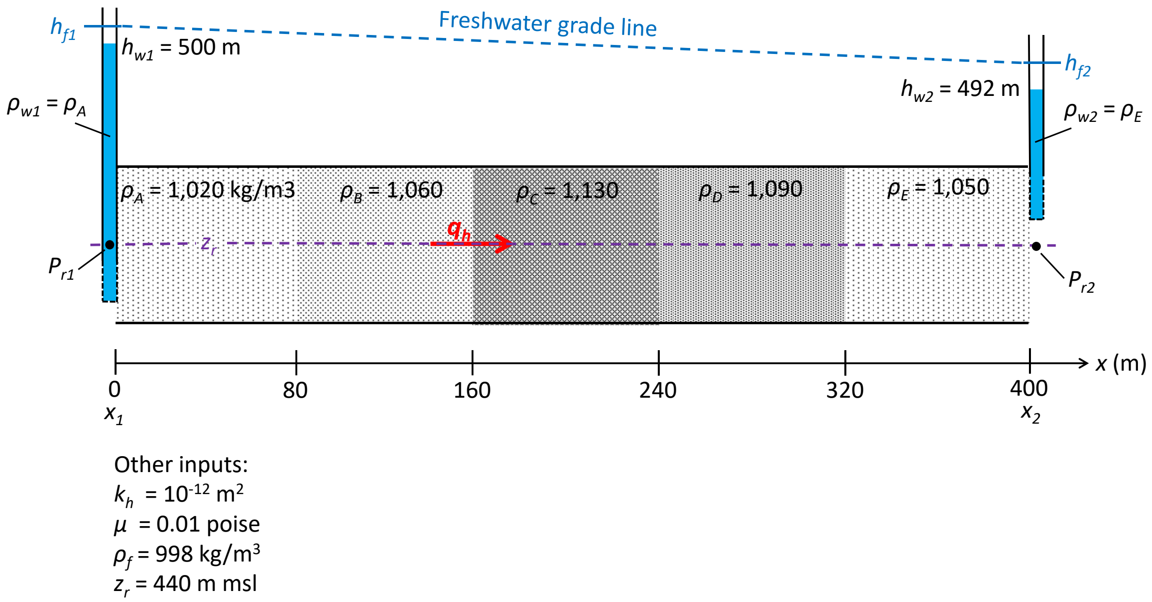


Figure 17 - Thought experiment for horizontal flow with lateral variations in groundwater density. The system has a uniform temperature of 20 °C, so groundwater density is controlled only by the variation in TDS.

Table 4 - Mathematical solution of the thought experiment shown in Figure 17. To provide good documentation, this problem is solved using MathCad® software. However, the equations could also be evaluated using a calculator or spreadsheet. The MathCad® output contains a semicolon before the equal sign and a dot to indicate multiplication of parameters and units (MathCad® treats units as parameters).

Inputs

$k := 10^{-12} \cdot m^2$	Intrinsic permeability
$\mu := 0.01 \cdot \text{poise}$	Pore water absolute viscosity
$x_1 := 0 \cdot m \quad x_2 := 400 \cdot m$	Coordinates at begining and end of flow system
$\rho_f := 998 \cdot \frac{kg}{m^3}$	Freshwater density
$z_r := 440 \cdot m$	Reference elevation
$h_{w1} := 500 \cdot m$	Well 1 fluid level elevation
$h_{w2} := 492 \cdot m$	Well 2 fluid level elevation

$$\rho(s) := \begin{cases} 1020 \cdot \frac{kg}{m^3} & \text{if } s < 80 \cdot m \\ 1060 \cdot \frac{kg}{m^3} & \text{if } 80 \cdot m \le s < 160 \cdot m \\ 1130 \cdot \frac{kg}{m^3} & \text{if } 160 \cdot m \le s < 240 \cdot m \\ 1090 \cdot \frac{kg}{m^3} & \text{if } 240 \cdot m \le s < 320 \cdot m \\ 1050 \cdot \frac{kg}{m^3} & \text{if } s \ge 320 \cdot m \end{cases}$$

Aquifer pore water density distribution

Calculations

$K_f := \frac{k \cdot \rho_f \cdot g}{\mu}$	Freshwater hydraulic conductivity		$K_f = 9.787 \times 10^{-4} \cdot \frac{cm}{sec}$
$P_{r1} := \rho(x_1) g \cdot (h_{w1} - z_r)$	Refrence level pressure at location of well 1		$P_{r1} = 5.923 \cdot atm$
$P_{r2} := \rho(x_2) \cdot g \cdot (h_{w2} - z_r)$	Refrence level pressure at location of well 2		$P_{r2} = 5.284 \cdot atm$
$P_r(x) := P_{r1} + \left(\frac{P_{r2} - P_{r1}}{x_2 - x_1}\right) \cdot x$	Reference level pressure distribution. If q_h is constant, then this distribution is linear		
$h_f(x) := z_r + \frac{P_r(x)}{\rho_f \cdot g}$	Freshwater head distribution	$h_{f1} := h_f(x_1)$	$h_{f1} = 501.32 m$
$h_w(x) := z_r + \frac{P_r(x)}{\rho(x) \cdot g}$	Well fluid level distribution	$h_{f2} := h_f(x_2)$	$h_{f2} = 494.71 m$
$q_h := -K_f \cdot \frac{(h_{f2} - h_{f1})}{x_2 - x_1}$	Specific discharge		$q_h = 5.11 \cdot \frac{m}{yr}$

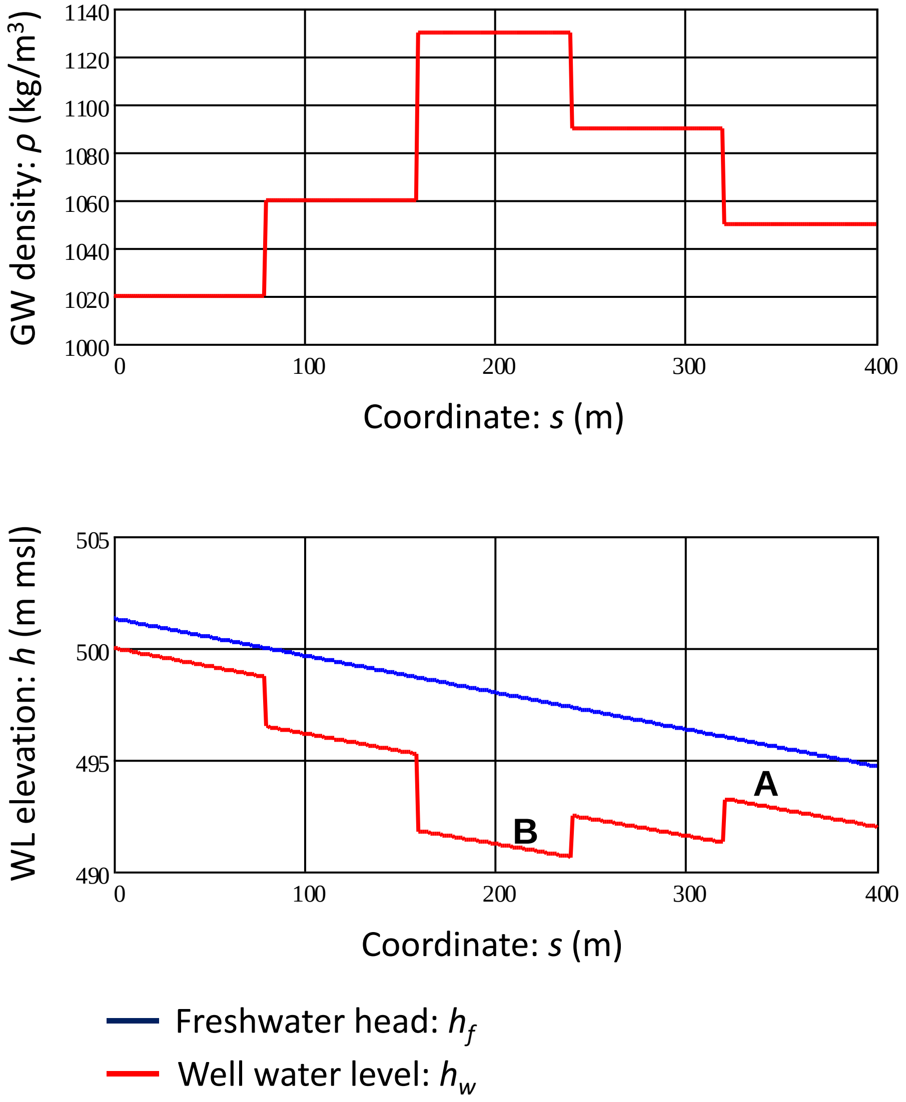


Figure 18 - Graphical results for the thought experiment for horizontal flow with lateral variations in groundwater density as shown in Figure 17 and evaluated using equations in Table 4.

