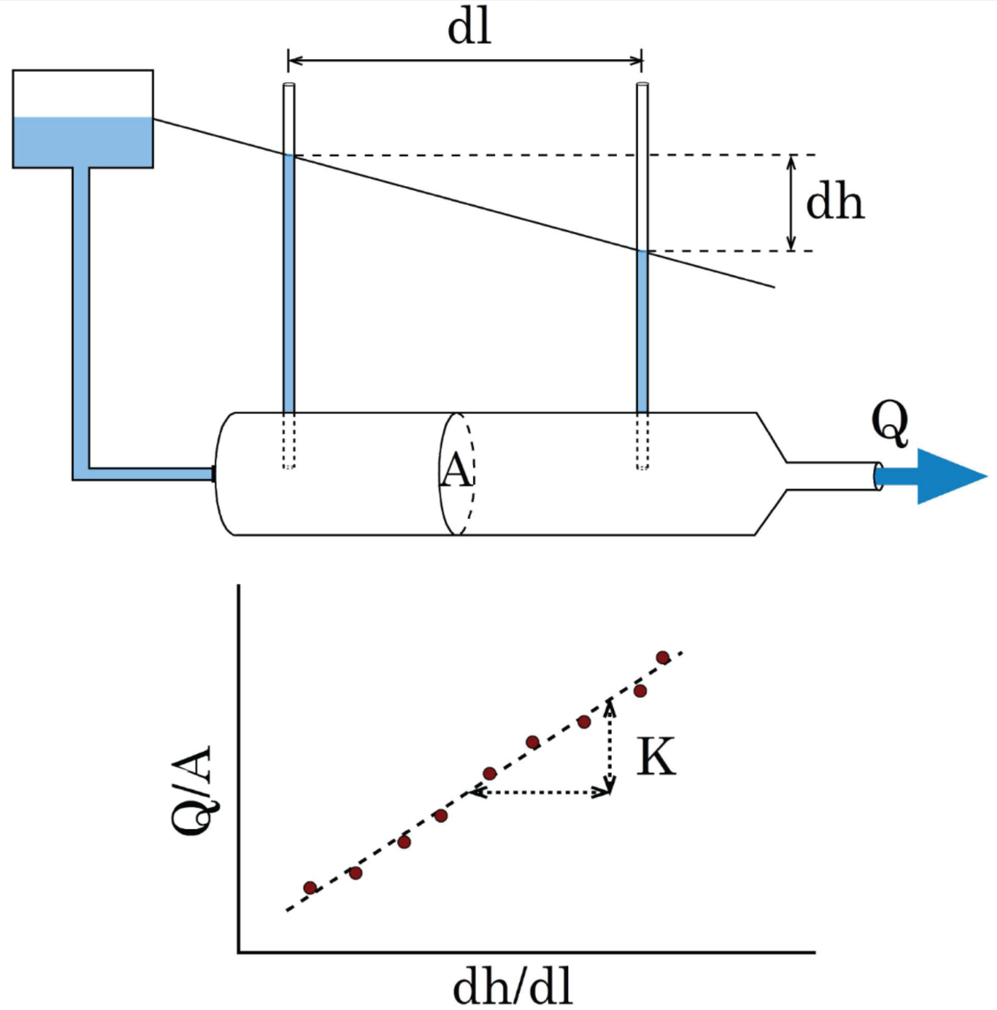

Basic Hydrogeology

An Introduction to the Fundamentals of Groundwater Science

Matthew M. Uliana

THE GROUNDWATER PROJECT

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Thank you.

Dedication

This book is dedicated to the hydrogeology community and all those who have committed at least some part of their lives to understanding and responsibly managing our groundwater resources.

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The Groundwater Project Foreword

The United Nations (UN)-Water Summit on Groundwater, held from 7 to 8 December 2022, at the UNESCO headquarters in Paris, France, concluded with a call for governments and other stakeholders to scale up their efforts to better manage groundwater. The intent of the call to action was to inform relevant discussions at the UN 2023 Water Conference held from 22 to 24 March 2023 at the UN headquarters in New York City. One of the required actions is *strengthening human and institutional capacity*, for which groundwater education is fundamental.

The UN-Water website↗ states that *more than three billion people worldwide depend on water that crosses national borders*. There are 592 transboundary aquifers, yet most do not have an intergovernmental cooperation agreement in place for sharing and managing the aquifer. Moreover, while groundwater plays a key role in global stability and prosperity, it also makes up 99 percent of all liquid freshwater—accordingly, groundwater is at the heart of the freshwater crisis. *Groundwater is an invaluable resource*.

The Groundwater Project (GW-Project), a registered Canadian charity with its beginnings in 2018, pioneers in advancing understanding of groundwater and, thus, enables *building the human capacity for the development and management of groundwater.* The GW-Project is not government funded and relies on donations from individuals, organizations, and companies. The GW-Project creates and publishes high-quality books about *all-things-groundwater* that are scientifically significant and/or relevant to societal and ecological needs. Our books synthesize knowledge, are rigorously peer reviewed and translated into many languages. Groundwater is 'hidden' and, therefore, our books have a strong emphasis on visualizations essential to support the spatial thinking and conceptualization in space and time of processes, problems, and solutions. Based on *our philosophy that high quality groundwater knowledge should be accessible to everyone*, The GW-Project provides all publications for free.

The GW-Project embodies a new type of global educational endeavor made possible by the contributions of a dedicated international group of over 1000 volunteer professionals from a broad range of disciplines, and from 70 countries on six continents. Academics, practitioners, and retirees contribute by writing and/or reviewing books aimed at diverse levels of readers including children, youth, undergraduate and graduate students, groundwater professionals, and the general public.

The GW-Project started publishing books in August 2020; by the end of 2024, we have published 55 original books and 77 translations (55 languages). Revised editions of the books are published from time to time. In 2024, interactive groundwater education tools and groundwater videos were added to our website, gw-project.org↗.

We thank our individual and corporate sponsors for their ongoing financial support. Please consider sponsoring the GW-Project so we can continue to publish books free of charge.

The Groundwater Project *Board of Directors*, January 2025

Foreword

This book: Basic Hydrogeology an Introduction to the Fundamentals of Groundwater Science is intended for use as the notes for a typical undergraduate, introductory-level course on groundwater science. The material in this book evolved from the lecture notes of the author prepared to support his teaching of an introductory course about groundwater. The author was the instructor of this course at a few universities and made the notes available for use by instructors at several other universities and colleges who provided feedback. It has been the standard practice in the teaching of university and college courses on most topics for the instructor to assign or recommend a particular commercially available textbook as the foundational material or framework for the course. However, these textbooks typically have 500-600 pages and include much more material than can be covered in an introductory course of the standard length—2 to 4 months with two or three hours of lecture and a laboratory session each week. The extra material in these textbooks can be distracting to the student and the cost can be a burden. Furthermore, the commercially available textbooks are not designed to be effective for the teaching of any particular type of introductory groundwater course but are aimed at the generic needs of many courses.

This book by Matthew Uliana is designed to be effective for understanding the materials that are typically covered in an introductory course with students from earth science departments as well as students from other disciplines such as geography, environmental science, and engineering. Over the past several decades groundwater science has expanded to include many topics. Consequently, introductory courses must prioritize parts of the subject matter in order to provide an overview. This book is focused on the physical aspects of hydrogeology, particularly the fundamentals of groundwater occurrence, flow, and aquifer hydraulics. However, the book also introduces basic concepts of groundwater chemistry and water quality, as well as contaminant behavior in groundwater systems. In addition, it familiarizes readers with the meaning of terms used in many other aspects of groundwater science.

The author of this book, Dr. Matthew Uliana is a senior hydrogeologist in a consulting firm that specializes in environmental science and engineering. He has selected the material with attention to its relevance to solving practical hydrogeology problems. The volume of material presented is limited to an amount that can be covered in a single course of typical length. For readers who want more in-depth explanations, the author provides links to other books that are available free-of-charge from the Groundwater Project. When coupled with the material in those books, many variations of introductory courses are supported.

John Cherry, The Groundwater Project Leader
Guelph, Ontario, Canada, August 2024

Preface

Hydrogeology is a science that deals with understanding the occurrence and movement of fluids in the subsurface, or underground. I have been a practicing hydrogeologist for just over three decades. During most of that time, including while I was a doctoral candidate at the University of Texas at Austin, USA, I worked in the industry as a consultant primarily on water resource and regulatory compliance projects.

Between 2001 and 2006, and again from 2014–2015, I taught university-level courses in hydrogeology at several different institutions. My classes included a wide variety of students, including geology students in an undergraduate geoscience department, both engineering and geoscience students at a Tier-1 research university, undergraduate students with little geoscience background, and graduate students in aquatic biology and environmental science. I realized very early in my teaching career that I did not like the way any of the available textbooks presented the material. Much of my dissatisfaction came from my experience working both in industry and in academia—I had a clear idea of what the key concepts were and in what order they should be provided. I concluded that even though all the textbooks I reviewed were good comprehensive resources, they were not appropriate for providing direct support for an introductory hydrogeology class.

Initially, I felt I owed it to my students to provide them with study materials that directly supported the concepts I was teaching, and I spent a tremendous amount of time preparing notes and graphics for each of my lectures so the students could focus on what I wanted them to learn. During a final exam review session at the end of one of those semesters, I noticed most of the students had printed out all the notes and put them in three-ring binders. I realized that those notes constituted the framework of a textbook.

Over the years, I refined my lecture notes, always using them as the primary teaching tool for my classes. At various times I considered formally publishing, but I never got much further than a few inquiries and discussions with potential co-authors. I have since decided that I can best serve the global hydrogeology community by developing my lecture notes into a formal textbook and publishing them for free access and use through The Groundwater Project.

My intent is not to produce another comprehensive treatment on the science of hydrogeology. Plenty of those already exist, including the classic *Groundwater*↗ by Freeze and Cherry (1979), as well the books by Fitts (2022), Fetter (2001), and Domenico and Schwartz (1998). Rather, my objective is to produce a textbook that fits my idea of what a textbook should be—a guide for teaching and learning the fundamentals. This book is therefore designed for university teachers and students who need an introduction to key concepts and for those practitioners who may want a simple desk reference. If you are one of those people, I hope that you find this manuscript useful.

Acknowledgments

Acknowledgments are provided in chronological order:

Thanks to my mentor and University of Texas at Austin (UT) graduate advisor, Dr. Jack Sharp, for letting me serve as lab instructor in his *Groundwater Hydrology* course, for letting me teach sections of his lecture, and for helping me develop the initial ideas about teaching hydrogeology that led to the development of this book. Additional thanks to all my fellow graduate students in Jack's research group for providing such a stimulating intellectual environment during my time as a student at UT.

Thanks to all my hydrogeology students for giving me the incentive to develop this work and for serving as a sounding board, focus group, and testing department as I revised the manuscript and refined my ideas.

Thanks to Suzanne Pierce for being the first (besides me) to use the early drafts of this book in a course and for convincing me that it was worth developing. I also thank the following for using early drafts of the book in their courses and for helping me realize there is a need for this type of textbook:

- Kristen Menking (Vassar College)
- Anna Martini (Amherst College)
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- Henry J. Ollegasagrem (Saskatchewan Institute of Applied Science & Technology)
- Jason Tuchelt (Lakeland College)
- Tom Timmermans (SES Environmental)
- Tej Gautam (Marietta College)
- Mohsin Tariq
- Celia Silverman
- Trent Garrison (Kentucky Community & Technical College System)
- Tom Ring (formally with the Yakama Nation)

Thanks to John Cherry and Eileen Poeter at The Groundwater Project for their enthusiastic response to my inquiry about publication and all their contributions since submission of the first draft. A separate thanks to them, the volunteer technical reviewers, and everyone at The Groundwater Project for putting so much energy and effort into developing a fantastic resource for our community and for letting me be a small part of that effort. Additionally, thanks go out to the wonderful editors and graphic support staff at The Groundwater Project for turning my notes and figures into a professional document.

Finally, I thank all of you who support The Groundwater Project and make use of the tremendous resource they have provided. I hope you can gain something by using my book, and I look forward to hearing from you with suggestions and other comments.

Matthew M. Uliana, PhD, PG

Citations are provided to acknowledge the sources of material presented in figures and tables. Where no citation appears, the figure or table material is original to this book.

1 Introduction to Hydrogeology

1.1 The Properties of Water

Pure water (H_2O)

- is clear, colorless;
- has no discernable taste or smell;
- at 1 atmosphere has a melting point of 0 °C and a boiling point of 100 °C; and
- has a density at 4 °C of 1.00 g/cc (1,000 kg/m^3).

Water is present throughout the solar system, but other than on Earth it generally exists as a gas or solid. Earth is the only known planet with extensive bodies of liquid water on the surface. Liquid water most likely occurs in the subsurface on Mars, and there are likely extensive oceans of saline liquid water beneath the frozen surfaces of several moons of Jupiter and Saturn. Liquid water is essential for life as we know it, and it is one of the reasons Earth is such a unique place in the solar system.

One of the things that makes water so unique is that it is a highly polar molecule (Figure 1a)—that is, the distribution of charges within the molecule results in a partial positive charge on one side and a partial negative charge on the other. Because of this, water is cohesive (Figure 1b), and water molecules like to stick to each other as well as to surfaces and other molecules.

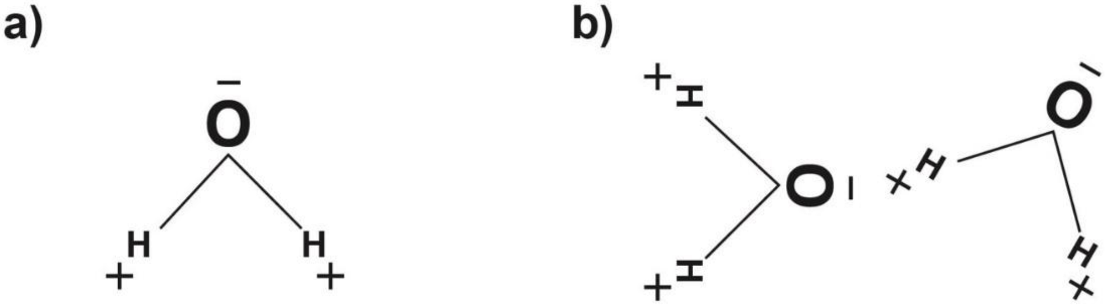

Figure 1 - Model of water molecules showing a) molecule polarity and b) attraction between water molecules responsible for cohesiveness and surface tension.

Polarity of the water molecule results in high surface tension and high solvent ability, which is why we say that water is the universal solvent.

1.2 Distribution of Water on the Earth

Over 75 percent of the Earth's surface is covered by water. The vast majority of that is in the oceans and is unfit for human consumption. Most of the freshwater is locked up in the ice caps and in glaciers. Usable, available freshwater—in lakes and rivers on the surface and in reservoirs underground—makes up less than one percent of the water in the world.

The distribution of the world's water is presented in Table 1.

Table 1 - Distribution of the world's water

Oceans	97.2%
Ice	2.14%
Groundwater	0.61%
Soil moisture	0.005%
Atmosphere	0.001%
Fresh surface water	0.0009%

1.3 The Hydrologic Cycle

To understand the hydrologic cycle, we must first understand systems and the components of a cycle.

1.3.1 Systems

Systems are a way of conceptualizing natural phenomenon based on the way that mass and energy move within and across the system boundaries. We can distinguish between three types of systems (Figure 2).

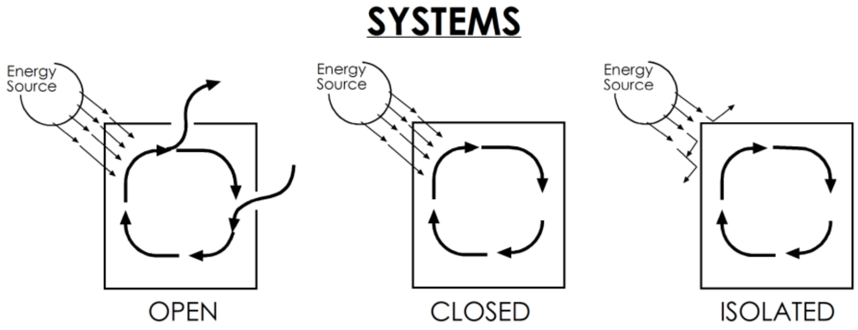

Figure 2 - Types of systems.

In an *open system*, mass and energy can move across the boundaries of the system. An example of an open systems is a lake, as mass (i.e., water) can flow into and out of the lake and energy can enter and leave the lake through sunlight and radiation of heat, respectively. In a *closed system*, mass stays constant and energy can move across the boundaries. We can consider the Earth as a closed system that receives energy from the sun and loses energy to space through radiation, but the overall mass of all elements on the earth is essentially constant (as long as we ignore the tiny addition of mass from meteorites and the loss of small amounts of atmosphere to space). In an *isolated system*, neither energy nor matter can move across the boundaries. The only truly isolated system is the entire universe, although the concept is useful for dealing with chemical reactions in sealed and thermally insulated flasks.

When we study and model natural systems, we can deal with them in two ways—as lumped systems or as distributed systems. *Lumped systems* deal only with inputs and outputs as a function of time. These systems ignore the details of what is going on within each reservoir or part of the system. When we deal with lumped systems, we are basically using some version of the *hydrologic equation*—also called the *continuity equation* or the *conservation of mass equation*—as shown in Equation (1).

$$I(t) - O(t) = \frac{\Delta Storage}{\Delta t} \tag{1}$$

where:

$I(t)$ = input to a system over time
$O(t)$ = output from a system over time
$\Delta Storage/\Delta t$ = change in storage during time Δt

Distributed systems deal with the particulars of how various parameters—such as temperature, pressure, and salinity—vary throughout the system with respect to space as well as time. We look at distributed models in more detail in Section 10 when we discuss modeling in general and groundwater flow models in particular.

1.3.2 Cycles

A cycle is defined as a dynamic system that contains the following four components:

1. a substance, element, or set of elements (not necessarily a chemical element) that are in flux, such as water;
2. a set of reservoirs in which the element resides, such as the oceans and the ice caps;
3. a set of fluxes, or processes that are moving the elements within reservoirs and from one reservoir to another, such as evaporation moves water from the ocean to the atmosphere and condensation creates precipitation that falls back to the ocean; and
4. some source of energy that is driving the cycle.

1.3.3 Overview of the Hydrologic Cycle

Water occurs throughout the Earth from the outer reaches of the atmosphere to deep within the mantle—possibly as deep as the core. Early in the history of the Earth's formation, around the time that the Earth's crust began to form, volcanic activity released lots of volatile gasses, including water, from the underlying mantle. In addition, comets and asteroids likely delivered a lot of water to the Earth's surface during that time. These volatile gasses made up the early atmosphere and oceans. It is thought that almost all the water that we find in the oceans, lakes, streams, atmosphere, and subsurface today was outgassed at this time, and essentially the same volume of water has been moving through the hydrologic cycle ever since.

The hydrologic cycle is defined as the set of reservoirs and fluxes that hold and move water through the atmosphere, on the surface, and in the subsurface of the Earth (Figure **3**). Except for minor amounts of extraterrestrial water brought in by comets, and small amounts of water vapor that are lost to outer space at the upper reaches of the atmosphere, there is a constant volume of water in the entire water cycle. Within the cycle, various reservoirs hold water and various processes that move water within reservoirs and from one reservoir to the next.

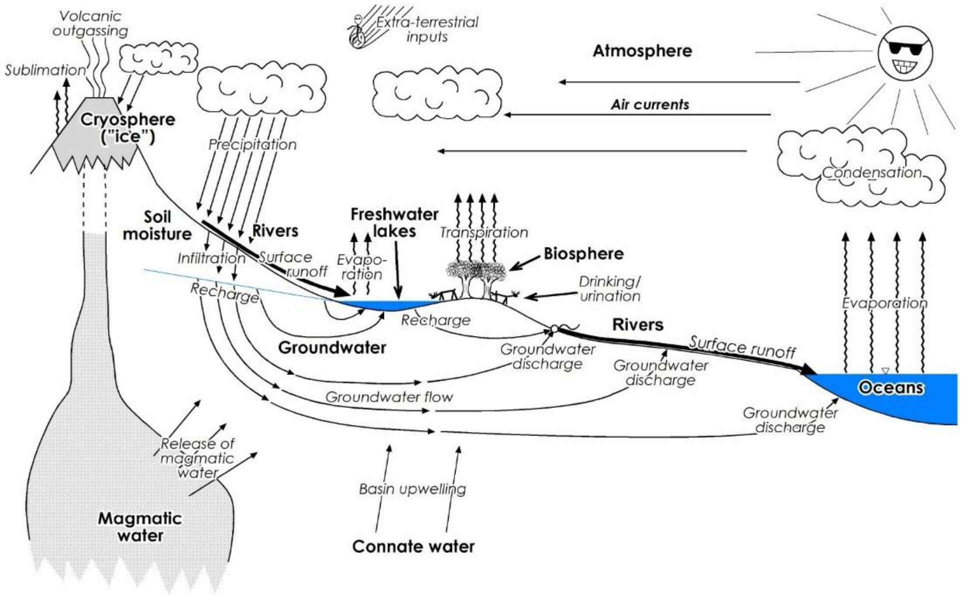

Figure 3 - A generalized and simplified diagram of the hydrologic cycle. Arrows show movement of water within and between reservoirs with straight lines representing movement of liquid and wavy lines represent movement of vapor .

Figure **3** shows the main reservoirs and fluxes, as well as the energy sources that drive the cycle. Reservoirs in the water cycle include the oceans, atmosphere, rivers, freshwater lakes, the unsaturated soil moisture, the saturated groundwater, *connate* water in deep sedimentary rocks, *magmatic* water from the mantle, water in the ice caps and glaciers (the *cryosphere*), and water in plants and animals (the *biosphere*). Fluxes of water are the result of processes that move water from one reservoir to the next—such as evaporation and infiltration—or within a reservoir—such as mechanical flow that moves groundwater and creates ocean currents.

Thought Question 1*: What is the primary source of energy driving the hydrologic cycle? What is a secondary source of energy?*

In terms of the systems discussed in Figure 2, we treat the hydrologic cycle like a closed system: The cycle has a set amount of water that does not change, however, energy enters the system in the form of sunlight. The amount of water gained from—or lost to—outer space is so small that we can consider it negligible. Within the hydrologic cycle, we can identify and delineate several open systems. The oceans are an example: water enters the oceans from rain, surface runoff, and groundwater discharge, and water leaves the oceans through evaporation, through mineral reactions, and at subduction zones; energy enters the ocean from solar radiation and leaves the ocean via evaporation and conduction. A lake is another example of an open system.

The science of hydrogeology deals with a specific part of the hydrologic cycle—the part that is underground. The primary focus of this book is on the saturated groundwater flow systems near the surface of the Earth, that is, within the first one or two kilometers below the surface. The water in the unsaturated zone (the moisture in the soil) and the flowing water on the surface of the Earth (the rivers and lakes) are also dealt with in separate sections. Magmatic water in the mantle and lower lithosphere, connate water (water that was trapped in sediments when they were first deposited), and petroleum-related fluids are not addressed in this book.

A more detailed overview of how groundwater fits into the overall hydrologic cycle is provided in *Groundwater in Our Water Cycle: Getting to Know Earth's Most Important Fresh Water Source*↗ by Poeter and others (2020) and can be downloaded for free or read online. In addition, the US Geological Survey (USGS) has a detailed water cycle diagram↗ that presents the same basic reservoirs and fluxes as Figure 3 but also includes the extensive anthropogenic aspects of the water cycle, such as groundwater pumping, surface water diversions, urban runoff, and agricultural return flows.

1.4 Groundwater

The strict technical definition of groundwater is any water that is found beneath the surface of the Earth. This definition includes:

a) moisture that is found in the pores between soil grains;

b) fresh to slightly saline water, found in saturated geologic units near the surface, which is used for drinking and irrigation;

c) extremely salty brines associated with petroleum deposits and deep sedimentary units; and,

d) water found in the lower lithosphere and in the mantle.

In this book, we are primarily concerned with (b), fresh to slightly saline water that is found near the surface and that is frequently used for domestic, agricultural, and industrial purposes. From this point on, when the book uses the term *groundwater*, it is referring to the water described by definition (b).

1.4.1 Distribution of Water in the Subsurface

Several terms, such as *porosity*, *permeability*, and *aquifer*, are introduced in this section and defined in a general way. These terms are discussed in much more detail in later sections. Readers are encouraged to also review the *Glossary of Hydrogeology*↗ by Sharp (2023) and *Hydrogeologic Properties of Earth Materials and Principles of Groundwater Flow*↗ by Woessner and Poeter (2020); both books are available as free downloads from The Groundwater Project.

Geologic materials (i.e., rock, soil, and sediment) always have some amount of empty space in them. This empty space is called pore space, and the percentage of pore space by volume in a rock or sediment is called the porosity. Within all naturally occurring geologic units, the pore space always contains some amount of moisture.

Below the surface of the Earth, the sediments and rock contain moisture; however, not all the pore space is full—that is, there is both air and moisture within the pore spaces. Just below the surface, the amount of moisture is small, and the amount of moisture tends to increase with depth. Eventually, the amount of moisture becomes so great that the pore space is completely filled with water and the soil or sediment is saturated (Figure 4).

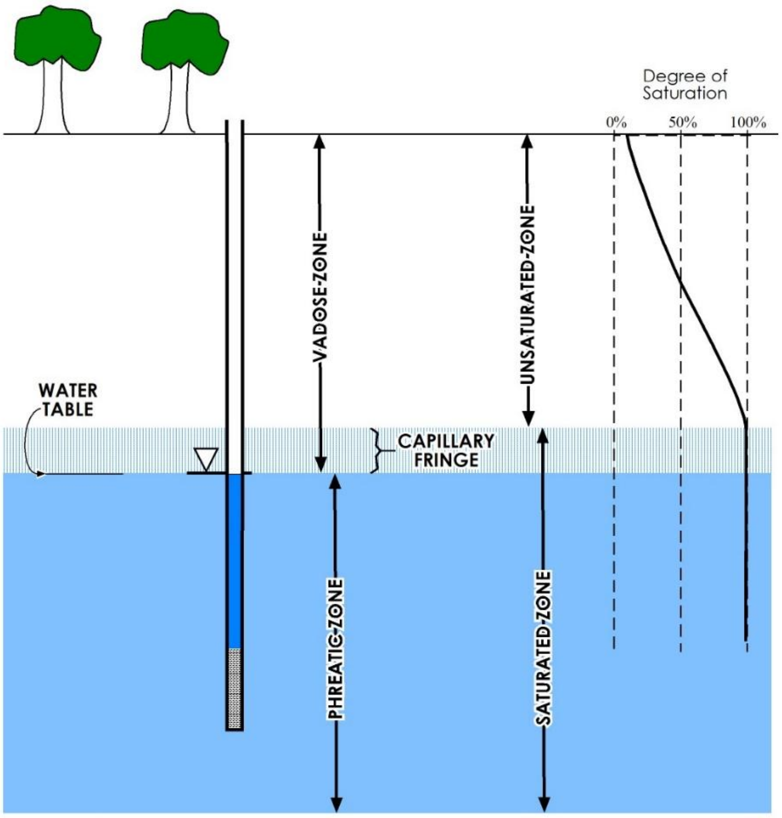

Figure 4 - Cross-section showing the distribution of water in the shallow subsurface.

Therefore, in the subsurface, we can distinguish between two basic zones: the unsaturated zone, and the saturated zone. The boundary between the two zones is often called the water table; in Section 14.2.2 the book it will be shown that the top of the saturated zone is not exactly at the same location as the water table.

1.4.2 Recharge and Discharge

As shown in Figure **3**, groundwater is a part of the dynamic hydrologic cycle, and water must somehow enter as well as leave the subsurface. Water entering the subsurface is called *infiltration*—percolation of surface water (from rain, perennial streams, melting snow, urinating dogs, and so on) downward into the soil. As the water percolates down through the soil, sediment, and rock, the percentage of the pore space that is filled with water (or the degree of saturation) increases until it reaches 100 percent (i.e., complete saturation). Once this water reaches the saturated zone, it is called *recharge*.

Just as water enters the saturated zone, it must eventually leave it. Movement of water out of the saturated zone is called *discharge*. Natural discharge can be through a spring, into the bed of a stream, lake, or ocean, or via evaporation directly from the water table. Pumping of groundwater through wells—holes drilled into the ground for the purpose of accessing subsurface fluids—is another way that water discharges from the saturated zone.

1.4.3 Groundwater Flow

Groundwater is always flowing, and the direction of flow is determined by the location of higher groundwater elevation. It is important to recognize that groundwater does not flow downhill; rather, it flows from higher hydraulic head (or higher water level elevation) to lower hydraulic head. We discuss hydraulic head in more detail in subsequent sections. Additional information can be found in *Conceptual and Visual Understanding of Hydraulic Head and Groundwater Flow*↗ by Cohen and Cherry (2020), which is available from The Groundwater Project as a free download or can be read online. The distribution of hydraulic heads in the saturated zone determines the direction in which the water will flow.

The speed with which groundwater flows, called the *velocity* or *flux*, is determined by the difference in hydraulic head and the permeability of the sediment or rock through which it flows.

Permeability is a number that describes the ease with which a fluid (like water) will move through a porous medium—that is, a rock, soil, or sediment that has enough pore space to allow water to move through it. In Section 3, it is shown how the difference in head from one point to the next, and the permeability, can be used to estimate how quickly groundwater moves. More information can be found in *Groundwater Velocity*↗ by Devlin (2020) and in Section 4 of *Hydrogeologic Properties of Earth Materials and Principles of Groundwater Flow*↗ by Woessner and Poeter (2020); both books are available from The Groundwater Project as a free download or can be read online.

1.4.4 Aquifers

An *aquifer* is a geologic unit that can store and transmit enough water to supply a well and/or spring. The factors that determine if a geologic unit can be called an aquifer include the following.

1. The permeability must be high enough that flow can be maintained at the required rates.
2. The aquifer dimensions must be great enough—that is, there must be a significant saturated thickness—to supply water to a well.
3. The quality of the water must be good enough for the intended use.

There are three basic types of aquifers: unconfined, confined, and perched as shown in Figure 5. Understanding the difference between unconfined and confined aquifers requires an understanding of hydraulic head and the storage characteristics of aquifer materials. In the Section 2, we introduce the concept of hydraulic head in detail and lay the groundwork for a quantitative understanding of groundwater storage and fluid flow in the subsurface. This is also discussed in *Groundwater Storage in Confined Aquifers*↗ by Wang (2020), which is available from The Groundwater Project as a free download or can be read online.

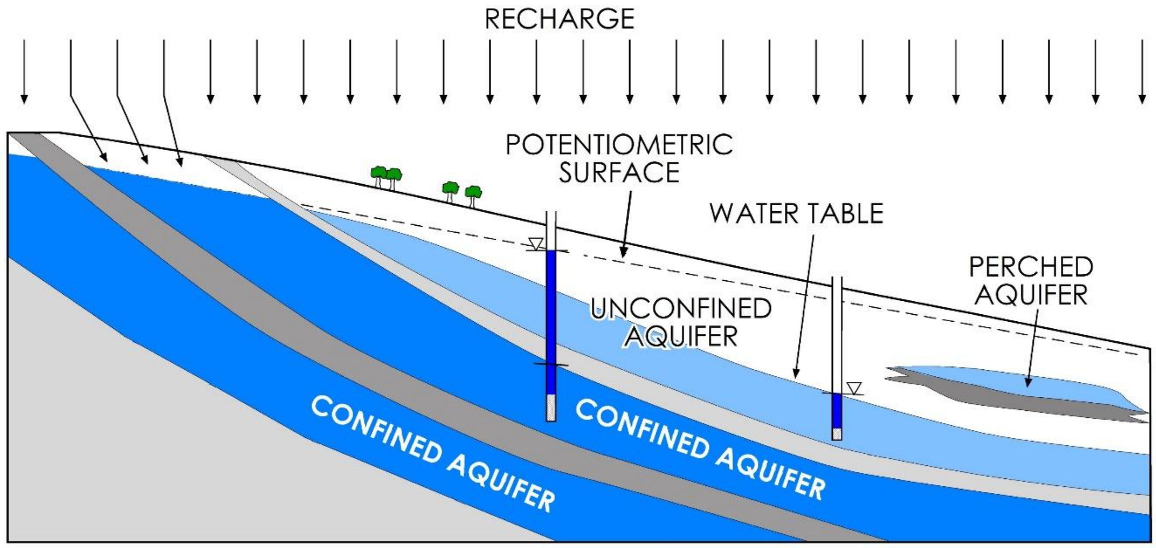

Figure 5 - Cross-section showing various types of aquifers. The arrows represent precipitation; the bent arrows at the upper left of the diagram represent infiltration of precipitation ultimately recharging the uppermost confined aquifer. All the straight arrows to the right of those represent precipitation recharging the unconfined and perched aquifers.

2 Potential and Kinetic Energy in Subsurface Fluids

2.1 Introduction

One of the fundamental equations that govern groundwater flow is called Darcy's law as shown by Equation (2).[1]

$$Q = -K\,A\,\frac{dh}{dl} \tag{2}$$

where (parameter dimensions are dark green font with mass as M, length as L, time as T):

Q = discharge (L^3T^{-1})

K = hydraulic conductivity (LT^{-1})

A = cross sectional area perpendicular to flow (L^2)

$\frac{dh}{dl}$ = hydraulic gradient, change in hydraulic head h over a distance l (:)

In plain words, this equation states that *discharge* (i.e., volumetric flow) through a *cross-sectional area* of a porous medium is directly and linearly proportional to the *hydraulic gradient,* and the constant of proportionality that relates discharge to the hydraulic gradient is a quantity called the *hydraulic conductivity*. The negative sign indicates that flow is in the opposite direction of the gradient.

We get into the details of the equation in other sections of this book, but for now, let us focus on the lowercase *h* in the hydraulic gradient term—that is, *hydraulic head*.

2.2 Fluid Energy

In simple, everyday terms, we think of hydraulic head as an elevation. More specifically, hydraulic head is the elevation of water in a manometer in a pressurized water pipe (Figure 6) or in a piezometer (Figure 7).

- A *manometer* is a vertical tube in a pressurized water pipe and is used to measure fluid pressure in the pipe.
- A *piezometer* is a vertical tube with an open or slotted interval—usually called the screened interval or the well screen—inserted into the ground and used to measure hydraulic head in an aquifer; it is basically a well that was constructed for the sole purpose of measuring groundwater levels.

The water level in a piezometer is a manifestation of the total fluid energy in a groundwater system so measuring a water elevation in a well is a measurement of the energy present in

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[1] Units for all equation variables in this book are expressed in general terms of length (L), mass (M), and time (T); for example, units of volumes are expressed as L^3, velocity as LT^{-1}, and so on.

the fluid at a certain point in a groundwater system—that is, the interval of the piezometer that is open to the aquifer (Figure 7).

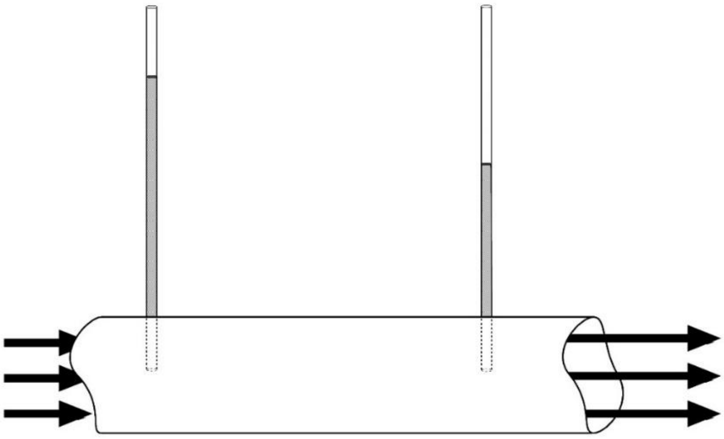

Figure 6 - Pipe of flowing water with manometers showing the loss of head along the flow path.

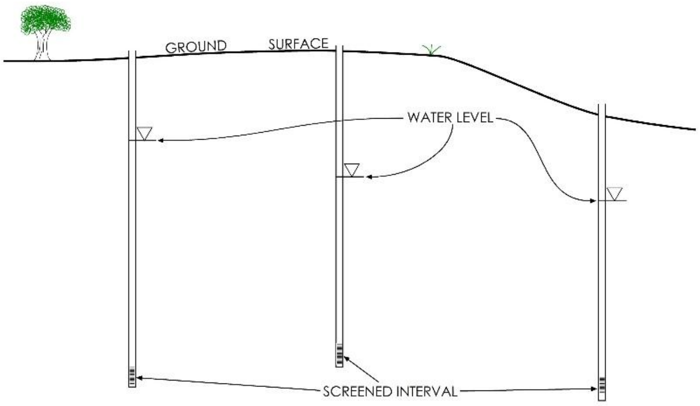

Figure 7 - Cross-section of aquifer showing hydraulic heads in three piezometers. The line with an inverted triangle represents the depth to which water rises within each piezometer.

The fluid energy in an aquifer varies throughout the aquifer and is different from one point to the next. This results in different water levels in piezometers located at different points throughout the aquifer, and these variations in water level represent variations in fluid energy that in turn drive the movement of water throughout the aquifer. As a result, an aquifer is a dynamic system of flowing water.

2.2.1 What do we mean by *Energy*?

Everything in the universe has some amount of energy associated with it, and that energy is present in various forms. Variations in energy are what drive all natural processes. The key to understanding physical processes is in understanding the distribution of energy in a system. We differentiate between potential energy and kinetic energy.

Potential energy is energy stored in a piece of matter or at a point in a system. It is generally associated with position within a system (measured as an elevation or distance above a datum) or with the thermodynamics of the system (measured as pressure, temperature, or chemical potential).

Kinetic energy is energy associated with motion and is generally measured as a velocity.

At every point in an aquifer, the fluid possesses some total amount of energy that is the *sum* of all the potential and kinetic energies in the fluid. The fluid energy at a point in an aquifer manifests itself as the water level in a piezometer. We can also say that the water level, or hydraulic head, represents the total energy in the aquifer at a given point, and we can use the various energy components of the hydraulic head—elevation, pressure, velocity, and so on—to understand the driving forces behind fluid motion in the subsurface.

2.2.2 The Bernoulli Equation

Fluid energies—and, subsequently, water levels—vary from one point in an aquifer to the next. Let us consider the first and second laws of thermodynamics. The first law of thermodynamics states that energy is conserved in any system; that is, it cannot be created or destroyed, and any changes in energy must be accounted for in any system as shown by Equation (3).

$$\text{energy added} - \text{energy subtracted} = \text{change in total energy} \tag{3}$$

This is the same as the conservation of mass, Equation (1), but stated for energy in a system. We could also express the first law in terms of the difference in energy at two points in a dynamic system as described by Equation (4).

$$\text{total energy}_{(\text{at point 1})} + \text{energy added or lost}_{(\text{betwen point 1 and 2})} = \text{total energy}_{(\text{at point 2})} \tag{4}$$

The second law of thermodynamics states that isolated systems tend to move towards increasing entropy. In a dynamic system like an aquifer, water moves from a point of higher energy (i.e., lower entropy) to a point of lower energy (higher entropy); in other words, groundwater spontaneously moves in the direction of decreasing hydraulic head. It is important to recognize that groundwater does NOT (necessarily) flow downhill relative to the land surface—it flows in the direction of decreasing hydraulic head.

The *Bernoulli equation* shown in Equation (5) describes the total energy of a fluid at all positions along a flow path in a closed system and is basically an expression of the first and second laws of thermodynamics.

$$z_1 + \frac{p_1}{\rho_w g} + \frac{v_1^2}{2g} + H_a = z_2 + \frac{p_2}{\rho_w g} + \frac{v_2^2}{2g} + H_L + H_E \tag{5}$$

where (parameter dimensions are dark green font with mass as M, length as L, time as T):

z = elevation (L)

p = pressure ($\mathrm{ML^{-1}T^{-2}}$)

ρ_w = fluid density ($\mathrm{ML^{-3}}$)

g = gravitational acceleration ($\mathrm{LT^{-2}}$)

v = velocity ($\mathrm{LT^{-1}}$)

H_a = heat energy added (L)

H_L = mechanical energy lost (L)

H_E = heat energy extracted (L)

subscripts = represent two different positions along the flow path

If we assume that no heat energy is added to or subtracted from the system, and that the only change is the loss of mechanical energy from one point to the next, we can rewrite the equation as Equation (6).

$$z_1 + \frac{p_1}{\rho_w g} + \frac{v_1^2}{2g} + I_1 = z_2 + \frac{p_2}{\rho_w g} + \frac{v_2^2}{2g} + I_2 \tag{6}$$

Where I is the internal energy at each point (i.e., the remainder of the potential energy not described by the other terms) and $I_2 - I_1$ is equal to H_L in Equation (5).