The lower graph in Figure 18 shows that standard evaluation of physical water levels in wells (h_w) could lead to uncertainties and contradictory interpretations. For example, if monitoring wells were installed at locations A and B, the physical water level would be higher in Well A. This could be misinterpreted to indicate flow to the left when the true flow direction is to the right. In contrast, equivalent freshwater heads (h_f) provide a consistent and correct representation of the hydraulics for this horizontal flow system.

How is this result useful? To assess its usefulness, we consider an aquifer with variable-density groundwater and a network of monitoring wells. Then we develop a contour map to interpret groundwater horizontal flow directions and migration velocities. This could be done by contouring pressure on a horizontal plane at an assigned reference elevation, but hydrologists tend to be more comfortable visualizing water-level elevations. If one constructs the contour map using the physical water levels in the wells (h_w), it is possible that flow directions and velocities could be incorrectly interpreted. An alternate approach is to convert the physical well levels to equivalent freshwater heads (h_f) for a chosen reference level elevation (z_r) and construct the contour map using these values. Standard analysis of the freshwater head contour map should provide a more representative interpretation of horizontal flow directions and specific discharges. The reference level elevation must be judiciously chosen to be consistent with the hydrologic issue being evaluated. This approach should be considered only when the density variation in the area of interest is relatively large and the flow is essentially horizontal. An example is the presence of a groundwater chemical plume having much higher TDS compared to the adjacent (unaffected) groundwater in the aquifer. If the density variation in the aquifer is relatively small, the head form of Darcy's is usually sufficiently accurate for practical application. Also note that in a variable-density system, freshwater head is not a hydraulic potential and *by itself* cannot be used to assess flow that is nonhorizontal.

Exercise 1↴, *Flow Through a Slurry Wall*, provides a practice problem with horizontal flow.

8 Vertical Flow Calculations

We now turn our attention to vertical flow in a homogeneous medium. In the pressure form of Darcy's law shown in Equation (10), we set the direction parameter (θ) to 90 degrees and designate the vertical distance variable as z. This leads to Equation (39).

$$q_v(z') = -\left(\frac{k_v}{\mu}\right)\left[\rho(z')\, g + \left.\frac{dP}{dz}\right|_{z'}\right] \tag{39}$$

where (parameter dimensions are in dark green font with mass as M, length as L, time as T, temperature as Θ):

z = vertical elevation above an arbitrary datum; positive upward (L)

z' = specified vertical coordinate (L)

q_v = vertical specific discharge, positive upward (LT^{-1})

k_v = vertical intrinsic permeability (L^2)

In this equation, groundwater density can be uniform or vary vertically, that is $\rho(z)$, and dP/dz is the vertical pressure gradient evaluated at z'. If the medium is anisotropic and the principal directions of permeability are horizontal and vertical, we can assign a specific value to the vertical intrinsic permeability, which is designated as k_v.

Figure 19 shows the geometry and relevant parameters for a groundwater piezometer. We use the term *piezometer* here to imply an installation similar to a monitoring well but with a short completion zone, so the pore fluid pressure is measured over a small vertical increment of the geologic medium. In comparison with Figure 15, new parameters to consider are shown here.

P_w = fluid pressure at the midpoint of the piezometer completion zone ($ML^{-1}T^{-2}$)

z_w = midpoint elevation of the piezometer completion zone (L)

$\rho(z)$ = vertical groundwater density distribution (ML^{-3})

ρ = density of the fluid column in the piezometer riser pipe (ML^{-3}); assumed equal to the groundwater density at the piezometer completion zone, $\rho(z_w)$

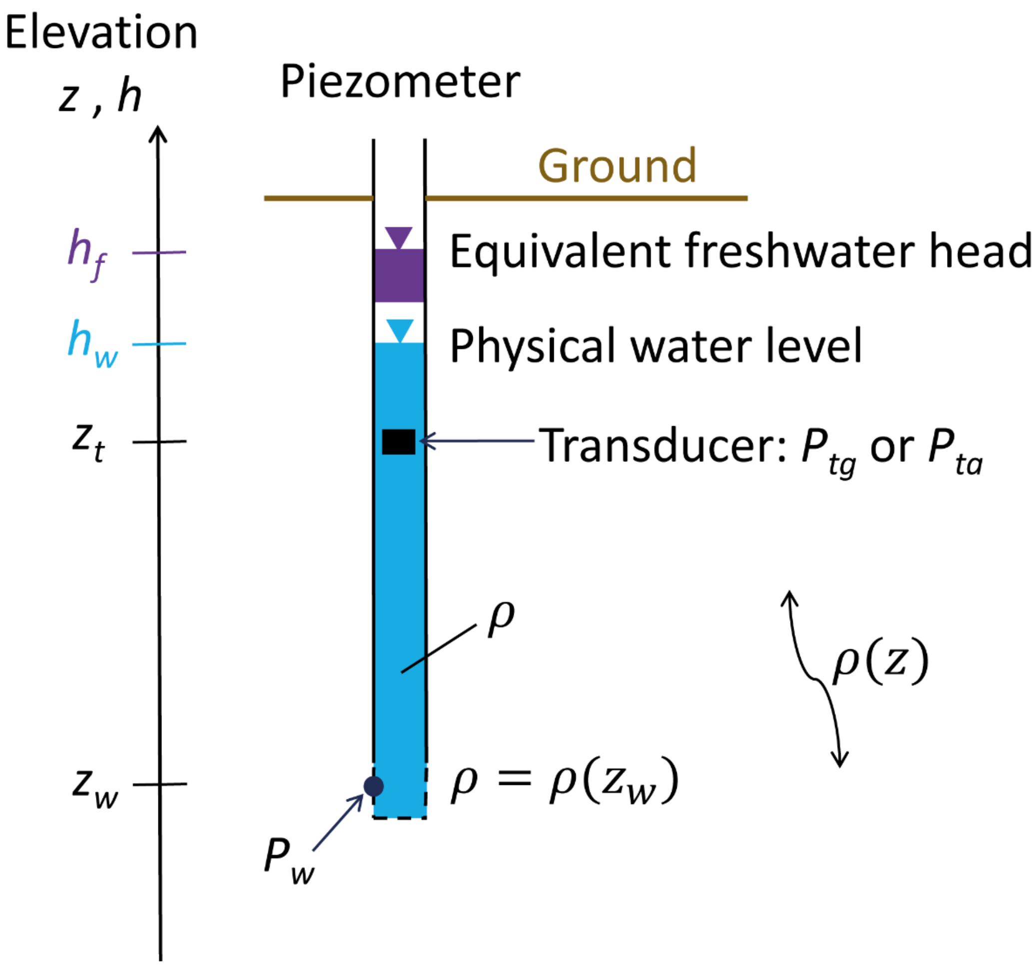


Figure 19 - Geometric and hydraulic parameters associated with a piezometer. A piezometer is typically designed to have a short completion zone so that P_w is the fluid pore pressure measured over a small vertical increment of the medium.

Using hydrostatics, we can now set up equations to compute the pore water pressure (P_w) at the midpoint of the piezometer completion interval for several types of field measurements. Equation (40) computes P_w when the physical water-level elevation (h_w) is measured in the piezometer riser:

$$P_w = \rho\, g(h_w - z_w) \tag{40}$$

Equation (41) computes P_w when using a vented transducer that measures gauge pressure (P_{tg}):

$$P_w = P_{tg} + \rho\, g(z_t - z_w) \tag{41}$$

Equation (42) computes P_w when using a unvented transducer that measures absolute pressure (P_{ta}):

$$P_w = P_{ta} - P_{atm} + \rho\, g(z_t - z_w) \tag{42}$$

If the piezometer is periodically purged for sampling, it is reasonable to assume that the density of the standpipe water column is the same as the groundwater density in the medium at the elevation of the completion zone, that is, $\rho = \rho(z_w)$. The density of the medium groundwater, $\rho(z_w)$, can be directly measured in the lab using a sample taken during purging. If the transducer is positioned at the midpoint of the completion zone, the term $(z_t - z_w)$ in Equations (41) and (42) goes to zero, and P_w can be measured directly without knowledge of the standpipe water column density, ρ.

A situation where two piezometers are located close together in map view but have completion zones at different elevations is commonly referred to as a *nested piezometer installation* (Figure 20). For two piezometers, the parameters in Equations (40), (41), and (42) are subscripted with "1" to indicate Piezometer 1, or "2" for Piezometer 2. Using one of the measurement methods discussed above, the fluid pressure is determined at the midpoint of each piezometer completion zone (P_{w1} and P_{w2}).

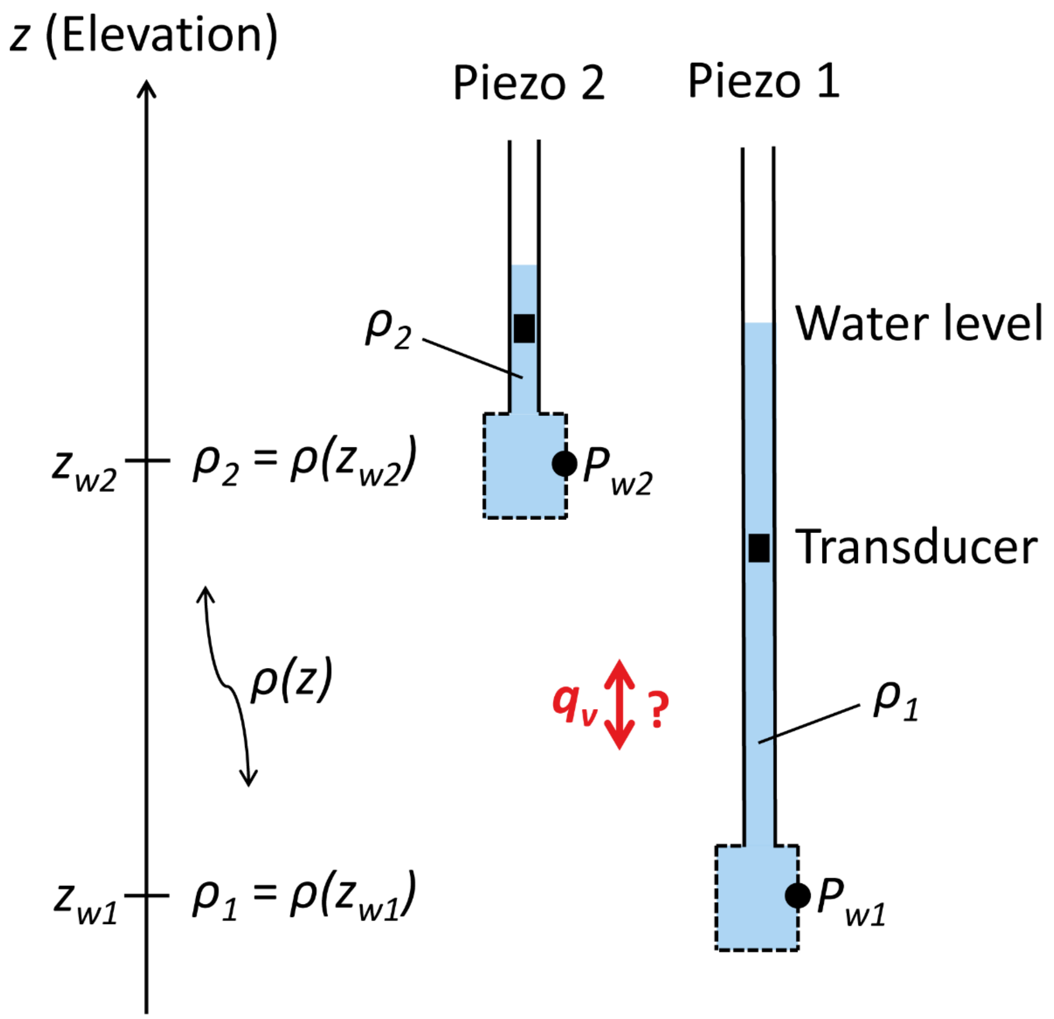


Figure 20 - Nested piezometer Installation. Subscripts 1 and 2 refer to Piezometer 1 and 2, respectively. Elevated groundwater density is uniform or varies vertically [$\rho = \rho(z)$]. P_{w1} and P_{w2} are fluid pressures within the completion zones of Piezometer 1 and 2, respectively.

The flow direction and magnitude of the vertical specific discharge (q_v) between the two piezometers is estimated using a finite difference approximation for the derivative in Equation (39), which leads to:

$$q_v \approx -\frac{k_v}{\mu}\left[\frac{P_{w2} - P_{w1}}{z_{w2} - z_{w1}} + \rho_c\, g\right] \tag{43}$$

where:

ρ_c = a characteristic groundwater density between the two piezometer completion zones (ML^{-3})

The characteristic groundwater density (ρ_c) has a value that best represents the density distribution existing between the two piezometers. In the absence of other information, ρ_c can be taken as the average of the groundwater densities at each piezometer completion zone as shown in Equation (44).

$$\rho_c = \frac{\rho_1 + \rho_2}{2} \tag{44}$$

where:

ρ_1 = groundwater density at the midpoint of Piezometer 1 completion zone (ML^{-3})

ρ_2 = groundwater density at the midpoint of Piezometer 2 completion zone (ML^{-3})

In Equation (43), flow is up for q_v positive and down for q_v negative. Because vertical intrinsic permeability, k_v, and fluid viscosity, μ, are always positive values, flow is up if the value within the parentheses is negative and down if the value is positive. Therefore, one can assess flow direction without knowing k_v and μ.

Some investigators (e.g., Post et al., 2007) have recast Equation (43) in terms of freshwater heads, which are used as surrogates for pressure. To do this, we use the definition of freshwater head in Equation (8) to develop two relationships, $P_{w1} = \rho_f\, g\,(h_{f1} - z_{w1})$ and $P_{w2} = \rho_f\, g\,(h_{f2} - z_{w2})$, which are substituted into Equation (43). Algebraic manipulation then leads to the equivalent vertical flux equation using freshwater heads as shown in Equation (45). This version of the flux equation uses the definition of standard vertical hydraulic conductivity provided in Equation (46).

$$q_v \approx -K_{stv}\left(\frac{\mu_{st}}{\mu}\right)\left[\frac{h_{f2} - h_{f1}}{z_{w2} - z_{w1}} + \frac{\rho_c - \rho_f}{\rho_f}\right] \tag{45}$$

$$K_{stv} = \frac{k_v\, \rho_f\, g}{\mu_{st}} \tag{46}$$

where:

K_{stv} = standard vertical hydraulic conductivity (LT^{-1})

K_{stv} is the hydraulic conductivity one would expect to measure in a laboratory permeameter containing minimally disturbed soil using pure water at 20 °C. As with Equation (43), if the value in the brackets of Equation (45) is negative, the flow is up. If the value is positive, the flow is down.

8.1 Vertical Flow Example 1

Figure 21 shows the mathematical setup for an example vertical flow problem, which is solved in Table 5.

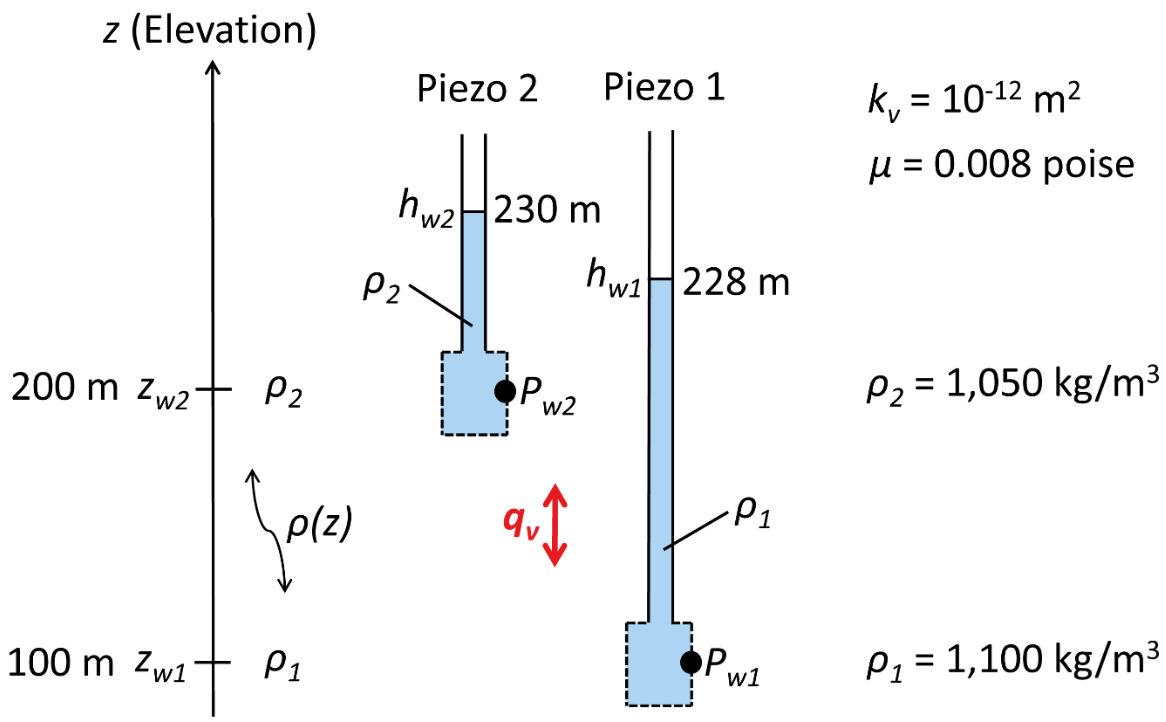


Figure 21 - Setup for example vertical flow problem. The physical water level (h_w) in the lower piezometer is 2 m lower than the level in the upper piezometer.