We can understand this equation by considering each individual term in the equation as representing a specific component of energy—that is:

- elevation,
- pressure,
- kinetic (velocity), and
- other internal energies (thermal, chemical).

The equation therefore describes the change in energy from one point along a flow path to the next. The interesting thing about it to us as hydrogeologists is that, since it expresses all the components of energy in the same units (i.e., length), we can use it to compare the relative magnitude of the individual components. When we do that, we see that for most groundwater situations the flow velocity is slow so *we can ignore the internal and the kinetic (velocity) components because they are so much smaller than the elevation and pressure components*. We can also use the equation to mathematically "convert" one form of energy to another—for example, we can calculate how much a certain change in elevation will result in a change in pressure.

2.2.3 Hydraulic Head and Hydraulic Potential

If we go with the assumption that we can ignore velocity and internal energy components when dealing with groundwater, we can drop all that out of the equation and express the fluid energy as the sum of the elevation and pressure components. That sum is what we call *hydraulic head*; in physical terms, it is the fluid energy per unit weight, and in mathematical terms, it is described by Equation (7).

$$h = z + \frac{p}{\rho_w g} \tag{7}$$

where (parameter dimensions are dark green font with mass as M, length as L, time as T):

h = hydraulic head (L)
z = elevation (L)
p = pressure ($ML^{-1}T^{-2}$)
ρ_w = fluid density (ML^{-3})
g = gravitational acceleration (LT^{-2})

If we multiply both sides of the equation by gravitational acceleration (g), we get a quantity called *hydraulic potential* (Φ; i.e., the Greek letter *phi*), which is the fluid energy per unit mass, as shown in Equation (8) and Equation (9).

$$\Phi = gz + p/\rho_w \tag{8}$$

such that

$$\Phi = gh \tag{9}$$

The hydraulic potential is simply a way of expressing the same fluid energy so that it is independent of gravity (in case you would want to compare a typical aquifer on Earth with, say, the liquid methane aquifers on Titan). We do not deal with hydraulic potential any further in this book; hydraulic head is sufficient for the problems that are addressed in this book.

2.2.4 Physical Description of the Components of Head

From Equation (7), we can see that hydraulic head is the sum of the elevation component (or *elevation head*) and the pressure component (or *pressure head*). The physical meaning of pressure head and elevation head are defined and described in Figure 8. This figure shows an idealized cross-section of a piezometer with a screened interval at the bottom. Figure 8a shows the depths and elevations measured in the field. Figure 8b shows the pressure, elevation, and total heads relative to a datum. In this case, the datum is mean sea level.

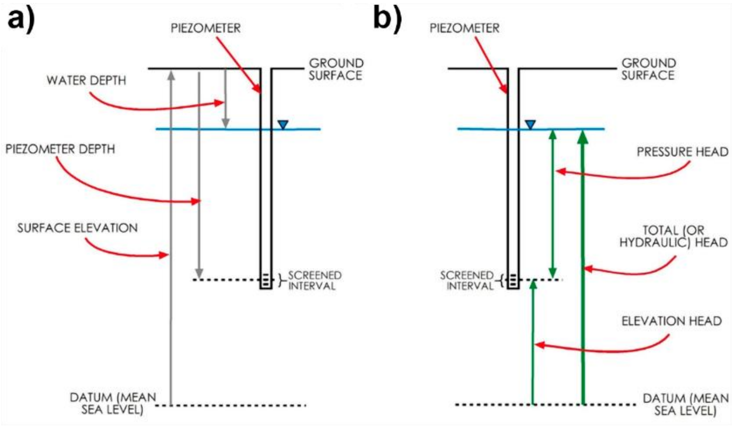

Figure 8 - Hypothetical well showing depths, elevations, and heads.

From Figure 8b, we see that the elevation head is the height of the screened interval above the datum. The datum is arbitrarily chosen—if we wanted, we could choose the middle of the screened interval as the datum, and the elevation head would be zero. However, it is important to realize that hydraulic head is only useful to us when we are looking at multiple wells in the same aquifer, and we need to have a constant datum for all those wells in order to make the comparison meaningful. Sea level is usually chosen, but it is not necessary that we choose that as the datum.

We see from Figure 8b that the pressure head is the length of the column of water in the well above the screened interval. Keep in mind that *pressure* and *pressure head* are two different things. The *pressure* at the screened interval is the force per unit area of the column of fluid above that point, while the *pressure head* is the pressure divided by the product of the density of the fluid and gravitational acceleration and is manifest as the length of the column of water above the screen. If we know the density of the fluid and the length of the column of water in the well, we can calculate the pressure at the well screen.

Finally, we see that the total head is just the sum of the other two heads or, more generally, the height of the water level in the well above the datum. This brings us back to the concept that the water level in a well is a direct measure of the fluid energy (Exercise 1↴).

2.2.5 Distribution of Head (i.e., fluid energy) in a 3-D Aquifer

There are three types of mathematical quantities:

- *scalar* quantities have magnitude only (e.g., temperature);
- *vector* quantities have both magnitude and direction (e.g., velocity); and,
- *tensor* quantities have magnitude that varies with respect to direction (e.g., stress).

Head is a scalar quantity; it is measured at a point, and it has a single magnitude that doesn't vary with respect to direction. Therefore, when we talk about hydraulic head from a theoretical standpoint, we are talking about the fluid energy at an infinitesimal point in an aquifer. An aquifer, however, exists in three dimensions and contains an infinite number of points, and the head within an aquifer varies from point to point.

The trends in that variation are what control the directions and magnitudes of groundwater flow in an aquifer. We call this variation the *hydraulic gradient*. The hydraulic gradient is defined as the change in hydraulic head over a change in length, and it is directly analogous to other physical gradients—for example, topographic slope, thermal gradients, concentration gradients, and so on.

Since hydraulic heads exist in three dimensions, hydraulic gradients are not necessarily horizontal. In reality, most groundwater flow is generally in a horizontal direction, and it is often a realistic assumption to ignore fluid movement in the vertical direction. However, there are many situations where vertical gradients—and, subsequently, vertical flow—are significant (Exercise 2↴).

2.2.6 Why Do We Care About This?

The important thing to realize here is that when we install a piezometer in an aquifer and measure a water level in that piezometer, we are measuring the fluid energy at a point (in three dimensions) that is located at the well screen. Figure 9 shows three wells; wells b and c are completed at approximately the same depth, while well a is completed in a much shallower interval. The two deep wells indicate a gradient pointing to the right of the page; however, the water level in the shallow well—located between the other two—is much lower than either of the other two wells. This indicates that there is some sort of vertical gradient in the system. It could also indicate that the wells are completed in different geologic units that are not well connected. This is discussed further in later sections of this book (Exercise 3↴).

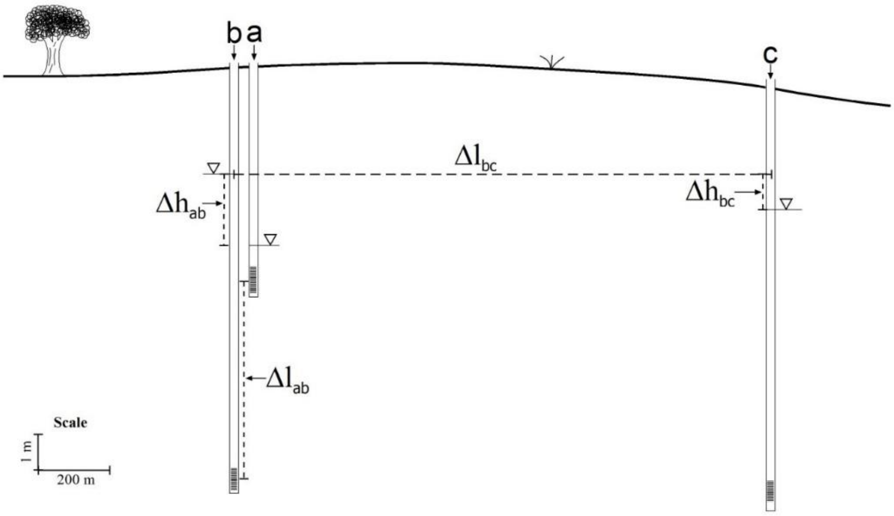

Figure 9 - Cross-section showing differences in head and their separation that can be used to calculate vertical and horizontal gradients. The large difference in water level of the shallow well (a) which is between wells (b) and (c) suggests there is a vertical gradient in the system, which could reflect that the wells are located in a zone of hydrodynamically-driven upward flow and/or that the wells are completed in different geologic units that are not well connected.

More information about hydraulic head and understanding the relationship between head and groundwater flow can be found in *Conceptual and Visual Understanding of Hydraulic Head and Groundwater Flow*↗ by Cohen and Cherry (2020), which is available from The Groundwater Project as a free download or can be read online.

3 Porous Media

3.1 Introduction

Up to this point, our discussion of hydrogeology has included the nature of water and the distribution of fluid energy (i.e., hydraulic head) in a flow system. Now we turn our attention to the material through which the water flows. This section deals with the various aspects and properties of porous media, including:

- porosity;
- permeability and hydraulic conductivity of porous media;
- variability of these parameters with respect to location and direction; and
- measurement of these parameters.

3.2 Porosity

All geologic materials have some amount of pore space, or empty space, in them. The term porosity (ϕ) refers to the fraction of the total volume of a rock or sediment that is pore space. More rigorously, it is defined as the volume of the voids divided by the total volume as shown by Equation (10).

$$\varphi = \frac{V_{\text{voids}}}{V_{\text{total}}} \tag{10}$$

where (parameter dimensions are dark green font with mass as M, length as L, time as T):

V_{voids} = volume of the voids (L^3)

V_{total} = total volume of the sample (L^3)

(The symbol *n* is sometimes used as a variable for porosity.)

Porosity is a dimensionless quantity and can be expressed as a percentage by multiplying the ratio by 100. We can also define a quantity called the *void ratio* (*e*), which is the volume of voids divided by the volume of solids, as expressed by Equation (11).

$$e = \frac{V_{\text{voids}}}{V_{\text{solids}}} \tag{11}$$

where (parameter dimensions are dark green font with mass as M, length as L, time as T):

V_{solids} volume of the solids (L^3)

In nature, geologic materials are almost never completely dry, and the void space will always contain some volume of water. We can think about geologic materials as composed of multiple phases: a solid phase, a water phase, and a gas phase (Figure 10). In turn, the solid phase can be further divided into its individual mineral phases (Figure 11).

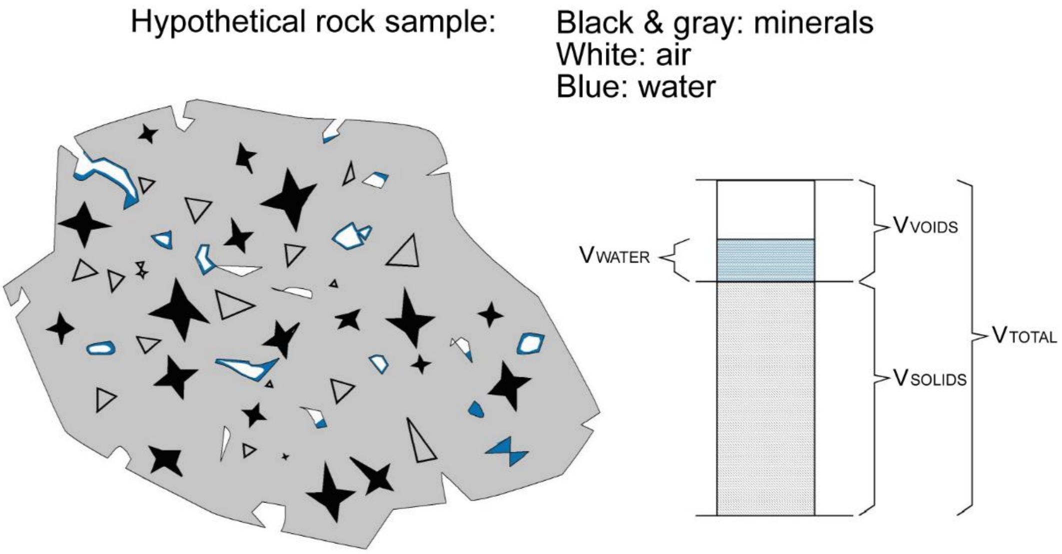

Figure 10 – Graphic depiction of solids (i.e., minerals) and partially and fully saturated pore space in a rock.

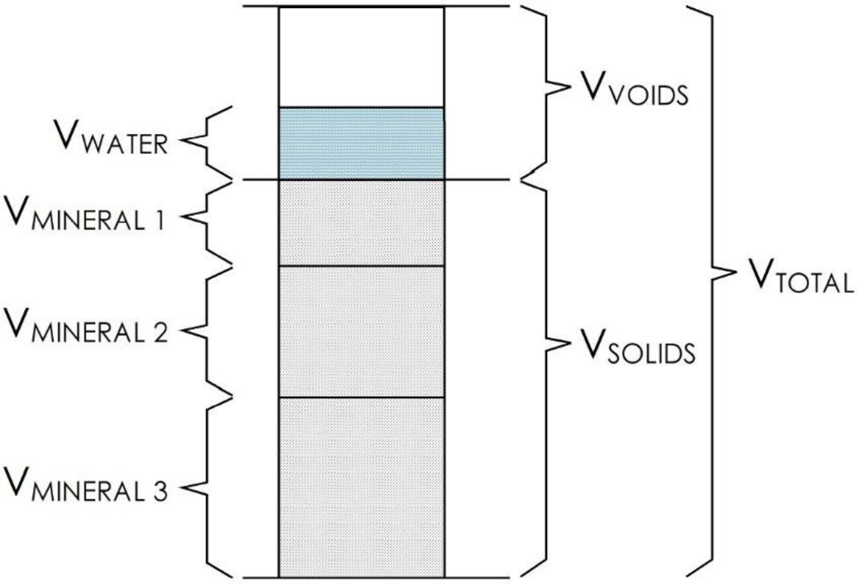

Figure 11 - Phase content (by volume).

Each of the mineral phases in a rock or sediment sample has a volume and mass associated with it. Every fluid and every mineral has a density associated with it.

3.2.1 Density

Density is defined as the ratio of an object's mass to its volume with dimensions of (ML^{-3}). We can therefore express it mathematically as shown by Equation (12) .

$$density = \frac{mass}{volume} \tag{12}$$

Often, we use the symbol ρ (i.e., the Greek letter *rho*) for density. Table 2 lists densities of some common substances.

Table 2 - Densities of some common substances at room temperature and pressure.

	g/cm^3	kg/m^3
Air	0.0012	1.2
Water	1.0	1,000
Quartz	2.65	2,650
Aluminum	2.70	2,700
Iron	7.86	7,860
Gold	19.3	19,300

Density is not a constant; it varies with temperature and pressure. Generally:

- higher temperature = lower density;
- lower temperature = higher density;
- higher pressure = higher density; and
- lower pressure = lower density.

Therefore, when density is reported, it is usually accompanied by a temperature and a pressure. If no temperature or pressure is given, it is usually implied that the density value is at *standard temperature and pressure* (STP), or 25 °C and 1 atmosphere (101,325 Pa). For naturally occurring water and other solutions, density also varies with salinity:

- higher salinity = higher density; and
- lower salinity = lower density.

The density of water behaves differently than most other substances. As liquid water cools, the density increases as expected. However, when water reaches about 4 °C, it attains its maximum density (1,000 kg/m^3), then as it cools further, the density decreases. This is why frozen water (ice) floats on liquid water, whereas the solid form of most other substances is denser than the liquid form and thus the solid form sinks in its liquid form.

We can also express density as the unit weight of a substance, where unit weight is equal to the density of a substance times gravitational acceleration (9.8 m/s^2). Unit weight is simply a way of expressing density in terms of weights that we measure on a scale. The unit weight of water is 9.807 kilonewtons per cubic meter (kN/m^3) at 4° Celsius (39° Fahrenheit). Or in imperial measure, 62.43 pounds-force per cubic foot (lb/ft^3).

Density can also be expressed in a non-dimensional way (i.e., without units) by taking the ratio of the density of the substance to the density of water. This non-dimensional ratio is called the specific gravity (γ; i.e., the Greek letter *gamma*), and is expressed mathematically by Equation (13).

$$\gamma = \frac{\rho_{\text{substance}}}{\rho_{\text{water}}} \tag{13}$$

Specific gravity allows us to record density values in a way that is independent of units. We can convert specific gravity values to density values in whatever units we want simply by multiplying by the density of water. Since the density of water is approximately

1 g /cm^3 (1,000 kg/m^3), specific gravity is basically equivalent to density in g/cm^3. Table 3 lists the specific gravities of some common rock forming minerals.

Table 3 - Specific gravities of some common rock-forming minerals.

Mineral name	Specific gravity
Quartz	2.65
Calcite	2.71
Dolomite	2.85
Na-feldspar	2.62
K-feldspar	2.57
Muscovite mica	≈2.80
Clay (kaolinite)	2.60

The majority of the rock-forming minerals that we are likely to encounter have a specific gravity of about 2.65 to 2.7. Since most rocks in the Earth's crust are made of minerals with specific gravities of about 2.7, we can usually assume a specific gravity of ≈2.7 when estimating the density of rocks.

However, all rocks have some amount of void (i.e., empty) space in them, and a certain percentage of this void space contains air and/or water. When determining the density of a sample of rock or soil, we need to consider the percentage of void space in the rock, and the percentage of that void space that contains water, to determine the bulk density of the sample. The bulk density is defined as the overall density of a sample of rock—including the void spaces, any water in the void spaces, and any other substances that might be incorporated into the rock, such as organic material—as opposed to the density of the individual minerals that make up the rock.

For example, if we consider a 1 cm^3 piece of solid quartz, we know that quartz has a specific gravity of 2.65 and a density of 2.65 g/cm^3 (2,650 kg/m^3); therefore, the piece of solid quartz has a mass of 2.65 grams (0.00265 kg). However, if we consider a 1 cm^3 (1x10^{-6} m^3) piece of quartz sandstone, the quartz sandstone contains some percentage volume of quartz grains, a percentage volume of empty space in between the quartz grains, and a percentage volume of water in the empty spaces. Since water has a specific gravity of 1.0 (at STP), and the mass of air is so small that it is negligible, the overall density (i.e., bulk density) of the sample of sandstone is less than 2.65 g/cm^3 (2,650 kg/m^3), and the mass of the 1 cm^3 (1x10^{-6} m^3) sample is less than 2.65 grams (0.00265 kg). Or, if our sandstone contains a large percentage of a heavier mineral, like magnetite ($\gamma = 5.2$) or galena ($\gamma = 7.5$), the sample may have a bulk density greater than 2.65 g/cm^3 (0.00265 kg/m^3). When calculating the mass of a sample, we need to first determine the bulk density of the sample, and then use that instead of the density of the minerals that make up the rock to calculate mass.

To calculate a bulk density of a sample of rock or sediment, we need to know the volume percentage of each *phase*—the water in the void space is a phase, each mineral is a phase, and so on—and the density of each phase in the sample. Bulk density is calculated by multiplying the volume percentage of each phase by the density of that phase, then summing up the products, expressed in mathematical terms by Equation (14). Exercise 4↴ provides an opportunity to work with the relationships between density, porosity, and saturation.

$$\begin{aligned}\rho_{\text{bulk}} = \left(\rho_{\text{MinA}} \times \left[\frac{\%volume_{\text{mineral-A}}}{100}\right]\right) \\ + \left(\rho_{\text{MinB}} \times \left[\frac{\%volume_{\text{mineral-B}}}{100}\right]\right) \\ + \left(\rho_{\text{water}} \times \left[\frac{\%volume_{\text{water}}}{100}\right]\right)\end{aligned} \tag{14}$$

As previously stated, there is usually some measurable quantity of water in the pore spaces of any rock or sediment sample. We have several ways of quantifying the moisture content of geologic materials.

3.2.2 Moisture Content

Moisture content can be expressed as gravimetric moisture content—that is, moisture content by weight—or volumetric moisture content (i.e., moisture content by volume). The gravimetric moisture content (ω ; i.e., the Greek letter *omega*) is the weight of water in the sample divided by the weight of the solids in the sample, as expressed in Equation (15).

$$\omega = \frac{weight_{\text{water}}}{weight_{\text{solids}}} \tag{15}$$

Volumetric moisture content (θ) is the volume of water in the sample divided by the total volume of the sample, as expressed in Equation (16).

$$\theta = \frac{volume_{\text{water}}}{volume_{\text{total}}} \tag{16}$$

Note that volumetric moisture content is always some value less than or equal to the porosity of the rock. We can also express the moisture content as the degree of saturation (S_d), which is the percentage of the void volume that is filled with water, as shown in Equation (17).

$$S_d = \frac{volume_{\text{water}}}{volume_{\text{voids}}} \tag{17}$$

All these quantities are expressed as fractions. We can also multiply the fraction by 100 to express the final value as a percentage.

If we combine the equations for porosity, moisture content, and degree of saturation, we see that they are all related such that the volumetric moisture content is equal to the porosity times the degree of saturation, as shown in Equation (18) and Equation (19):

$$\theta = S_d \, \varphi \tag{18}$$

$$\frac{volume_{\text{water}}}{volume_{\text{total}}} = \left(\frac{volume_{\text{water}}}{volume_{\text{voids}}}\right)\left(\frac{V_{\text{voids}}}{V_{\text{total}}}\right) \tag{19}$$

3.2.3 Geologic Materials and Implications for Fluid Flow

Now that the basic concepts have been described, let's discuss porosity in the context of geologic materials.

Primary Versus Secondary

We distinguish between two main types of porosity in geologic materials:

Primary Porosity is void space that forms in the rock during deposition and diagenesis:

- for example, empty space between sand grains in a sandstone, or within vesicles in a basalt; and
- generally primary porosity provides more storage but supports less flow than secondary porosity.

Secondary Porosity is void space that forms post-diagenesis:

- for example, fractures or dissolution features; and
- generally secondary porosity provides less storage but more flow than primary porosity.

The idea of fluid storage versus fluid flow is discussed later in this book along with dual porosity flow that occurs in fractured rocks and karst.

Effective Porosity

We defined the porosity of a rock as the volume of voids over the total volume. However, in the field, not all the porosity is available for fluid flow. Some voids are too small, or the pore throats are too narrow, or the pore is a dead end, or is isolated from the rest of the pores. In addition, some amount of moisture adheres to the solid by surface tension and not move with the groundwater flow. The porosity available for fluid flow is called the effective porosity (ϕ_{eff}). Figure 12 illustrates the relationship between the total porosity and the effective porosity in a hypothetical rock.

Figure 12 - Effective porosity—only the connected pores (light blue) can transmit water; the unconnected pores (dark blue) are not a part of the effective porosity.

Examples of materials with potentially high total, but low effective, porosity include:

- clay (lots of saturated void space, but pores are very small); and,
- vesicular basalts (many of the vesicles are not connected).

Thought Question 2*: Is porosity a scalar, vector, or tensor quantity? How about effective porosity?*

Specific Yield

If we saturate a rock and let it drain by gravity, some portion of the water in the pore spaces will be held in by surface tension and narrow pore throats and in pores that are not connected. The fraction that drains is the specific yield (S_y) and the fraction that is retained is called the specific retention (S_r). More rigorous definitions are shown in Equation (20) and Equation (21). Their sum equals the total porosity, i.e., $S_y + S_r = \phi$.

$$S_{\mathrm{y}} = \frac{volume_{\mathrm{drained}}}{volume_{\mathrm{total}}} \tag{20}$$

$$S_{\mathrm{r}} = \frac{volume_{\mathrm{retained}}}{volume_{\mathrm{total}}} \tag{21}$$

Thought Question 3*: What is the relationship between* S_y *and* ϕ_{eff}*?*

Grain Size and Packing

For an ideal situation where grains are perfectly round with uniform size and packing, porosity does not vary with grain size. Grain packing affects porosity (Figure 13a and b), as does grain sorting (Figure 13c and d). Figure 13a and c have the same porosity because they have the same packing and the same sorting; the geometry of the pore space and relative volume of pore space is the same for these two figures.

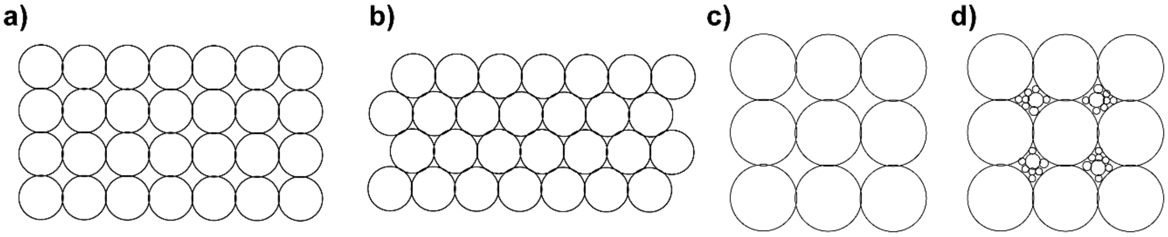

Figure 13 - Affect of packing and sorting on porosity: a) cubic packing; b) rhombic packing; c) well-sorted sediments have higher porosity, but size does not impact porosity for the same packing so the porosity of (a) and (c) are the same; and d) poor sorting fills up the pore space between larger grains with smaller grains and lowers the total porosity.

3.3 Permeability

Porosity is an important parameter in hydrogeology; however, it is not the only quantity that we need to understand to characterize groundwater systems. Different materials transmit fluids with varying degrees of difficulty. The ease with which a porous medium transmits fluid is called the *permeability*.

3.3.1 Darcy's Law

Henry Darcy (1856) experimented with fluid flow through a pipe filled with sand (Figure 14a) as part of a larger project to design filtration systems for the public fountains of Dijon, France. He forced water through the pipe under a hydraulic head—that is, he filled up a reservoir at a higher elevation than the pipe and allowed gravity to drive water through the sand—then he measured:

- the rate of discharge through the pipe (Q); and
- the loss in head (dh) across some length of the pipe (dl).

He concluded that discharge of water through the pipe is directly proportional to the energy loss and inversely proportional to the length of the path of flow (Figure 14b). Or, if we call the loss in head over a length the hydraulic gradient, he found that the discharge is directly proportional to the hydraulic gradient. This statement is expressed mathematically as Equation (22).

$$\frac{Q}{A} \propto \frac{dh}{dl} \qquad (22)$$

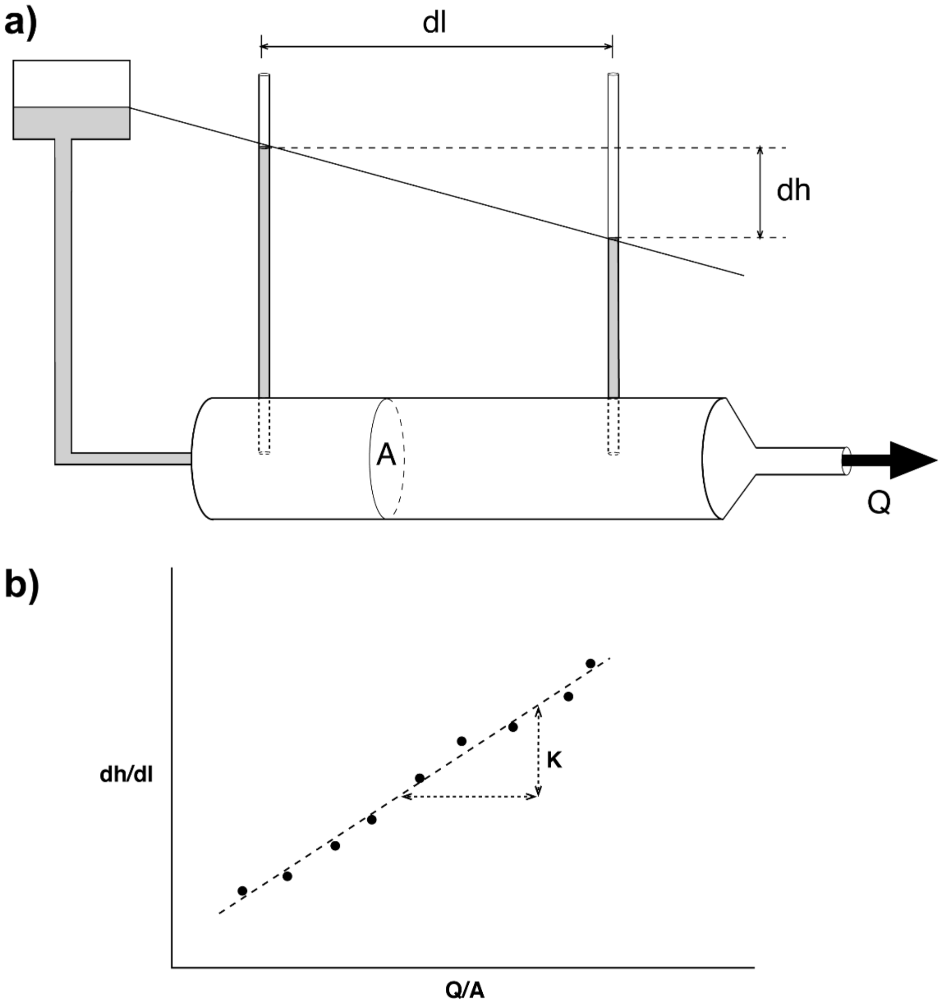

Figure 14 - a) Darcy tube; b) graph showing experimental results.

3.3.2 What Does This Mean?

Notice that the data fall on a relatively straight line when graphed, meaning that the proportionality is linear. Whenever you have a relationship where one quantity is linearly proportional to another, the slope of the line allows you to set the two quantities equal to each other. In mathematical terms, this slope is a *constant of proportionality* or a *coefficient of proportionality*. In this specific equation, we call the constant the *hydraulic conductivity* (K). Inserting K and rearranging the equation to solve for Q gives us Darcy's law, as expressed by Equation (23).

$$Q = -KA\frac{dh}{dl} \tag{23}$$

where (parameter dimensions are dark green font with mass as M, length as L, time as T):

Q = volumetric discharge (L^3T^{-1})

K = hydraulic conductivity (LT^{-1})

A = area perpendicular to flow (L^2)

dh = difference in head (L)

dl = distance over which dh occurs (L)

If we divide through by the cross-sectional area, the volumetric discharge reduces to a 1-dimensional discharge (i.e., velocity) called the *Darcian velocity* (q), as shown by Equation (24).

$$q = -K\frac{dh}{dl} \tag{24}$$

Darcian velocity is also called the *specific discharge*. This value should not be considered to represent the actual velocity of groundwater flow as not all of the cross-sectional area perpendicular to flow is actually open and available for fluid flow. This is discussed in more detail in Section 3.3.7.

When we deal with aquifers, we often use a quantity called the transmissivity (T), which is defined as the hydraulic conductivity multiplied by the saturated thickness of the aquifer (b), as expressed by Equation (25).

$$T = Kb \tag{25}$$

The importance of this value will become evident in later sections of this book; for now, it is useful to be aware of the definition of the term.

In summary, hydraulic conductivity is a quantity that characterizes the ease with which water moves through a porous medium. Darcy's study indicated that different porous media (i.e., different types of sand with different grain size distributions) have different K values, indicating that each media has a distinct value of K. It is important to realize, however, that this relationship is for water specifically; if we are studying some other fluid—petroleum, high-salinity brine, molasses, and so on—we need to use a different coefficient.

3.3.3 Permeability for Other Fluids

The ease with which a fluid moves through a porous medium is dependent on:

- properties of the media (size of pores and connectivity); and
- properties of the fluid (density and viscosity).

The hydraulic conductivity, as measured and characterized by Darcy's law, is for water. This quantity not only characterizes the properties of the media, but it also contains the properties of the fluid moving through that media. Because of this, a hydraulic conductivity obtained for a sand by using water as the fluid in Darcy's apparatus cannot be used to calculate the flow rate of a different fluid—for example, motor oil or mercury. This is not a big deal in most hydrogeology problems, since we generally deal with water in near-surface environments, but it does concern us if we are dealing with multiple phases, or types of fluids—for example, oil and water—in an aquifer. To handle such multi-phase flow problems we use a quantity called the *intrinsic permeability* (k). This value is a quantity that characterizes the ease with which any fluid flows through a porous medium, and only characterizes the porous medium itself. Equation (26) shows how intrinsic permeability can be used in Darcy's law.

$$Q = k\frac{\rho_{\text{fluid}}\, g}{\mu_{\text{fluid}}} \tag{26}$$

where (parameter dimensions are dark green font with mass as M, length as L, time as T):

k = intrinsic permeability (L^2)

ρ_{fluid} = density of the fluid (ML^{-3})

g = gravitational acceleration (LT^{-2})

μ_{fluid} = viscosity of the fluid ($ML^{-1}T^{-1}$)

This equation requires that the user specify the properties of the fluid—that is, density and viscosity. Otherwise, it is the same as Darcy's law. The hydraulic conductivity and the intrinsic permeability are related as shown by Equation (27).

$$k = K\frac{\mu_{fluid}}{\rho_{fluid}\, g} \tag{27}$$

3.3.4 Other Famous Equations Analogous to Darcy's Law

The basic ideas conveyed by Darcy's law are:

a) discharge is driven by a gradient;

b) discharge is linearly proportional to a gradient; and

c) there is a constant of proportionality that characterizes the ease with which water flows through the system (a *conductivity* parameter).

Throughout the natural sciences we find numerous other physical processes that are mathematically identical to groundwater flow in that they have the same three properties. Examples including:

- heat flow (Fourier's Law);
- electrical flow (Ohm's Law); and
- diffusive flow of solutes (Fick's Law).

Each one of these equations has a discharge rate, a gradient or potential, and a constant of proportionality that represents conductivity. The importance of this is that we can use solutions from these other sciences and apply them to groundwater flow problems. This is discussed using a specific example in the section on Well Hydraulics (Section 7).

3.3.5 The Limits of Darcy's Law

Although Darcy's law is regarded as a fundamental relationship in the Earth sciences, it has limitations, and there is a range of conditions over which it is valid.

Upper limit

One of the assumptions inherent in Darcy's law is that the flow is laminar. Darcy's law is not valid above the onset of turbulence in the system because the discharge is no longer linearly proportional to the gradient.

Lower limit

The lower limit of Darcy's law occurs at the *threshold gradient*, where fluid viscosity is too strong for the gradient to overcome the resistance to flow.

Applicability

Darcy's law is still widely used because

1) groundwater systems rarely become turbulent, and
2) at gradients near the threshold gradient, flow is generally so small that it can be ignored.

Thought Question 4*: In what geologic situations might we be concerned about groundwater flow exceeding the limits of Darcy's law?*

3.3.6 Measuring Permeability

Understanding permeability is one thing; measuring it is another matter. Unfortunately, no one has developed a simple permeability meter with a digital display that you can point at a rock or sediment and instantly measure the permeability. Instead, we have a variety of ways of measuring permeability, both in the field and in the laboratory. These methods generally involve forcing fluid through a sample of the material, measuring the head loss and discharge rate, and calculating permeability from the measurements.

Permeameters or *Darcy tubes* are like the apparatus shown in in Figure 14a. The procedure involves filling the tube with a sample, running water through the sample under a constant head in the reservoir, reading the values of head loss (dh) and discharge (Q), then using those measurements and Darcy's law to calculate hydraulic conductivity. Permeameters are generally used for unconsolidated (i.e., loose) material like sand or gravel that can be packed into a column or minimally disturbed cores of the material. Some permeameters are designed to measure permeability in small cores taken from consolidated (i.e., solid) rock.

Gas permeameters are like regular permeameters except they use a gas (usually nitrogen) as the fluid instead of water. These can be portable for field use or set up in a laboratory. Gas permeameters require additional mathematical adjustments to account for the compressibility of air.

Well/piezometer tests, also known as pumping tests or hydraulic tests, are tests performed in the field on wells or piezometers. These tests generally involve pumping water at a known rate while simultaneously monitoring water levels in the pumping well and, if possible, any nearby wells. Wells can also be tested by displacing a known volume of water from the well and monitoring the recovery of the water levels over time; this is known as a slug test. More information on pumping and slug testing can be found in *An Introduction to Hydraulic Testing in Hydrogeology: Basic Pumping, Slug, and Packer Methods*↗ by Woessner and others (2023), which is available as a free download from The Groundwater Project. Well testing is discussed in more detail in Section 8.

Numerous researchers have developed empirical equations to calculate permeability—that is, equations expressing relationships based on experimental data, as opposed to equations derived from first principles. The empirical equations are based on observed correlations between permeability and properties of the rock, primarily grain size distributions, average grain size, or *effective* grain size. In general, larger grain size is correlated with higher permeability; however, the relationship is not that simple. As indicated by Figure 13c and Figure 13d, the distribution of grain sizes can be just as significant as overall size, with well-sorted sediments—that is, very little variation in grain size—correlated with greater permeability.

Permeability can also be estimated using inverse methods, which involve using data collected in the field—for example, water levels in wells, groundwater flow rates from discharge to springs and streams, and dimensions of the aquifer—to estimate, or "back out", permeability values using numerical models and automated model calibration techniques.

3.3.7 Fluid Velocity

We have stated that dividing the Darcy equation by the cross-sectional area reduces the volumetric discharge (Q) down to a one-dimensional discharge (q) that we call the specific discharge or Darcian velocity. In reality, however, groundwater is not flowing through the entire cross-sectional area; it is only flowing through the pore spaces, which make up a percentage of the total area. We must therefore modify the specific discharge by dividing it by the effective porosity to get a value called the average linear velocity (v), which is a more accurate characterization of the groundwater velocity. Calculation of specific discharge is shown by Equation (28).

$$v = \frac{q}{\varphi_{\text{eff}}} \tag{28}$$

It is common for practicing hydrogeologists to use estimates of hydraulic conductivity, effective porosity, and hydraulic gradient to calculate an average linear velocity and then use that velocity to develop estimates about travel times of contaminants in the aquifers that they study. These calculated travel times are then used to develop plans for remediation (i.e., cleaning up) of groundwater contamination at sites that have been impacted by leaking tanks or accidental releases of contaminants. Several decades of experience in assessing and remediating groundwater contamination has shown that travel time estimates based on the average linear velocity are almost always much longer than observed travel times, which means that contaminants generally move much faster than the average linear velocity estimates. Exercise 5⤵ provides an opportunity to make calculations related to groundwater velocity.

These observed discrepancies are likely associated with the fact that the true velocity of the groundwater varies from point to point in an aquifer depending on the size of the pores, the tortuosity of the flow paths, and constriction at the pore throats. If one were able to measure flow velocities at many points in an ideal aquifer, one would see very fast velocities at a few locations, very slow to even stagnant water in other locations, with a wide range of velocities in between. Plotting a frequency histogram of these velocities would likely show us a normal distribution centered around the average linear velocity calculated by Equation (28). However, given the observation that contaminants arrive sooner than expected, we can conclude it is common for contaminants to move along paths with higher velocities. Thus, we need to be cautious about using the average linear velocity to estimate travel times. A discussion of alternative methods for developing meaningful estimates of groundwater velocity is beyond the scope of this book. Readers are encouraged to review *Groundwater Velocity*↗ by Devlin (2020), available from The Groundwater Project as a free download or can be read online.

Thought Question 5*: How is it that the variations in actual groundwater velocity can result in faster than expected arrival of contaminants at a potential receptor (i.e., a downgradient water well) versus estimates derived from calculating the average linear velocity?*

3.3.8 Permeability Distributions in Aquifers

As mentioned, the definitions of scalar, vector, and tensor quantities in mathematics are:

- scalar quantities have magnitude only;
- vector quantities have magnitude and direction; and
- tensor quantities have magnitude that varies with respect to direction.

Permeability is a tensor quantity. Not only does it vary from place to place, but it also varies with respect to direction. We can use two vectors to graphically represent a permeability tensor and to describe the variations in permeability throughout a system.

When we talk about variations of permeability in an aquifer, we use the terms heterogeneity and anisotropy. *Heterogeneity* is variation in a parameter with respect to *location* within a system. *Anisotropy* is variation in a parameter varies with respect to *direction*.

Figure 15 graphically shows the relationship between these two terms. It is important not to use these terms interchangeably—each has a specific meaning.