Table 5 - Mathematical solution for the vertical flow problem shown in Figure 21. However, the equations could also be evaluated using a calculator or spreadsheet. The MathCad® output contains a semicolon before the equal sign and a dot to indicate multiplication of parameters and units (MathCad® treats units as parameters).

Inputs

Upper Piezometer	Lower Piezometer			
$k_v := 1\cdot 10^{-12}\cdot m^2$		Vertical intrinsic permeability		
$\mu := 0.008\cdot poise$		Groundwater absolute viscosity at prevailing system temperature		
$z_{w2} := 200\cdot m$	$z_{w1} := 100\cdot m$	Piezometer elevation (z positive upward)		
$h_{w2} := 230\cdot m$	$h_{w1} := 228\cdot m$	Physical fluid elevation in piezometer		
$\rho_2 := 1050\cdot \frac{kg}{m^3}$	$\rho_1 := 1100\cdot \frac{kg}{m^3}$	Groundwater density within the formation at the piezometer completion depth (for this analysis, this is also the fluid density inside the piezometer standpipe).		
$\rho_f := 998.2\cdot \frac{kg}{m^3}$		Freshwater density at standard temperature (20 degrees C)		
$\mu_{st} := 0.01\cdot poise$		Water viscosity at standard temperature (20 degrees C)		

Calculations

Based on pressures

$P_{w2} := \rho_2\cdot g\cdot (h_{w2} - z_{w2})$	Water pressure in upper piezometer	$P_{w2} = 308.91\cdot kPa$	
$P_{w1} := \rho_1\cdot g\cdot (h_{w1} - z_{w1})$	Water pressure in lower piezometer	$P_{w1} = 1380.78\cdot kPa$	
$\rho_c := \frac{\rho_1 + \rho_2}{2}$	Charateristic (average) density between the two piezometers	$\rho_c = 1075\,\frac{kg}{m^3}$	
$q_v := \frac{-k_v}{\mu}\left(\frac{P_{w2} - P_{w1}}{z_{w2} - z_{w1}} + g\cdot \rho_c\right)$	Vertical specific discharge using pressures (positive upward)	$q_v = 6.96\cdot \frac{m}{yr}$	Up

Based on freshwater levels

$h_{f2} := z_{w2} + \frac{\rho_2}{\rho_f}(h_{w2} - z_{w2})$	Piezometer 2 freshwater head	$h_{f2} = 231.56\,m$	
$h_{f1} := z_{w1} + \frac{\rho_1}{\rho_f}(h_{w1} - z_{w1})$	Piezometer 1 freshwater head	$h_{f1} = 241.05\,m$	
$K_{vf} := \frac{k_v\cdot \rho_f\cdot g}{\mu_{st}}$	Freshwater vertical hydraulic conductivity	$K_{vf} = 9.79\times 10^{-6}\cdot \frac{m}{sec}$	
$q_{vf} := -K_{vf}\cdot \frac{\mu_{st}}{\mu}\cdot\left(\frac{h_{f2} - h_{f1}}{z_{w2} - z_{w1}} + \frac{\rho_c - \rho_f}{\rho_f}\right)$	Vertical specific discharge using freshwater heads (positive upward)	$q_{vf} = 6.96\cdot \frac{m}{yr}$	Up

The computed vertical specific discharge in Table 5 is 6.96 m/yr upward regardless of whether the calculation is based on pressures or freshwater heads. In the problem setup, the physical water level in the lower piezometer is 2 m lower than the level in the upper piezometer. If the elevated groundwater density is not considered, these levels could be incorrectly interpreted to indicate downward flow when in fact the flow direction is up.

Exercise 2↴, *Leakage Through a Pond Liner*, provides a practice problem with vertical flow.

8.2 Vertical Flow in an Aquitard

Next, we consider a relatively low-permeability aquitard that is bounded above and below by two aquifers as shown in Figure 22. In each aquifer, the groundwater density is uniform, and the flow is horizontal, so the vertical pressure distribution is hydrostatic. Flow within the aquitard is vertical, and we assume a characteristic groundwater density (ρ_c) that reasonably represents the vertical density distribution. A piezometer is installed in each aquifer relatively close to, but not necessarily at, the aquifer/aquitard boundary.

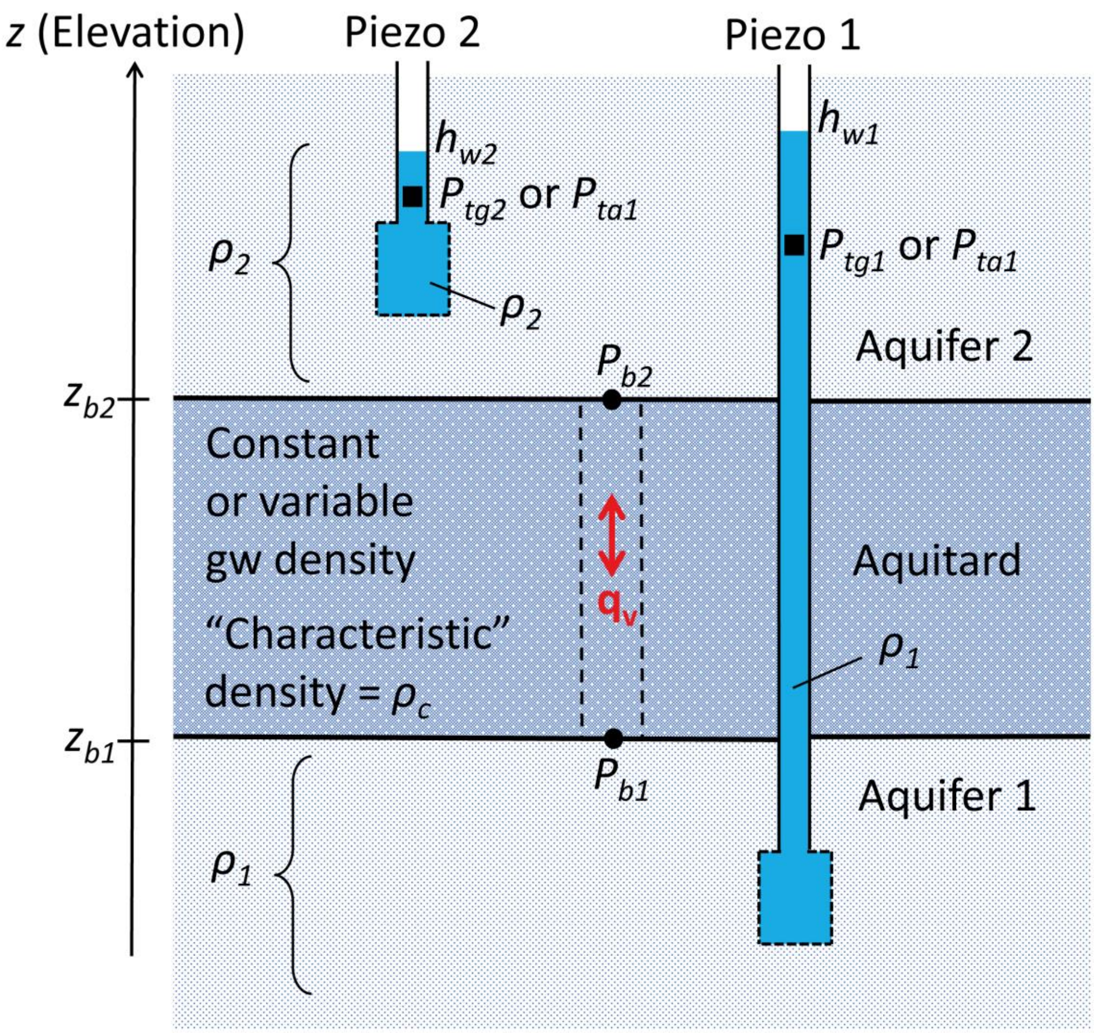


Figure 22 - Vertical flow in an aquitard that is sandwiched between two permeable aquifers. Subscript 1 represents the lower portion of the aquifer/aquitard system and subscript 2 represents the upper portion of the system. The abbreviation 'gw' stands for groundwater.

Depending on how the measurement is made, we use Equation (47), (48), or (49) to estimate the fluid pressure at an aquitard boundary.

If the physical water-level elevation (h_w) is measured:

$$P_b = \rho\, g(h_w - z_b) \tag{47}$$

If a vented transducer measures true gauge pressure (P_{tg}):

$$P_b = P_{tg} + \rho\, g(z_t - z_b) \tag{48}$$

If an unvented transducer measures absolute pressure (P_{ta}):

$$P_b = P_{ta} - P_{atm} + \rho\, g(z_t - z_b) \tag{49}$$

where (parameter dimensions are in dark green font with mass as M, length as L, time as T, temperature as Θ):

P_b = pressure at an aquitard boundary ($ML^{-1}T^{-2}$)

ρ = groundwater density in the aquifer (ML^{-3})

z_b = elevation of the aquitard boundary (L)

The parameters in Equations (47), (48), or (49) are subscripted "1" to indicate the lower piezometer and the bottom boundary of the aquitard. Subscript "2" indicates the upper piezometer and the top boundary of the aquitard.