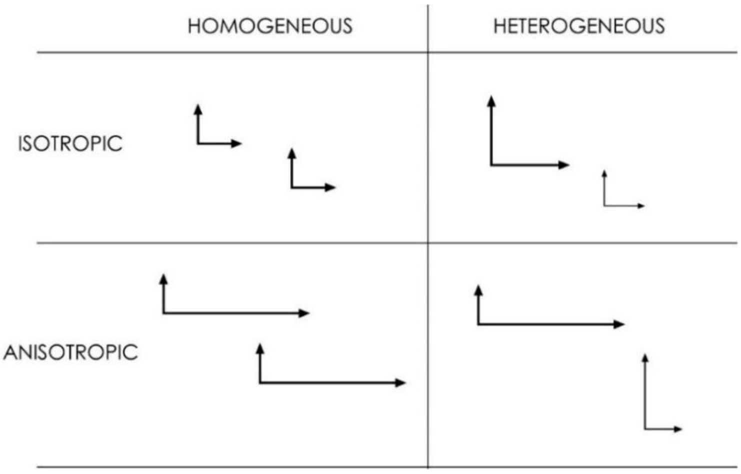

Figure 15 - Graphical representation of the relationship between heterogeneity and anisotropy.

It should be noted that *all* aquifers and groundwater systems have some degree of heterogeneity with respect to permeability. It is essentially impossible to determine the actual distribution of permeabilities in an aquifer, so it is common for practicing hydrogeologists to assume that the system can be approximated as a homogeneous system with a single permeability or a heterogeneous system with a small number of homogenous permeability zones. Anisotropy in aquifer systems is generally associated with the natural layering of geologic formations such that permeability in the vertical direction is generally less than that in the horizontal direction. Anisotropy is also associated with systematic fractures (e.g., fault zones, joint sets) and is also associated with certain depositional environments (e.g., fluvial). More information on the impact of fractures on groundwater flow can be found in *Fractures and Faults in Sandstone and Sandstone-Shale/Mudstone Sequences and Their Impact on Groundwater*↗ by Aydin and others (2023), section 5.6 of *Hydrogeologic Properties of Earth Materials and Principles of Groundwater Flow*↗ by Woessner and Poeter (2020), and *Structural Geology Applied to Fractured Aquifer Characterization*↗ by Fernandes and others (2023). Each book is available as a free download from The Groundwater Project.

3.3.9 Statistical Distribution

Permeability is a quantity that varies over 19 orders of magnitude! When we are studying an aquifer and we measure many permeabilities in that aquifer, our measurements typically cover several orders of magnitude. When multiple measured aquifer permeability values are available it is common to take an average of those values and use that average to perform calculations or develop models. Since permeability generally ranges across several orders of magnitude, we must be careful when averaging permeabilities to get a single value to characterize an aquifer.

In mathematics there are several ways of calculating a mean (i.e., average) for a set of data. The relevant methods for us are listed here.

- *Arithmetic mean:* Add up the values and divide by the number of measurements.

$$\frac{\sum_{i=1}^{n}(k_i)}{n} \tag{29}$$

- *Geometric mean:* Add up the logs of each value and divide by number of measurements and raise ten to that power.

$$10^{\left(\frac{\sum_{i=1}^{n}\left(log(k_i)\right)}{n}\right)} \tag{30}$$

- *Harmonic mean:* Divide the number of measurements by the sum of the reciprocal of each measurement.

$$\frac{n}{\sum_{i=1}^{n}\left(\frac{1}{k_i}\right)} \tag{31}$$

We are generally used to dealing with quantities that follow normal distributions, in which the values are distributed around an arithmetic mean value. Examples of normally distributed quantities include the heights of people in a population, the grades of students in a large class, and variations in air temperatures over time at a single location. Permeability is a log-normally distributed value (Figure 16). A log-normal distribution means that the logarithm of permeability is distributed around a mean, and if we calculate an arithmetic mean for log-normally distributed data we are likely over-estimating the value that could serve as a representative value for the entire formation. The implication is that we should not use the arithmetic mean of multiple measured permeability values to characterize the permeability of an aquifer; instead, it is more appropriate to use a geometric mean—that is, average the logarithms rather than the actual values.

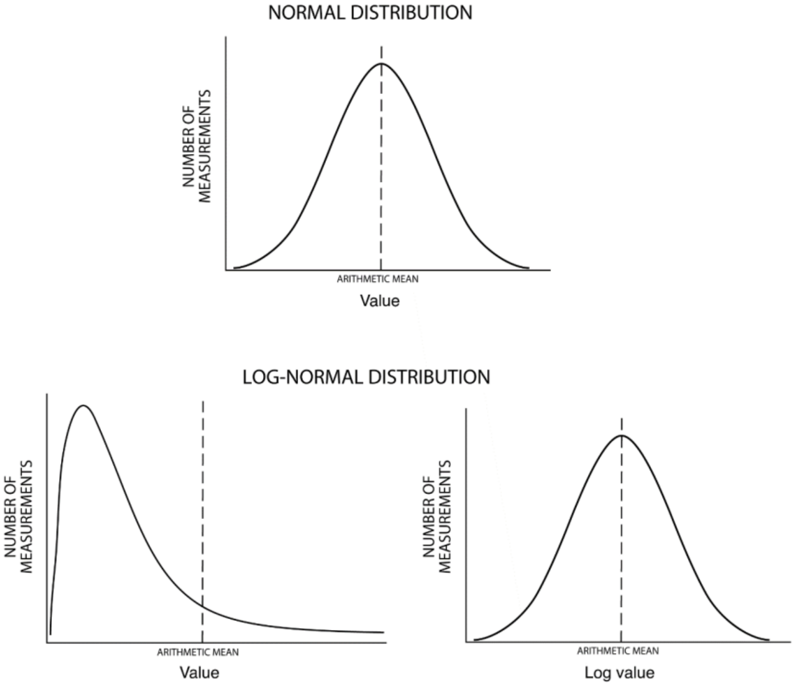

Figure 16 - Normal versus log-normal distribution.

A more detailed discussion of the properties of porous media and the relationships between those properties and groundwater flow can be found in *Hydrogeologic Properties of Earth Materials and Principles of Groundwater Flow*↗ by Woessner and Poeter (2020) which is available from The Groundwater Project as a free download or can be read online.

4 Flow Equations

4.1 Introduction

Mathematics is a language that we use to describe natural phenomenon. The heart of this language is the equation. An equation is a way of expressing a quantity as a function of one or more other quantities. This section describes a simple method for reading and interpreting equations, then presents and discusses the basic equations that we use to understand groundwater systems.

4.2 Equations: A General Discussion

For someone not used to dealing with equations and mathematical statements, the key to understanding them is to think about the physical and conceptual relationships that the equation expresses. Let's take a famous equation—shown in Equation (32)—and think about what it means conceptually.

$$E = mc^2 \tag{32}$$

Where E represents energy, m represents mass, and c^2 is a very large number: the square of the speed of light. What this equation means, in Einstein's words (Mayne, 1948), is:

> *"It followed from the special theory of relativity that mass and energy are both but different manifestations of the same thing—a somewhat unfamiliar conception for the average mind. Furthermore, the equation E is equal to m c-squared, in which energy is put equal to mass, multiplied by the square of the velocity of light, showed that very small amounts of mass may be converted into a very large amount of energy and vice versa. The mass and energy were in fact equivalent, according to the formula mentioned before."*

The point of the equation is that matter (i.e., mass) and energy are fundamentally the same thing. The equation tells us that energy is equal to mass times a very large number ($[3.0 \times 10^8 \text{ m/s}]^2$), indicating that a small amount of mass is equal to a large amount of energy. This is a very significant concept, especially to the residents of Hiroshima and Nagasaki, Japan, during the mid-1940s (where 64 kg of enriched uranium was converted to 16 kilotons of energy).

A useful approach to understanding equations and mathematics is to look at the relationships between the various parts of the equation and determine how those parts relate to reality. Here are basic steps for that approach.

1. Look at each item in the equation and determine what it represents (e.g., length; density; velocity). It also helps to figure out the dimensions of each variable (e.g.,

length, L; density ML^{-3}; velocity, LT^{-1}) and their associated units (e.g., meters; kg/m^3; m/s).

2. Determine if each item varies or is constant. Constants are items with a value that does not change such as gravitational acceleration on Earth, the speed of light in a vacuum, and the number of molecules in a mole of a substance.
3. Compare the variables on one side of the equation to those on the other side. Look for numerator–denominator relationships. Quantities in the numerator on both sides of the equal sign are directly proportional whereas quantities in the numerator on one side and the denominator on the other side are inversely proportional.
4. Notice how the variables are modified (e.g., by exponents, logarithms, numerical constants) and determine what that indicates about the relationships between the variables.

Let's apply this procedure to understanding Einstein's equation. Answer the following questions related to Equation (32).

1. What does each variable (E, m, c) represent?
2. Which is variable and which constant?
3. What are the dimensions for energy?
4. What are the dimensions for mass?
5. How are energy and mass related to each other?
6. How are the variables modified? Are there any exponents?

When we put all of this together, we see that energy and mass are directly proportional because both are in the numerator, and that the constant of proportionality is the square of the speed of light. Since the square of the speed of light is such a large number, we can conclude that a *small amount of mass* is equivalent to a *large amount of energy*.

The advantage of the equation over a sentence that says basically the same thing is that the equation allows us to directly calculate how much energy we can get from a given mass of matter. This gives us the two things that make equations important to us.

1. Equations express natural phenomena in a language that is widely understood and easy to communicate.
2. Equations allow us to calculate and make predictions about the magnitude of one parameter based on the values of other parameters.

Next, let's look at some equations that are a little closer to the subject matter of this book.

4.3 Darcy's Law

As previously described, Darcy's law is shown again in Equation (33).

$$Q = -KA\frac{dh}{dl} \tag{33}$$

Go through the four steps and try to understand what the equation means.

Steps 1 and 2. The variables and constants are as follows.

Q = volumetric discharge: variable with dimensions of L^3T^{-1}

K = hydraulic conductivity: constant with dimensions of LT^{-1}

A = cross-sectional area perpendicular to flow: variable with dimensions of L^2

dh = change in head: variable with dimensions of L

dl = change in length along flow path: variable with dimensions of L

Step 3. The numerator-denominator relationships to discharge (Q) are:

- cross-sectional area (A), directly proportional;
- change in head (dh), directly proportional;
- change in length (dl), inversely proportional;
- hydraulic gradient (dh/dl), directly proportional; and
- hydraulic conductivity, directly proportional.

Step 4. There are no modifiers on the variables so the relationships are linear.

Using the above, we can express Darcy's law in a sentence as "The volumetric discharge is directly proportional to the cross-sectional area and the hydraulic gradient, and the proportionality is regulated by a linear constant called the hydraulic conductivity" (Exercise 6⤵).

4.4 Groundwater Flow Equations

Darcy's law is the basic equation that governs the one-dimensional flow of water in a porous medium. So far, we have treated the gradient term as a simple quantity that can be calculated by dividing the difference in head from one point to the next by the distance between the two points (Figure 17). Also, we have assumed that wells A and B are along a line oriented with steepest head decline for the point of interest.

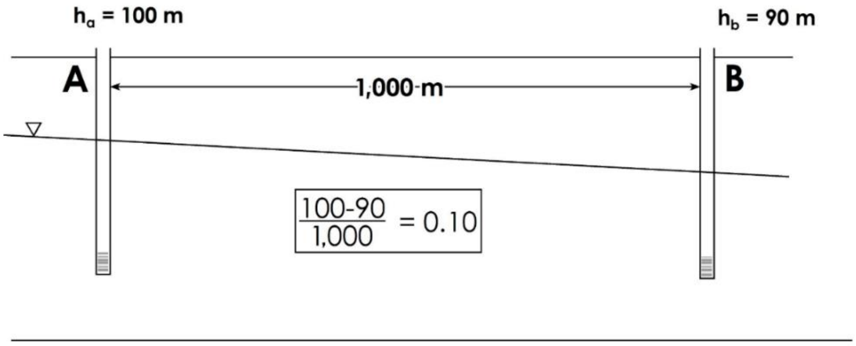

Figure 17 - Cross section showing the overall gradient between two wells.

It is not inappropriate to do this in practice; however, it may not be entirely accurate. The water table could look something like Figure 18. In this case, the gradient is smaller near well A and larger near well B.

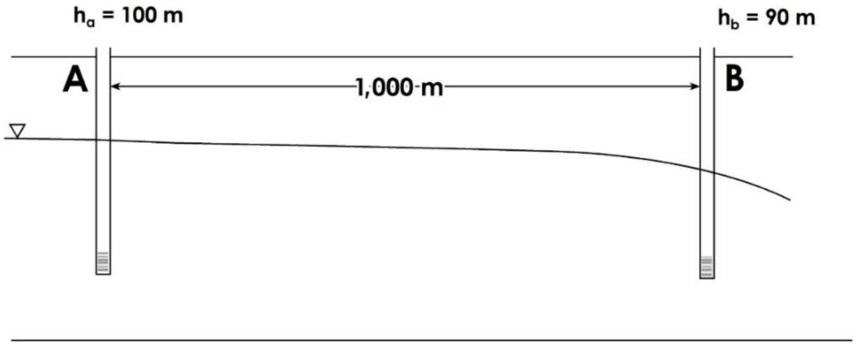

Figure 18 - Cross section showing the water table between two wells.

The hydraulic gradient represents the slope of the water table or potentiometric surface at a point, and that slope varies from one point to the next. In practice, we can occasionally ignore the type of variations shown in Figure 18 and assume that they are not significant. However, it is important to have a basic understanding of the theory and mathematics underlying the hydraulic gradient so that you can decide when it is appropriate to ignore those variations and when it is important to acknowledge them.

4.4.1 Differential Equations

Natural, dynamic, three-dimensional systems (like aquifers) contain an infinite number of points, and the conditions in those systems vary from point to point. Groundwater flow is driven by a gradient. at each point. A gradient is a slope at a point (i.e., a rise of a line on a graph divided by the run), which is also known as a derivative. Derivatives are the underlying basis of differential equations. So, we use differential equations to describe and understand groundwater systems.

A derivative is the instantaneous change in a parameter with respect to some independent variable. In Darcy's law, the hydraulic gradient is a derivative. Figure 19 graphically shows the slope of the water table over progressively smaller intervals (Figure 19a through c) with Figure 19d showing the slope at an infinitesimally small distance (i.e., at a point). We use the symbol Δ to indicate a finite amount of change in a parameter; for example, Δh refers to a specific change in head over a given lateral distance (Δl).

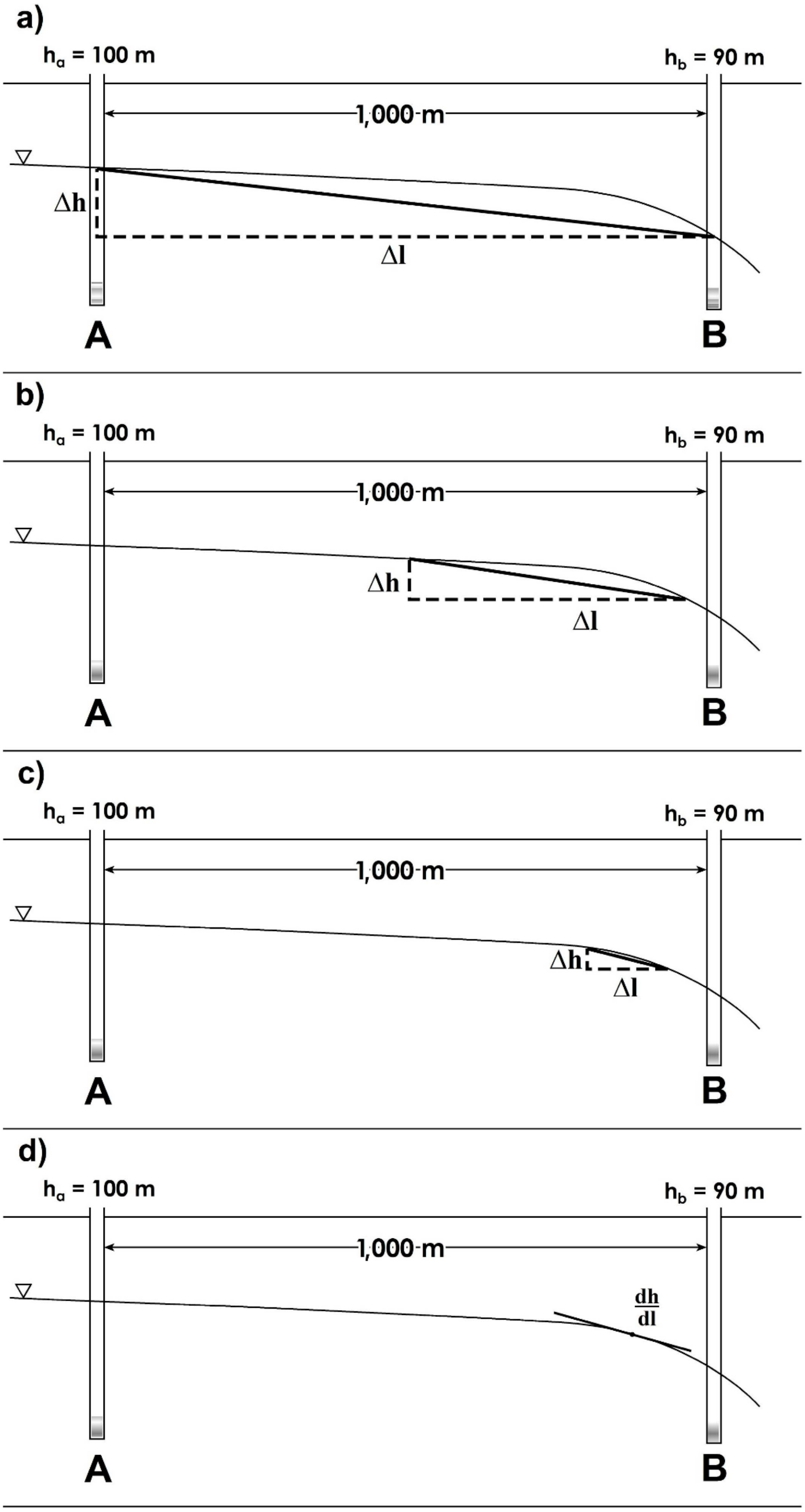

Figure 19 - a), b), c) Illustration of hydraulic gradient as a slope along progressively smaller finite distances ($\Delta h/\Delta l$) and d) as an infinitesimal derivative at a single point (dh/dl).

In a differential equation, we use the symbol d to signify the infinitesimal change in magnitude of a parameter (e.g., head) with respect to some independent variable (e.g., length along the flow path). So, for Darcy's law, we are looking at the hydraulic gradient at a single point in a single direction (along the length of the flow path), and the derivative dh/dl represents the gradient in that direction at that single point.

Since groundwater flow (i.e., q) is driven by the gradient, q also varies from one point to the next. So, we can think about the change in flow with respect to length as $\frac{\mathrm{d}q}{\mathrm{d}l}$.

An aquifer, however, exists in three dimensions and contains an infinite number of points, and at each point groundwater is flowing in some direction. Let's choose a single point in our aquifer and consider how groundwater is flowing at that point (Figure 20). At this point groundwater is flowing in some unique direction. We can represent the flow direction and magnitude with a vector (i.e., an arrow), and define a three-dimensional *coordinate system* (the x, y, and z arrows) to provide a frame of reference. Any vector within a coordinate system can be resolved into *component vectors* in the principal (x, y, and z) directions. What this means is that the three component vectors add up to the groundwater flow vector.

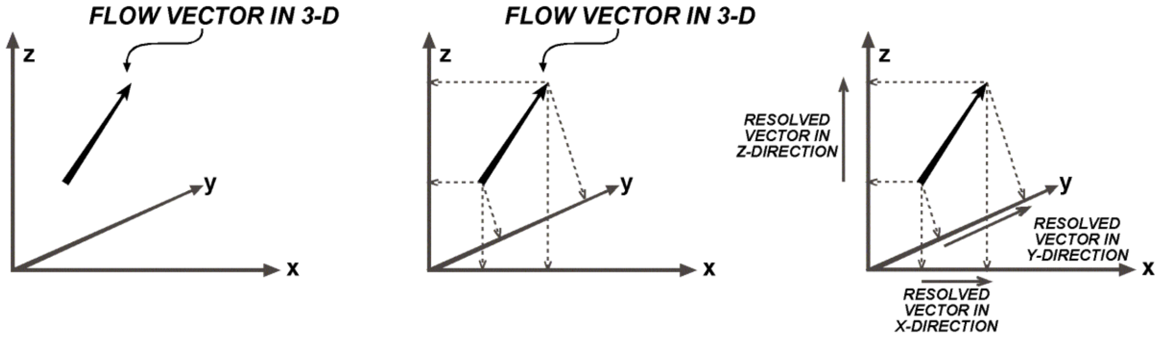

Figure 20 - Vector resolved into component vectors in three dimensions.

Each one of the component vectors represents the flow (i.e., q) in each of the principal directions. There is a q in the x direction (q_x), in the y direction (q_y), and in the z direction (q_z). When we have a derivative that has multiple independent variables (i.e., the three directions), we use a partial differential equation, which uses the symbol ∂ instead of d to indicate the change in magnitude of a variable with respect to the independent variable in each direction. We can then express that flow vector using partial differentials as $\left(\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z}\right)$.

Generally, groundwater flow varies with time and we can equate the flow vector to the change in water stored in the system over an infinitesimally small time (i.e., a derivative of water level, h, with time, t) as expressed by the partial differential equation shown as Equation (34).

$$\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} = S_s \frac{\partial h}{\partial t} \tag{34}$$

where (parameter dimensions are dark green font with mass as M, length as L, time as T):

$\boldsymbol{S_s}$ = the *specific storage* (L^{-1}), which is a coefficient that relates changes in hydraulic head to changes in volume of water stored in a unit volume of aquifer. This term is defined in more detail in Chapter 5.

Darcy's law (expressed as the specific discharge) is shown again here in Equation (35).

$$q = -K \frac{dh}{dl} \tag{35}$$

So, we can substitute it for q, to obtain Equation (36).

$$\frac{\partial \left(K_x \frac{\partial h}{\partial x} \right)}{\partial x} + \frac{\partial \left(K_y \frac{\partial h}{\partial y} \right)}{\partial y} + \frac{\partial \left(K_z \frac{\partial h}{\partial z} \right)}{\partial z} = S_s \frac{\partial h}{\partial t} \tag{36}$$

This is the primary groundwater flow equation, which describes the flow of groundwater at *every point* in a flow system. The left side of this equation is called the divergence of flux because it represents the amount of flux entering or leaving a point.

4.4.2 What Does This Mean in Conceptual Terms?

This is just a more complex version of the continuity equation; each term on the left is:

$I - O$ (*what goes in* minus *what goes out*)

for each direction (x, y, and z) for a given point in the flow system, and the term on the right is:

$\Delta Storage$ (*change in storage*)

for that point in the system.

In a 3-D flow system, there are an infinite number of points, and an infinite number of equations that describe flow through each point. We cannot simply plug a few values into the equation and directly solve it; instead, we must use some indirect method to solve the equation.

4.4.3 How Do We Solve These Equations?

We use one of several methods to solve the partial differential equations that represent natural systems.

Flow nets are a graphical representation of a groundwater flow system that can be used to identify flow paths in an aquifer and estimate hydraulic conductivity. More information on constructing and using flow nets can be found in *Graphical Construction of*

Groundwater Flow Nets↗ by Poeter and Hsieh (2020), which is available from The Groundwater Project as a free download or can be read online.

Analytical equations are solutions of the groundwater flow equation for specific situations, such as a system with a simple geometry. An example of an analytical solution to the 3-D groundwater equation—i.e., Equation (36)—is the Theis equation, which solves for radial flow to a pumping well with a specific set of assumptions. The Theis equation is discussed in more detail in Section 7.

Numerical solutions involve discretizing a system—that is, reducing the infinite number of points/equations to a finite number and simultaneously solving that set of equations. For numerical solutions, each individual equation is generally dependent on the values for the surrounding equations, often in a non-linear manner; therefore, the equations need to be solved using an iterative method that requires a computer and specialized software. These types of solutions, and the software used to develop them, are generally called *computer models*. Popular software packages used for developing groundwater flow models include MODFLOW and FEFLOW. Computer models are discussed in more detail in Section 10.

A more detailed discussion of the equations governing groundwater flow can be found in the seventh section of *Hydrogeologic Properties of Earth Materials and Principles of Groundwater Flow*↗ by Woessner and Poeter (2020), which is available from The Groundwater Project as a free download or can be read online. In addition, a good introduction to using MODFLOW can be found in *Getting Started with MODFLOW*↗ by Winston (2023), which is also available as a free download from The Groundwater Project.

5 Storage Parameters and Aquifer Conditions

5.1 Introduction

In Section 4 we discuss equations in general and introduced the main groundwater flow equation, which is shown here as Equation (37).

$$\frac{\partial q_x}{\partial x}+\frac{\partial q_y}{\partial y}+\frac{\partial q_z}{\partial z}=S_s\frac{\partial h}{\partial t} \tag{37}$$

where (parameter dimensions are dark green font with mass as M, length as L, time as T):

q = specific discharge (i.e., groundwater flow) in the x, y, or z direction (LT^{-1})

h = hydraulic head (L)

t = time (T)

x, y, z = lengths in the principal directions in our coordinate systems

We also substituted Darcy's law, shown as Equation (38) for q, and to get Equation (39).

$$q=-K\frac{dh}{dl} \tag{38}$$

where (parameter dimensions are dark green font with mass as M, length as L, time as T):

K = hydraulic conductivity (LT^{-1})

$\frac{dh}{dl}$ = hydraulic gradient, or change in head with length (:)

$$\frac{\partial\left(K_x\frac{\partial h}{\partial x}\right)}{\partial x}+\frac{\partial\left(K_y\frac{\partial h}{\partial y}\right)}{\partial y}+\frac{\partial\left(K_z\frac{\partial h}{\partial z}\right)}{\partial z}=S_s\frac{\partial h}{\partial t} \tag{39}$$

S_s is a storage parameter called the *specific storage*. It is a coefficient that relates changes in head in an aquifer to the change in the amount of water stored in the aquifer at a certain point. In other words, it is a quantity that tells us how much water that a unit volume of aquifer can yield for a unit decline in head. The term to the right of the equal sign in Equation (39) is equivalent to $\Delta S/\Delta T$ of Equation (1)—that is, it represents the *change in storage with respect to time*. As stated in Section 5, Equation (39) is another way of expressing the continuity equation.

In this section, we discuss storage parameters in general and how they relate to confined versus unconfined aquifer conditions. We also discuss transient versus steady-state conditions in dynamic systems like aquifers.

5.2 Aquifers

Before we discuss storage parameters, let's review what an aquifer is and discuss aquifer conditions and the occurrence of groundwater in nature.

An aquifer is a geologic unit that can store and transmit enough water to supply a well. The factors that determine if a geologic unit is an aquifer include:

- the formation must be capable of storing sufficient volumes of water;
- the rock or sediments in the aquifer must have a sufficiently high permeability that flow to a well or spring can be maintained; and
- the aquifer must have sufficient areal extent and saturated thickness to supply water to the well.

If the aquifer unit is close to the Earth's surface, and the sediments that comprise it are relatively permeable, the primary source of recharge to the aquifer will probably be infiltration of groundwater from precipitation that falls directly on the ground. In this case, a water table will form, and the aquifer is called a water-table aquifer or an unconfined aquifer (Figure 21). In a water-table aquifer the water table is defined as the point where the pore-water pressure and the atmospheric pressure are equal. Under those conditions, water in a well will rise to the point where the water table was encountered during drilling.

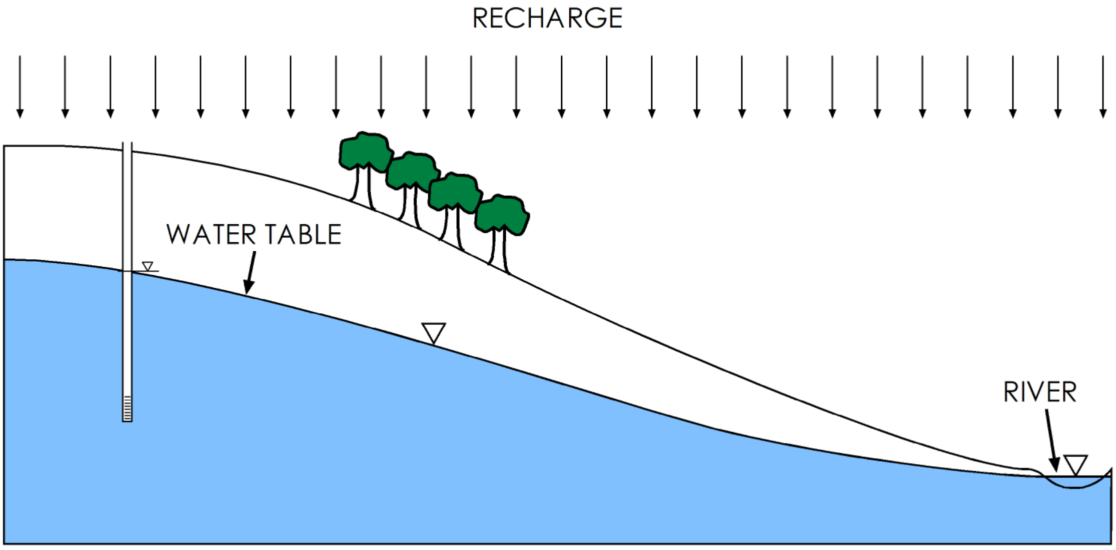

Figure 21 - Unconfined aquifer.

If the regional geology consists of alternating layers of low- and high-permeability units (e.g., interbedded sand and clay), then the permeable layers that are overlain by the low-permeability units can form confined aquifers (Figure 22). Confined aquifers contain an *additional component of pressure head* that causes the water in a well to rise above the top of the aquifer unit. The low-permeability units are called *confining layers*.

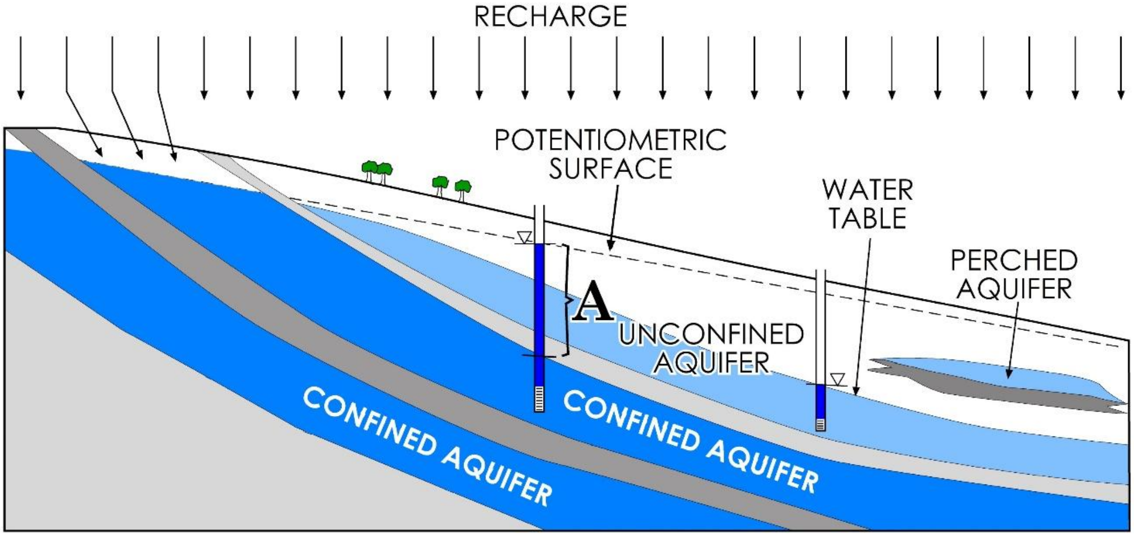

Figure 22 - Interbedded sand and clay units creating both confined and unconfined aquifers. The bent lines on the left indicate recharge to the outcrop of the confined aquifer.

This additional component of pressure head (A in Figure 22) occurs because water enters at a higher elevation in the recharge area and, even though groundwater loses energy as it flows to the point in the aquifer where the well is completed, the remaining energy in the water is greater than that of the water in the overlying unconfined aquifer.

This is the same as what we see in the Darcy tube—the reservoir to the left represents the recharge area, and the change in head from one manometer to the other represents the potentiometric (or piezometric) surface (Figure 23).

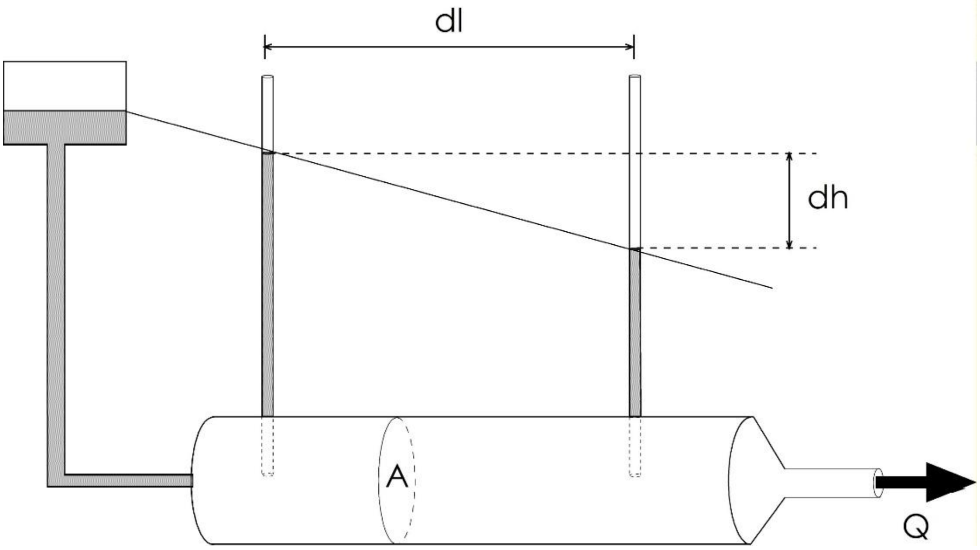

Figure 23 - Darcy tube experiment.

In addition to unconfined and confined aquifers, there are also perched aquifers, which are saturated zones above isolated low-permeability units (like clay lenses) as illustrated on the right end of Figure 22. These are unconfined aquifers that are not laterally continuous.

The important concept established here is that there are two types of groundwater conditions. Unconfined conditions exist where the groundwater is not restricted by a confining layer and the pore fluid pressures at the top of saturation are equal to atmospheric pressure. Confined conditions exist where there is a confining layer and the water under the confining layer has an additional component of pressure that causes it to rise in a well to a point above the top of the confined aquifer. This point represents a point on the potentiometric (or piezometric) surface of the confined aquifer.

The reason we bring this up now is because the storage parameters that characterize an aquifer depend on the aquifer condition. Now we move on to a discussion of storage parameters, focusing first on storage of water in a confined aquifer.

5.3 Storage Parameters

5.3.1 Confined Aquifers—A Conceptual Understanding

Let's consider a confined aquifer that consists of a layer of sand overlain by a layer of clay that acts as a confining layer. If we install a well into the aquifer and pump water out of it, the water level in the well will decline. This reduction in water level caused by pumping is called *drawdown*. As pumping continues the water levels in the aquifer around the well also begin to decline in a way that results in the formation of a *cone of depression* in the potentiometric surface (Figure 24). The more we pump from the well, the larger the depth and areal extent of the cone of depression. The example in Figure 24 shows that a volume of water has been removed from the aquifer—reflected by the volume in the barrel—and has lowered the water level in the well and in the surrounding aquifer. However, we have not changed the saturated thickness of the aquifer itself—that is, the aquifer is still saturated up to the base of the confining layer. What is the source of the water in the barrel—that is, where does that water come from if all the pore space in the aquifer is full? It is released as the aquifer framework compresses and the sand grains and water in the aquifer pores expand slightly.

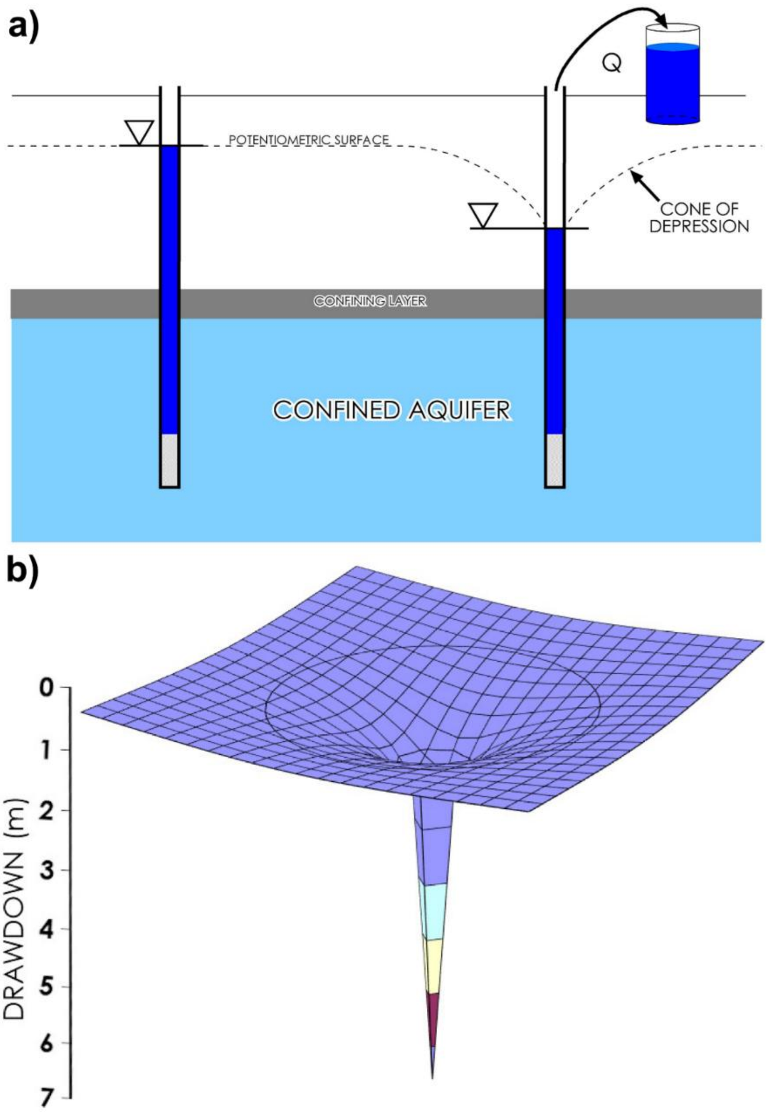

Figure 24 - a) Changes in the potentiometric surface in response to pumping from a well in a confined aquifer; b) 3-D view of a typical cone of depression centered around a pumping well.

An unconfined aquifer is made up of rock or sediments with pore spaces that are full of water. When there is no confining layer and the aquifer is recharged by water infiltrating from above, the pore spaces are full of water and the pore fluid pressure at any depth in the aquifer is equal to atmospheric pressure plus the pressure created by the weight of the overlying column of water. Therefore, in an unconfined aquifer, you can fit a volume of water into the pore space that is equal to the volume of the pore space.

However, if the aquifer is confined and an additional component of pressure is added to the water (as shown in Figure 25), the additional pressure will effectively force a greater volume of water into the pores of the sand. It does this by pushing the sand grains around so that pore volume is increased, and slightly compressing the water and the sand grains. This increased pressure and volume causes water levels in the aquifer to rise above the top of the aquifer—unlike an unconfined aquifer in which the water level defines the top of the aquifer. Increasing the pore fluid pressure causes the aquifer to undergo elastic deformation—that is, the change in volume is recoverable—so that a subsequent decrease in fluid pressure will cause an associated decrease in overall deformation of the pore spaces as illustrated in Figure 25.