We can insert the appropriate parameters into Equation (43) to get the approximate equation for vertical specific discharge in the aquitard as shown by Equation (50).

$$q_v \approx -\frac{k_v}{\mu}\left[\frac{P_{b2} - P_{b1}}{z_{b2} - z_{b1}} + \rho_c\, g\right] \tag{50}$$

where:

ρ_c = a characteristic groundwater density for the aquitard [kg/m^3]

Since it is unlikely that a sampling well would be installed in the aquitard, three values can be considered for the characteristic groundwater density in the aquitard (ρ_c):

- $\rho_c = (\rho_1 + \rho_2)/2$, the average of the two aquifer densities. This might be reasonable if the density is diffusion controlled and a linear vertical density distribution exists in the aquitard.
- $\rho_c = \rho_1$ might be reasonable for significant upward flow in the aquitard.
- $\rho_c = \rho_2$ might be reasonable for significant downward flow in the aquitard.

8.3 Vertical Flow Example 2

The setup for an example aquitard problem is shown in Figure 23. The conditions depicted in this example are similar to those observed at a field site located close to Great Salt Lake in Utah, USA. The problem is solved in Table 6.

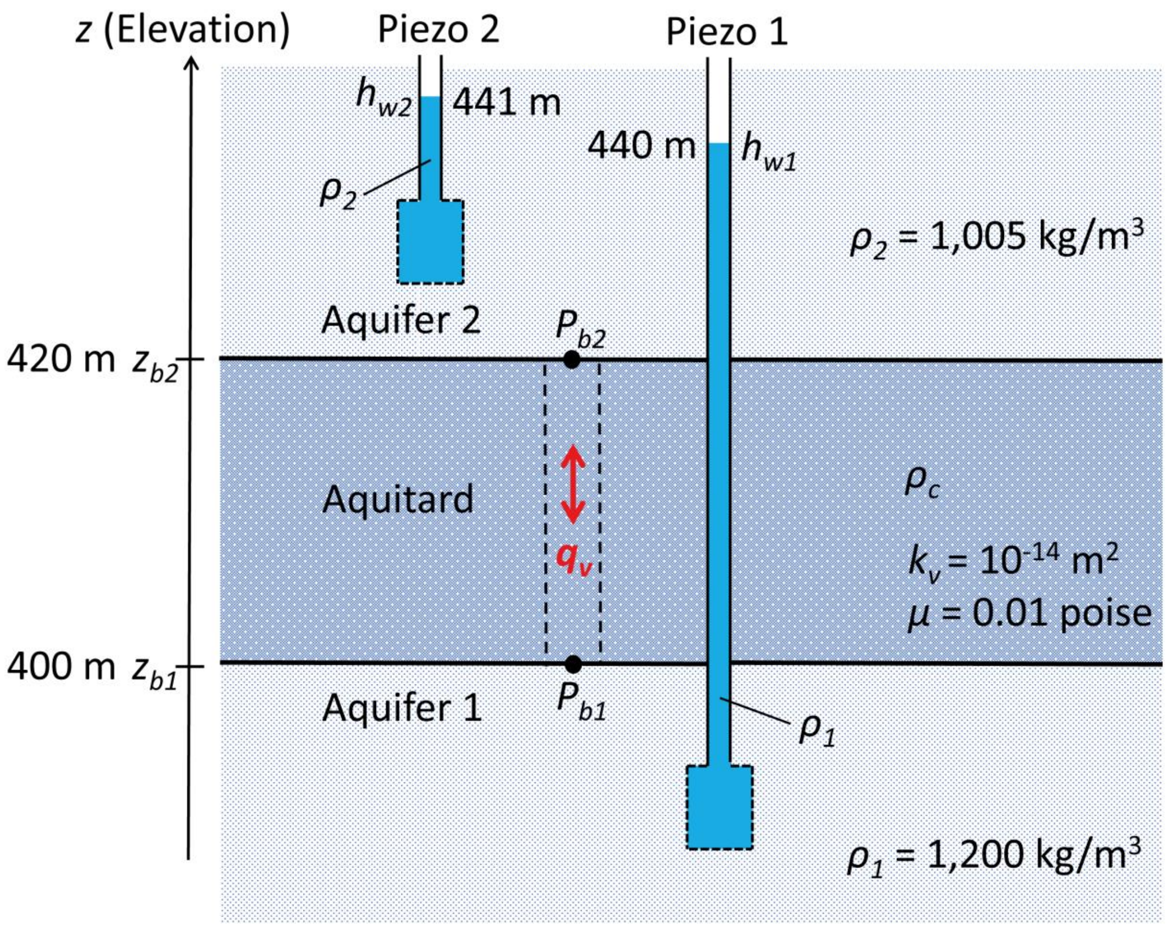


Figure 23 - Example problem for vertical flow in an aquitard. The direction and magnitude of vertical specific discharge (q_v) is to be evaluated.

Table 6 - Mathematical solution for the example aquitard problem. To provide good documentation, the problem is solved using MathCad® software. However, the equations could also be evaluated using a calculator or spreadsheet. The MathCad® output contains a semicolon before the equal sign and a dot to indicate multiplication of parameters and units (MathCad® treats units as parameters).

Inputs

Note: 1 kPa = 1000 Pascals

$k_v := 1 \cdot 10^{-14} \cdot m^2$	Vertical intrinsic permeability		
$\mu := 0.01 \cdot poise$	Water absolute viscosity		
$h_{w2} := 441 \cdot m$	Physical water level in upper piezometer		
$h_{w1} := 440 \cdot m$	Physical water level in lower piezometer		
$\rho_2 := 1005 \cdot \frac{kg}{m^3}$	Groundwater density in the upper aquifer		
$\rho_1 := 1200 \cdot \frac{kg}{m^3}$	Groundwater density in the lower aquifer		
$z_{a2} := 420 \cdot m$	Top of aquitard elevation		
$z_{a1} := 400 \cdot m$	Bottom of aquitard elevation		

Calculations

$P_2 := \rho_2 \cdot g \cdot (h_{w2} - z_{a2})$	Water pressure at top of aquitard	$P_2 = 206.97 \cdot kPa$	
$P_1 := \rho_1 \cdot g \cdot (h_{w1} - z_{a1})$	Water pressure bottom of aquitard	$P_1 = 470.72 \cdot kPa$	
$\rho_{av} := \frac{\rho_1 + \rho_2}{2}$	Average groundwater density		
$q_{v1} := \frac{-k_v}{\mu}\left(\frac{P_2 - P_1}{z_{a2} - z_{a1}} + g \cdot \rho_1\right)$	Specific discharge if groundwater density in the aquitard is the same as in the lower aquifer.	$q_{v1} = 0.448 \cdot \frac{m}{yr}$	Up
$q_{v2} := \frac{-k_v}{\mu}\left(\frac{P_2 - P_1}{z_{a2} - z_{a1}} + g \cdot \rho_{av}\right)$	Specific discharge if groundwater in the aquitard varies linearly between the densities in the upper and lower aquifers	$q_{v2} = 0.750 \cdot \frac{m}{yr}$	Up
$q_{v3} := \frac{-k_v}{\mu}\left(\frac{P_2 - P_1}{z_{a2} - z_{a1}} + g \cdot \rho_2\right)$	Specific discharge if groundwater density in the aquitard is the same as in the upper aquifer. ***This result is not reasonable as the computed flow direction is Up.***	$q_{v3} = 1.051 \cdot \frac{m}{yr}$	Up

For all cases shown in Table 6, the computed flow direction is up (q_v positive) even though the physical water level is lower in the lower piezometer. The computed magnitude of specific discharge varies depending on the characteristic groundwater density considered in the aquitard, ρ_c. With upward flow, it is unlikely that the groundwater density in the aquitard is equal to density in the upper aquifer (the third case). The other two cases give vertical specific discharges that range from 0.448 to 0.750 m/yr.

9 Concluding Remarks

In most groundwater systems with slight to moderate variations in pore water density, the traditional hydraulic head form of Darcy's law is sufficiently accurate for practical application. However, in systems with large variations in pore water density, the head form of Darcy's law can lead to misinterpretation of flow directions and inaccurate estimates of specific discharge. In these situations, one should use the pressure form of Darcy's law, which is more universal and not subject to restrictions contained in the head form. The more general applicability of the pressure form of Darcy's law has been recognized in other disciplines, such as petroleum engineering, but its practical use may be unfamiliar to many practicing groundwater hydrologists.