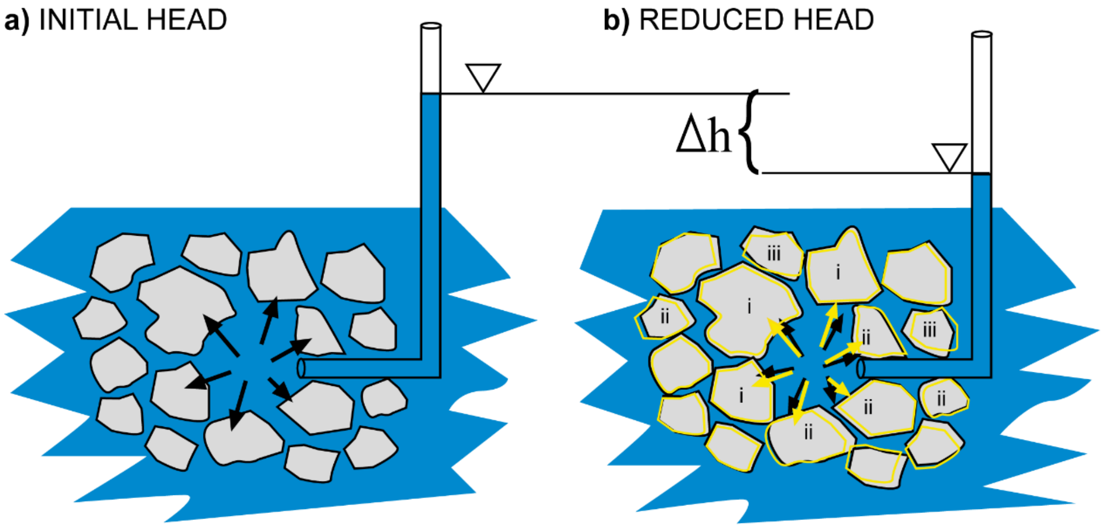

Figure 25 - Schematic of elastic deformation in a confined sand aquifer related to changes in fluid pressure. Gray polygons represent grains of sand, blue represents saturated pores between the sand grains, black arrows and the height of the blue column represent the fluid pressure in the large pore in the middle of the diagram. a) High pressure head pushes grains of sand apart (greatly exaggerated in this image), compresses water molecules by a small but measurable amount, and compresses individual sand grains by a nearly imperceptible amount. b) When head decreases, pressure is lower and sand grains move closer together, water molecules expand slightly, and sand grains expand a nearly imperceptible amount. The yellow grain outlines in (b) show the original sizes and locations of the sand grains as depicted in (a). The yellow arrows in (b) show the magnitude of pressure in (a). The lowercase letters in (b) represent examples of changes due to the reduction in fluid pressure: i = individual grain size increase, ii = sand grains that moved closer together, and iii = sand grains that rotated to accommodate reduced pore space.

The tiny changes in pore volume associated with movement and rotation of sand grains is the source of the volume of water that is removed from a confined aquifer. Pumping water from the confined aquifer lowers the fluid pressure—that is, the pressure head above the top of the aquifer—causing the aquifer to 'relax' and yield water that is stored by the excess pressure.

5.3.2 Confined Aquifers—A Mathematical Understanding

Now that we have a basic conceptual understanding of elastic storage of water in a confined aquifer, let's describe this concept mathematically. What we as hydrogeologists are interested in knowing is what volume of water is associated with a given decline in head in the aquifer, or conversely, how much head loss will be associated with a given volume withdrawn from an aquifer.

Let's consider a unit volume of our confined aquifer—by *unit volume* we mean a cube with some convenient dimensions like 1 meter cubed. This unit volume is saturated with water, and the water has an additional component of pressure that causes the water to rise above the top of the cube (Figure 26). If we remove some volume of water from the cube, we will reduce the additional component of pressure and lower the head in the cube. Removing more water will cause more head loss; less water will cause less head loss. Therefore, by trial-and-error we can determine the volume of water that needs to be removed to cause a unit decline in head—in this case, a head decline of one meter.

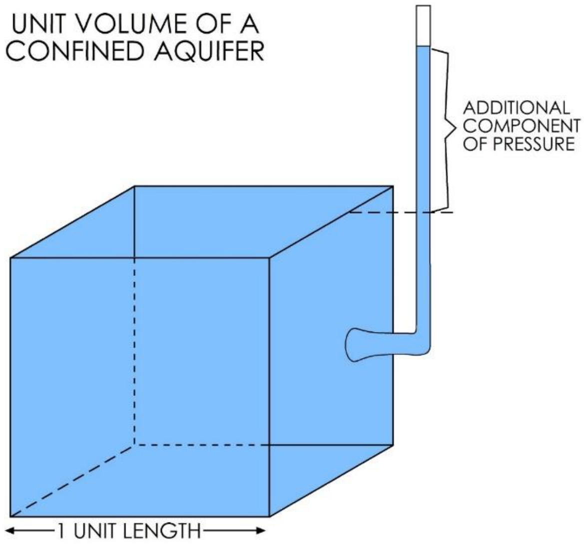

Figure 26 - Unit volume of a confined aquifer.

This number that we measure—the volume of water per unit volume of aquifer that results in a unit decline in head—is the specific storage, given as S_s in the groundwater flow equation. The specific storage is an aquifer storage parameter that mathematically describes the volume of water that can be withdrawn from a unit volume of a confined aquifer for a unit decline in head. This value is also called the elastic storage coefficient, as it deals only with the amount of water removed from elastic storage as described in the previous section and depicted in Figure 25. The specific storage is a function of the compressibility of the aquifer matrix—in this case, the sand—and the compressibility of water. It is rigorously defined as shown by Equation (40).

$$S_s = \rho_w g \varphi (\alpha + \beta) \tag{40}$$

where (parameter dimensions are dark green font with mass as M, length as L, time as T):

ρ_w = density of water (ML^{-3})

g = gravitational acceleration (LT^{-2})

φ = porosity (:)

α = compressibility of the aquifer matrix ($LT^{-2}M^{-1}$)

β = compressibility of water ($LT^{-2}M^{-1}$)

Because we are dealing with a unit volume of aquifer and a unit decline in head, the specific storage refers to a point in the aquifer. However, when we are dealing with aquifers in a field setting, the thickness of the aquifer has an influence on how much water the aquifer will yield; given the same aquifer material, hydraulic conductivity, and porosity, a thicker aquifer will transmit and yield more water than a thinner aquifer. If we multiply the hydraulic conductivity of the aquifer by the aquifer thickness, we get a quantity called the transmissivity (T), which describes the amount of water that can move through a unit width of the fully saturated thickness of the aquifer under a hydraulic gradient of one. Similarly, we have a quantity called the storativity (S), which for a confined aquifer is the specific storage multiplied by the saturated thickness of the aquifer, as shown in Equation (41).

$$S = S_s b \tag{41}$$

where (parameter dimensions are dark green font with mass as M, length as L, time as T):

b = saturated thickness of the aquifer (L)

In other words, the storativity describes the volume of water removed from a unit *area* of aquifer for a unit decline in head.

Thought Question 6*: Use dimensional analysis to determine the dimensions (or the units) of specific storage and of storativity.*

A more detailed discussion of the theory behind confined aquifer storage parameters can be found in *Groundwater Storage in Confined Aquifers*↗ by Wang (2020) which is available from The Groundwater Project as a free download or can be read online.

5.3.3 Unconfined Aquifers

In a confined aquifer, all the water pumped out via a well is provided by the release of water in elastic storage. In an unconfined aquifer, we see the same thing—the changes in head result in a release of a small amount of water from elastic storage in the aquifer. However, as the water level declines, we also remove water from pore spaces as the water table declines which dewaters a section of the aquifer (Figure 27). This drainage of the aquifer's pores produces a greater volume of water for an equal decline of head as compared to a confined aquifer.

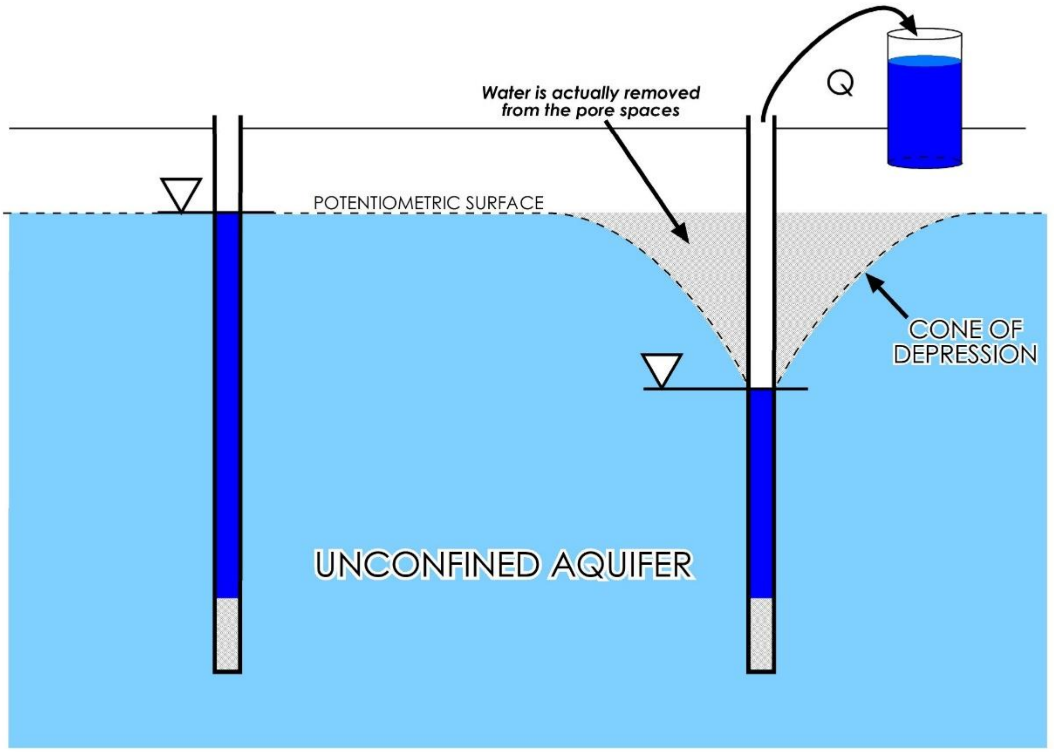

Figure 27 - Changes in the potentiometric surface (i.e., water table) in response to pumping from a well in an unconfined aquifer.

This additional volume of water that drains from the pore spaces is characterized by a parameter called specific yield. As described in Section 3.2.3, specific yield is the volume of water that drains from a rock under the force of gravity and is essentially equivalent to the effective porosity of the aquifer matrix. The volume of water released from a unit volume of aquifer due to dewatering of the aquifer is therefore just the unit volume times the specific yield. For example, a 1 cubic meter volume of aquifer with a specific yield of 0.1 will yield 0.1 cubic meters of water for each 1-meter decline in hydraulic head.

In unconfined aquifers, therefore, the volume of water released from a unit volume of aquifer is a function of both the *specific yield*—that is, the water removed from the pore space—and the *specific storage*—the water released from elastic storage in the aquifer. Therefore, in an unconfined aquifer, the storativity of the aquifer is the sum of the specific yield and the thickness times the specific storage, as shown in Equation (42).

$$S = S_y + S_s\, b \tag{42}$$

The magnitudes of these parameters, however, are drastically different: Specific yield (i.e., effective porosity) values in an aquifer generally range from 1 percent to 50 percent, or 0.01 to 0.5, while specific storage values generally range from $1\text{x}10^{-6}$ to $1\text{x}10^{-4}$ m^{-1} or 0.000001 to 0.0001 m^{-1}. This means that, in an unconfined aquifer, the volume of water provided by release from elastic storage is orders of magnitude less than that provided by dewatering the pores, so we generally assume that the specific storage is negligible and such that unconfined storativity as only the specific yield. In fact, when aquifer are tested the value obtained for specific yield incorporates what is yielded from elastic storage.

Thought Question 7: *Consider two aquifers; each has the same porosity, permeability, and saturated thickness, but one is confined while the other is unconfined. If identical wells are installed in each and pumped at the same rate for the same period of time, in which aquifer would you expect to observe greater drawdown? Why?*

6 More on Flow Equations

6.1 Introduction

Let's review our previous discussion of the groundwater flow equations. We have the main equation that governs three-dimensional flow of water in a porous medium, assuming that flow is within the limits of Darcy's law—that is, not so high that flow becomes turbulent, and not so low that it is below the gradient threshold, as expressed in Equation (43).

$$\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} = S_s \frac{\partial h}{\partial t} \tag{43}$$

where (parameter dimensions are dark green font with mass as M, length as L, time as T):

q = specific discharge (i.e., groundwater flow) in the $x, y,$ or z direction (LT^{-1})
x, y, z = lengths in the principal directions in our coordinate systems
h = hydraulic head (L)
t = time (T)
S_s = specific storage (L^{-1})

We substituted Darcy's law—shown in Equation (44)—for q, to obtain Equation (45).

$$q = -K \frac{dh}{dl} \tag{44}$$

where (parameter dimensions are dark green font with mass as M, length as L, time as T):

K = hydraulic conductivity (LT^{-1})
dh/dl = hydraulic gradient, or change in head with length (:)

$$\frac{\partial \left(K_x \frac{\partial h}{\partial x}\right)}{\partial x} + \frac{\partial \left(K_y \frac{\partial h}{\partial y}\right)}{\partial y} + \frac{\partial \left(K_z \frac{\partial h}{\partial z}\right)}{\partial z} = S_s \frac{\partial h}{\partial t} \tag{45}$$

This equation is another way of expressing the continuity equation, or $I - O$ (*what goes in minus what goes out*) $= \Delta Storage$ (*change in storage in the system*).

In Section 4, we established that the left side of the equation describes the divergence of flux at any given point in the aquifer, which is a fancy way of saying $I - O$ along each of the principal directions in our x, y, z coordinate system. Then, in Section 5, we put forth the idea that the right side of the equation describes changes in storage in the system, and we discussed the specific storage and other related storage parameters. The storage parameters relate changes in head to changes in the volume of water stored in the aquifer. More specifically, the specific storage describes the volume of water that can be removed from (or added to) a unit volume of aquifer that will result in a unit decline (or rise) in hydraulic head.

The hydraulic head describes the total fluid energy at a specific point in an aquifer. This head changes from point to point, creating a tensor quantity called the *hydraulic gradient*, which reflects the second law of thermodynamics—systems spontaneously move from higher to lower energy—for example, groundwater flows from high head to low head. Therefore, the hydraulic gradient describes how head changes with respect to *location* in the aquifer. But heads in natural aquifers do not only change from one point to the next; they also change through time.

This brings us to the final part of the equation for our discussion; specifically, the partial derivative of head versus time, or $\partial h/\partial t$, on the right side of the equation. This term describes changes in head (h) at a point in the aquifer with respect to time (t) and combines with the specific storage to describe volume changes—that is, changes in the volume of water stored in the aquifer—with time. In this section, we discuss time as it relates to groundwater, and then discuss some applications of the flow equations.

6.2 Steady State versus Transient Groundwater Flow

6.2.1 Steady-State Flow

Consider the continuity equation; otherwise known as the *law of mass conservation*, or the *continuity principle*:

$I - O$ (*what goes in minus what goes out*)

$= \Delta Storage$ (*change in storage in the system*) / Δt (*change in time*).

This principle states that mass is conserved in a system, and any change of mass entering a system must be balanced by a corresponding change in either the mass leaving the system, or the mass stored in the system. In other words, if the mass going in equals the mass going out, then the mass in storage will not change, and $\Delta S/\Delta t$ will equal zero. In the case where I is equal to O, and $\Delta Storage/\Delta t$ equals zero, we say that the system is at steady state. If the system is at steady state, heads at any given point in the aquifer do not change through time, the ∂h on the right side of the equation equals zero, as shown by Equation (46) and Equation (47).

$$\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} = 0 \tag{46}$$

$$\frac{\partial \left(K_x \frac{\partial h}{\partial x}\right)}{\partial x} + \frac{\partial \left(K_y \frac{\partial h}{\partial y}\right)}{\partial y} + \frac{\partial \left(K_z \frac{\partial h}{\partial z}\right)}{\partial z} = 0 \tag{47}$$

If we assume the aquifer is homogeneous—that is, the permeability at one point is the same as at any other point—then the hydraulic conductivity (K) will not change with respect to location, and we can remove K_x, K_y, and K_z from inside the differentials as shown in Equation (48).

$$K_x \frac{\partial \left(\frac{\partial h}{\partial x}\right)}{\partial x} + K_y \frac{\partial \left(\frac{\partial h}{\partial y}\right)}{\partial y} + K_z \frac{\partial \left(\frac{\partial h}{\partial z}\right)}{\partial z} = 0 \tag{48}$$

If we also assume that the permeability of our aquifer is *isotropic*, then the hydraulic conductivity (K) at any point in the aquifer will be the same in all directions, and $K_x = K_y = K_z = K$. In that case, we can factor out K and rewrite it as shown by Equation (49).

$$K\left(\frac{\partial \left(\frac{\partial h}{\partial x}\right)}{\partial x} + \frac{\partial \left(\frac{\partial h}{\partial y}\right)}{\partial y} + \frac{\partial \left(\frac{\partial h}{\partial z}\right)}{\partial z}\right) = 0 \tag{49}$$

Also, we can use a simpler notation for the derivatives and rewrite it as shown by Equation (50).

$$K\left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2}\right) = 0 \tag{50}$$

Each one of the derivatives in the parentheses is a 2nd order partial differential. This simply means that it isn't only that head changes with distance, but the change in head with distance changes with distance (or, in other words, the gradient changes with distance). If we then divide both sides by K, the equation reduces to Equation (51).

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0 \tag{51}$$

Equation (51) is called the *Laplace equation*. This equation describes the steady-state flow of groundwater (i.e., no changes in storage) in a homogeneous and isotropic aquifer. As we can see, the assumptions of homogeneity and isotropy greatly simplify the equation.

6.2.2 Transient Flow

Steady state means that the amount going in equals the amount going out. In groundwater systems, this means that the amount recharging the system—infiltration of rain, loss from streams, leakage from other formations—is equal to the amount discharging the system—spring flow, discharge to streams, pumping by wells. In field settings, recharge is never exactly equal to discharge—although, in some cases, they may be pretty close over the long term, such that it is often appropriate to assume a system is in a steady state when dealing with regional flow systems. That is discussed more in subsequent sections of this book. Aquifers are in a constant state of dynamic change; recharge varies, discharge varies, the volume in storage varies, and head varies. If it rains more in one year than it usually does, recharge increases and heads increase. If we pump a water from a well, we remove water from storage and heads decline. These scenarios depict transient flow conditions—conditions where heads and the volume of water in storage are changing through time.

The key point to consider here is time. Continuously pumping water from a well will result in ongoing changes in head, and we as hydrogeologists are interested in knowing things like how much we can pump from a well in a day, how fast heads will decline when we pump, and how long it will take for heads to return to their original level if we stop pumping. The right side of the equation essentially represents this aspect of aquifers, so the right side is zero for steady state flow conditions.

When flow varies with time, conditions are transient and the right side of Equation (50) is replaced with Equation (52) which describes the change in storage in the aquifer with time, becoming the transient groundwater flow equation shown in Equation (53).

$$\frac{\Delta Storage}{t} \approx S_s \frac{\partial h}{\partial t} \tag{52}$$

$$K\left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2}\right) = S_s \frac{\partial h}{\partial t} \tag{53}$$

It is useful to remember that specific storage is a coefficient, just like K is a coefficient. A coefficient simply relates two quantities to one another:

- K relates discharge to the hydraulic gradient. If we increase the gradient, we increase the discharge, and vice versa; K tells us how much change in discharge we get for a given change in gradient.
- S_s relates changes in head to changes in storage in the aquifer. If we remove a volume of water from storage, we lower the head in the aquifer, and vice versa; S_s tells us how much head changes for a given change in storage.

The $\partial h/\partial t$ refers to the change in head with time, and the S_s is the coefficient that states how much change in storage will occur given that change in head. In summary:

- the three-dimensional groundwater flow equation is comprised of two parts: 1) the flux (the left side), and 2) the change in storage in the aquifer with time (the right side);
- the left side is composed of 2 parts, 1) the divergence of groundwater flux in each of the three principal directions, and 2) a hydraulic conductivity parameter that describes the ease with which groundwater moves through the medium in response to head gradient; and
- the right side is composed of two parts as well; 1) the change in head with time, and 2) a storage parameter that describes the change in storage resulting from the change in head.

6.3 Flow Calculations and Applications of the Flow Equation

The following is a general overview of various ways of quantifying groundwater flow and solving, either directly or indirectly, the groundwater flow equation.

6.3.1 Steady-State Flow in a Confined Aquifer

In the case of steady-state flow through a confined aquifer (Figure 28), a change in head does not change the saturated thickness.

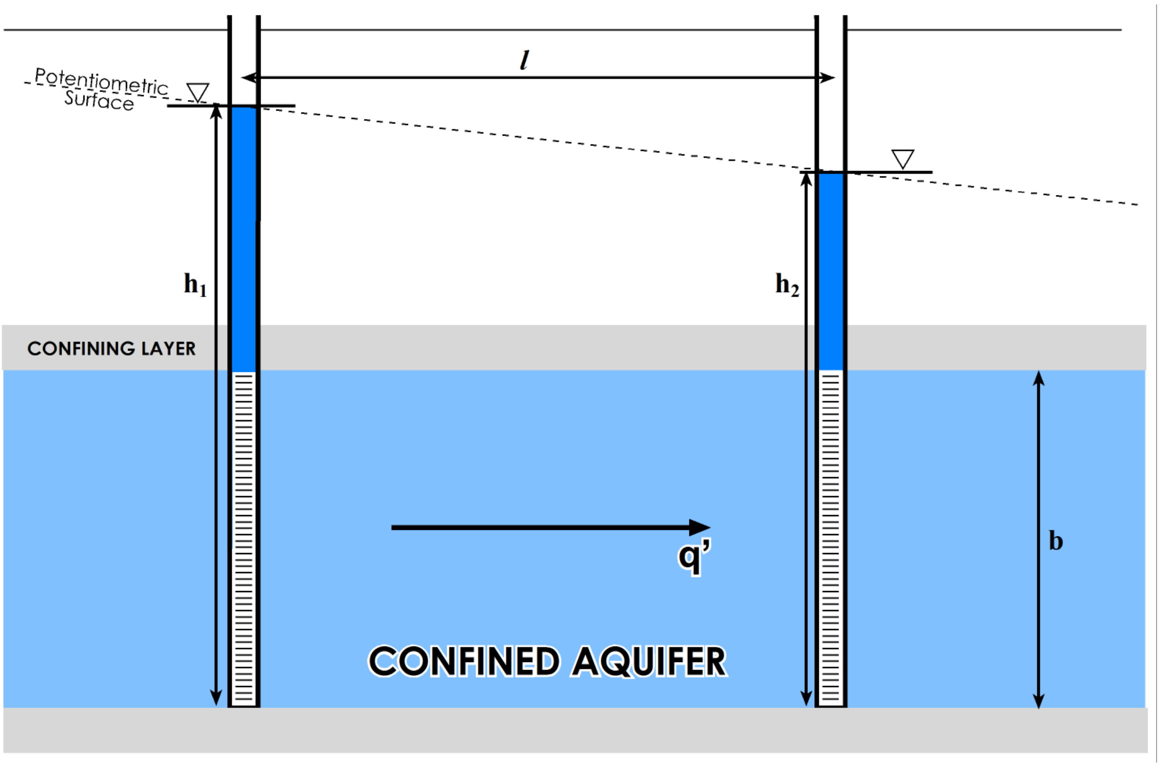

Figure 28 - Steady flow through a confined aquifer of uniform thickness.

We can use a direct variation of Darcy's law to calculate volumetric flow per unit width of aquifer (q') as shown in Equation (54).

$$q' = Kb\frac{dh}{dl} \tag{54}$$

where (parameter dimensions are dark green font with mass as M, length as L, time as T):

q' = flow per unit width of aquifer (L^2T^{-1})

Given that $Kb = T$, we could substitute T into the equation for Kb.

The head (h') at some intermediate distance (x) along the flow path can be calculated by Equation (55).

$$h' = h_1 - \frac{q'}{Kb}x \tag{55}$$

where (parameter dimensions are dark green font with mass as M, length as L, time as T):

x = some distance from h_1 (L)

6.3.2 Steady-State Flow in an Unconfined Aquifer

Unconfined aquifers are a bit more mathematically complicated because the water table is the upper boundary, and since the water table changes from one point to the next, the saturated thickness (b) changes as well (Figure 29). Since we cannot treat b as a constant, we must use some integral calculus and specific assumptions to develop an equation (based on Darcy's law) that can be solved directly.

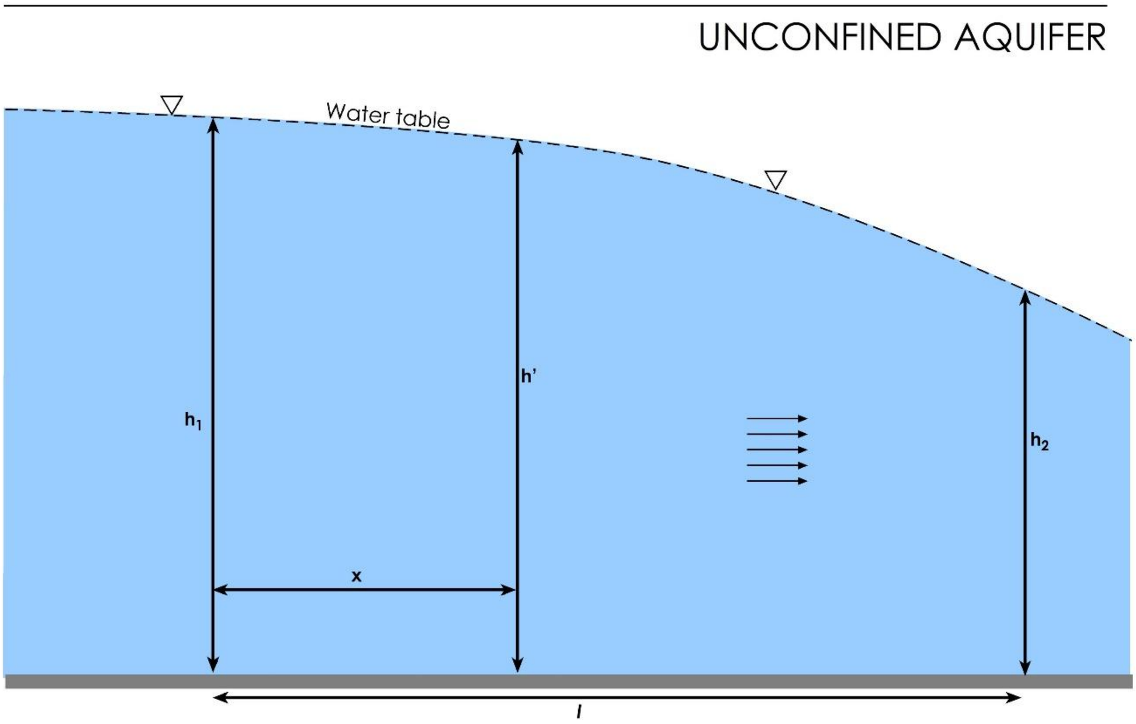

Figure 29 - Steady flow through an unconfined aquifer.

We won't go through the derivation here, but we discuss the assumptions. This solution was first derived by Dupuit in 1863; hence they are called the Dupuit assumptions. This solution assumes:

1. the hydraulic gradient is equal to the slope of the water table, and
2. flow is horizontal.

Incorporating these assumptions results in the Dupuit equation that is shown as Equation (56).

$$q' = \frac{1}{2}K\left(\frac{h_1^2 - h_2^2}{L}\right) \tag{56}$$

Calculating intermediate heads is a problem in an unconfined aquifer because unconfined aquifers usually receive recharge from above the water table (Figure 21).

Equations have been derived to calculate heads in an unconfined aquifer with areal recharge. These can be explored using the interactive educational tool called WTR (Water Table Recharge) by visiting https://interactive-education.gw-project.org/wtr/↗ on the Groundwater Project web site (https://gw-project.org/↗).

A more detailed discussion of the equations governing groundwater flow can be found in the seventh section *Hydrogeologic Properties of Earth Materials and Principles of Groundwater Flow*↗ by Woessner and Poeter (2020) which is available from The Groundwater Project as a free download or can be read online.

6.3.3 Flow Lines and Flow Nets

Another way of solving steady-state groundwater flow problems is by using flow lines and equipotential lines. A *flow line* is an imaginary line that traces the path that a particle of groundwater would follow as it flows through an aquifer. *Equipotential lines* are lines that connect points of equal head in the aquifer. In an isotropic aquifer, the flow lines are perpendicular to the equipotentials and point in the direction of decreasing head. In an anisotropic aquifer, the flow lines will be deflected according to the direction of maximum K (Figure 30).

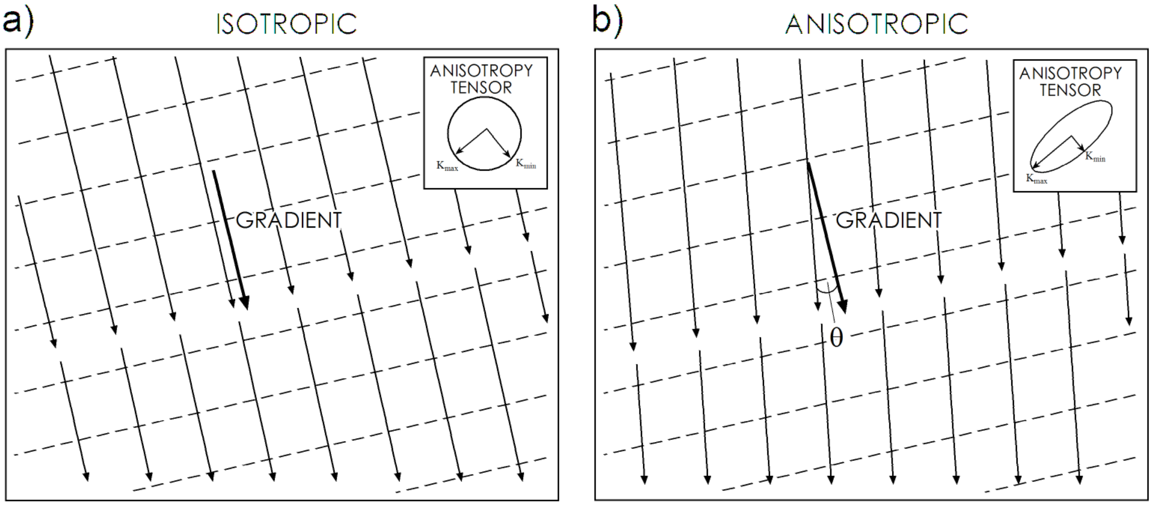

Figure 30 – Equipotentials (dashed) and flow lines (arrows) in isotropic and anisotropic aquifers.

If the degree of anisotropy is known with certainty, there are ways that the amount of deflection (angle θ in the anisotropic diagram, Figure 30b) can be calculated.

A *flow net* is a graphical representation of the steady-state groundwater flow equation that depicts equipotentials and flow lines and, if properly drawn, represents a graphical solution to the two-dimensional Laplace equation—that is, steady-state flow in a homogeneous and isotropic aquifer. Flow net construction requires the following assumptions.

1. The aquifer is homogeneous, isotropic, and fully saturated.

2. The aquifer is at steady-state—heads are not changing with time.
3. Flow is laminar and Darcy's law is valid.
4. All *boundary conditions*[2] are known. The three basic boundary conditions that we need to understand to use flow nets are:
    a) no-flow boundary—no gradient or flow across the boundary; the boundary represents a flow line;
    b) constant-head boundary—no gradient or flow parallel to the boundary, boundary represents an equipotential line; and
    c) water table boundary—head is equal to elevation along this boundary—the water table can be a flow line or it can be intersected by both flow lines and equipotential lines.

Flow nets are two-dimensional and can be drawn to represent either map-view or cross-sectional problems. Flow nets must be drawn according to a specific set of rules so they can then be used to calculate the total discharge through the area. The key to drawing correct flow nets is to draw them in such a way that:

1. there is an equal change in head from one equipotential to the next, called *contour interval;*
2. there is a constant discharge between flow lines;
3. flow lines cross equipotentials at right angles; and
4. the intersections of flow and equipotential lines create *curvilinear squares*, meaning that the area within the grids is basically square.

The reason for this is because the area between the flow lines represents a *flow tube*, and the discharge in that flow tube (ΔQ) is constant from one equipotential to the next. Therefore, according to Darcy's law, if the gradient decreases, then the distance between consecutive equipotentials increases and the flow lines must be further apart (maintaining the curvilinear square) to accommodate the constant discharge at the lower velocity, and vice-versa (i.e., if gradient increases the distance between equipotential lines is shorter so the flowlines must be closer (maintaining the curvilinear square) to encompass the constant discharge).

Now let's think about this: if the gradient decreases, then the equipotentials get further apart and the velocity decreases. The discharge over the entire area decreases and there are fewer flow tubes in the same area, so each flow line gets further away from its neighbor. In the lower right corner of the flow net shown in Figure 31 the equipotentials get further apart (gradient decreases), discharge per unit width also decreases, and the flow lines are further apart to maintain curvilinear squares. Now, since gradient and discharge are *linearly* related, changing one will change the other by the same magnitude—for

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[2] Boundary conditions are discussed in more detail in Section 9

example, double the gradient results in double the discharge—so the *grids* will always be curvilinear squares.

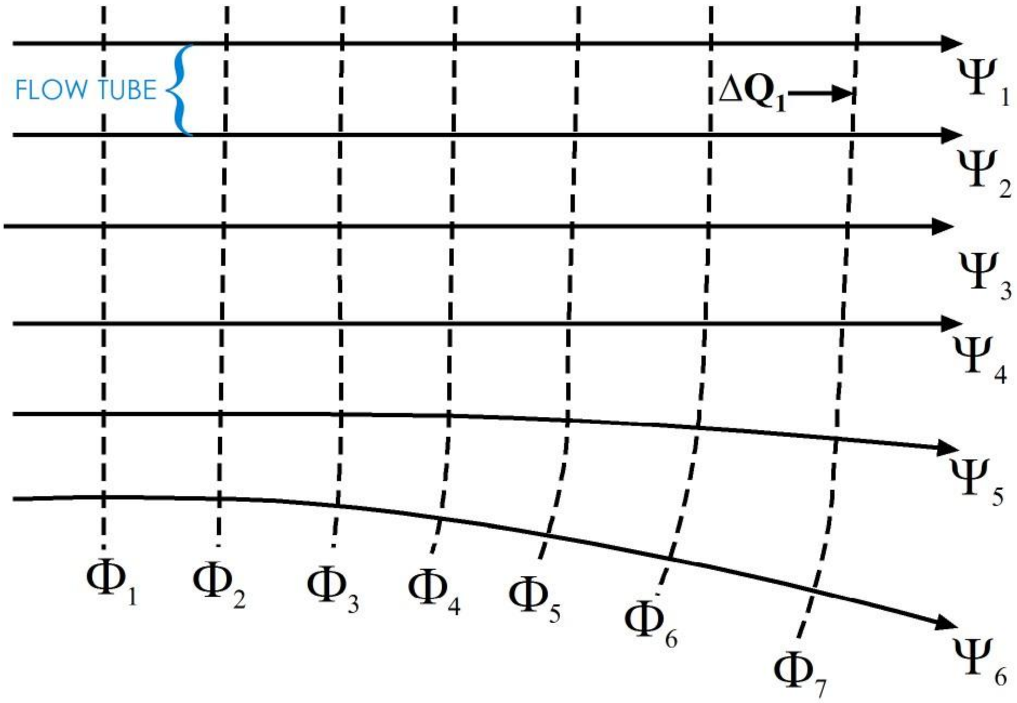

Figure 31 - Sample flow net showing the relationship between gradient and flow tube width. The solid lines (Ψ) are flow lines. The dashed lines (Φ) are equipotentials. ΔQ_1 is the discharge in one flow tube defined by two flow lines.

When drawing a flow net, the first thing to do is identify the boundary conditions and draw those to scale on the area where the flow net is to be drawn. Usually, when creating a map-view flow net the flow net will be drawn directly onto a base map; if so, the boundary conditions will likely be readily apparent as indicated by the position of water bodies and low-permeability materials. Boundary conditions must be dealt with as shown in Figure 32a,b, with no-flow boundaries being flow lines and constant head boundaries being equipotential lines.

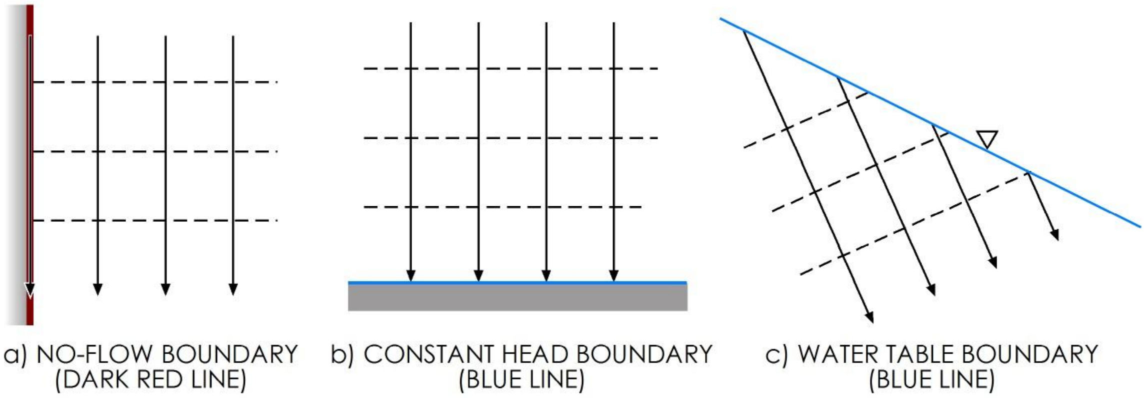

Figure 32 - Rules for dealing with boundaries in a flow net: a) no-flow boundaries are flow lines; b) constant head boundaries are equipotential lines, equipotential lines intersect the water table at the elevation equal to the equipotential line value.

The no-flow boundary represents a flow line—flow along the boundary is parallel to the boundary, and there is no gradient perpendicular to the boundary. No-flow boundaries on a flow net represent known flow lines; therefore, those flow lines should be drawn in first and used to constrain subsequent flow lines.

Constant head boundaries, like lakes and streams, are points where the head does not change. For this reason, a constant-head boundary represents either a single equipotential value (in the case of a lake or pond), or it represents a set of known elevation points (in the case of a stream or river). For lake/pond boundaries, the flow lines always intersect the boundary at right angles, and the nearest equipotentials should be nearly parallel to the boundary. For streams, the equipotentials come into the stream at a tangent—at the point where the line and stream meet, they are parallel. Streams can be difficult to deal with when constructing a rigorous flow net. For simple groundwater contour maps, we need to remember the rules concerning gaining and losing streams. That is, equipotential lines form a vee in the upstream direction for gaining streams, and they form a vee in the downstream direction for losing streams

Water table boundaries (Figure 32c) are often used in constructing a flow net on a cross section. A classic cross-sectional flow net is flow through an earthen dam (Figure 33).

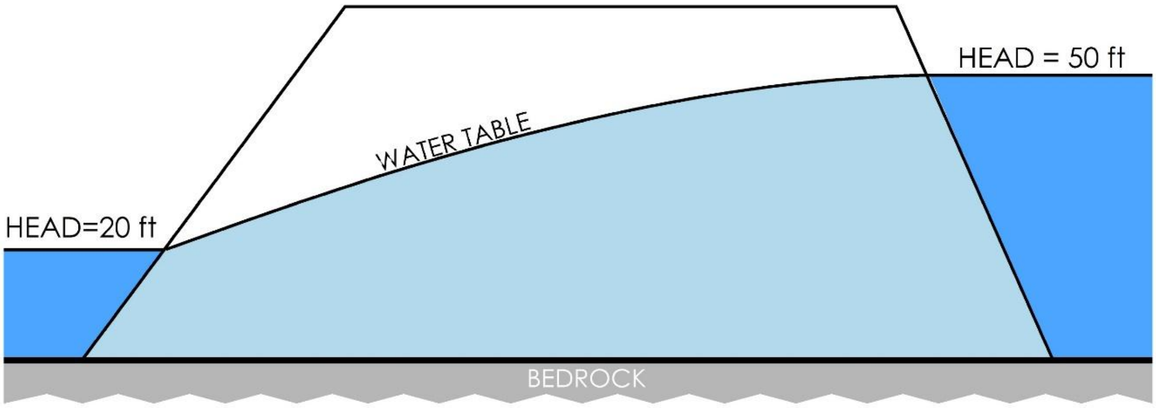

Figure 33 - Cross section showing flow through an earthen dam.

Once the flow net is drawn, the discharge through the flow net can be calculated by the formula shown as Equation (57).

$$q^1 = \frac{Kdh}{f} \tag{57}$$

where (parameter dimensions are dark green font with mass as M, length as L, time as T):

q^1 = total volumetric discharge per unit width of aquifer (L^3T^{-1})

d = number of flow tubes (areas bounded by adjacent pairs of flow lines) (:)

h = total head loss over the length of the flow lines (L)

f = number of equipotential drops (:)

If reliable estimates of the total discharge through the flow net are available, then the flow net can potentially be used to estimate the hydraulic conductivity for the aquifer (Exercise 7⤵).

More detailed instructions for constructing flow nets, including examples and exercises, are provided in *Graphical Construction of Groundwater Flow Nets*↗ by Poeter and Hsieh (2020) which is available from The Groundwater Project as a free download or can be read online.

7 Well Hydraulics

7.1 Introduction

We have discussed the flow equations so as to understand, both conceptually and mathematically, how groundwater flows in a porous medium. The main equation that we developed and discussed is a partial differential equation that governs flow at a given point under all conditions (assuming that Darcy's law is valid). In essence, it is the all-purpose equation that covers all situations. In order to solve this differential equation for a natural groundwater system, we need to apply it to a specific situation and make some simplifying assumptions. One specific situation that the equation has been solved for is flow of water in an aquifer to a well. In this section, we discuss the concepts and mathematics of well hydraulics, or flow to a pumping well.

7.2 Background Information on Wells

A *well* is a general, all-encompassing term for a hole drilled into the ground for the purpose of accessing fluids in the subsurface. Wells can be constructed to remove fluids (oil wells, water supply wells), to add fluids (waste and brine injection), or to sample and monitor the naturally occurring fluids (monitoring wells). Wells have also been used to recover mineral resources in the subsurface; for example, subsurface sulfur deposits can be accessed by injecting extremely hot water, dissolving the sulfur, and pumping the concentrated sulfur solution up to the surface for recovery. Wells can also be used to access energy resources directly; for example, geothermal energy can be collected by injecting water into thermally active zones and recovering the heated water.

A well that is designed and installed for the sole purpose of measuring water levels in an aquifer is called a *piezometer*. Piezometers can be much smaller in diameter than water wells and are generally 1.25 to 2 inches (3 to 5 cm, i.e., 0.03 to 0.05 m) in diameter. Piezometers can also be called *monitoring wells*. Monitoring wells are usually at least 2 inches (0.05 m) in diameter and are generally constructed for the purpose of periodic sampling of water to determine water quality; for example, in contaminated aquifers that require monitoring through time.

Wells are an important aspect of applied hydrogeology, both from a practical standpoint (e.g., water supply wells, dewatering wells) as well as for protection of humans and ecosystems and for scientific study (e.g., water levels, water sampling). Wells are the way we look into the subsurface and gather bits of information that we can use to understand the groundwater and the aquifer through which it flows.

7.2.1 Basic Well Construction

Historically, wells were dug by hand and the walls were lined with rocks to keep the well clear and open. These days, there are several safer and more efficient ways to drill a hole in the ground and install a well. Basic drilling methods include the following.

- Cable tool: Uses a weighted drill bit that is repeatedly raised and dropped by a cable, like a pile driver. It may seem archaic, but it is an effective method and results in minimal disturbance in the subsurface.
- Water or mud rotary: The drilling rig turns a drill bit and circulates a drilling fluid (either water or *drilling mud*[3]) into the subsurface through the borehole. The drilling fluid lubricates and cools the bit and carries drill cuttings to the surface.
- Air rotary: Like the fluid rotary methods, except air is used, and the bit (called a *down-hole hammer*) acts like a jackhammer run on compressed air. The air carries the cuttings to the subsurface.
- Auger: The drilling rig rotates either a hollow-stem or solid-stem continuous flight auger without drilling fluids. Good for installation of shallow monitoring wells and piezometers.
- Direct push: Uses the weight of the drill rig and hydraulic power to push the bit into the subsurface without rotation.
- Jet drilling: Water is pumped under pressure down a small diameter pipe with a special bit; the pressurized water loosens sediments and rock that are carried out by the circulating water. The bit can also be lifted and dropped (like a cable tool rig) during drilling to help the bit advance.
- Sonic drilling: Uses high-frequency vibrations to advance the drill bit into the formation. The advantage to this method is that no fluids are needed and there is minimal damage to the formation along the sides of the borehole.

Each one of these methods has advantages and disadvantages and is appropriate for certain applications. Auger, direct push, and jet drilling methods are generally only used in unconsolidated deposits (i.e., loose sediments) rather than solid rock.

Once the borehole is drilled, a well can be installed. In certain situations, it may be acceptable to leave the borehole open and start using it—for example, if you drilled a borehole in the middle of competent bedrock directly through the outcrop. In most cases, however, there is a possibility that the walls of the borehole may collapse inward and block the well. Generally, at a minimum, a short pipe called a *surface casing* is installed and sealed into the top of the borehole to prevent water, surface materials, and biota entering the hole from the surface. In addition, there may be several *hydrostratigraphic units* in one borehole; and it may be preferable to keep the upper ones isolated from the producing zone. Sections

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[3] Drilling mud is a thick, heavy, and viscous fluid that is circulated through boreholes during drilling to cool and lubricate the drill bit, bring drill cuttings to the surface, and keep the open borehole from collapsing.

of pipe, called *casing*, are then placed in the well to keep the well open and isolate the upper units. If the producing zone is composed of unconsolidated, or poorly consolidated, sediments, or if it has some fraction of fine sediment associated with it, a *well screen* and/or *filter pack* must be placed in the producing zone to keep fine particles out and let the water in. Figure 34 shows examples of several geologic scenarios and different casing/screen setups.

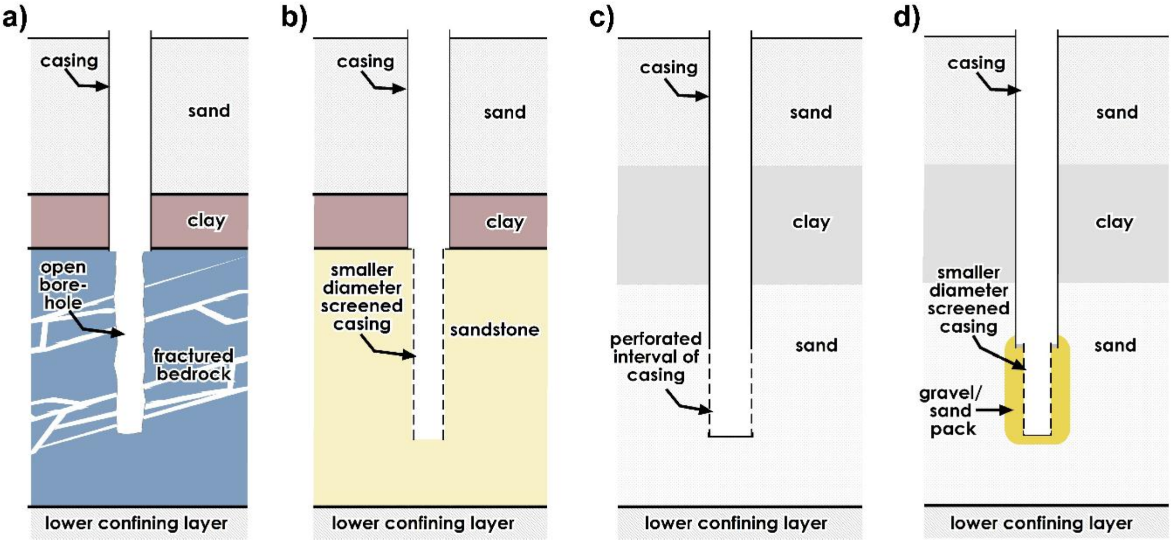

Figure 34 - Examples of some typical well construction scenarios.

Construction of monitoring wells is more rigorous than construction of water wells. In a water supply well, the objective is to produce as much water as possible; therefore, it is advantageous to have a long screen length that covers as much of the saturated thickness of the aquifer as possible. In a monitoring well, however, it is important to design the well to be precisely completed in the hydrostratigraphic unit which needs monitoring. Several decades of shallow groundwater environmental assessments and remediation projects have generated relatively strict state and federally mandated protocols concerning the installation of monitoring wells.

Figure 35 shows typical monitoring well construction. Monitoring wells and piezometers usually consist of 1.25- to 2-inch (0.03 to 0.05 m) diameter PVC pipe with threaded fittings and a cap at the bottom of the hole. The PVC pipe will have a section in the aquifer that has thin slots cut into it; this section of pipe is called the *screened interval* or *slotted interval*. Screened interval is a general term that we use to describe the length of the well that is open to the aquifer. The open borehole around the screened interval is backfilled with clean, well-sorted sand, called the *sand pack* or *filter pack*. The sand pack should fill the *annulus* (the space between the well casing/screen and the edge of the borehole) to a point at least a foot above the top of the screen. The sand pack is then covered with a seal, usually a smectite clay called *bentonite*, which is purchased as dry pellets or a granular powder that swell when hydrated. This seals off the well screen from the upper part of the borehole and keeps surface water or water from overlying formations from moving down the borehole into the monitored zone. The remainder of the borehole is then backfilled with either cement, drill cuttings, or some combination of the two. The well must then be capped with a secure access cover.

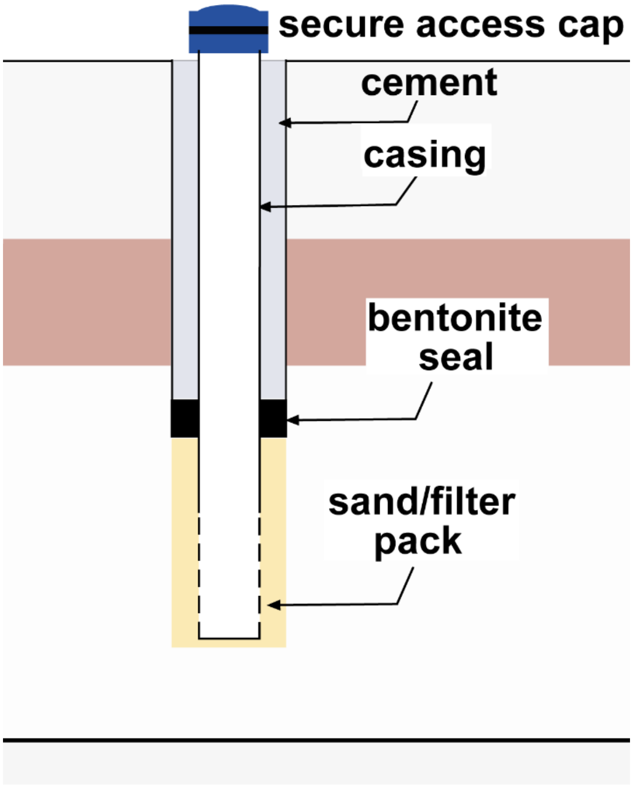

Figure 35 - Typical monitoring well construction.

Shallow monitoring wells are usually installed using hollow-stem augers. Once the required depth is reached, the PVS pipe is put together and inserted down the center of the auger. Sand is then poured in around the pipe, and the augers are pulled up a little bit so that the sand falls into the open borehole. This process is repeated until the screen is covered and the sand is in place. Then the bentonite pellets are added, and the augers are pulled out of the ground. If the bentonite seal is emplaced above the water table, water can be poured in to hydrate the bentonite. The rest of the hole is typically backfilled with cement, and the top of the well is secured with a cap and frequently enclosed within a manhole or protective metal vault.

7.2.2 Well Development

One of the big concerns with well construction is the amount of disturbance to the subsurface caused by drilling. Regardless of the method, the aquifer formation is always affected in some way. Problems related to disturbance include the following.

- Lots of fine material may get into well screen and filter pack, which can clog or fill (known as silting up) the well.
- Drilling fluids (especially muds) flow into the formation and may clog pores.
- The drill bit can smear the walls of the borehole, closing off pores.

- A large volume of water and drilling fluids must be removed from the formation before samples representative of the formation can be obtained.

Following drilling and installation of the well, the effects of this disturbance must be corrected as best as possible. This process is called *development* of the well. Well development in water supply wells with relatively high yields generally entails pumping water from the well for an extended period, until the water runs clear. In low-yield monitoring wells, if there is not enough water to sustain pumping then it may not be an option. Plungers can be put into wells to force water in and out of the screen for a period. Compressed air can also be forced down into the well to lift water and fine material out of the well screen. Usually, with monitoring wells, any water that was introduced during drilling must be removed so that groundwater samples accurately represent the formation.

US Geological Survey Water-Resources Investigations Report 96-4233 (Lapham et al., 1996) is a good general source of information concerning well construction and development. This report is available as a free download from the USGS.

7.3 Well Hydraulics

Now that we have discussed wells, we discuss how groundwater flows to wells. To understand the hydraulics behind flow to wells, we need to first simplify the system and start with some basic assumptions that are listed here.

1. The aquifer is homogeneous and isotropic.
2. The aquifer is bounded on the bottom and top by confining layers.
3. All geologic formations are horizontal and have infinite horizontal extent.
4. The potentiometric surface of the aquifer is horizontal prior to the start of pumping, and water levels are at steady state.
5. Any changes to the position of the potentiometric surface are caused only by the pumping well.
6. All flow in the aquifer is horizontal.
7. All flow to the well is radial.
8. Darcy’s law is valid.
9. Groundwater has a constant density and viscosity.
10. All wells are fully penetrating.
11. The diameter of the pumping well is negligible.

Generally, in practice, well hydraulics solutions can be applied even when these assumptions are not rigorously met, but they are needed to be precise with respect to the mathematical development. We now use the groundwater flow equation that we developed in previous sections to describe groundwater flow towards and into a pumping well.

7.3.1 Horizontal Radial Flow

We have discussed and developed the equation that governs the flow of groundwater in three dimensions; as shown by Equation (58).

$$\frac{\partial q_x}{\partial x}+\frac{\partial q_y}{\partial y}+\frac{\partial q_z}{\partial z}=S_s\frac{\partial h}{\partial t} \tag{58}$$

where (parameter dimensions are dark green font with mass as M, length as L, time as T):

q = specific discharge (i.e., groundwater flow) in the x, y, or z direction (LT^{-1})

x, y, z = lengths in the principal directions in our coordinate systems

h = hydraulic head (L)

t = time (T)

S_s = specific storage (L^{-1})

We substituted Darcy's law—shown as Equation (59)—for q.

$$q=-K\frac{dh}{dl} \tag{59}$$

where (parameter dimensions are dark green font with mass as M, length as L, time as T):

K = hydraulic conductivity (LT^{-1})

dh/dl = hydraulic gradient, or change in head with distance, l (:)

The result is Equation (60).

$$\frac{\partial\left(K_x\frac{\partial h}{\partial x}\right)}{\partial x}+\frac{\partial\left(K_y\frac{\partial h}{\partial y}\right)}{\partial y}+\frac{\partial\left(K_z\frac{\partial h}{\partial z}\right)}{\partial z}=S_s\frac{\partial h}{\partial t} \tag{60}$$

Now, if we assume the aquifer is homogeneous and isotropic (as discussed in Section 5—*More on Flow Equations*), but not necessarily steady-state, the result appears as Equation (61).

$$K\left(\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}+\frac{\partial^2 h}{\partial z^2}\right)=S_s\frac{\partial h}{\partial t} \tag{61}$$

Assumption #6 states that all flow is horizontal. This means that there is no vertical component of flow, and therefore, the z term can be eliminated to obtain Equation (62).

$$K\left(\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}\right)=S_s\frac{\partial h}{\partial t} \tag{62}$$

Now, recalling that *transmissivity* (T) is defined as $T = Kb$ (where b is the saturated thickness of the aquifer) and *storativity* (S) is defined as $S = S_s b$, we multiply both sides by b, to get Equation (63).

$$Kb\left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2}\right) = S_s b \frac{\partial h}{\partial t} \tag{63}$$

Then, substituting with S and T, and dividing both sides by T, we obtain Equation (64).

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = \frac{S}{T}\frac{\partial h}{\partial t} \tag{64}$$

If the aquifer is receiving recharge, either from infiltration of rain in an unconfined aquifer, or from leakage through a confining layer in a confined aquifer, we can add that directly to the equation and get Equation (65).

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{w}{T} = \frac{S}{T}\frac{\partial h}{\partial t} \tag{65}$$

where (parameter dimensions are dark green font with mass as M, length as L, time as T):

w = rate of vertical leakage or recharge (LT^{-1})

Now, the thing about flow to a well is that it is *radial*; in other words, flow lines are converging on the well from all sides (Figure 36). Because the flow lines converge, the equipotential lines—indicated by dashed lines on Figure 36 get closer together near the well. This represents the *cone of depression* that forms around the well.

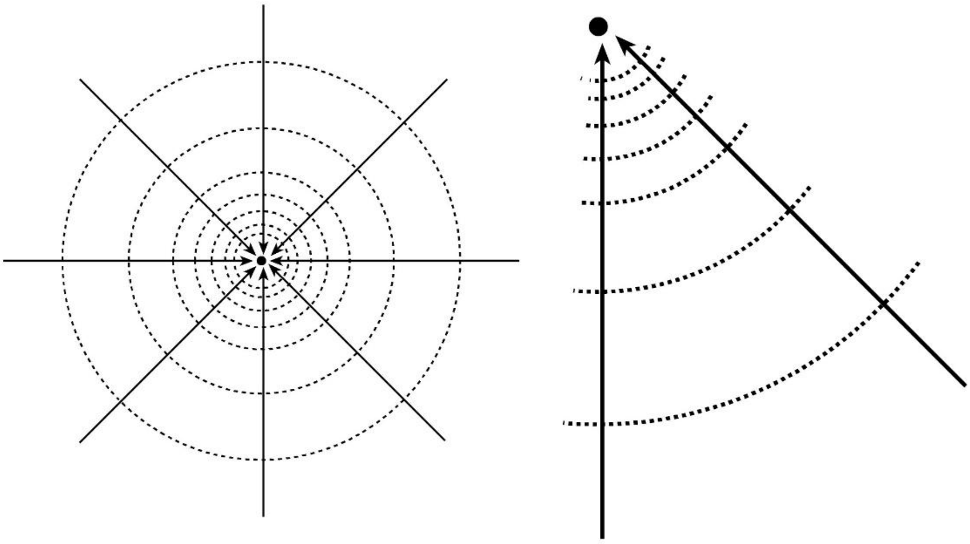

Figure 36 - Radial flow to a well. The dashed lines are equipotential lines that are lowest closest to the well, which is represented by the dot in the center.

Now, because we are dealing with radial flow to the well, it helps to express the equations in terms of a *polar (or cylindrical) coordinate system* rather than the simple Cartesian

(x, y) coordinate system. In polar coordinates, a point is defined by its location with respect to a fixed central pole (in this case, the well). The location of a point is defined by its radial distance from the pole (r) and the angle (θ) between the radial line and some fixed polar axis (for example, north). Polar coordinates are graphically described in Figure 37.

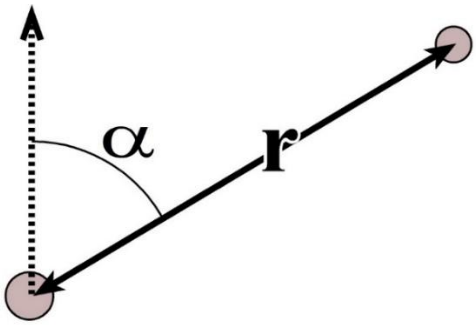

Figure 37 - Polar coordinate system.

We can use the Pythagorean Theorem and some basic geometry to re-state our two-dimensional flow equations in polar coordinates (Box 1↴ shows a complete derivation). When we do so, we have Equation (66) and Equation (67).

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} = \frac{S}{T}\frac{\partial h}{\partial t} \tag{66}$$

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} + \frac{w}{T} = \frac{S}{T}\frac{\partial h}{\partial t} \tag{67}$$

Equation (67) is a differential equation that describes radial horizontal flow to a well based on a polar coordinate system. Since it is a differential equation, it requires some complex mathematical operations—like Fourier transforms, Bessel functions, and Laplace transforms—to solve. Fortunately for us, these solutions have already been done, and all you need at this point is some understanding of what they mean and how they are used.

7.4 The Theis Equation

In Section 3, we discussed that the mathematics governing the flow of groundwater—that is, the mathematics showing that groundwater discharge is proportional to a gradient—are identical to those that govern many other physical processes like the flow of heat in a solid, the diffusion of solutes in solution, and electrical current through a conductive solid. In the 1930s a civil engineer, C. V. Theis, noted the similarity between groundwater flow and heat flow, and he noted that pumping a well was mathematically analogous to heating a block of metal with a thermally conductive rod (Figure 38).

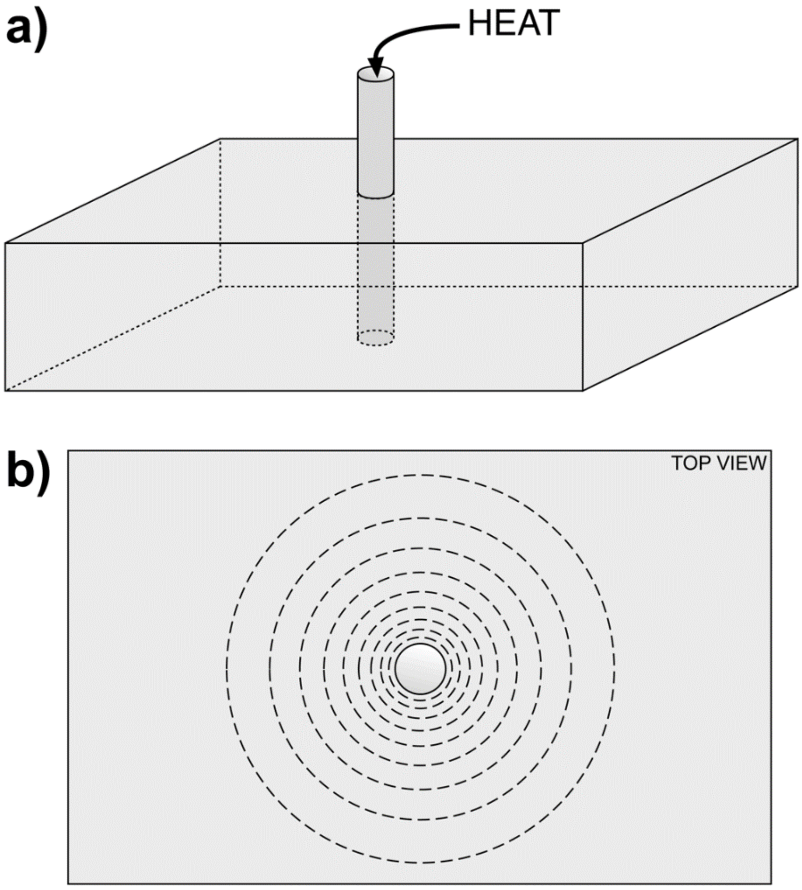

Figure 38 a) Metal rod heating a metal plate, and b) contour lines of equal temperature with higher temperature near the rod.

This situation assumes that the block has a constant thickness and an infinite extent—even though it is drawn here as a block with finite proportions. If we start with the block at a constant temperature, then begin to heat the rod with a constant temperature, heat will be transferred to the block. It will take time for heat to move through the block, and the rate of movement of heat is dependent upon the thermal conductivity of the metal.

The part of the block immediately adjacent to the rod will heat up first, then heat will flow through the block and, over time develop a distribution of temperatures like the contours shown in Figure 38b. Conversely, if we start with a warm block of metal and cool the end of the rod, heat will flow out of the metal into the rod and set up a similar distribution of temperatures with lower values near the rod. Mathematically this is the same thing that happens in an aquifer pumped by a well, except that water is flowing instead of heat, and a distribution of heads form instead of a distribution of temperatures. That distribution of heads in an aquifer is the cone of depression.

Theis (1935) developed this idea mathematically as shown in Equation (68).

$$s = \frac{Q}{4\pi T}\mathrm{W}(u) \tag{68}$$

The value of u is defined in Equation (69).

$$u = \frac{r^2 S}{4Tt} \tag{69}$$

$W(u)$ is the well function, which is the integral of u from u to infinity, as shown in Equation (70).

$$W(u) = \int_u^\infty \frac{e^{-u} du}{u} \tag{70}$$

$W(u)$can also be expressed as the infinite series in Equation (71).

$$W(u) = -0.5772 - \ln u + u - \frac{u^2}{2 * 2!} + \frac{u^3}{3 * 3!} - \frac{u^4}{4 * 4!} + \frac{u^5}{5 * 5!} - \mathit{and\ so\ on} \ldots \tag{71}$$

where (parameter dimensions are dark green font with mass as M, length as L, time as T):

s = drawdown (L), defined as the initial head minus the head at time t

Q = pumping rate from the well (L^3T^{-1}); assumed to be constant

T = aquifer transmissivity (where $T = Kb$) (L^2T^{-1})

r = radius (distance from pumping well) (L)

S = storativity (where $S = S_s b$) (:)

t = time after the start of pumping (T)

An important thing to note at this point is that we are describing a transient situation; in other words, we are interested in understanding how *heads* in an aquifer change *with respect to time* when water is *pumped from a well at a constant rate*. That is what the Theis equation calculates—the drawdown at a point in the aquifer some distance from the pumping well (r) at some time (t) after pumping begins in response to a well pumping at a constant rate (Q). If one knows the transmissivity and storativity of a confined aquifer, then the equation can be used to predict how much drawdown a pumping well will generate in that aquifer at some distance from the pumping well and at some time after the start of pumping (Exercise 8⤵).

Solving the Theis equation requires values of W(u) for corresponding values of u. A table of values of W(u) for values of u is provided as Table 4 (Exercise 9⤵).

Table 4 **-** Values of W(u) for values of u. For example, for u=4x10^{-7}, W(u)=14.15.

u	**1.0**	**2.0**	**3.0**	**4.0**	**5.0**	**6.0**	**7.0**	**8.0**	**9.0**
1x10^{-0}	0.219	4.9x10^{-2}	1.3x10^{-2}	3.8x10^{-3}	1.1x10^{-3}	3.6x10^{-4}	1.2x10^{-4}	3.8x10^{-5}	1.2x10^{-5}
1x10^{-1}	1.82	1.22	0.91	0.70	0.56	0.45	0.37	0.31	0.26
1x10^{-2}	4.04	3.35	2.96	2.68	2.47	2.30	2.15	2.03	1.92
1x10^{-3}	6.33	5.64	5.23	4.95	4.73	4.54	4.39	4.26	4.14
1x10^{-4}	8.63	7.94	7.53	7.25	7.02	6.84	6.69	6.55	6.44
1x10^{-5}	10.94	10.24	9.84	9.55	9.33	9.14	8.99	8.86	8.74
1x10^{-6}	13.24	12.55	12.14	11.85	11.63	11.45	11.29	11.16	11.04
1x10^{-7}	15.54	14.85	14.44	14.15	13.93	13.75	13.60	13.46	13.34
1x10^{-8}	17.84	17.15	16.74	16.46	16.23	16.05	15.90	15.76	15.65
1x10^{-9}	20.15	19.45	19.05	18.76	18.54	18.35	18.20	18.07	17.95
1x10^{-10}	22.45	21.76	21.35	21.06	20.84	20.66	20.50	20.37	20.25
1x10^{-11}	24.75	24.06	23.65	23.36	23.14	22.96	22.81	22.67	22.55
1x10^{-12}	27.05	26.36	25.96	25.67	25.44	25.26	25.11	24.97	24.86
1x10^{-13}	29.36	28.66	28.26	27.97	27.75	27.56	27.41	27.28	27.16
1x10^{-14}	31.66	30.97	30.56	30.27	30.05	29.87	29.71	29.58	29.46
1x10^{-15}	33.96	33.27	32.86	32.58	32.35	32.17	32.02	31.88	31.76

8 Aquifer Testing

8.1 Introduction

Section 7 discusses the radial flow of water to a well and introduces the Theis equation. We show how, given a transmissivity and storativity, the Theis equation can be used to calculate the amount of drawdown that would occur in the aquifer at some distance from a well (pumping at a constant rate) after some period. The Theis equation, however, is more commonly used to do the opposite—that is, to use drawdown data from a pumped well to estimate transmissivity and storativity of the formation. This section focuses on *aquifer tests*[4]: tests that involve pumping from a well, measuring drawdown in the aquifer during pumping, then using those data and the Theis equation to estimate aquifer parameters.

8.2 Well Hydraulics—A Quick Review

Flow to a well is a special case because flow is *radial*—that is, converging on a specific point in the aquifer (the well). To handle this mathematically, C.V. Theis derived the equation that governs 2-D radial flow to a well, as shown in Equation (72).

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} + \frac{w}{T} = \frac{S}{T}\frac{\partial h}{\partial t} \tag{72}$$

Theis then *'stole'* an analytical solution for this equation from the heat flow literature to develop his eponymous equation, which is shown in Equation (73) and where u is defined by Equation (74), and $\mathrm{W}(u)$—the *well function*—is as shown in Equation (75).

$$S = \frac{Q}{4\pi T} W(u) \tag{73}$$

$$u = \frac{r^2 S}{4Tt} \tag{74}$$

$$W(u) = \int_u^\infty \frac{e^{-u} du}{u} \tag{75}$$

where (parameter dimensions are dark green font with mass as M, length as L, time as T):

s = drawdown (L) is defined as the initial head minus the head at time t

Q = pumping rate from the well ($\mathrm{L^3T^{-1}}$) is assumed to be constant

T = aquifer transmissivity ($T = Kb$) ($\mathrm{L^2T^{-1}}$)

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[4] It is acceptable to refer to these types of tests as *aquifer test*, *hydraulic test*, or *pumping test*. It is not, however, appropriate to use the term *pump test*, as that implies that one is testing the *pump* rather than the *aquifer*.

r = radius (distance from pumping well) (L)

S = storativity (where $S = S_s b$) (:)

t = time after the start of pumping (T)

The Theis equation calculates *drawdown* (s) at some distance (r) from the pumping well at some time (t) after pumping began for a constant pumping rate (Q). This equation describes the *cone of depression* that forms around a pumping well. If we know T and S, we can use this equation to directly calculate the drawdown at any distance from the well at any time after the start of pumping.

But the problem is how do we measure T and S in the first place? It turns out that one of the best ways that to measure T and S in an aquifer is to pump water from a well at a constant rate, measure drawdown in the aquifer at various times after pumping starts, and then use these data and the Theis equation to calculate T and S. Doing this is called an *aquifer test*, or more specifically a *constant rate aquifer test*.

8.3 Aquifer Tests and Ideal Drawdown Curves

8.3.1 Multi-Well Aquifer Test

The following is a general procedure for conducting a multi-well constant rate aquifer pumping test which, if properly performed, can be used to develop reliable estimates of T and S for the producing formation. To do this, you need the following.

- A *pumping well* with a pump that allows you to control and measure the pumping rate.
- One or more *observation wells* close enough to the pumping well to experience a measurable drawdown induced by pumping from the pumping well.
- Some means of measuring water levels in the pumping and observation wells at specific times throughout the course of the test; these should be some combination of automatic water level meters and manual measurements.
- A sufficient amount of time to pump the well that will result in a measurable change in water level in the observation wells. Typically, tests are run for at least 12 to 24 hours, though it is not uncommon for pumping tests to be as short as 2 hours or as long as 30 days.
- A plan for properly disposing of the water discharged from the well. Care must be taken to avoid having discharged water flow back into the well, as this will compromise the test. Discharge should drain away from pavements and buildings to avoid flooding and damage to infrastructure. In addition, some areas may require permits or other regulatory compliance approval for discharge to existing streams or bodies of standing water.

The general procedure for running an aquifer test is presented here.

- Measure the *static water level*—that is, the initial water level before pumping begins—in all the wells for a period at least as long as the planned test to determine whether the water levels are stable.
- Start the pump, and immediately begin taking water level measurements in the wells at various intervals—starting with something on the order of every 30 seconds.
- As time progresses, the rate of drawdown will decrease, and measurements can be taken less frequently.
- Continue pumping for the pre-designated period while continuing to take water level measurements.
- Shut off the pump and measure the *recovery* of water in the pumping well—measure water levels at increments of time until the water level approaches the original static water level.

A properly documented multi-well aquifer test will produce the following data set:

- pumping rate;
- distance to each observation well from the pumping well; and a
- table of time versus drawdown measurements for the pumping well and for each observation well.

The time versus drawdown data, when plotted on a scatter plot and fit with a line, will form shape that depends on the type of aquifer and presence of boundary conditions within the cone of depression. These curves are used to calculate estimates of T and S. Before we get into that, there are some basic concepts that we need to cover concerning the drawdown response of different aquifers to pumping wells.

8.3.2 Drawdown Curves

When we discussed storage parameters, we presented the concept of *elastic storage*. This is the response of a confined aquifer to pumping; pressure in the aquifer is released, the aquifer structure compresses, grains and water molecules expand, and water is released from aquifer storage into the well. In a fully confined aquifer, all the water pumped out through the well is released from elastic storage, with no interformational leakage, recharge from above, or changes in saturated thickness. This will result in a curve on a plot of drawdown versus time that looks like Figure 39.

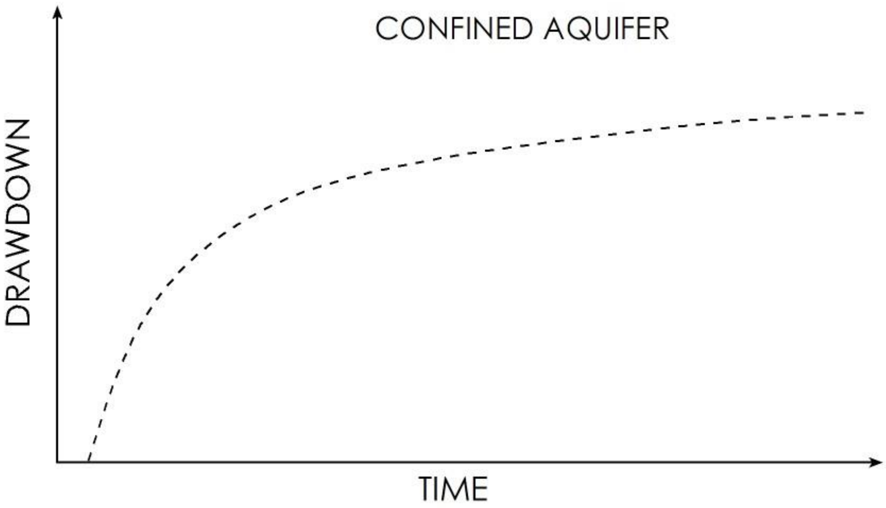


Figure 39 - Plot of drawdown versus time in an observation well installed in a confined aquifer in response to a well pumping from the same aquifer at a constant rate.

In Figure 40, the confined aquifer curve of Figure 39 (dashed line) is compared to the response for a confined aquifer with a leaky confining layer (solid line). In this case, the water that is leaking through the confining layer is adding water to the aquifer, replacing water that would otherwise need to be removed from storage and cause drawdown, so the drawdown is less than it is in the fully confined case. The initial part of the curve is like a fully confined curve, but when the leakage becomes significant, drawdown is reduced, and the drawdown curve deviates from the fully confined response. A similar response would occur in an aquifer in connection with a constant head boundary (e.g., a river or lake).

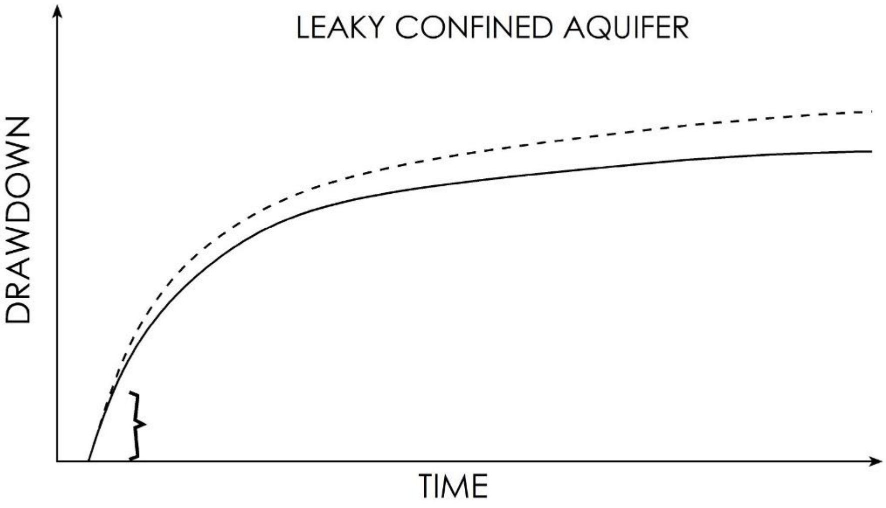


Figure 40 - Plot of drawdown versus time in an observation well in a leaky confined aquifer (solid line) in response to a well pumping at a constant rate. The dashed line is for all the same conditions except it is a fully confined aquifer (Figure 39). The bracket represents the period when nearly all water is derived from elastic storage before leakage from the recharge boundary is significant.

The second type of boundary previously discussed is the no-flow boundary. Figure 41 shows the response in a confined aquifer with a lateral no-flow boundary (solid line) compared to the fully confined, unbounded, aquifer response (dashed line). In an aquifer with a lateral no-flow boundary, the same amount of water moves toward the well but a smaller area contributes water from storage so there is more drawdown, especially in later-stages of the test. Like the previous scenario, the early stage of drawdown matches the fully confined curve, and then deviates from it when the influence of the boundary becomes significant.

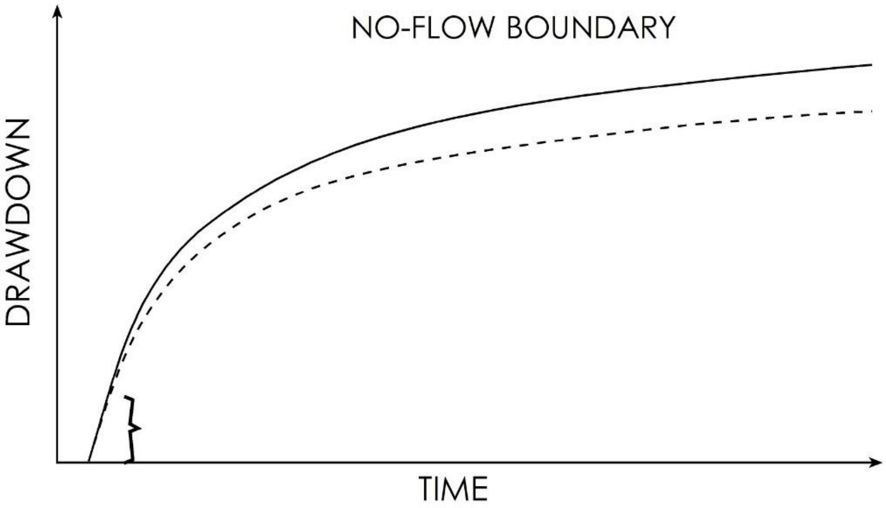


Figure 41 - Plot of drawdown versus time in an observation well in an aquifer influenced by a no-flow boundary (solid line) in response to a well pumping at a constant rate. The dashed line is for all the same conditions except it is a fully confined aquifer (Figure 39). The bracket represents the period when nearly all water is derived from elastic storage from a complete circle around the well before the zone between the well and the no-flow boundary decreases its contribution from storage.

The unconfined aquifer is a more complex system, in that the drawdown response (solid line in Figure 42) is affected by several processes.

1. The first stage represents release from elastic storage.
2. In the second stage, pores are dewatered and the declining water table creates vertical gradients in the flow system; these are equivalent to additional water leaking in from above.
3. In the third stage, vertical gradients decrease and depression of the water table significantly reduces the saturated thickness, and therefore, the transmissivity.

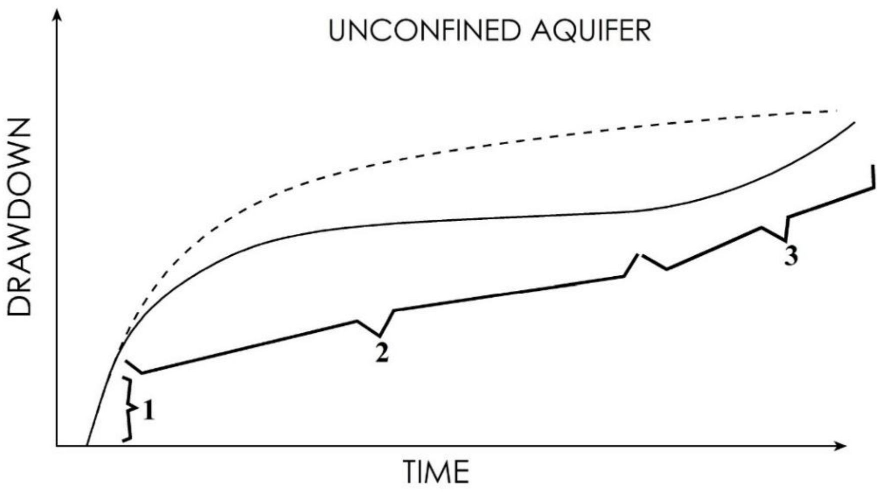


Figure 42 - Plot of drawdown versus time in an observation well in an unconfined aquifer (solid line) in response to a well pumping at a constant rate. The dashed line is for the fully confined aquifer of similar properties (Figure 39). Bracket 1 represents the period when nearly all water is derived from elastic storage near the well. Bracket 2 indicates the period of significant pore dewatering and downward flow from the water table. Bracket 3 shows the stage when pore dewatering slows and the lower water table decreases the aquifer transmissivity.