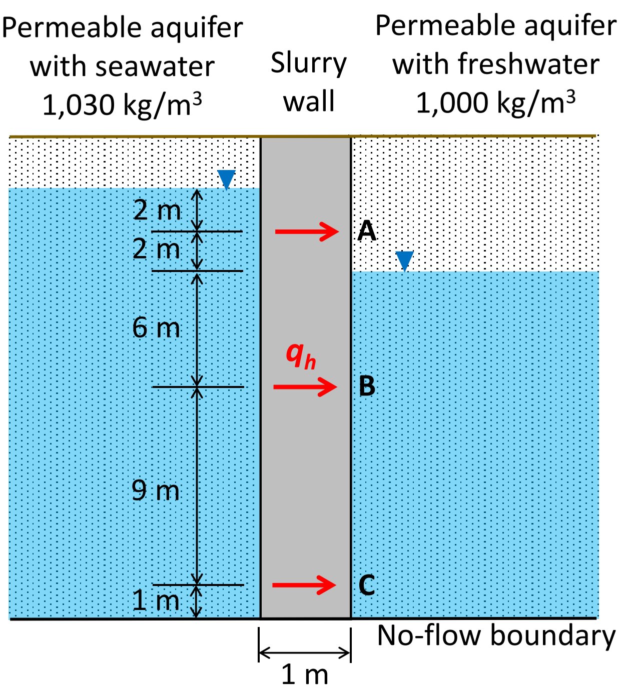
10 Exercises

These two practice problems are provided to supplement the four solved problems that are presented in Sections 7 and 8 of this book.

Exercise 1 - Flow Through a Slurry Wall

The image below shows a vertical slurry wall constructed in a permeable aquifer to minimize the encroachment of saline groundwater into portions of the aquifer with low TDS groundwater. The saline groundwater is chemically similar to seawater and has a fluid density of 1,030 kg/m^3. The low TDS groundwater (freshwater) has a density of 1,000 kg/m^3. The physical water table in the seawater aquifer is 4 m higher than the water table in the freshwater aquifer. Samples of the slurry wall material were tested in a laboratory permeameter, and the average intrinsic permeability was 5x10^{-12} cm^2. The system temperature is 20 °C, the vertical pressure distributions on each side of the wall are hydrostatic, and the pore fluid pressure is approximately zero where the aquifer is unsaturated; that is, above the water table.

Our objective is to estimate the horizontal specific discharge through the slurry wall at three different locations (A, B, and C) as shown in the following image.



Problem description. This figure is not to scale.

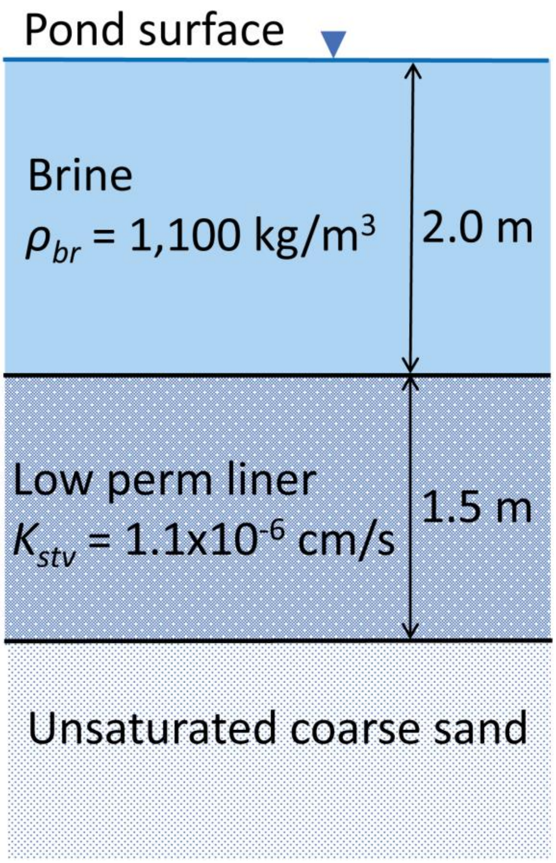
Solution to Exercise 1↴

Return to where text linked to Exercise 1↥

Exercise 2 - Leakage Through a Pond Liner

The image for this exercise illustrates a brine wastewater pond with a temperature of 25 °C and a fluid density of 1,100 kg/m^3. At the bottom of the pond is a compacted soil liner that is 1.5 m thick and has a footprint area of 0.5 hectares. During construction, recompacted samples of the liner material were tested in a laboratory permeameter using tap water at 20 °C. The average hydraulic conductivity measured from these laboratory tests (K_{stv}) is 6.1x10^{-7} cm/sec. Below the liner is an unsaturated coarse sand that is assumed to have zero pore water pressure.

We need to estimate the vertical specific discharge through the liner and the total flow rate leaking from the pond bottom. For this exercise, we disregard leakage through the pond sides.



Problem description. This figure is not to scale.

Solution to Exercise 2↴
Return to where text linked to Exercise 2↥

11 References

CRC. (2023). *CRC handbook of chemistry and physics* (104th ed.). CRC Press. https://www.routledge.com/blog/article/crc-handbook-of-chemistry-and-physics-104th-edition↗.

Domenico, P. A., & Schwartz, F. W. (1990). *Physical and chemical hydrogeology*. John Wiley & Sons.

Drefahl, A. (2023). Dens1: Aqueous solution densities—A calculator in the Axeleratio chemistry online calculator (ACOC) suite. http://www.axeleratio.com/calc/solution_density/form/dens1.htm↗.

Freeze, R. A., & Cherry, J. A. (1979). *Groundwater*. Prentice-Hall. https://gw-project.org/books/groundwater/↗.

Hubbert, M. K. (1940). The theory of groundwater motion, part 1. *Journal of Geology*, *48*(8), 785–944. https://doi.org/10.1029/TR021i002p00648-1↗.

Hubbert, M. K. (1956). Darcy's law and the field equations of flow of underground fluids. *Transactions of the American Institute of Mining, Metallurgical, and Petroleum Engineers (AIME)*, *207*(1), 222–239. https://doi.org/10.2118/749-G↗.

Kell, G. S. (1975). Density, thermal expansivity, and compressibility of liquid water from 0.deg. to 150.deg. *Journal of Chemical & Engineering Data*, *20*(1), 97–105. https://doi.org/10.1021/je60064a005↗.

McWhorter, D. B., & Sunada, D. K. (1977). *Ground-water hydrology and hydraulics*. Water Resources Publications, LLC. Groundwater Hydrology and Hydraulics - The Groundwater Project (gw-project.org)↗.

Post, V. E. A., & Simmons, C. T. (2022). *Variable-density groundwater flow*. The Groundwater Project. https://doi.org/10.21083/978-1-77470-046-4↗.

Post, V., Kooi, H., & Simmons, C. (2007). Using hydraulic head measurements in variable-density ground water flow analyses. *Groundwater*, *45*(6), 664–671. https://doi.org/10.1111/j.1745-6584.2007.00339.x↗.

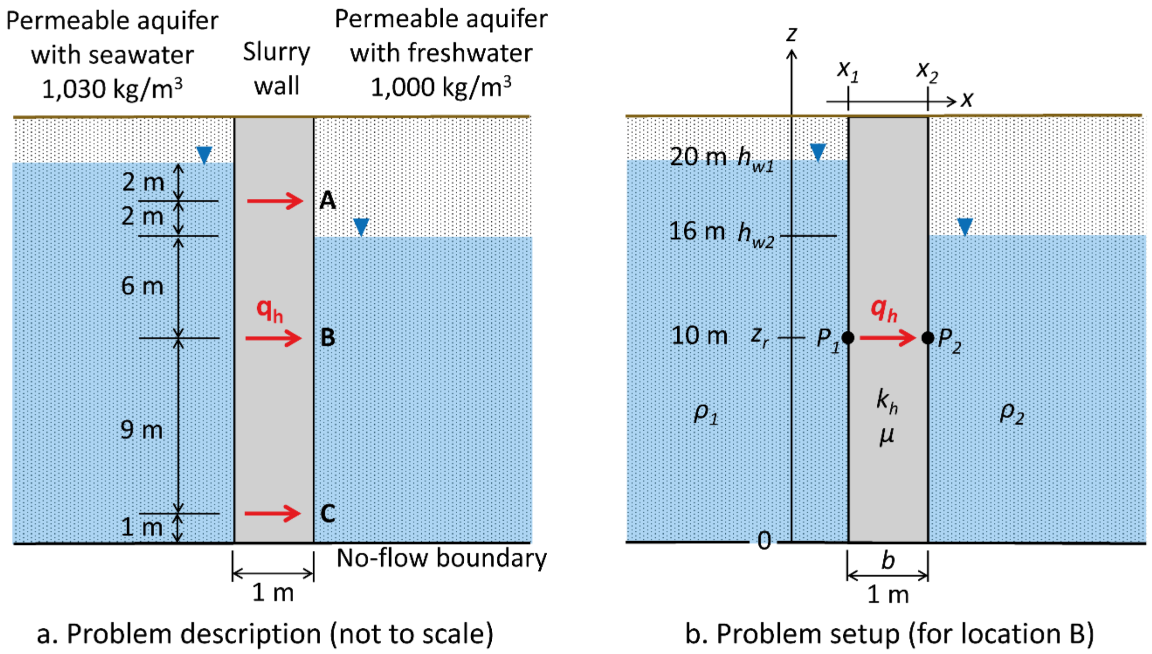
12 Exercise Solutions

Solution to Exercise 1

Answer:

1. At location A, specific discharge (q_h) = 0.319 m/yr
2. At location B, specific discharge (q_h) = 0.665 m/yr
3. At location C, specific discharge (q_h) = 0.707 m/yr

The following image illustrates the problem and the mathematical setup that leads to the solution for location B.