Thought Question 8*: The dashed curve in Figure 39 represents the Theis curve for drawdown versus time for an ideal confined aquifer. On a copy of Figure 39 add at least 15 points that represent time-drawdown data taken from an observation well during a constant-rate aquifer test such that the first 7-8 points match the Theis curve during the early part of the test, but the remaining points indicate drawdown increasing more rapidly with time than predicted by the Theis equation. How would you interpret this image – in other words, what does your graph tell you about the aquifer that you are testing?*

8.4 Aquifer Test Analysis

The following is a general, simplified discussion of the analysis of aquifer test data. A detailed and comprehensive overview of aquifer testing procedures and data analysis methods is provided in *An Introduction to Hydraulic Testing in Hydrogeology: Basic Pumping, Slug, and Packer Methods*↗ by Woessner and others (2020), which is available as a free download from The Groundwater Project. Professionals and students who are interested in aquifer testing are encouraged to download and review that book.

8.4.1 The Theis Curve-Matching Method for Non-Leaky (i.e., fully confined) Aquifers

The data set produced by a multi-well aquifer test with a single observation well includes a pumping rate, a distance from the pumping well to the observation well, and a set of time-drawdown data—that is, values for drawdown at various times after the start of pumping. Analyzing the data using the traditional Theis curve-matching method requires the following steps.

1. Obtain a Theis equilibrium type curve graph and a blank log-log graph that has the same scale as the Theis type curve. A blank log-log graph is provided as Figure 43 and the type curve with the same scale is provided as Figure 44.
2. Plot the time-drawdown data on the blank log-log graph. An example is shown in Figure 45a.
3. Overlay the time-drawdown data graph on the type curve—it is best to use a light table or tape the type curve on a window or white computer screen—and move the graph until the data line up with the type curve. It is essential that the axes on both graphs are parallel during the matching procedure; otherwise, the match will not be correct. An example is shown in Figure 46a
4. Select a corresponding point on each graph and determine the values of u and W(u) for that point on the time-drawdown graph. The point does not have to fall on the Theis curve; it only needs to be in an area where both graphs overlap so a common match point can be determined. An example is provided in Figure 46b.
5. Re-arrange the Theis equation—Equation (73)—to solve for T and substitute the values of s and $W(u)$ from the curve match point to determine T. The re-arranged equation is shown as Equation (76).

$$T = \frac{Q}{4\pi s} W(u) \tag{76}$$

6. Re-arrange the equation for u shown as Equation (74) to solve for S and substitute the values of t and u from the curve match point, and the value of T determined in step 5, to calculate S. The re-arranged equation is shown as Equation (77).

$$S = \frac{4Ttu}{r^2} \tag{77}$$

Figure 46 shows an example of a time-drawdown data set with a correct curve match. Figure 46a shows the data plotted on a log-log graph with a type curve plotted to the same scale. In Figure 46b the data graph has been overlain on the type curve such that the axes are parallel and the data form a best fit to the drawdown curve. Note that the data do not exactly fall on the type curve; there will always be some "noise" in a time-drawdown data set due to measurement limitations and variations in water level related to barometric pressure changes, surface stresses (e.g., vehicular traffic), and distant wells pumping the aquifer. In Figure 46b match point t (time) = 1 and s (drawdown) = 1 results in u = 0.31 and W(u) = 7. Figure 47 shows an example of an incorrect curve match with non-parallel axes (Exercise 10↴ and Exercise 11↴).

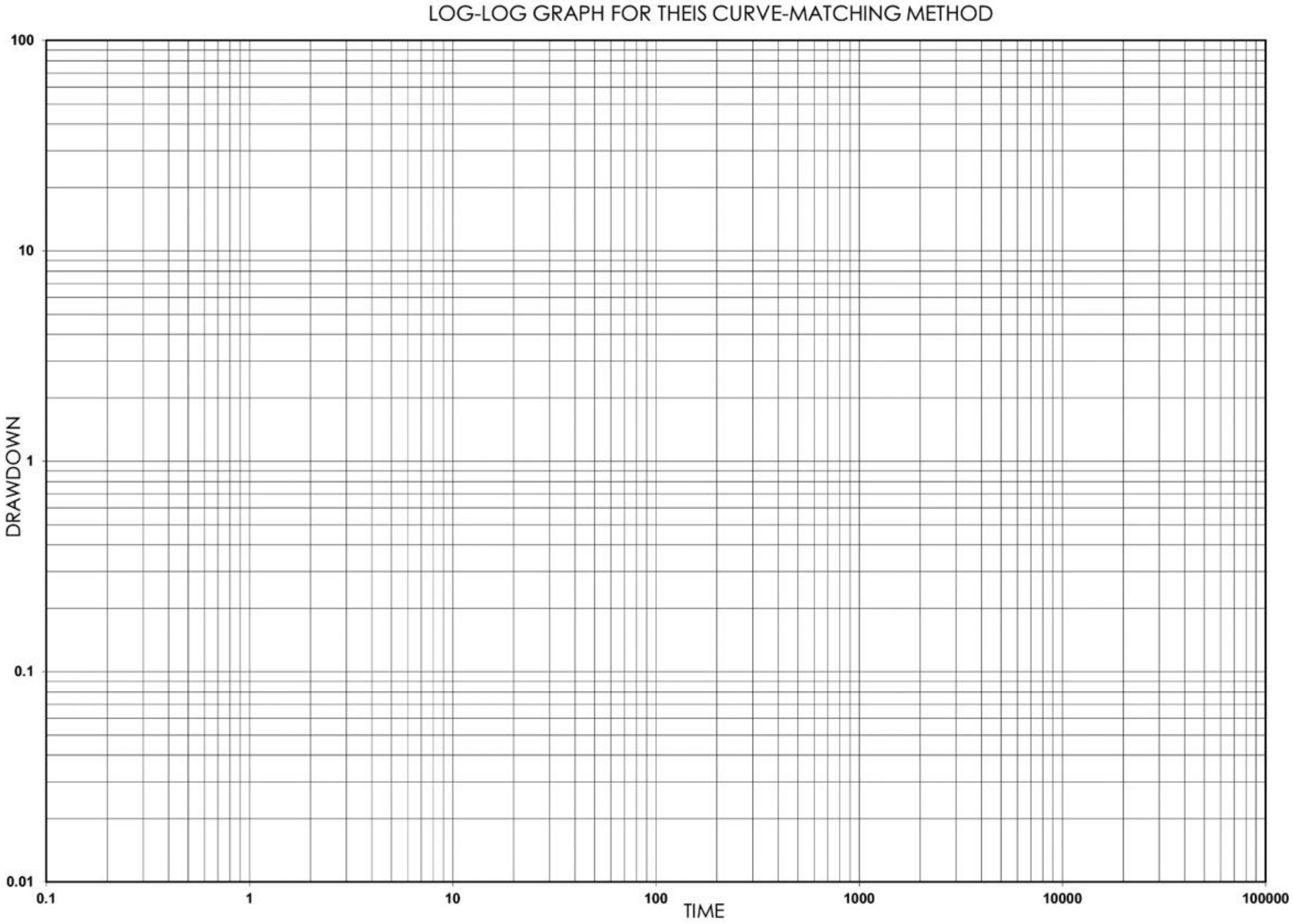


Figure 43 - Blank log-log graph paper with same scale and extent as type curve shown in Figure 44 for curve-matching.

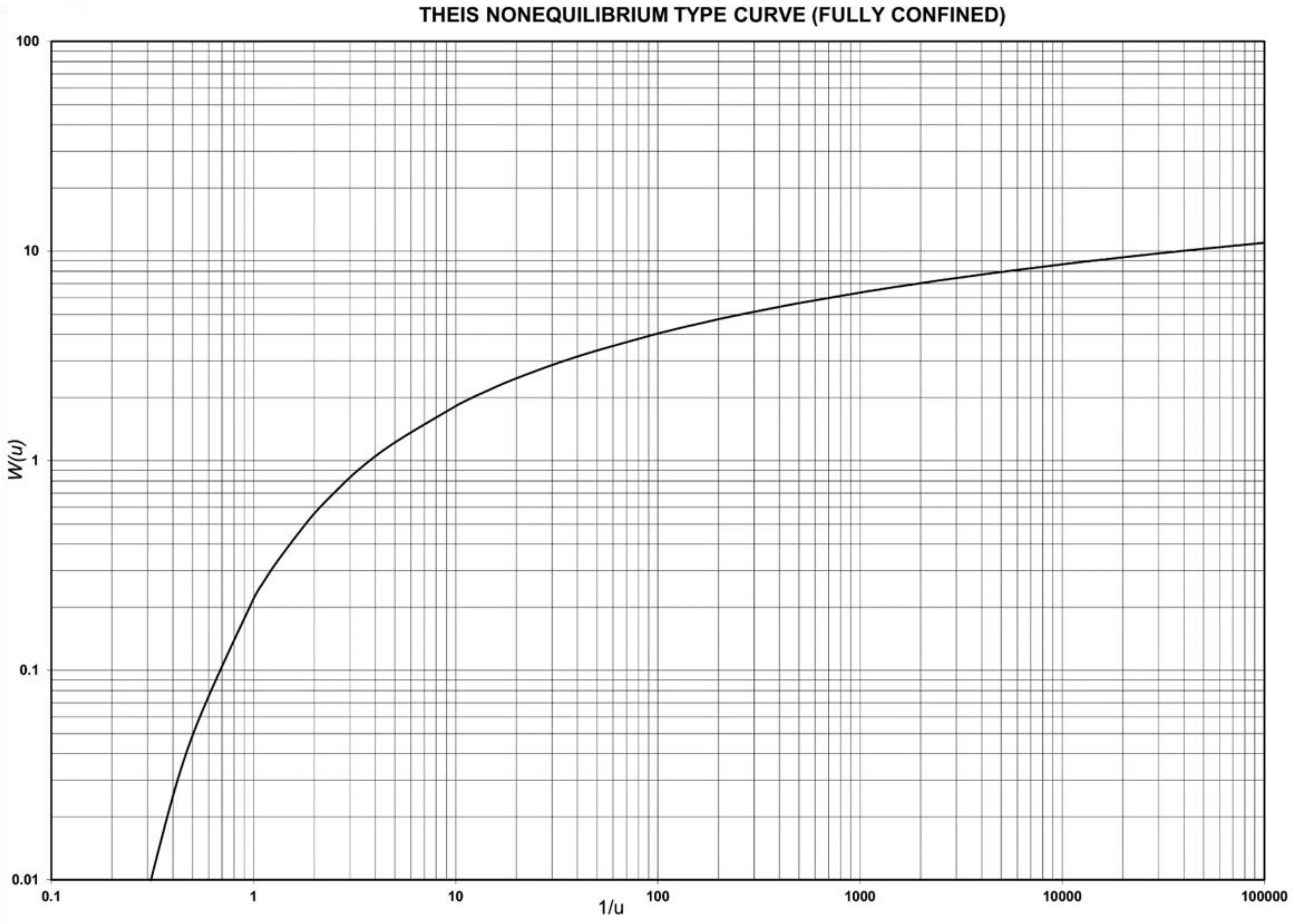


Figure 44 - Theis non-equilibrium type curve for non-leaky (i.e., fully) confined aquifers.

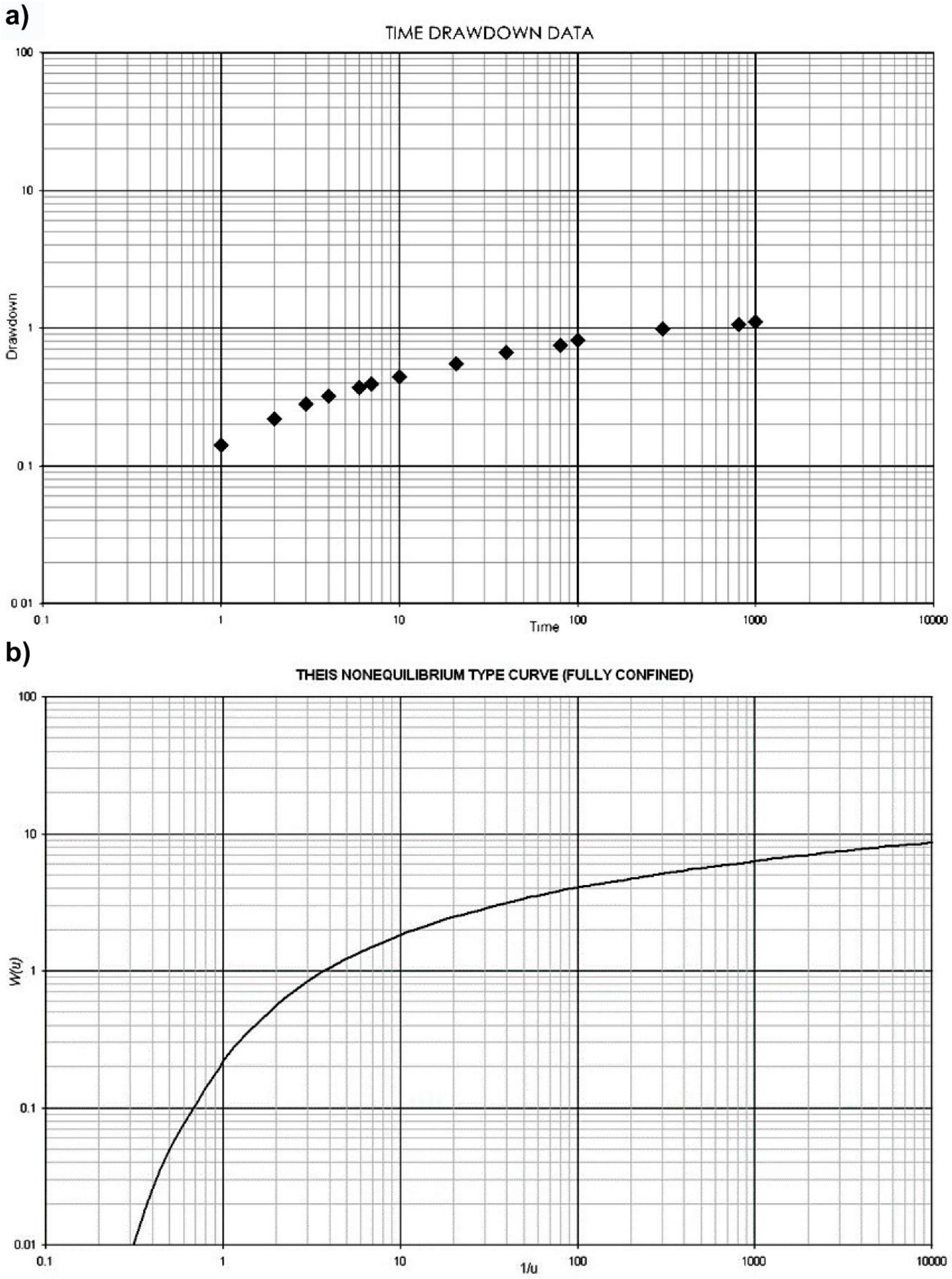


Figure 45 - Example data for curve-matching method: a) time-drawdown data from an aquifer test; b) Theis non-equilibrium type curve for non-leaky (i.e., fully) confined aquifers.

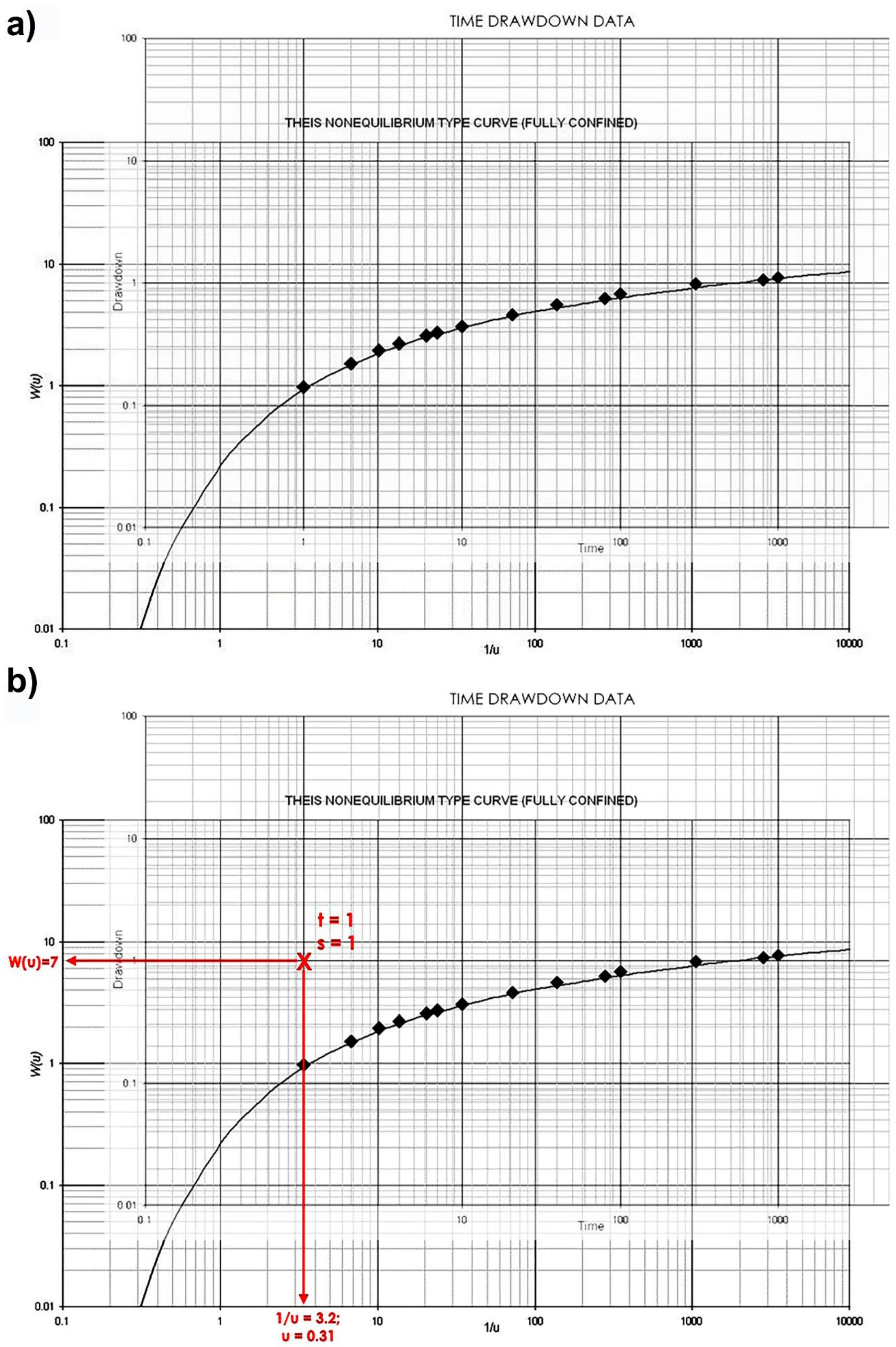


Figure 46 - Example of curve-matching method: a) plotted data overlain on type curve with best match of the data to the type curve; b) match point *t* (time) = 1 and *s* (drawdown) = 1 on data graph corresponds to *W(u)* = 7 and *u* = 0.31.

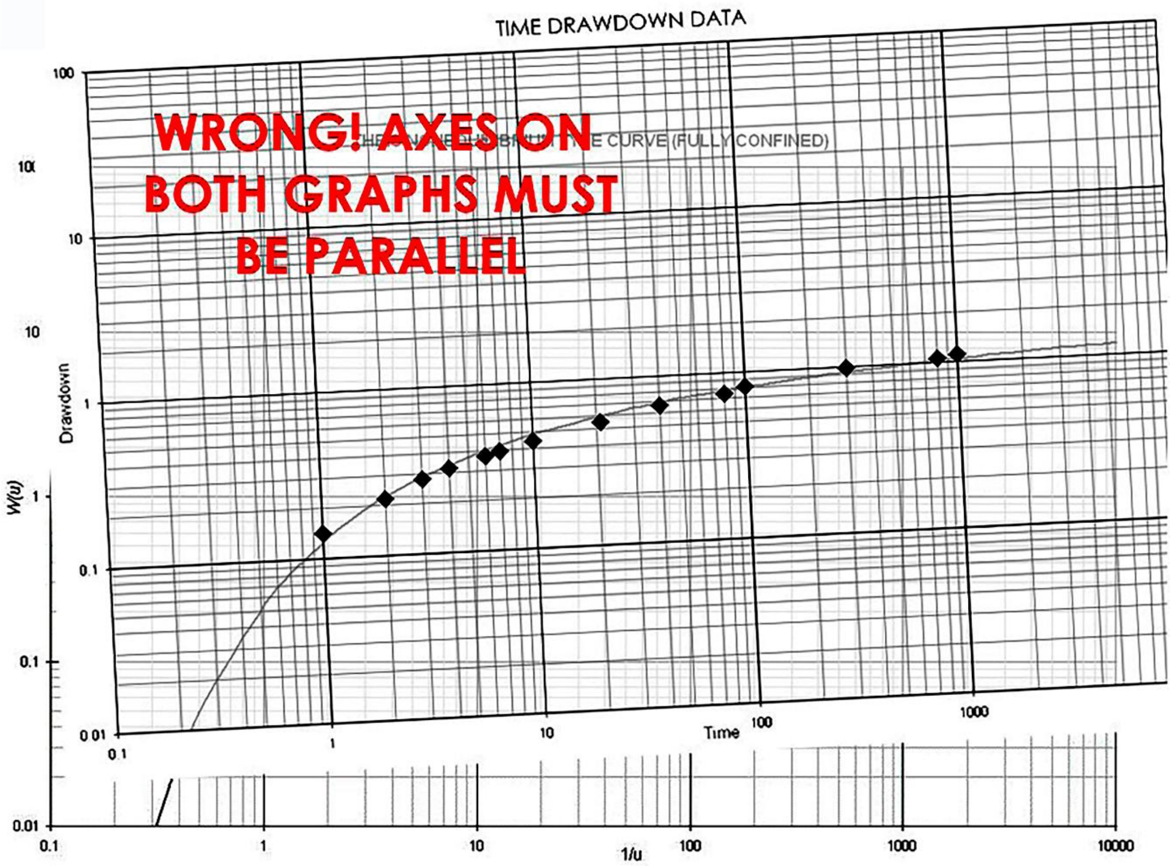


Figure 47 - Example of incorrect curve-matching due to failure to properly align the axes.

8.4.2 Leaky Confined and Unconfined Methods

As shown in Figure 40, the drawdown curve for a leaky confined aquifer will not follow the same shape as a non-leaky confined aquifer; therefore, drawdowns from a leaky confined aquifer will not match the Theis non-leaky type curve due to reduced drawdowns caused by the addition of water to the aquifer through the leaky confining layer. Hantush and Jacob (1955) and Walton (1960) developed curve-matching methods that are like the Theis method but include additional type curves and an additional term in the well function that accounts for the contribution of interformational leakage. A version of the Walton type curves is provided as Figure 48.

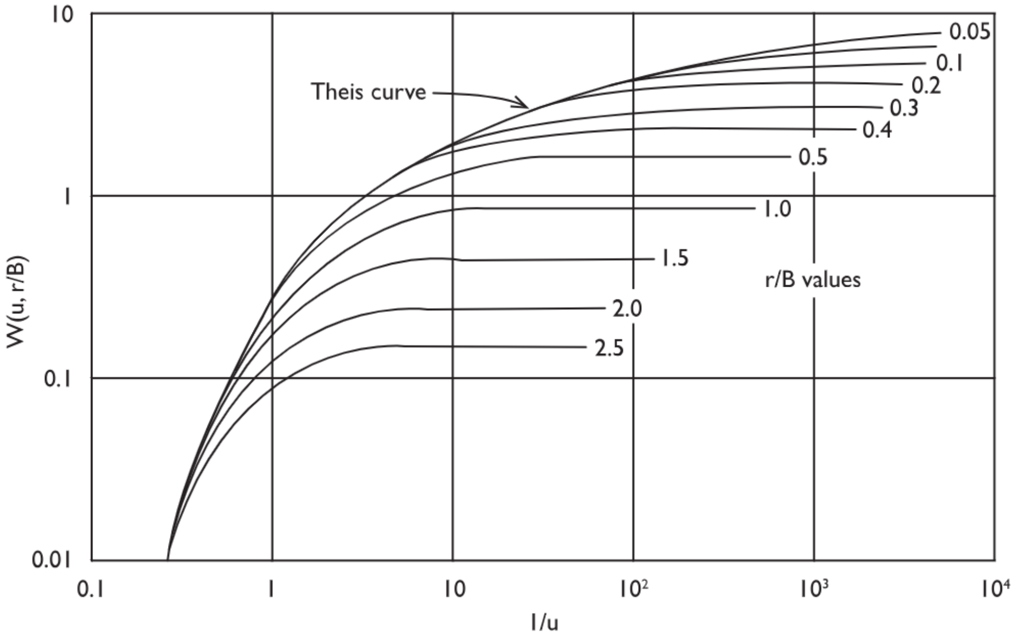


Figure 48 - Theoretical type curves of W(*u,r/B*) versus 1/*u* for a leaky aquifer (from Freeze & Cherry, 1979; modified from Walton, 1960).

In an unconfined aquifer, the drawdown response has three separate parts to it which result in a distinctive shape shown in Figure 42. Neuman (1975) developed a similar curve-matching method in which the well function is composed of three separate arguments, one for each of the drawdown stages. A version of the Neuman type curves is presented as Figure 49.

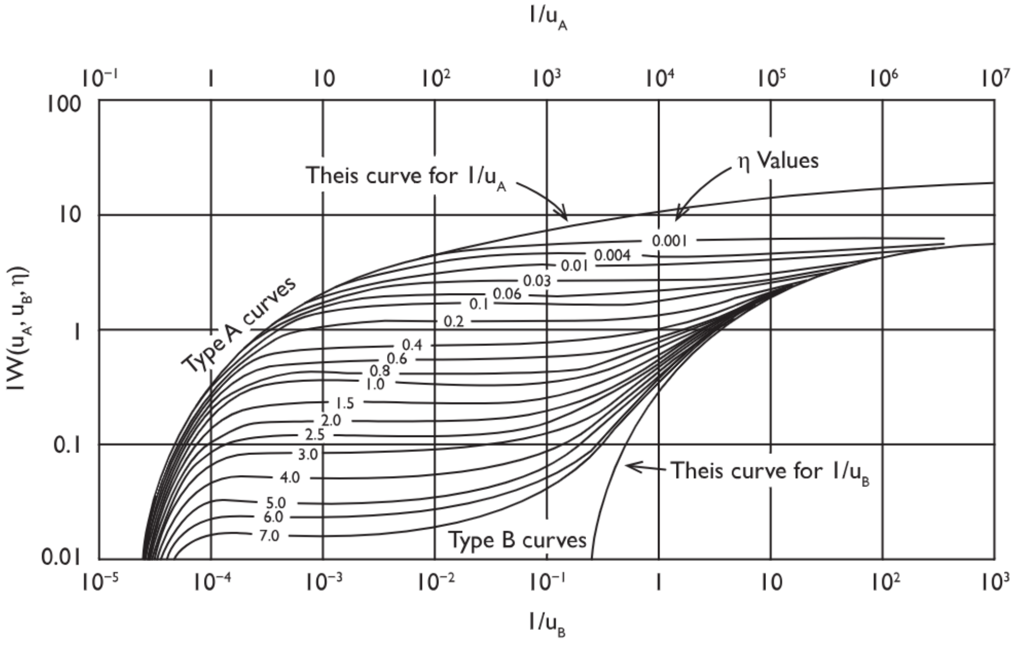


Figure 49 - Theoretical type curves of W(*uA, uB, η*) versus 1/*uA* and 1/*uB* for an unconfined aquifer (from Freeze & Cherry, 1979; modified from Neuman 1975).

The leaky and unconfined methods are discussed in more detail in sections 9 and 10 of *An Introduction to Hydraulic Testing in Hydrogeology: Basic Pumping, Slug, and Packer Methods*↗ by Woessner and others (2020), as well as in *Analysis and Evaluation of Pumping Test Data (2nd edition)*↗ by Kruseman and others (2000). Both books are available as free downloads from The Groundwater Project. Readers are encouraged to review these books for more information.

8.5 Simplification of the Theis Equation

The well function in the Theis equation is shown in Equation (78).

$$\mathrm{W}(u) = -0.5772 - \ln u + u - \frac{u^2}{2*2!} + \frac{u^3}{3*3!} - \frac{u^4}{4*4!} + \frac{u^5}{5*5!} - \textit{and so on} \ldots \tag{78}$$

where:

$$u = \frac{r^2 s}{4Tt} \tag{79}$$

Cooper and Jacob (1946) noted that, when u becomes small, everything to the right of the term $ln(u)$ in the well function becomes negligible and can be omitted. Omitting this term reduced the well function to a simple term, which in turn reduces the Theis equation to that shown in Equation (80).

$$s = \frac{2.3Q}{4\pi} \log \frac{2.25Tt}{r^2 S} \tag{80}$$

This is called the *Cooper–Jacob simplification* of the Theis non-equilibrium equation. This means that, in certain situations, we can use this simplified version rather than the more complex Theis equation. More specifically, the quantity u becomes small when t becomes large—that is, at long times after the start of pumping—or when r becomes small—that is, very close to the pumping well. This concept can be applied to analyzing aquifer test data to estimate T and S using the methods described in the following two sections.

8.5.1 Time-Drawdown Method

If time-drawdown data are available from a single observation well, we can use the Cooper-Jacob simplification to estimate T and S. The method is as listed here.

- Plot drawdown versus time on semi-log paper with drawdown on the arithmetic scale and time on the log scale.
- Draw a straight line to fit the data, focusing on the later data, and extend the line to the point where it intersects the time axis.
- Choose two points on the line that are one log-cycle apart in time—for example, 10 and 100, 1 and 10, or 100 and 1,000 so that the slope of the line is divided by 1.

- Determine the change in drawdown over that interval (Δs).
- Plug that into Equation (81) and solve for T.

$$T = \frac{2.3Q}{4\pi\Delta s} \tag{81}$$

- Determine the value of time where the line crosses the x-axis (t_0).
- Plug t_0 into Equation (82) and solve for S.

$$S = \frac{2.25Tt_0}{r^2} \tag{82}$$

This procedure is illustrated by Exercise 12↴.

8.5.2 Distance-Drawdown Method

If two or more observation wells located at different distances from the pumping well have drawdown data, the distance-drawdown method can be used to estimate T and S. The method is as follows.

- Plot the drawdown versus distance of each well at a given time after pumping on semi-log paper (with distance on the log scale).
- Draw a straight line to fit the data and extend the line through the drawdown = zero axis.
- Choose two points on the line that are one log-cycle apart in distance—for example, 10 and 100, 1 and 10, or 100 and 1,000, so that the slope of the line is divided by 1.
- Determine the change in drawdown over that interval (Δs).
- Substitute Δs) into Equation (83) and solve for T .

$$T = \frac{2.3Q}{4\pi\Delta s} \tag{83}$$

- Determine the value of distance where the line crosses the x-axis (r_0).
- Substitute r_0 into Equation (84) and solve for S.

$$S = \frac{2.25Tt}{r_0^2} \tag{84}$$

This procedure is illustrated by Exercise 13↴.

8.6 Single Well Aquifer Tests

Our discussion of aquifer testing so far has dealt with collecting and analyzing data from an observation well located at distance (r) from the pumping well. It is always better to use data from an observation well to determine aquifer parameters. However, practicing hydrogeologists quickly learn that observation wells might not be available and they may be too expensive to install specifically for testing purposes, so we often have no choice but to rely on analyzing the data from pumping wells to determine aquifer parameters.

Since the Cooper–Jacob simplification is valid for very small values of r, we can use the radius of the borehole of the well as r and apply the time-drawdown method to water level data collected from the well during the pumping test. This analysis can also be applied to analyzing the recovery data—that is, the recovery of water levels after pumping has ceased.

8.6.1 Limitations of Single Well Tests

Single-well aquifer tests are useful and are often relied-upon for characterization of aquifers; however, the following issues with analyzing time-drawdown data from pumping wells should be acknowledged and understood.

Wellbore Storage

The objective of aquifer testing is to put a constant pumping stress on the aquifer; in other words, to continuously remove water from the aquifer at a constant rate and monitor changes in the aquifer during pumping. During the initial stages of pumping, the pump is removing water that is stored within the well under static conditions, and the removal of this water will affect the initial part of the drawdown curve. This is called the wellbore storage effect The portion of the drawdown curve affected by wellbore storage can be determined by:

1. calculating the volume of water stored in the well above the drawdown depth using the radius of the well and the distance between the static water level and the drawdown water level;
2. calculating the amount of time required to remove that volume of water given the pumping rate applied to the well for the test; and
3. only analyzing the part of the drawdown curve after that time.

Care should be taken not to analyze the part of the drawdown curve that is affected by wellbore storage, as this can produce erroneous results.

Unreliable for Storage Parameters

The issues with wellbore storage generally do not affect the determination of T, if the proper part of the drawdown curve is analyzed. The wellbore storage effect, however, has a great impact on the aquifer storage response as perceived by the pumping well; therefore, aquifer storativity (S) cannot be directly or reliably determined from the pumping well time-drawdown data without complicated numerical modeling approaches. In general, reliable storativity cannot be determined from single well pumping tests.

Partial Penetration

The Theis equation assumes that all flow in the aquifer is horizontal and radial, which requires that the pumping well be completed across the entire saturated thickness of the aquifer. In many cases, the pumping well is only completed across the upper part of the aquifer. This is referred to as *partial penetration* (Figure 50). Partial penetration can affect the response of the aquifer to pumping and result in drawdowns that are greater than for a fully penetrating well. More information about partial penetration and methods for correcting the data and properly analyzing wells with partial penetration can be found in section 12.1.1 of *An Introduction to Hydraulic Testing in Hydrogeology: Basic Pumping, Slug, and Packer Methods*↗ by Woessner and others (2020) which is available as a free download from The Groundwater Project.

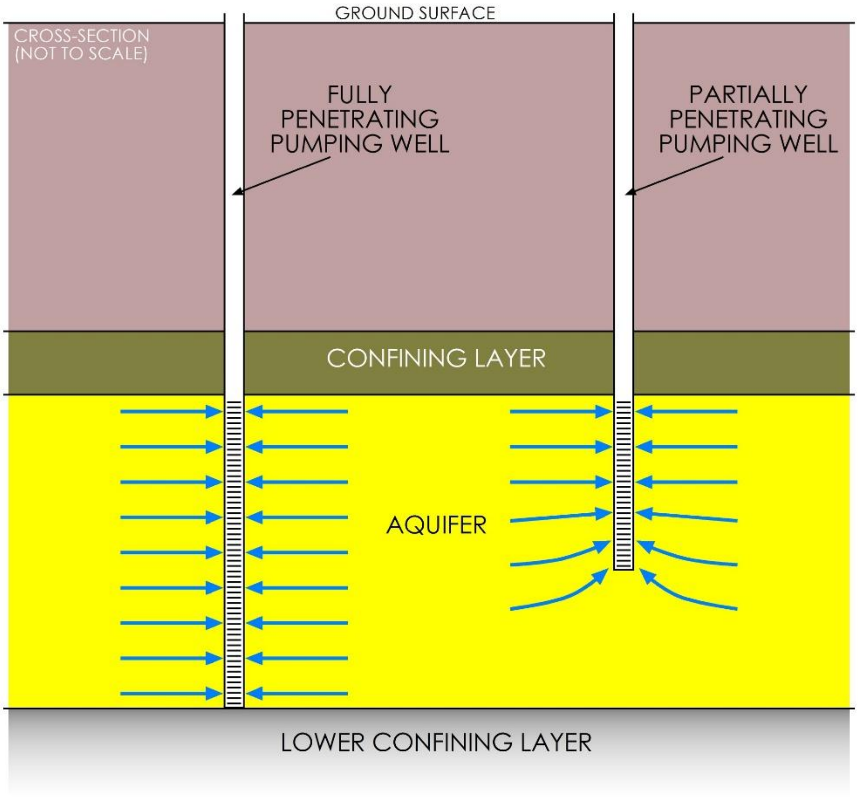


Figure 50 - Schematic cross-section showing two pumping wells completed in a confined aquifer. Pumping from the fully penetrating well on the left induces horizontal flow to the well (blue arrows) across the entire thickness of the aquifer with no vertical flow. Pumping from the partially penetrating well induces horizontal flow across the upper part of the aquifer and vertical flow from deeper portions.

Drawdown in a Pumping Well

Pumping from a well removes water from the aquifer, which in turn causes a reduction in hydraulic head—that is, it causes drawdown—which in turn causes a decline in water level in the well. If we were able to install a piezometer right at the edge of the borehole of a pumping well or otherwise monitor the change in hydraulic head in the aquifer right at the edge of the pumping well, we would see that the drawdown measured within the pumping well is greater than the drawdown in the aquifer next to the well, and the difference between the head in the pumping well and the head in the aquifer is related to the *well efficiency*. Efficiency is less than 100% because energy is lost as water enters the well bore, flows through the sand pack and the screen, then flows vertically in the well to the pump. Because the drawdown measured in the pumping well is greater than that measured in the aquifer, the drawdowns calculated by analyzing drawdown data from a pumping well will result in T estimates that are not as reliable as those determined from analyzing data from an observation well. We discuss well efficiency in greater detail in Section 8.7.

Thought Question 9*: If measured drawdowns in the pumping well are greater than those in the adjacent formation, how would that affect the transmissivity values estimated from a pumping well versus those from an observation well in the same aquifer during the same test? Would the T estimates from the pumping well be greater or less than those from the observation well?*

8.7 Well Loss and Well Efficiency

Well efficiency refers to the difference between the expected performance of a well (i.e., the expected drawdown in response to pumping at a specific rate for a given amount of time) and the actual performance. In other words, it refers to the difference between the drawdown we expect to see in the aquifer at the pumping well and the actual drawdown measured in the well during pumping. To understand well efficiency, we have to revisit the idea that hydraulic head is a measure of fluid energy, and that drawdown is actually measuring a reduction in fluid energy—that is, a decline in head—resulting from stressing an aquifer—that is, pumping water from the aquifer.

8.7.1 Drawdown and Fluid Energy

Pumping from a well draws water into the well from the aquifer, which in turn causes water to flow from farther away in the aquifer towards the well. This reduces pressure in the aquifer, which in turn reduces the pressure head component of energy in the formation causing a drop in hydraulic head—that is, drawdown—and formation of a cone of depression in the aquifer around the well. The amount of energy loss for a given pumping rate at a given distance from the pumping well is associated with the laminar flow of water through the aquifer and is related to the properties of the formation—specifically the transmissivity and storativity. The loss in head that the aquifer experiences is called *formation loss* (s). The formation loss is what we measure in an observation well near a pumping well.

As water flows into the pumping well, it is affected by the pump and well screen/filter-pack in ways that result in additional loss of energy. Water entering the well will typically transition from laminar flow to turbulent flow, which results in greater head loss within the well. In addition, if the well screen openings and grain sizes in the filter pack are not appropriate, they can also induce additional turbulence if the openings are too large or cause greater head loss if they are too small. All these factors cause an additional component of head loss within the well called *well loss* (s_w). The total drawdown that occurs within the well (s_t) is the sum of these two drawdowns, as expressed by Equation (85).

$$s_t = s + s_w \tag{85}$$

A well that does not experience any well loss would have a well efficiency (E) of 100 percent. Wells that experience well loss have E less than 100 percent, with E defined as shown in Equation (86).

$$E = \frac{s_{calculated}}{s_{measured}} 100 \tag{86}$$

The relationship between well loss and discharge for turbulent flow is not linear; therefore, a well does not have a single efficiency. Instead, the efficiency is dependent upon the discharge with higher discharge resulting in lower efficiency. Every well does, however, have a value called the *well loss constant* (C), which is a coefficient that relates well loss to discharge. Well loss is proportional to some power (a) of the discharge as shown by Equation (87).

$$s_w = CQ^a \tag{87}$$

We cannot directly calculate the well loss constant for a well, but we can use aquifer test results to estimate the well efficiency for a given pumping rate if we have an observation well. We do this by comparing the expected drawdown in the pumping well—calculated using T and S from the observation wells along with r = the radius of the pumping well borehole—to the measured drawdown in the well. The overall performance of the well can also be determined by performing a *step-rate test* and evaluating the *specific capacity* at various pumping rates (Exercise 14⤵).

8.7.2 Specific Capacity and Step-Rate Tests

The specific capacity of a well is the ratio of a well's pumping rate to the drawdown in the well after the well has been pumped for a given period. Measuring specific capacity involves pumping from a well and monitoring the water levels until the drawdown stabilizes, then dividing the final drawdown by the pumping rate. The resulting number is the pumping rate per unit drawdown (Exercise 15⤵). A specific capacity test is usually done by drillers at the completion of drilling a water supply well, and the data from those tests are frequently included in driller's logs or other public records.

The drawdown observed in a pumping well, as previously stated, is a function of the formation loss and the well loss; therefore, the specific capacity is affected by the well efficiency. Since well efficiency is inversely related to pumping rate—that is, higher pumping rates will cause more well loss and lower efficiency—the overall performance of the well can be evaluated by performing a *step-rate*, or *step-drawdown*, test. A step-rate test involves pumping the well at a range of pumping rates—generally 25 percent of, to greater than 100 percent of, the desired production rate of the well—and determining a specific capacity for each pumping rate. The range of specific capacities can then be used to determine an optimal pumping rate for the well and to develop estimates of the well efficiency at the optimal rate. For a well that is properly constructed and appropriately designed for the aquifer lithology, a step-rate test should produce relatively consistent specific capacities across the tested range of pumping rates.

8.7.3 Example step-rate test with analysis

An example of a step-rate test is shown in Table 5 and Figure 51. This test was performed in a well completed in an unconfined aquifer composed of unconsolidated quartz sand deposited on a very stiff, low permeability clay (Figure 52). The pump used in this test was rated to sustain a maximum pumping rate of about 545.1 cubic meters per day (cmd) (0.00631 m^3s^{-1}); therefore, pumping rates of about 25 percent, 45 percent, 70 percent, and the maximum that the pump could produce (101 percent) were used in the test. The static water level in the well is about 12.1 m below ground surface (bgs). The top of the screened interval for the well is at about 21.3 m bgs (Figure 52). Since the well is unconfined, it is important to not let water levels in the well drop below the top of the screened interval, as this can introduce oxygen into the gravel pack and potentially induce oxidation of the well or microbial growth, which can in turn clog the well screen. The consulting hydrogeologist therefore decided that the available drawdown in the well is the water column above a point about 1.5 m above the top of the well screen—in this case, 19.8 m bgs—to maintain saturation of the screened interval with a sufficient margin to allow for greater pumping over short time periods. Based on this assumption, the available drawdown—presented in Table 5 as the *Remaining Available Head* for the static water level—is 7.7 m.

Table 5 - Data from a step-rate test performed in an unconfined aquifer in an unconsolidated clastic sedimentary aquifer.

Step	**Pump Rate**		**Water Level**	**Drawdown**	**Specific Capacity**	**Remaining Available Head**
	cmd	**m^3s^{-1}**	**m bgs**	**m**	**cmd/m**	**m**
STATIC WATER LEVEL			**12.1**	**0.0**		**7.7**
STEP 1	**120**	**0.0014**	**13.8**	**1.7**	**70.3**	**6.0**
STEP 2	**245**	**0.0028**	**16.1**	**3.9**	**62.4**	**3.7**
STEP 3	**382**	**0.0044**	**18.4**	**6.3**	**60.8**	**1.4**
STEP 4	**551**	**0.0064**	**21.6**	**9.5**	**58.1**	**-1.8**
FINAL YIELD:	**452**	**0.0052**	**19.8**	**7.7**	**58.9**	**0**

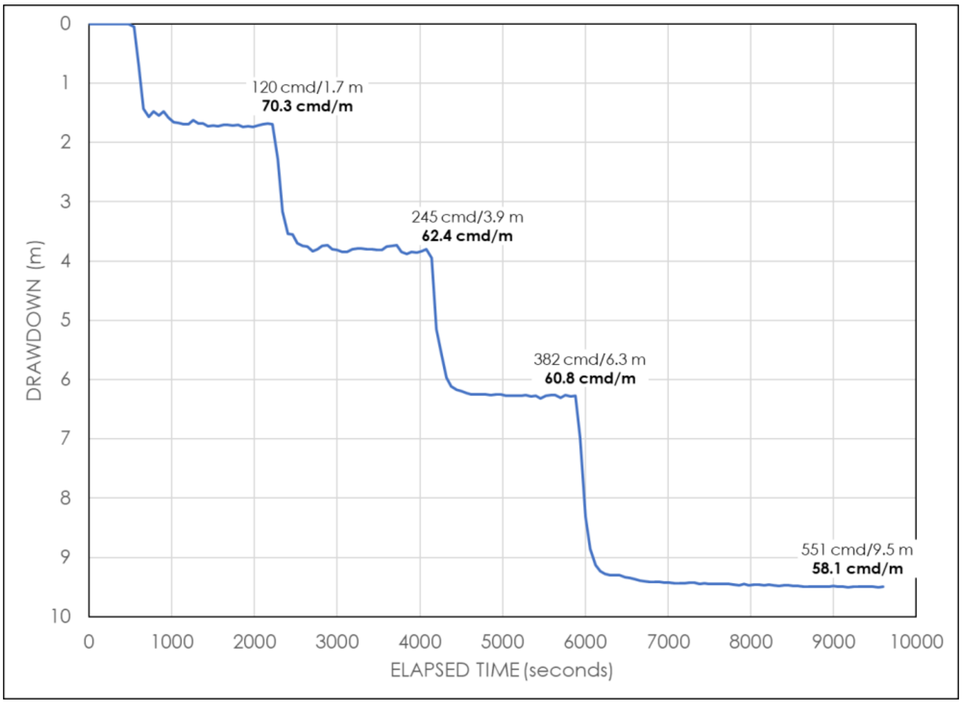


Figure 51 - Time-drawdown data for a step-rate test with four pumping rates performed on a well in an unconfined aquifer in unconsolidated clastic sedimentary material.

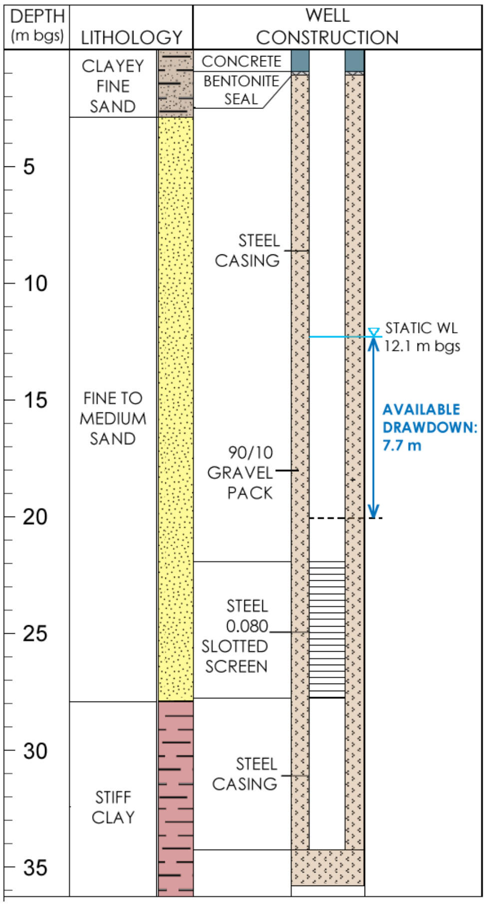


Figure 52 - Lithology and well construction diagram showing the static water level and the available drawdown in the well, which in this case is defined as the water column above a point 1.5 m above the top of the screened interval.

Figure 51 shows time-drawdown data for each step and indicates the stabilization of the water levels at each step. The specific capacity at each step is calculated based on the step pump rate and the final drawdown at each step; these are presented in Table 5 and on Figure 51. The specific capacity values range from 70.3 cmd/m at the lowest pumping rate (120 cmd) to 58.1 cmd/n at the highest pumping rate (551 cmd), which is consistent with the expectation of lower efficiency at higher pumping rates. The specific capacity at the highest pumping rate (58.1 cmd/m) is about 80 percent of the specific capacity at the lowest pumping rate (70.3 cmd/m), which indicates that the well construction is reasonably good but there is some inefficiency at desirable pumping rates.

The *Remaining Available Drawdown* column of Table 5 is the difference between the water level at the various pumping rates and the 19.8 m bgs drawdown limit established for the well. The table indicates that pumping at 551 cmd exceeds the drawdown limit; therefore, the optimal pumping rate is somewhere between 400 cmd and 551 cmd. The optimal pumping rate, or well yield, can therefore be determined by graphing the pumping rates versus the drawdowns at each step, establishing a best fit trendline to the data, and calculating the pumping rate (in this case, 452 cmd) for a drawdown that equals the available drawdown with no pumping (7.7 m) and results in zero remaining drawdown (Figure 53).

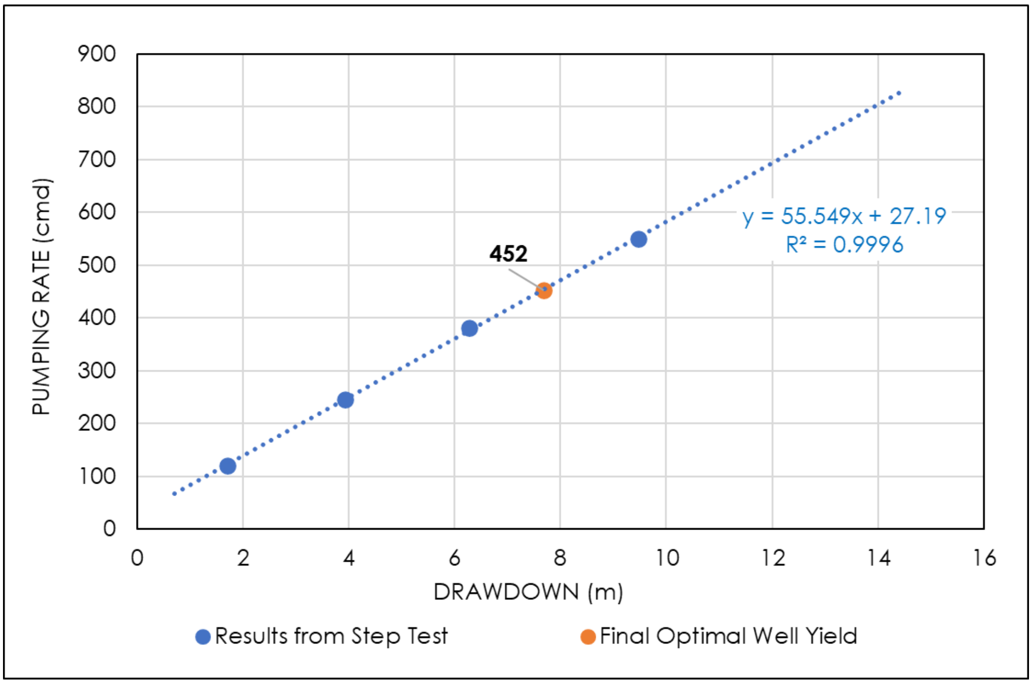


Figure 53 - Graph of test pumping rate versus drawdown. The final optimal well yield is determined by interpolation to find the pumping rate for a drawdown of 7.7 m.

More details about step-test procedures and data analysis can be found in *An Introduction to Hydraulic Testing in Hydrogeology: Basic Pumping, Slug, and Packer Methods*↗ by Woessner and others (2020), which is available as a free download from The Groundwater Project.

8.8 Slug Tests

Pumping tests are not always feasible for various reasons. It could be that a suitable pump is not available or is beyond the budget for the project. Or it could be that the aquifer is not very productive, and the available wells are not capable of pumping long enough at sufficient rates to collect adequate data for analysis. It could also be that the aquifer is highly contaminated, in which case running a pumping test would result in the production of large volumes of contaminated water that require treatment and disposal that is too expensive for the project budget. For such cases there are alternative methods called slug tests. These tests involve displacing a known quantity—that is, a "slug"—of water to or from a well or piezometer and monitoring water levels in the well as the water level recovers to the initial static water level. Water can be added to or removed from the well, or the water level can be displaced using a slug made of a cylinder (usually a sealed length of pipe that is filled with sand or cement) that is lowered into the well below the water table. The basic theory behind slug testing is that the well is in hydraulic equilibrium with the aquifer at the start of the test, then the slug displaces the water level and essentially creates a hydraulic gradient between the water level in the well and the hydraulic head in the aquifer. This gradient then drives the water level in the well back to hydraulic equilibrium, which manifests as a recovery of water levels in the well over time.

8.8.1 Test Procedure

The general procedure for slug tests involves measuring the static water level in the well, then introducing or removing a slug or a volume water, taking an initial measurement at the start of recovery (h_0), then measuring the heads (h) at various times during the recovery period. The quantities of h/h_0 at each measurement time, called the *residual drawdown*, are generally the key values used to analyze the slug testing results.

There are many methods for analyzing slug test data, including manually matching to type curves as well as sophisticated numerical computer methods. A detailed overview of slug test analysis is presented in section 14 of *An Introduction to Hydraulic Testing in Hydrogeology: Basic Pumping, Slug, and Packer Methods*↗ by Woessner and others (2023) and in *Analysis and Evaluation of Pumping Test Data (2nd edition)* ↗ by Kruseman and others (2000). Both books are available as free downloads from The Groundwater Project. Students and working hydrogeologists who would like more information about conducting and analyzing slug tests should download and review those books. An exercise related to slug tests is provided as Exercise 7 in the in the book by Woessner and others (2023).

8.8.2 Potential Problems with Slug Tests

One limitation of slug tests is that they are generally not capable of determining storage parameters—that is, storativity, specific yield—for the aquifer. There are complex numerical methods that can derive reasonable estimates; however, these are difficult to use and generally not widely available. This can be a major limitation for aquifer studies that require prediction of the impacts of future pumping, as those predictions require reliable storage parameters.

A second limitation of slug tests, and single well tests in general, is related to the scale of the volume of aquifer measured. For multi-well aquifer tests with a pumping well and a single observation well, the drawdown response in the observation well reflects a somewhat integrated "average", or representative, transmissivity and storativity for the part of the aquifer between the two wells. For any single well test, the test is at best measuring a very small volume of the aquifer around the well. Aquifers always have some amount of heterogeneity, or spatial variability in the permeability and storage parameters, and multi-well tests generally integrate this variability better than single well tests. On the other hand, slug tests can help investigate heterogeneity when multiple wells are available for testing on a study site.

Finally, for slug tests and single well tests in general, the well loss and influence of the filter pack have a significant impact on test results. Care must be taken when performing slug tests to ensure that a sufficient stress—that is, water level displacement—is applied to the well to adequately stress the aquifer outside of the well. Most piezometers and wells have a gravel pack or filter pack (Figure 34 and Figure 35), which is a zone of high-permeability sand and gravel emplaced in the annulus between the well screen and the intact aquifer formation. When performing a slug test, the displacement of the water level and resulting re-equilibration of the well with the aquifer will be affected by the filter pack in the well, and if an insufficient stress is applied to the well, then the test will measure the permeability of the filter pack rather than the aquifer. Since a properly designed filter pack is intended to be more permeable than the aquifer, the results of the test will likely over-state the permeability of the aquifer and make the test results unreliable.

8.9 Multiple Wells, Boundaries, and Image Wells

In Section 6 we introduced the concept of *boundary conditions* relative to how boundaries affect flow lines and equipotential lines on flow nets. In this section we discuss how boundary conditions affect drawdowns induced by pumping wells and relate that to the influence of multiple pumping wells on a single point in an aquifer.

8.9.1 Superposition

As we previously demonstrated, pumping from a well at a constant rate for a certain amount of time will result in a certain amount of drawdown at an observation well located some distance away from the pumping well. Now let's consider what would happen to the water levels at the observation well if there are multiple pumping wells in the general vicinity of the observation well (Figure 54).

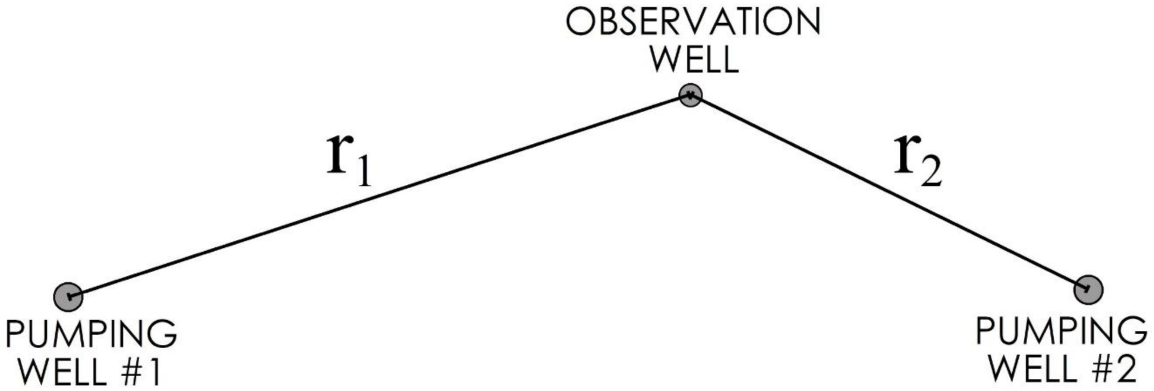


Figure 54 - Map view of observation well influenced by multiple pumping wells.

In this case, each pumping well induces a cone of depression in the aquifer, and the cones of depression overlap at the observation well and throughout the aquifer. Since each pumping well is affecting the observation well, the resulting drawdown at the observation well will be greater than it would for a single pumping well. The effect of pumping from one well on the cone of depression developed by a second pumping well is called *well interference*. In this case, the total drawdown experienced by the observation well is the sum of the drawdowns induced by each pumping well. This is the principal of superposition. It is applicable for near-linear conditions such as those in a confined aquifer, or in unconfined aquifers when the drawdown is relatively small compared to the saturated thickness of the aquifer.

It is therefore relatively easy to calculate the drawdown in an observation well influenced by multiple pumping wells, provided that the aquifer T and S are known with confidence and if one is calculating drawdowns at a specific point in time. The procedure involves calculating the drawdown from pumping well #1 at distance r_1, then calculating the drawdown from pumping well #2 at r_2, then summing the two values to get a total drawdown. The procedure is illustrated by Exercise 16↴.

8.9.2 Use of Image Wells to Represent Boundaries

This concept of superposition is useful for calculating the drawdowns induced by multiple pumping wells. It can also be used to evaluate drawdowns affected by hydraulic boundaries in the aquifer. Let us consider two basic types of hydraulic boundaries—constant head boundaries and no-flow boundaries—and the specific influence that each of these have on the drawdown response in an observation well (Figure 40 and Figure 41). As previously discussed, constant head boundaries essentially *add* water to the system when head inside the boundary declines so less water is removed from storage and thus drawdowns are not as large (Figure 40). In the case of no-flow boundaries, less area is available around the well to contribute water from storage so the same volume of pumped water comes from a smaller area of the aquifer thus drawdowns are larger. These boundary impacts are equivalent to the effect a second pumping well would have on an observation well in an unbounded aquifer. This equivalence can be used to calculate the effect of a boundary on drawdowns from a pumping well through the application of *image wells*.

If the location of a linear boundary and the transmissivity and storativity of the aquifer are known with some confidence, the drawdown at an observation well can be calculated by mathematically placing an imaginary pumping well, the *image well*, in the aquifer an equal distance from the boundary, but on the *opposite* side of the boundary. To represent a hydraulic boundary the image well is an injection well (Q is negative), while a no-flow boundary is represented with a pumping well (Q is positive). The drawdown at the observation well is then calculated as the sum of the calculated drawdowns due to the pumping well and the image well.

An example of a pumping well near a no-flow boundary is shown in map view (Figure 55) and in cross-sectional view (Figure 56a). The pumping well is located some distance $r_{boundary}$ away from the edge of the aquifer. The boundary created by the uplifted block of granite reduces the area around the well where water can be removed from storage to flow to the pumping well. This can be represented as a second pumping well in an unbounded aquifer an equal distance from the boundary and pumping at the same rate (Figure 56b). This image well creates the exact same drawdown as the pumping well along the line between them. With the same water level on both side of the boundary, the gradient is zero which is the definition of a no-flow boundary. The drawdown at the observation well located near the boundary is the sum of the calculated drawdown from the *pumping well*, at distance r_{real} and the calculated drawdown from the imaginary well, or image well, at distance r_{image}.

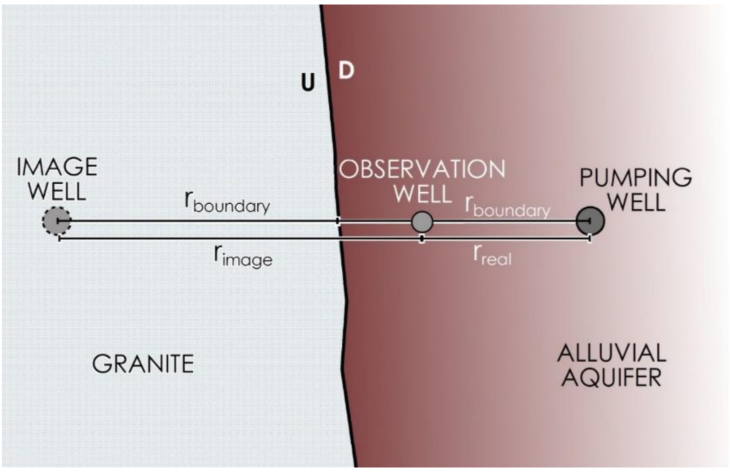


Figure 55 - Map view of pumping well at a no-flow boundary with an associated image well.

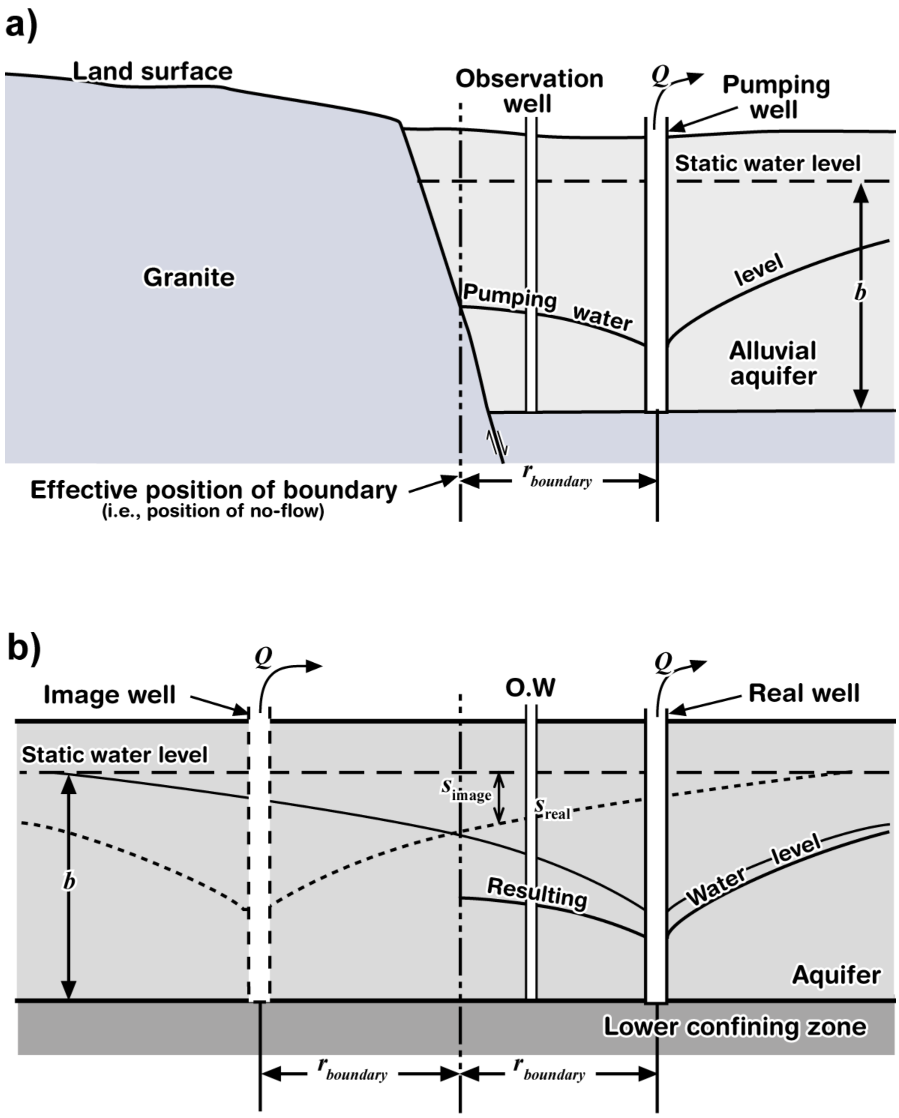


Figure 56 - a) Cross-section of the system shown in Figure 55; b) equivalent system with unbounded aquifer and an image well.

The same procedure can be done to evaluate drawdowns near a constant head boundary (Figure 57 and Figure 58a); the only difference is that a constant head boundary is supplying additional water to the system which results in less drawdown because less water is removed from storage to satisfy the pumping well. In that case, the image well is an injection well with a negative Q (Figure 58b) that produces a negative s so it produces "drawup" such that the calculated drawdown associated with the image well is *subtracted* from the pumping well drawdown (Exercise 17↴).

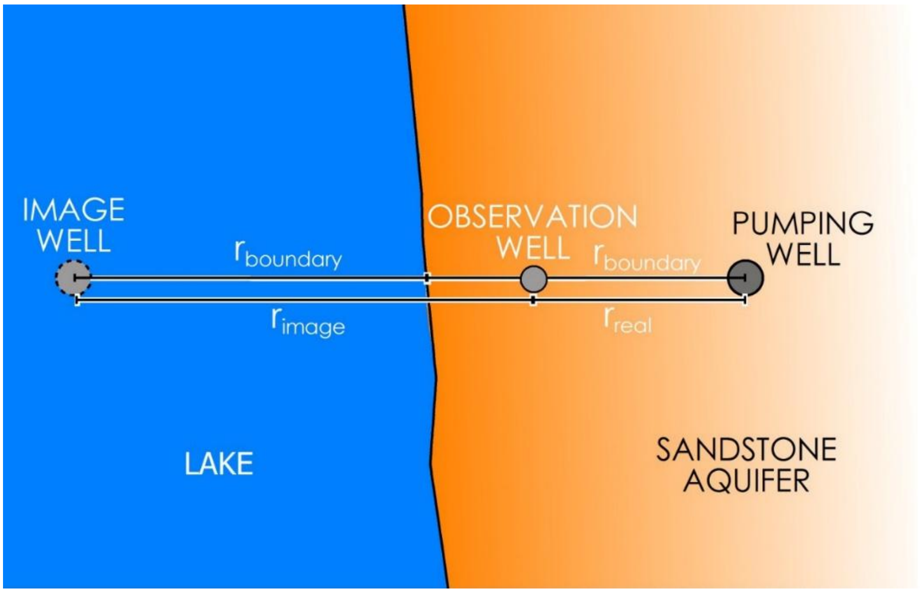
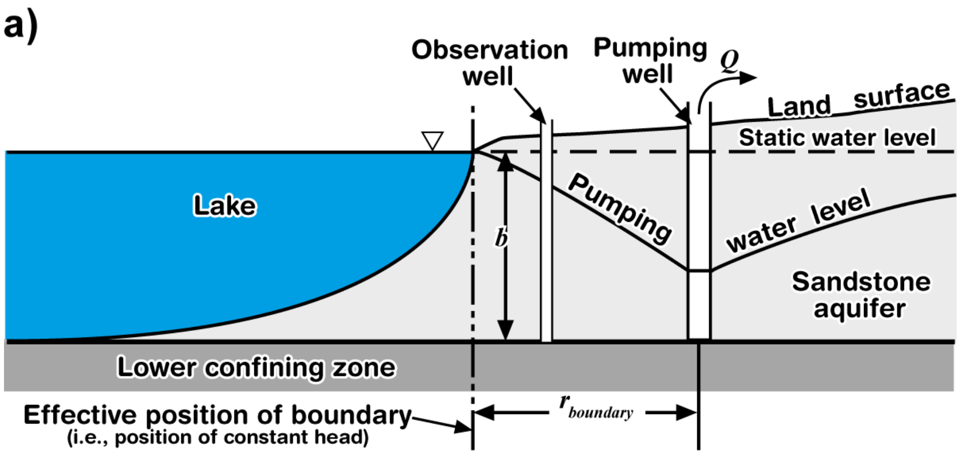


Figure 57 - Map view of pumping well at a constant head boundary with an associated image well.



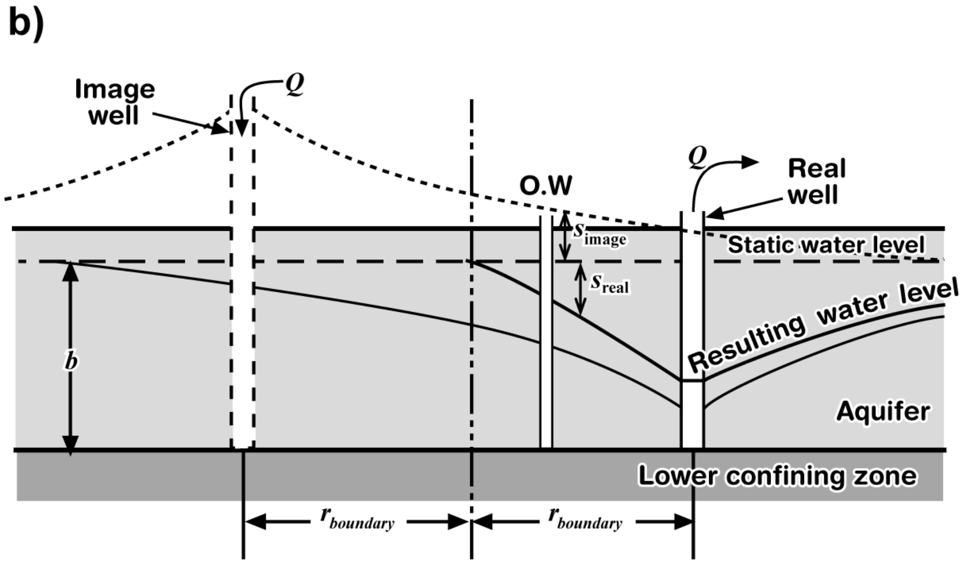


Figure 58 - a) Cross-section of a system shown in Figure 57; b) equivalent system with unbounded aquifer and an image well.

9 Groundwater Flow Systems

9.1 Review of Sections 1 to 8

This section provides a general review of the concepts presented in the first part of this book. We started by considering the properties of water itself and presenting the *hydrologic equation*, which is also known as the conservation of mass or the continuity equation. We defined the term aquifer as a subsurface geologic formation that is capable of producing economically viable quantities of water to wells. We then discussed the concept of fluid energy, how energy is distributed throughout a groundwater system, and how the distribution of fluid energy creates *hydraulic gradients* that drive the flow of fluid through a porous medium. This is a fundamental aspect of hydrogeology—that groundwater discharge is driven by a hydraulic gradient, and that groundwater flows in the direction of decreasing fluid energy—that is, decreasing hydraulic head. We then looked at the relationship between the discharge and the gradient, and showed that the two are linearly proportional to each other with a constant of proportionality called *hydraulic conductivity*—as defined by Darcy's law. Hydraulic conductivity is a type of *permeability*, which is a mathematical term that quantifies the ease with which a fluid flows through a porous medium.

We then looked at the properties of the porous medium itself, including porosity, density, moisture content, and the relationships between these properties. We next discussed permeability in more detail and showed how permeability refers to the relationship between the discharge and the gradient as in other physical processes, including heat flux, electrical current flow, and molecular diffusion. We then reviewed Darcy's law in more detail and used it to calculate discharge through an aquifer as a volumetric discharge (Q), as a 1-dimensional discharge (q), and as a more representative average linear velocity (v).

Then we looked at the fundamental equations that govern the flow of groundwater in three-dimensions. We presented the groundwater flow equation and showed how the equation is a more complex version of the continuity equation expressed as a partial differential equation. The left side of the equation is the *divergence of flux* which is the $I - O$ part of the continuity equation resolved in three dimensions. The right side of the equation describes the $\Delta Storage/\Delta t$ part of the continuity equation in terms of the storage parameter and the changes in head with time.

We then discussed storage parameters in detail as related to the difference in the behavior of confined versus unconfined aquifers. In confined aquifers, the addition or removal of water to/from storage is controlled by the *elastic response* of the aquifer and is governed by the *specific storage*. In unconfined aquifers, addition or removal of water comes from *dewatering the pore spaces,* which is governed by the *specific yield*. Then we looked at the time differential on the right side of the equation and discussed the difference between steady state and transient flow. We also discussed heterogeneity and anisotropy, described how those factors affect the relationship between flow lines and equipotential lines on a flow net, then introduced the concept of *boundary conditions* in aquifers.

After we discussed the basic physics and mathematics behind groundwater flow in three dimensions, we considered the specific situation of radial flow to a pumping well. We discussed the concept of radial flow, then presented the Theis equation, which calculates the changes in hydraulic head throughout an aquifer in response to a well pumping at a constant rate. We developed a conceptual understanding of aquifer response to a pumping well and looked at the difference in that response with different types of aquifers—confined, leaky-confined, and unconfined—and with various boundary conditions. We then discussed multi-well aquifer tests and how to use the Theis equation and data collected from aquifer tests to estimate the transmissivity and storativity of an aquifer. We reviewed other ways of using the Theis equation and then discussed well efficiency, specific capacity, well yields, and testing of individual wells without observation wells. Finally, we revisited the concept of boundary conditions and looked at the influence of boundary conditions on aquifer drawdowns.

The next step is to expand the scope of our study up to the scale of entire groundwater *flow systems.* In this section, we define flow systems, differentiate between local and regional flow systems, and discuss the way we study these systems. We will use the more generic term *flow system* rather than *aquifer*, as the term aquifer has a specific meaning related to the production of usable quantities of groundwater. A flow system includes aquifers as well as subsurface units that cannot produce usable quantities of groundwater. When studying groundwater flow systems, we will often classify the various stratigraphic components of the flow system as *hydrostratigraphic units*. For example, in Figure 22 the uppermost unconfined aquifer might be considered as one hydrostratigraphic unit, the underlying uppermost confining layer as a second hydrostratigraphic unit, the uppermost confined aquifer as a third, and so on to include the entire system. It is important to not necessarily refer to these units as "formations", as formation has a specific geological definition that may correspond to our hydrostratigraphic units, but may not as a single geologic formation may contain multiple hydrostratigraphic units and vice versa.

***Thought Question 10**: Why would hydrogeologists be interested in studying or characterizing groundwater flow systems that are not considered to be aquifers—that is, are not capable of producing viable quantities of useful water?*

9.2 Characteristics of Flow Systems

A groundwater flow system is a three-dimensional entity with the following components:

- volume and boundary conditions: dimensions encompassing a finite volume and physical boundaries to flow;

- recharge: an area and set of processes that allow water to enter; and
- discharge: an area and set of processes that allow water to exit.

Next, we discuss each of these components.

9.2.1 Boundary Conditions

Flow systems are three dimensional bodies that have physical boundaries. There are two basic types of boundaries, or boundary conditions, that we use to characterize the limits of flow systems: specified flux (or Neumann) boundary—in particular the special case of no-flow; and specified head (or Dirichlet)—in particular the special case of constant head). Other types of boundaries are variations and combinations of the Neumann and Dirichlet boundaries that get their names from people who first described them.

No-Flow Boundaries

A no-flow boundary is a boundary that possesses qualities that do not allow for the flow of groundwater across the boundary. It is a special case of a specified flux boundary with zero flux. Since there is no flow across the boundary, there is also no hydraulic gradient across the boundary—that is, $dh/dx = 0$, where x is perpendicular to the boundary. Some examples are provided here.

- If permeable aquifer units are directly in contact with low-permeability formations—for example, alluvial deposits adjacent to an upthrown block of granite or other low-K crystalline rock at a fault—then there will be no significant flow from the low-K rock into the flow system, or vice versa (Figure 59). This is the same as the situation described in Figure 55 and Figure 56a.

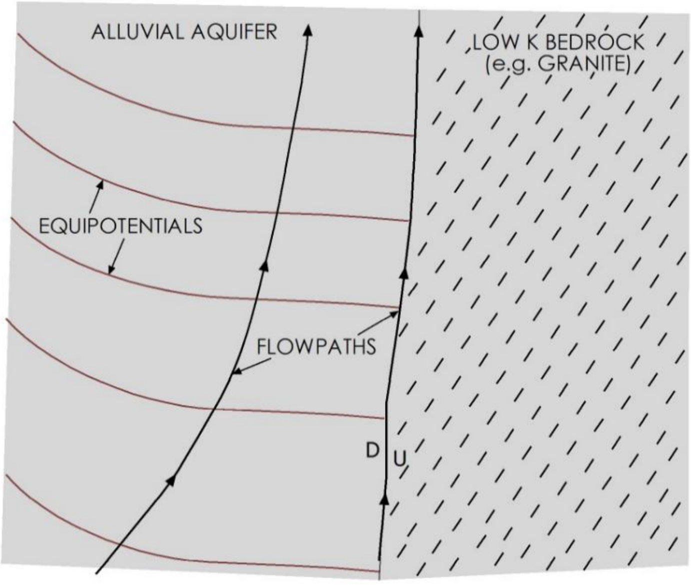


Figure 59 - No-flow boundary.

- Locally high groundwater levels can produce a *groundwater divide*—analogous to a surface water or watershed divide. At a groundwater divide, all hydraulic gradients—and groundwater flow—are away from the boundary; therefore, there is no hydraulic gradient or flow perpendicular to the boundary (Figure 60). Since flow is moving away from the boundary, water must be coming into the flow system at that point; therefore, groundwater divides are typically associated with recharge areas.

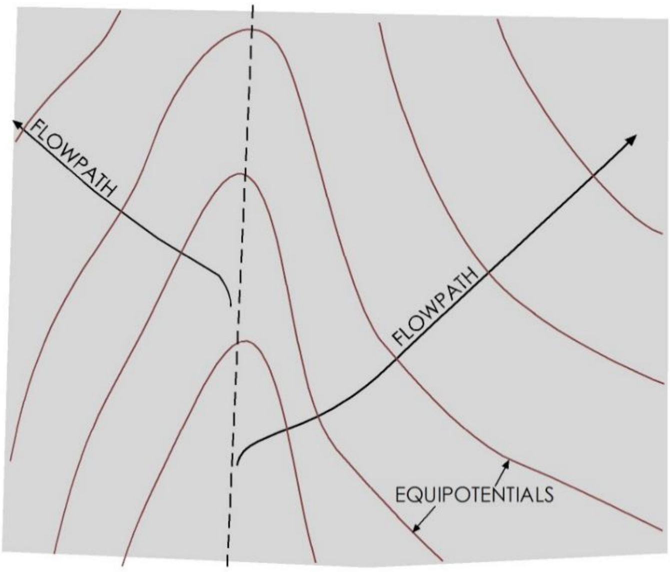


Figure 60 - No-flow boundary at a groundwater divide.

- Wherever parallel flow lines exist in an aquifer, there is no hydraulic gradient, and therefore no flow occurs in the direction perpendicular to the flow lines. This case is called a *symmetry boundary* (Figure 61).