Problem description and setup. The direction and magnitude of horizontal specific discharge (q_h) is to be evaluated at locations A, B, and C.

General input parameters:

k_h = horizontal intrinsic permeability of the slurry wall material ($5\text{x}10^{-12}$ cm^2)

μ = dynamic viscosity of the pore fluid; water at 20 °C (0.001 kg/m/s)

g = acceleration of gravity (9.807 m/s)

Assuming hydrostatic conditions, the fluid pressure on the left (seawater) side is calculated as follows.

$$P_1 = \rho_1 \, g \, (h_{w1} - z_r)$$

where:

P_1 = porewater pressure in the seawater aquifer at the reference elevation

ρ_1 = seawater density (1,030 kg/m^3)

h_{w1} = water level (water table) elevation in the seawater aquifer (20 m)

z_r = reference elevation at location B (10 m)

$$\text{At location B: } P_1 = 1030\frac{\text{kg}}{\text{m}^3}\ 9.807\frac{\text{m}}{\text{s}^2}(20\text{ m} - 10\text{ m}) = 101{,}012\ \frac{\text{kg}}{\text{m s}^2}$$

The fluid pressure (P_1) on the left side at location B is computed to be 101,012 Pa. On the right (freshwater) side, the pressure is calculated as follows.

$$P_2 = \rho_2\ g\ (h_{w2} - z_r)$$

where:

P_2 = porewater pressure in the freshwater aquifer at the reference elevation

ρ_2 = freshwater density (1,000 kg/m^3)

h_{w2} = water level (water table) elevation in the freshwater aquifer (16 m)

$$\text{At location B: } P_2 = 1000\frac{\text{kg}}{\text{m}^3}\ 9.807\frac{\text{m}}{\text{s}^2}(16\text{ m} - 10\text{ m}) = 58{,}842\ \frac{\text{kg}}{\text{m s}^2}$$

Assuming a linear pressure gradient, the horizontal specific discharge (q_h) through the slurry wall at location B is estimate as follows.

$$q_h = -\frac{k_h}{\mu}\left(\frac{P_2 - P_1}{b}\right)$$

$$b = x_2 - x_1$$

where:

q_h = horizontal specific discharge at location B [m/s]

b = slurry wall width (1.0 m)

$$q_h = -\frac{5\text{x}10^{-12}\ \text{cm}^2\frac{\text{m}^2}{10{,}000\ \text{cm}^2}}{0.001\frac{\text{kg}}{\text{m s}}}\left(\frac{58{,}842\ \frac{\text{kg}}{\text{m s}^2} - 101{,}012\ \frac{\text{kg}}{\text{m s}^2}}{1\text{ m}}\right)\frac{3.1557\text{x}10^7\text{s}}{\text{yr}} = 0.665\frac{\text{m}}{\text{yr}}$$

Using appropriate inputs and unit conversions, the specific discharge at location B is estimated to be 0.665 m/yr. The positive value indicates that flow in the slurry wall is in the positive x direction; that is, from left to right.

Using the same equations and appropriate inputs, the aquifer pressures and specific discharges at locations A, B and C are shown in the following table. Note at location A, the pressure on the right side of the slurry wall (P_2) is assumed zero because the soil is unsaturated.

Summary of calculations.

Location	**h_{w1} (m)**	**h_{w2} (m)**	**z_r (m)**	**P_1 (Pa)[(a)]**	**P_2 (Pa)[(a)]**	**q_h (m/yr)**
A	20	16	18	20,202	0[(b)]	0.319
B	20	16	10	101,012	58,842	0.665
C	20	16	1	191,916	147,100	0.707

[(a)] Pressure in Pascals (Pa). 1 Pa = 1 kg/m/s^2

[(b)]The freshwater aquifer is unsaturated at Location A; it is reasonable to assume that the pore fluid pressure P_2 is approximately zero.

Return to Exercise 1↥

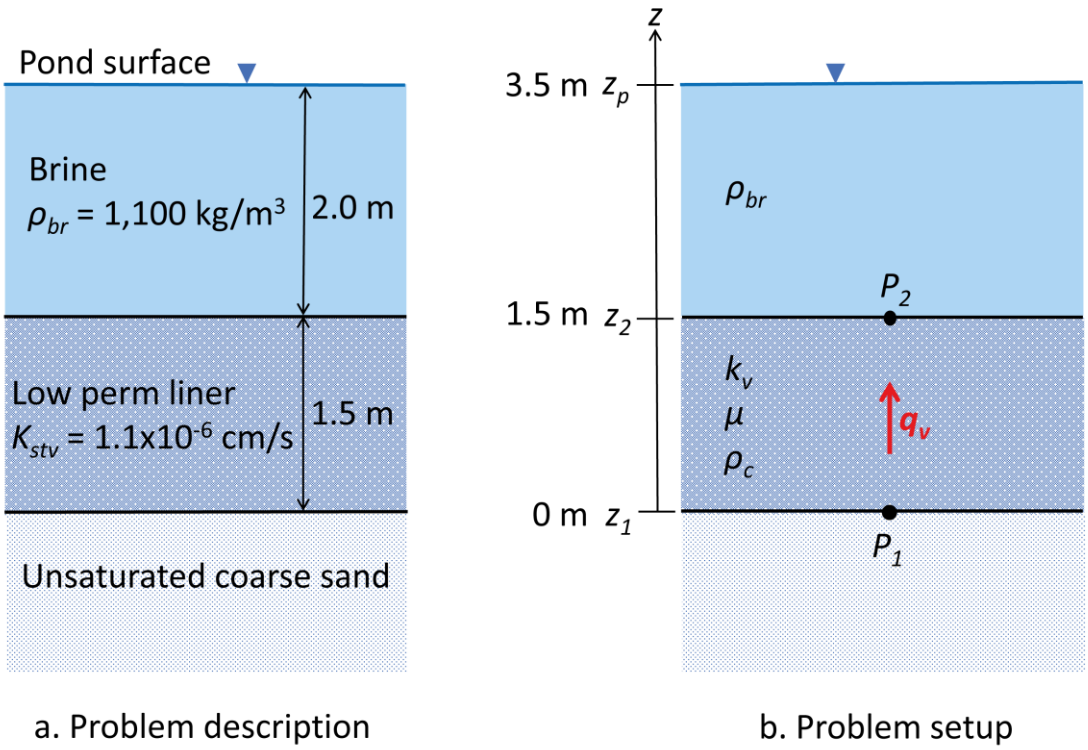
Return to where text linked to Exercise 1↥

Solution to Exercise 2

Answer:

1. Specific discharge (q_v) = 0.556 m/yr (down)
2. Total pond leakage (Q) = 13,710 liters/day

The image shown here illustrates the problem and the mathematical setup that leads to the solution.



Example 2 problem description and setup. The direction and magnitude of vertical specific discharge through the liner (q_v) is to be evaluated.

Compute the vertical intrinsic permeability (k_v) of the liner material:

$$k_v = \frac{K_{stv}\ \mu_{st}}{\rho_f\ g}$$

where:

K_{stv} = laboratory measured vertical hydraulic conductivity using pure water at 20 °C (6.1×10^{-7}cm/sec)

ρ_f = density of laboratory permeant; tap water at 20 °C (998.2 kg/m^3)

μ_{st} = dynamic viscosity of laboratory permeant; water at 20 °C (0.001 kg/m/s)

g = acceleration of gravity (9.807 m/s^2)

$$k_v = \frac{6.1\text{x}10^{-7}\,\dfrac{\text{cm}}{\text{s}}\,\dfrac{1\text{ m}}{100\text{ cm}}\,1\text{x}10^{-3}\,\dfrac{\text{kg}}{\text{m s}}}{998.2\,\dfrac{\text{kg}}{\text{m}^3}\,9.807\,\dfrac{\text{m}}{\text{s}^2}} = 6.231\text{x}10^{-16}\text{ m}^2\ \ \frac{10{,}000\text{ cm}^2}{1\text{ m}^2} = 6.231\text{x}10^{-12}\text{ cm}^2$$

Using the values given above, the intrinsic permeability (k) of the liner material is computed to be 6.231×10^{-12} cm^2.

By choosing the datum for the vertical (elevation) axis at the liner bottom, values of the elevation parameters are given here.

z_1 = elevation at the liner bottom (0 m)

z_2 = elevation of the liner top (1.5 m)

z_p = elevation of pond surface (3.5 m)

The fluid pressure at the liner bottom (P_1) is assumed to be zero, a reasonable approximation because the underlying coarse-grained sand is unsaturated.

Based on hydrostatics, the pressure at the liner top is:

$$P_2 = \rho_{br}\, g\,(z_p - z_2)$$

where:

ρ_{br} = brine density (1,100 kg/m^3)

$$P_2 = 1{,}100\,\frac{\text{kg}}{\text{m}^3}\,9.807\,\frac{\text{m}}{\text{s}^2}\,(3.5\text{ m} - 1.5\text{ m}) = 21{,}575\,\frac{\text{kg}}{\text{m s}^2}$$

Using values given above and appropriate unit conversions, the fluid pressure at the liner top (P_2) is computed to be 21,575 kg/m/s^2.

Using the pressure form of Darcy's law, and assuming a linear pressure gradient, the specific discharge (q_v) through the liner is estimated:

$$q_v = -\,\frac{k_v}{\mu}\left(\frac{P_2 - P_1}{z_2 - z_1} + \rho_c\, g\right)$$

where:

μ = dynamic viscosity of water at 25 °C (0.00089 kg/m/s)

ρ_c = characteristic density of pore fluid in the liner (1,100 kg/m^3). We expect the flow direction in the liner to be down; thus, it is reasonable to assume that the pore water in the liner is brine $(\rho_c = \rho_{br})$

$$q_v = -\,\frac{6.231\text{x}10^{-12}\text{ cm}^2\,\dfrac{\text{m}^2}{10{,}000\text{ cm}^2}}{0.00089\,\dfrac{\text{kg}}{\text{m s}}}\left(\frac{21{,}575\,\dfrac{\text{kg}}{\text{m s}^2} - 0}{1.5\text{m} - 0} + 1{,}100\,\frac{\text{kg}}{\text{m}^3}\,9.807\,\frac{\text{m}}{\text{s}^2}\right)\frac{3.1557\text{x}10^{7}\text{s}}{\text{yr}}$$

$$= -0.556\,\frac{\text{m}}{\text{yr}}$$

Using values given above and appropriate unit conversions, the estimated specific discharge (q_v) is -0.556 m/yr. The negative value indicates that the flow direction is down.

The magnitude of the pond leakage flow rate (Q) is:

$$Q = |q_v|\ A$$

where:

A = footprint area of bottom liner (0.5 hectare)

$$Q = \left|-0.556\,\frac{\text{m}}{\text{yr}}\right| \frac{\text{yr}}{365.25\ \text{day}}\ 0.5\ \text{hectare}\ \frac{10{,}000\ \text{m}^2}{\text{hectare}}\ \frac{1000\ \text{L}}{\text{m}^3} = 7{,}611\ \frac{\text{L}}{\text{day}}$$

Using appropriate inputs and unit conversions, the leakage flow rate (Q) is computed to be 7,611 liters/day.

Return to Exercise 2↥

Return to where text linked to Exercise 2↥

13 Notations

Parameter dimensions are in dark green font with mass as M, length as L, time as T, temperature as Θ).

A	=	area (L^2)
g	=	acceleration of gravity (LT^{-2}), 9.807 m/s^2
H	=	hydraulic head existing as a hydraulic potential (L)
h_f	=	freshwater head (L)
h_{fmi}	=	freshwater head at midpoint of convection cell leg i (L)
h_p	=	pointwater head (L)
h_{pmi}	=	pointwater level at midpoint of convection cell leg i (L)
h_w	=	physical water-level elevation in a well or piezometer (L)
i	=	index number (subscript) to identify each of the four convection cell legs (1, 2, 3, 4) or index number to identify each of two wells or piezometers (1, 2) (dimensionless)
j	=	mass flux (MT^{-1}L^{-2})
K	=	hydraulic conductivity (LT^{-1})
K_{sth}	=	standard horizontal hydraulic conductivity; pure water at 20 °C (LT^{-1})
K_{stv}	=	standard vertical hydraulic conductivity; pure water at 20 °C (LT^{-1})
k	=	intrinsic permeability of the porous medium (L^2)
k_h	=	horizontal intrinsic permeability (L^2)
k_v	=	vertical intrinsic permeability (L^2)
L	=	length of the convection cell leg (L)
μ	=	groundwater dynamic viscosity at the prevailing system temperature (ML^{-1}T^{-1}); useful conversion: 1 poise = 1 gm/cm/s = 0.1 kg/m/s
μ_{st}	=	standard dynamic viscosity of pure water at 20 °C (ML^{-1}T^{-1}), 0.001 kg/m/s = 0.01 poise
P	=	pore fluid gauge pressure (ML^{-1}T^{-2}); useful conversion: 1 Pascal = 1 kg/m/s^2 = 1 Newton/m^2
P'	=	pore fluid pressure at a specified location of interest (ML^{-1}T^{-2})
P^*	=	arbitrary pressure for the reference state (ML^{-1}T^{-2})

P_{atm}	=	prevailing atmospheric pressure ($ML^{-1}T^{-2}$)
P_b	=	pressure at an aquitard boundary ($ML^{-1}T^{-2}$)
$P_i(O)$	=	water pressure at the upstream end of convection cell leg i ($ML^{-1}T^{-2}$)
$P_i(L)$	=	water pressure at the downstream end of convection cell leg i ($ML^{-1}T^{-2}$)
P_r	=	groundwater pressure at a chosen reference level ($ML^{-1}T^{-2}$)
P_{ta}	=	absolute pressure measured by an unvented submersible transducer ($ML^{-1}T^{-2}$)
P_{ta}	=	absolute pressure measured by an unvented submersible transducer ($ML^{-1}T^{-2}$)
P_{tg}	=	gauge pressure measured by a vented submersible transducer ($ML^{-1}T^{-2}$)
P_w	=	fluid pressure at the midpoint of a well or piezometer completion zone $ML^{-1}T^{-2}$)
ϕ	=	mechanical potential: work per unit mass of fluid to transform it from an arbitrary reference state to the current state at a location of interest (L^2T^2), useful conversion: Jule/kg = Newton m/kg = m^2/s^2
q	=	specific discharge; volumetric flow rate per unit cross-sectional area normal to the direction of flow (LT^{-1})
q_h	=	horizontal specific discharge (LT^{-1})
q_v	=	vertical specific discharge, positive upward (LT^{-1})
Q	=	volumetric flow rate (L^3T^{-1})
ρ	=	pore fluid (groundwater) density (ML^{-3})
ρ'	=	pore fluid density at a specified location of interest (ML^{-3})
ρ_1	=	groundwater density at the midpoint of Piezometer 1 completion zone (ML^{-3})
ρ_2	=	groundwater density at the midpoint of Piezometer 2 completion zone (ML^{-3})
ρ_c	=	characteristic or average groundwater density (ML^{-3})
ρ_f	=	freshwater density; pure water at 20° C (ML^{-3}), 998.2 kg/m^2
s	=	distance (L)
s'	=	specified distance coordinate (L)

s_i	=	distance coordinate at the location of well i (L)
T	=	temperature (Θ), °C for Celsius or °F for Fahrenheit
θ	=	direction of flow from horizontal; counterclockwise positive (dimensionless), radians or degrees
x	=	horizontal distance variable; in this book positive to right (L)
x'	=	specified horizontal coordinate (L)
z	=	vertical elevation variable positive upward (L)
z'	=	specified vertical coordinate (L)
z^*	=	elevation of the reference state datum (L)
z_b	=	elevation of the aquitard boundary (L)
z_{mi}	=	midpoint elevation of convection cell leg i (L)
z_r	=	reference elevation (L)
z_t	=	pressure transducer elevation (L)
z_w	=	midpoint elevation of well or piezometer completion zone (L)

14 About the Author

Fred Marinelli has over 40 years of experience in groundwater hydrology with a split between university research and private-sector consulting. After three years as a field-mapping geologist, he obtained an MS in hydrology from the University of Arizona (1984) and a PhD in civil engineering (hydrology) from Colorado State University (1995). His research pursuits have included well hydraulics and aquifer testing, distribution of floating petroleum products at the water table, and analysis of two-phase and unsaturated flow in layered porous media. His emphasis in the private sector has been mining hydrology including borehole testing, mine dewatering, chemical migration at mine facilities, hydraulics of heap leaching, and tracing groundwater migration in flooded underground mines. He lives in Erie, Colorado, USA.

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