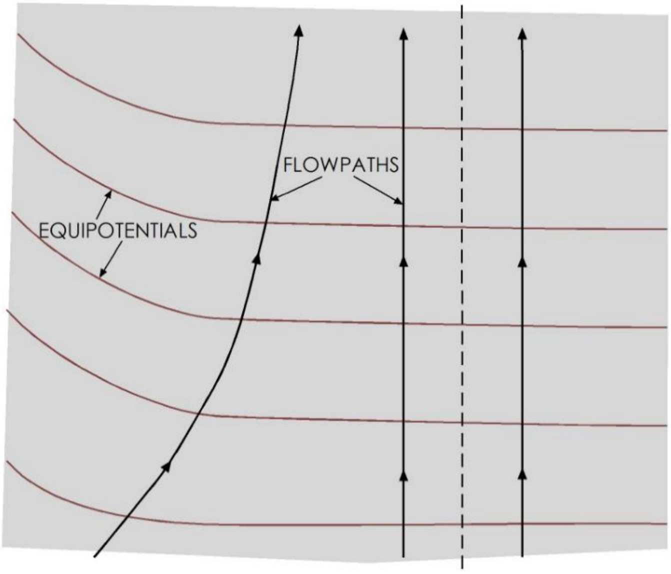


Figure 61 - No-flow symmetry boundary.

Constant Head

A constant head boundary is a boundary at which the hydraulic head within the aquifer does not change. It is a special case of a specified head boundary with unchanging head. Constant head boundaries form where a groundwater flow system is in direct hydraulic communication with a body of surface water that has a constant water level. At the boundary, the hydraulic head in the groundwater system is equal to the water level elevation in the surface water body. An example of this is along the coast of a non-flowing body of water—for example, a lake or a pond—such as that depicted in Figure 62. In this case, the shore of the lake represents a single equipotential line within the aquifer that is equivalent to the elevation of the lake surface. If the water levels in the groundwater flow system are greater than those in the lake, then water will flow into the lake perpendicular to the shoreline.

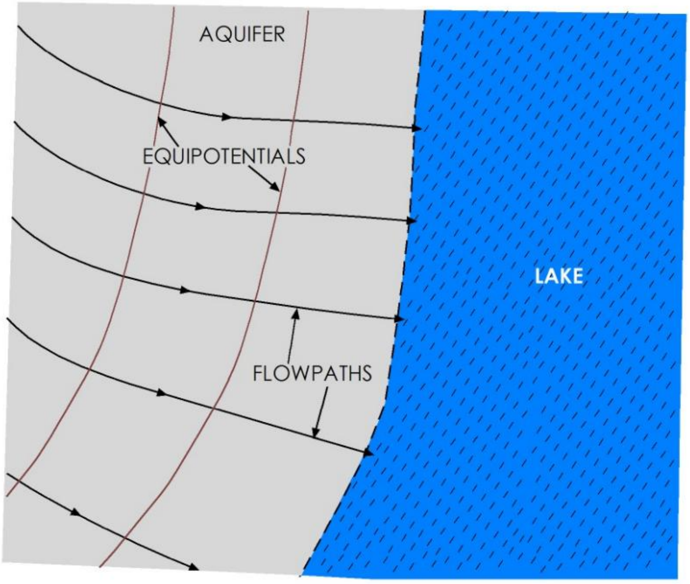


Figure 62 - Constant head boundary at a lake.

If the boundary is along a river, the heads will vary along the length of river as its stage decreases downstream, but at any point along the stream channel the heads in the aquifer are equivalent to the river stage at that point. Each point along the river therefore represents a point on an equipotential line in the groundwater flow system (Figure 63). It should be noted that, as lake and river stage elevations change in response to droughts or large precipitation events, the heads within the groundwater at the boundary do so too.

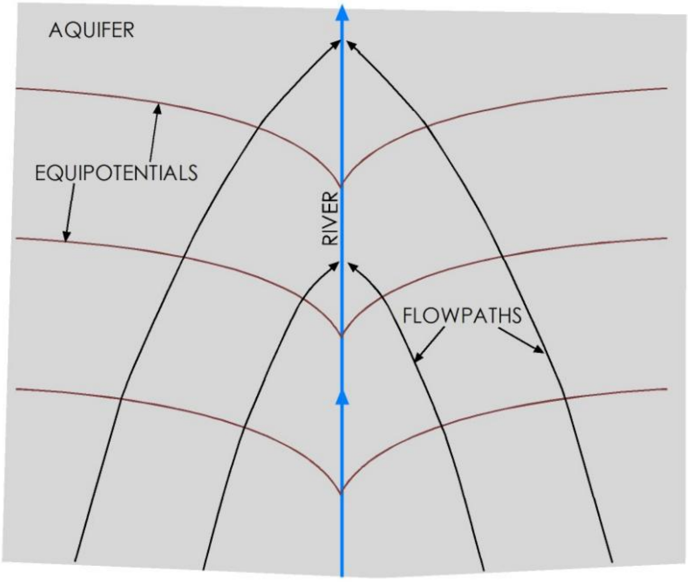


Figure 63 - Constant head boundary along a gaining stream.

Other Boundary Conditions

The two boundary conditions presented in this section represent the boundaries most frequently dealt with in understanding groundwater flow systems. Other boundary conditions exist, including head-dependent flux (Cauchy) boundary, Robin boundary (a weighted sum of Neumann and Dirichlet conditions), Mixed boundary (with varying boundary types along its length), a free-surface boundary that describes a water table or salt-water interface, and a seepage face boundary that is a free-surface boundary confined to a plane on the earth's surface. A discussion of these is beyond the scope of this book.

9.2.2 Recharge

Recharge refers to the set of processes by which water enters the saturated part of a groundwater flow system. Areas where recharge is occurring are called *recharge areas* or *recharge zones*. There are several ways that recharge gets into a flow system.

- Direct infiltration on the outcrop with some percentage of rainwater seeping into the ground and ultimately flowing through the water table into the saturated zone.
- Infiltration through the beds of losing streams or reservoirs. In some parts of the world, especially in extremely arid regions, this is the dominant form of recharge to the groundwater.
- Interformational flow, or subsurface flow, from one formation/flow system directly into another system, usually as flow through leaky confining layers or zones where two flow systems are in direct hydraulic communication.

The primary source of water for recharge is precipitation. When it rains, some percentage of the water runs off over the surface, some evaporates back into the atmosphere, some is taken up by plants and animals (*transpiration*), and the remainder gets to the water table and recharges the aquifer. In addition, some amount of surface runoff may flow into and through surface water bodies that are actively losing water to the underlying groundwater system. Finally, accumulation of snow at higher elevations in winter can result in additional surface water flow and recharge during spring thaw. Recharge is therefore frequently quantified as some percentage of precipitation. We can measure precipitation in a given area with a high level of confidence. We can also measure stream discharge and runoff and get a good estimate of how much of a rainstorm ends up as runoff. We can measure *potential evaporation* by placing a pan of water outside and seeing how long it takes to evaporate. However, it is more difficult to estimate transpiration and actual evaporation. Transpiration is difficult to quantify because any given area has a wide range of plants, and the overall transpiration is a function of the various percentages of each type of plant present in the area. Actual evaporation is difficult to quantify because it depends upon how much water is available on the surface at any given time; plus, evaporation rates vary with conditions such as temperature, humidity, windspeed, and solar radiation. Because of the many processes involved and uncertainties associated with their magnitudes, it is always difficult to put a number on recharge to any aquifer. Generally, it is accepted that annual recharge rates in most aquifers are somewhere between 1 and 10 percent of annual precipitation, though it can be as high as 30 percent in humid regions.

9.2.3 Discharge

Discharge is the set of processes by which water leaves the saturated part of a groundwater flow system. Discharge processes include:

- discrete discharge to a spring or seep;
- discharge into a gaining stream, wetland, or lake;
- interformational flow (leakage through a confining layer);
- evaporation—in some arid areas, this is the primary mechanism for discharge from flow systems; and
- pumping from a well.

Discharge can also be difficult to quantify, especially in areas dominated by well pumping or evaporation. Interformational flow is generally small compared to the other mechanisms. Springs and gaining streams can be gaged, and changes in their flow can be attributed to either recharge or discharge. Much of the discharge from actively developed aquifers is via pumping from wells, and many aquifers suffer from a lack of reliable records for volumes of water pumped over time.

Readers who are interested in learning more about recharge and discharge processes, specifically those associated with the interactions between groundwater and surface water, are encouraged to download and review *Groundwater-Surface Water Exchange*↗ by Woessner (2020) which is available from The Groundwater Project as a free download or can be read online.

9.2.4 Heterogeneity and Anisotropy

Heterogeneity refers to variations in the characteristics of a system with respect to *location*. A parameter like permeability will vary from place to place in a flow system, depending upon the rock types and variations in sediment characteristics—such as grain size. A common example of heterogeneity in a flow system is interbedded sand and clay units in a fluvial depositional system.

Heterogeneity affects flow paths in two ways. One, if the interface between the different zones is parallel to the flow directions, water will tend to move in the path of least resistance. This affects the localized flow paths water takes in a system; for example, in Figure 64 the overall direction of flow is left to right on the diagram, but in some areas the heterogeneity results in flow lines with vertical components.

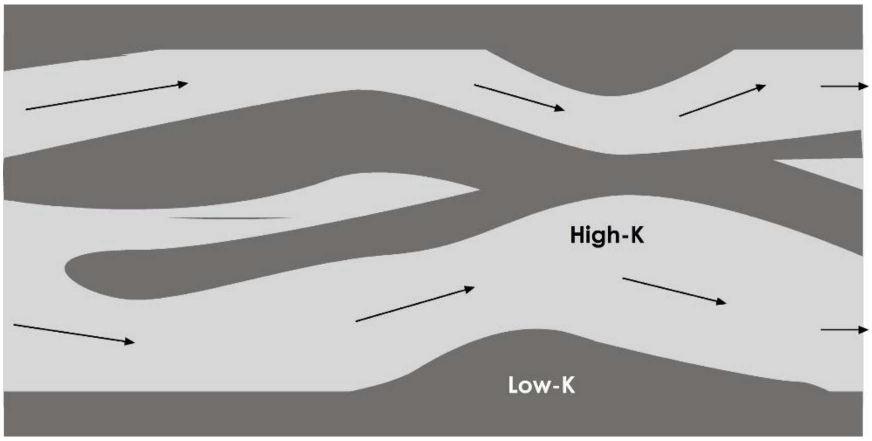


Figure 64 - Influence of heterogeneities on flow paths.

The second way that heterogeneity affects flow directions is associated with refraction of flow lines at the boundaries between units with different hydraulic conductivities. When groundwater moves from a formation with a given hydraulic conductivity into a formation with a different conductivity, the discharge between each flow lines stays the same. However, since K is different, the gradient must change, and therefore the flow lines must move either together or apart, depending on whether the change in K is increasing or decreasing. Therefore, when flow lines cross into areas with different conductivities, they *refract* or bend—like light moving through a substance with one optical velocity into one with a different optical velocity. If the conductivities of the two formations are known, the angles of refraction can be calculated using Equation (88) with K and α defined in Figure 65.

$$\frac{K_1}{K_2} = \frac{tan(\alpha_1)}{tan(\alpha_2)} \tag{88}$$

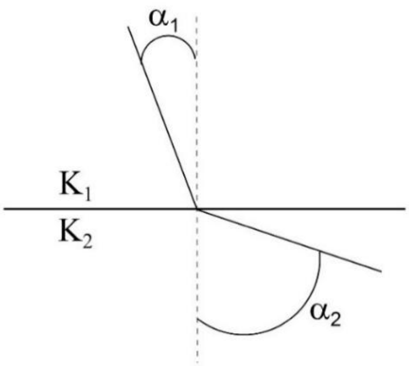


Figure 65 - Refraction of flow lines at the interface of two formations with different conductivities.

The amount of refraction, and the direction in which the flow lines are refracted—relative to the line normal to the interface—is determined by the degree of difference in hydraulic conductivity and by which way the water is flowing (Figure 66).

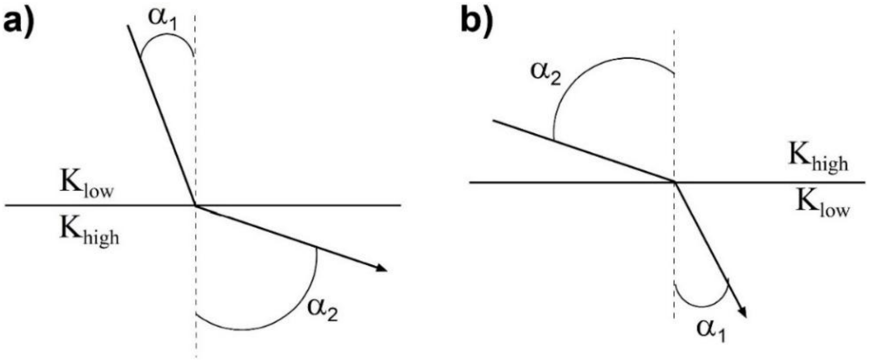


Figure 66 - Refraction of flow lines a) low to high conductivity; b) high to low conductivity.

As a result, low permeability formations tend to have vertical gradients and vertical flow, whereas higher permeability formations tend to have horizontal flow lines. In a layered system of interbedded sand and clay, the horizontal permeability will be much greater than the vertical permeability, and as a result, in layered systems the overall flow direction tends to be horizontal direction and concentrated in the higher K units. Flow in lower K units tends to be vertical (Figure 67).

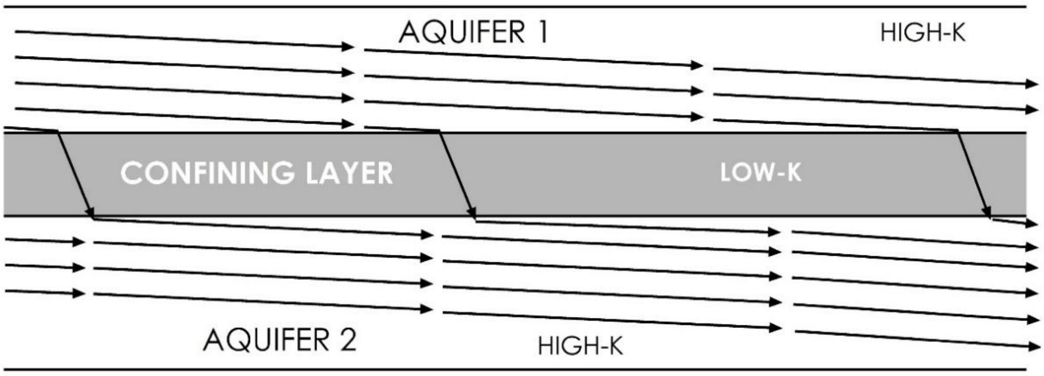


Figure 67 - Cross-section showing effect of heterogeneity in the form of layered sediments on flow paths which creates anisotropy for the system of layers.

Anisotropy refers to variations with respect to *direction*. Layering of interbedded high and low K formations, as shown in Figure 67, is one form of anisotropy. Anisotropy can also result from sedimentary structures formed during deposition, both on a large scale—like channel sands interspersed with overbank deposits in a fluvial system—and on a smaller scale—cross-beds, graded bedding. More significantly, geologic structures like fractures are frequently responsible for anisotropic aquifers.

As shown in Figure 30, if the direction of maximum permeability in an anisotropic system is not parallel to the gradient, the flow paths will be deflected and will not follow the gradient. In moderately anisotropic aquifers, a groundwater map showing the equipotentials does not indicate the primary flow directions. For example, in the Edwards Aquifer of central Texas the overall hydraulic gradient points towards the Gulf Coast. However, extensive fracturing associated with the Balcones Fault Zone, and later karstification, has created an anisotropy that deflects the flow paths so that they are almost parallel to the equipotentials. For more information, *The Edwards Aquifer*↗ by Sharp and Green (2022) is available as a free download from The Groundwater Project as a free download or can be read online.

9.2.5 Relationship between Topography and Flow Systems

King (1899) and Hubbert (1940) noted the relationship between topography and groundwater flow patterns. Both noticed that the water table tends to be a subdued replica of the topography (Figure 68).

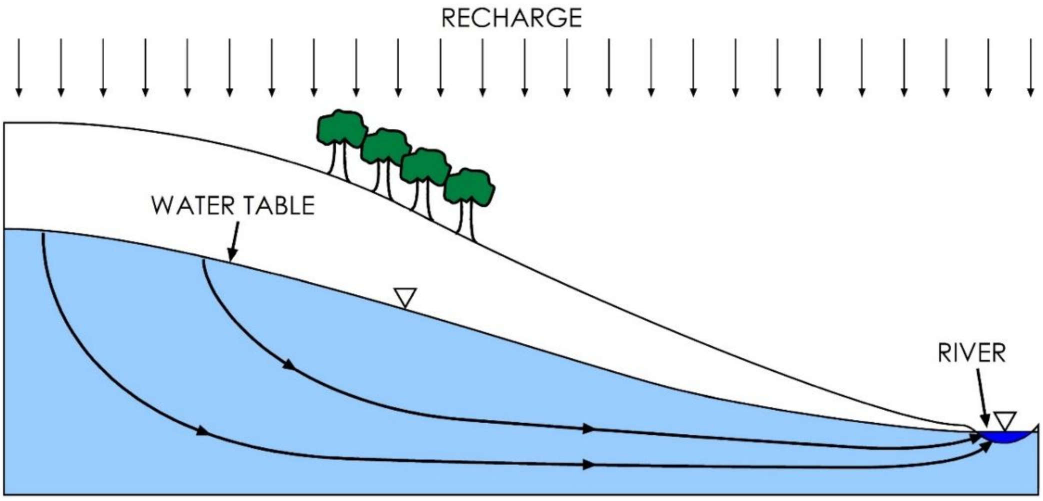


Figure 68 - Cross section of water table aquifer showing the relationship between topography and the orientation of the water table.

Tóth (1963) then developed a computer model, using the same groundwater flow equations that we presented in the previous sections. This is important because Tóth's model was one of the first numerical computer groundwater models and it paved the way for a lot of later groundwater modeling approaches and techniques. His model looked at how flow systems are related to topography. He set up his model as a two-dimensional cross section and used a mathematical equation to define a specified head boundary that represented a water table mimicking uneven topography on the surface. When he varied the topography, he noted that, in some places, smaller local flow systems would develop within the larger regional flow systems. This was the first mathematical evidence that these types of systems exist, and it confirmed the findings of King and Hubbert. Exercise 18↴ provides an opportunity to create a representation of a topographically driven flow system and track groundwater flowlines and particles through the system.

The relationship between topography and groundwater flow patterns is generally only valid in humid areas with sufficient rainfall to support recharge to the water table. For example, Uliana (2000) used naturally occurring isotopes and major ion chemistry, in conjunction with analytical modeling to show that regional groundwater flow patterns in a subtropical arid region (the Chihuahuan desert of west Texas, USA) flow beneath one of the mountain systems and cross the surface drainage divides.

10 Flow Modeling

10.1 Introduction

The word ‘model’ is used frequently in hydrogeology as in all the natural sciences. It is used both as a noun (“… develop a *model* of a system…”) and as a verb (“… use an equation to *model* a system…”). This word means a lot of different things to a lot of different people; to some, it is a computer program that simulates a natural process, to others it a scaled-down physical representation of an object—like a model car or a model solar system used in a classroom—and to some it refers to tall, attractive, exotic-looking people that display the latest clothing styles on runways in Paris. We won’t be dealing with the latter in this section. However, the concept of *models* is something that has applications in all the sciences, and it is important that we define what it means in general and what it means to hydrogeologists in particular.

In this section, we discuss the idea of modeling groundwater flow systems at the highest level with the objective of providing the most general overview of the subject. At the end of the section, we present references and suggestions for additional training and education related to developing and using computer groundwater flow models.

10.2 Models: A General Definition and Specific Details

There are a lot of different definitions of the word *model*; so many that I came up with my own. A model is *a simplified representation of an object, structure, or process that cannot be studied directly because it is too large, too small, or too complicated, or because direct study or observation is, for other reasons, not possible or feasible*.

One of the keys to this definition is that we can represent a *process* as well as a structure or object. We traditionally think of models in the sense of things that represent the structure or appearance of something, like a plastic model airplane or a stick model of a benzene molecule. Model airplanes can be very sophisticated in their representation of the appearance and detail of a real airplane—everything is scaled down correctly, and the details can even include the rivets on the wing and the instrumentation in the cockpit. It is important to recognize, however, that the model just *looks* like a plane; it can’t actually *act* like a plane—that is, it can’t fly. The model of an organic molecule is the same—it may have the correct bond angles and the relative size of the atoms is correct, but it does not exhibit the same behavior as a real molecule—for example, it doesn’t bond with other molecules, it doesn’t have a melting or boiling point. These types of models are called *geometric models* because they depict the *geometry*—that is, structure, appearance, distribution, etc.—of whatever it is they represent. Some examples of geometric models are:

- plastic toy figures of WWII fighter planes;
- blueprints;
- geologic maps and cross sections; and

- molecular symbols and diagrams (Figure 69).

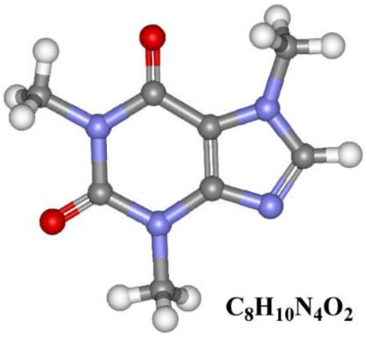


Figure 69 - A model of the author's favorite molecule.

It is of more importance to us as scientists and hydrogeologists to be able to develop *process models*, or models that simulate physical processes. These models must contain some component of the geometry of the system, but they also have some component that describes how the system behaves—that is, how it responds to changes in certain conditions, how it changes with time, and so on. This component can be a physical object, or it could be purely mathematical—that is, an equation or a set of equations.

Process models can simulate objects and processes such as the examples listed here.

- Weather and climate
    - Make forecasts about weather conditions in the near future.
    - Simulate the response of global climate to changes in atmospheric CO_2.
- Stress and strain
    - Simulate stresses in a bridge or building foundation.
    - Simulate strain in deformed geologic units.
- Watersheds
    - Simulate flooding in response to large storms.
    - Calculate flow rates at various points in watershed.
- Aquifers and groundwater flow systems
    - Simulate changes in head with response to pumping.
    - Calculate well yields and long-term availability.

10.2.1 Why Do We Use Process Models?

The main reasons for developing and using process models for groundwater flow systems—as well as other natural phenomena—are outlined below.

Make Predictions

Forecasting and making predictions about the future state of a system is the most common and most widely recognized reason for developing process models. More specifically, groundwater flow models are widely used to make predictions, generally about how the system will change in the future in response to a change or a *stress* applied to it. One example of a change to a groundwater system could be reduction in precipitation, and a corresponding reduction in recharge, in response to future climate change. An example of a stress applied to a groundwater system could be the addition of a large amount of pumping associated with a proposed industrial development. These models can be very useful tools for managing water resources; however, one must always remember that models are only as good as the data and assumptions used to develop them, and caution must always be taken when making management decisions based on model predictions.

Test Hypotheses and Investigate Processes

In some cases, we may not be sure why a natural process behaves the way it does, and a model can help us constrain our hypotheses concerning that behavior. For example, we may be studying the structure of a set of folded and faulted rocks. We don't know exactly what stresses caused the strain that we observe, so we develop hypotheses about compression and tension and the principal stress directions. We then put together a model that simulates those stresses and compare the model results to the structure of the rock in the field. If the model results don't match, then we need to refine or reformulate the hypotheses. If the model results do match, then the model supports the hypotheses. Keep in mind when model results match field observations it doesn't *prove* anything; however, model results can provide robust support for a hypothesis when models are properly developed.

Evaluate System Sensitivity

Another use of models is to help the investigator identify areas where more data is needed. Models can be used to test the sensitivity of a system to certain parameters, and a model may show that the variations of one parameter don't make much difference in system behavior, while variation of another parameter makes a big difference. For example, a hydrogeologist designing a model for evaluating groundwater pumping from a regional aquifer may have a lot of reliable transmissivity values from several single well aquifer tests but only a few storativity values. A sensitivity analysis of the aquifer may indicate that the model results do not vary much over the range of available storativity estimates, which means that the model results are acceptable and there is likely no need to perform additional aquifer tests to improve the model results.

Estimate Parameters (Inverse Models and Model Calibration)

If we know a lot about our system but are uncertain about the values of a specific parameter, we can use a model to estimate that parameter by applying the information that we know and solving for what we don't know. Models developed for the purpose of estimating input parameters are called *inverse models*. Figure 70 and Figure 71 provide an example of simple inverse modeling to estimate a representative transmissivity for a regional groundwater flow system. Using the calculations shown in Figure 70 with the system parameters outlined in Figure 71, along with recharge values ranging from 0.5 to

5 percent of annual precipitation, results in transmissivity values ranging from 1.07 m^2/s to 10.7 m^2/s. These values are within the range of values estimated from aquifer tests and other methods for the area shown in Figure 71.

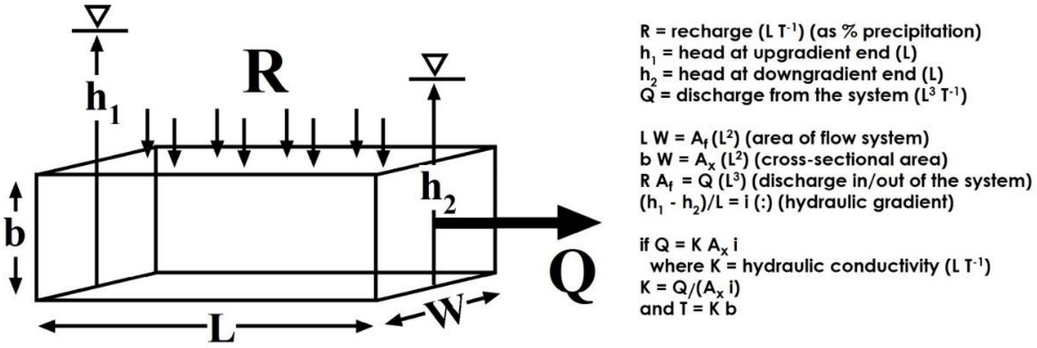


Figure 70 - Schematic diagram illustrating inverse model determination of transmissivity (after Uliana, 2000).

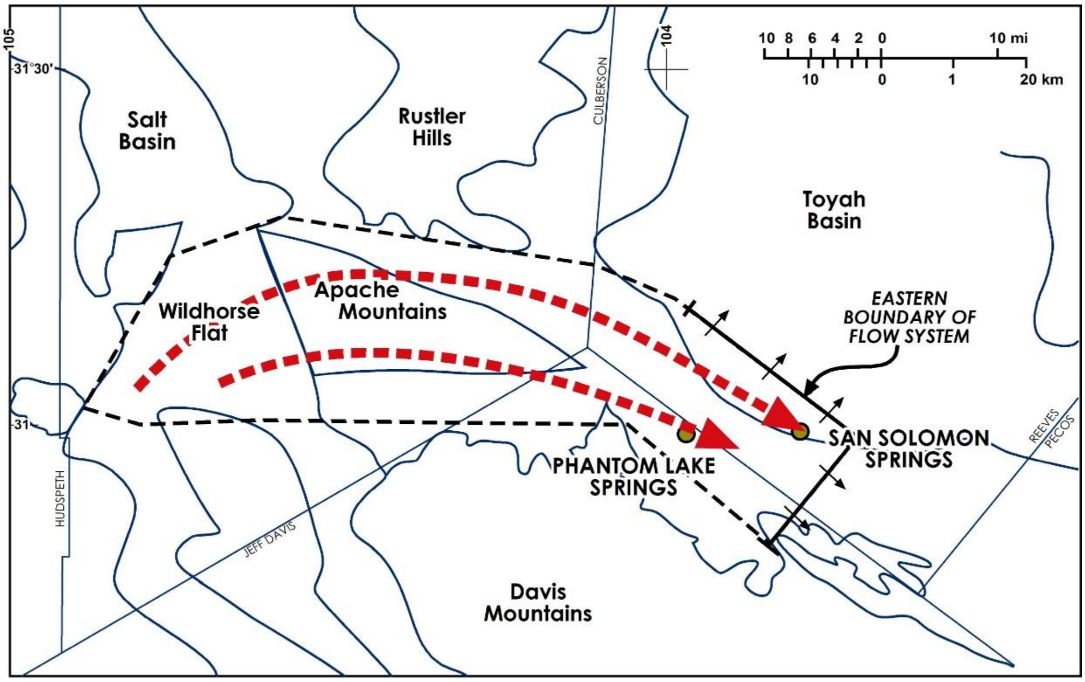


Figure 71 - Map of an area used in regional transmissivity and residence time calculations. The polygon enclosing the regional flow system (dashed lines) has an approximate area of 240 square kilometers. The solid line north and east of the springs represents the eastern boundary of the flow system. The small arrows represent discharge from the regional system into the Toyah Basin, Texas, USA (after Uliana, 2000).

This example is a simple analytical approach based on geometry and an overall water balance. Parameter estimation can also be done using a process called *calibration*, which involves comparing model results to historical aquifer data—for example, water levels, pumping rates from wells—and changing select input parameters—for example, permeability—to match model results to actual data. Figure 72 shows model results—in this case, model-calculated drawdowns in response to pumping from a simulated well—matched to historic drawdown data from an aquifer test. In this case, the model transmissivity and storativity were varied over reasonable ranges of values to optimize the match between the model results and the drawdown values measured during the test. This process can be done manually through trial and error for simple problems. For more complex models there are more sophisticated methods that involve automated software packages (e.g., PEST, https://pesthomepage.org/↗) that can consider a very large number of possible combinations of input parameters and determine the best fit to the data.

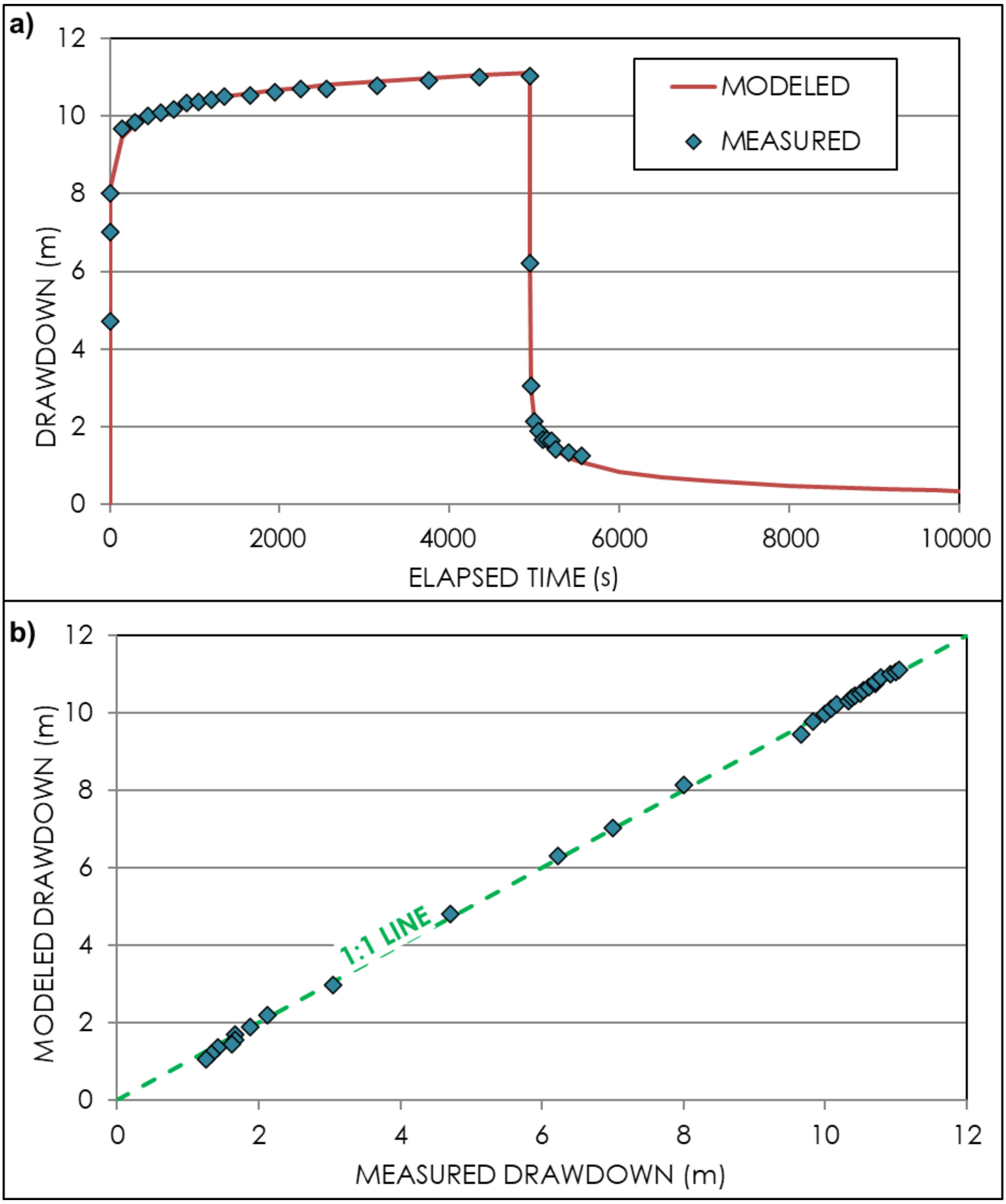


Figure 72 - Example of a calibrated model results matching measured aquifer test results. a) measured drawdowns versus time (blue diamonds) compared to the final calibrated model results (red line); b) a direct comparison of individual modeled drawdowns to the corresponding measured drawdowns with comparison to a 1:1 line.

One challenge associated with calibrating groundwater models is that, in some cases, a variety of model configurations can match the available data equally well. When this happens, model predictions are not reliable. This issue can be alleviated by having a variety of field measurements to calibrate the model (i.e., not only water level measurements but also flow measurements). Exercise 19↴ illustrates this problem using the model of Dupuit flow that was discussed in Section 6.3.2.

Evaluate and Quantify Uncertainty

As previously defined, a model is a *simplified* representation of a natural system. Aquifers are complex, heterogeneous systems that are extremely difficult to characterize and understand. In addition, aquifers are underground, and even when we have extensive networks of boreholes and related data, we are only directly observing a vanishingly small fraction of the overall volume of the aquifer. When hydrogeologists and engineers make groundwater flow models for predictions and forecasting, they generally make many simplifying assumptions about the input parameters—for example, assuming that the entire aquifer can be simulated with a single permeability—with very little verification of the validity of the final model results. To state it in another way, groundwater models always have various levels of uncertainty that range from the input parameters chosen to the final output and predictions.

Rather than develop models to simply make predictions with the hope that the input parameters are appropriate, models can be developed to evaluate and determine the level of predictive uncertainty associated with the available input parameters. This approach typically involves applying ranges of input parameters—for example, permeability—that are based on the ranges of available data, then using specialized software to develop large numbers of model simulations that are matched to available data associated with the predictions of interest—for example, drawdowns or water levels. The results of these models can be used to quantify the amount of risk associated with various management strategies—for example, installing a proposed well field—which can then be used to inform project managers and project investment.

A more detailed overview of model uncertainty analysis is beyond the scope of this book. Readers are encouraged to review the papers and other instructional materials at the Groundwater Modelling Decision Support Initiative (GDMSI) website at https://gmdsi.org/↗.

10.3 Specific Types of Groundwater Flow Models

We have defined the word model and presented reasons for using models. Now, we discuss the various types of models used to simulate groundwater flow in more detail.

10.3.1 Qualitative Models

Simple groundwater contour maps and flow nets are geometric models that provide qualitative information about groundwater flow processes. For example, a groundwater contour map can be used to infer the direction of groundwater flow and identify changes in hydraulic gradient throughout the mapped area, which can in turn be used to identify heterogeneities in aquifers.

10.3.2 Conceptual Hydrogeological Models

All the aspects of a groundwater flow system described in Sections 1 through 9 can be used to develop a summary of the relevant aspects of an aquifer or groundwater flow system. This summary is called a *conceptual hydrogeological model* (CSM). A CSM contains information about aquifer structure; aquifer boundaries; recharge and discharge mechanisms along with estimates of recharge and discharge rates; hydrostratigraphic units that make up the various components of the aquifer and associated confining units along with their properties; interpretations of groundwater flow paths; and projections about future changes to the aquifer (e.g., pumping from wells, effects of climate change). A CSM is the first objective of major aquifer studies, and developing a reliable CSM is a prerequisite for developing the process models described in the subsequent sections of this book.

10.3.3 Physical or Analog Models

Physical models that simulate groundwater flow include tanks full of sand or other sediments with input pipes and drains that can be used to simulate flow through porous media. Physical models made from layers of sand or clay have also been used extensively in structural geology problems. Another physical model simulating fluid flow used two glass plates sandwiched together with small channels etched on the surface of the plates.

Analog models use the similarity between Darcy’s law and other physical processes to model fluid flow. The most popular way of doing this is with electrical circuit models in which the flow of current represents the flow of groundwater, the changes in voltage represent the changes in head, and the resistance of the conductors represents the hydraulic conductivity, as in Equation (89)and Equation (90). Capacitors can be added to the circuit to simulate aquifer storage.

$$\text{Darcy's law: } q = -K\frac{dh}{dl} \tag{89}$$

$$\text{Ohm's law: } I = -\frac{1}{R}\frac{dV}{dl} \tag{90}$$

where (parameter dimensions are dark green font with mass as M, length as L, time as T):

q = water flow through a conductive porous medium

I = current flow through an electrical conductor

$K \text{ and } 1/R$ = proportionality coefficients (K is hydraulic conductivity and R is electrical resistance, which is the inverse of electrical conductance)

dh/dl = head drop across the conductive porous medium

dV/dl = voltage drop across the electrical conductor

10.3.4 Mathematical Models

Any equation that represents a physical process is a model of that process. Darcy's law is a model that represents the flow of groundwater in one dimension. Because it is a model, it tells us something about how the system behaves—that is, flow is linearly proportional to the hydraulic gradient—and it can be used to make predictions about the system—for example, to calculate travel times and flow velocities.

A good deal of this book talks about the basic equations that govern the flow of groundwater in a three-dimensional, transient system. Specific solutions to this equation can be developed and used simulate groundwater flow in flow systems and aquifers. The basic groundwater flow equation is shown as Equation (91).

$$\frac{\partial\left(K_x\frac{\partial h}{\partial x}\right)}{\partial x}+\frac{\partial\left(K_y\frac{\partial h}{\partial y}\right)}{\partial y}+\frac{\partial\left(K_z\frac{\partial h}{\partial z}\right)}{\partial z}=S_s\frac{\partial h}{\partial t} \tag{91}$$

This equation describes the flow of groundwater and change in storage with time at a single point in a flow system. If we apply boundary conditions to this partial differential equation, then for simple geometry and homogenous properties we can solve the equation for a specific flow system using an analytical solution.

The problem with using this equation for practical groundwater flow problems is that parameters and boundary conditions vary in space and time, and we have an infinite number of points in any system—and therefore an infinite number of equations to solve—and the heads and fluxes at each point are dependent upon the heads and fluxes at all the surrounding points. We can't simultaneously solve an infinite number of equations, so we must simplify the system in some way and then use that simplification to solve the problem and model the system. There are many methods for solving equations like these, but they are variations of two basic approaches to solving partial differential equations: analytical and numerical.

Analytical Solutions

One can consider a specific scenario, develop some assumptions that simplify the system, and then use those assumptions to develop an *analytical solution* to the equation. The Theis equation is an example. Theis considered a specific situation—radial flow to a well—with several simplifying assumptions—homogeneity, constant thickness, infinite extent, a fully penetrating well of infinitesimal radius, and so on—and derived an equation that directly solves for the drawdown in the aquifer at some distance from a pumping well and at some time after pumping started.

Analytical solutions are advantageous because they are easy to use—since they can be solved directly—and they give unambiguous results at any of the infinite number of locations within the system. However, they are limited because they require simplification of the systems, and so they are not always applicable to solving meaningful problems. Also, we may have a specific situation for which we want to derive a solution, but the situation may be so complex that a direct solution isn't possible. In such cases we turn to numerical approaches.

Numerical Solutions

If an analytical solution is not possible, an alternate approach to solving the equation is to develop an approximation called a *numerical solution*. The basic procedure for developing a numerical solution is to first determine the problem domain—the geometry and volume of the system— and its boundary conditions, then reduce the infinite number of points—and corresponding equations—in the system to a finite number of points, and then use an *iterative* method to solve the equations at those points.

The process of reducing the system to a finite number of points is called *discretization*—that is, making a continuous function into a discrete function. An example is shown in Figure 73. In this example, an alluvial aquifer that contains an infinite number of points is discretized by superimposing a regular grid of six columns and eight rows containing 48 points, or *nodes*. Each point contains a version of the groundwater flow equation that calculates hydraulic head at each point based on the values of head at the surrounding points. The grid also has boundary conditions based on the physical boundaries of the model—the river and the granite fault block—and the northern and southern extent of the study area that are symmetry boundaries (Figure 61).

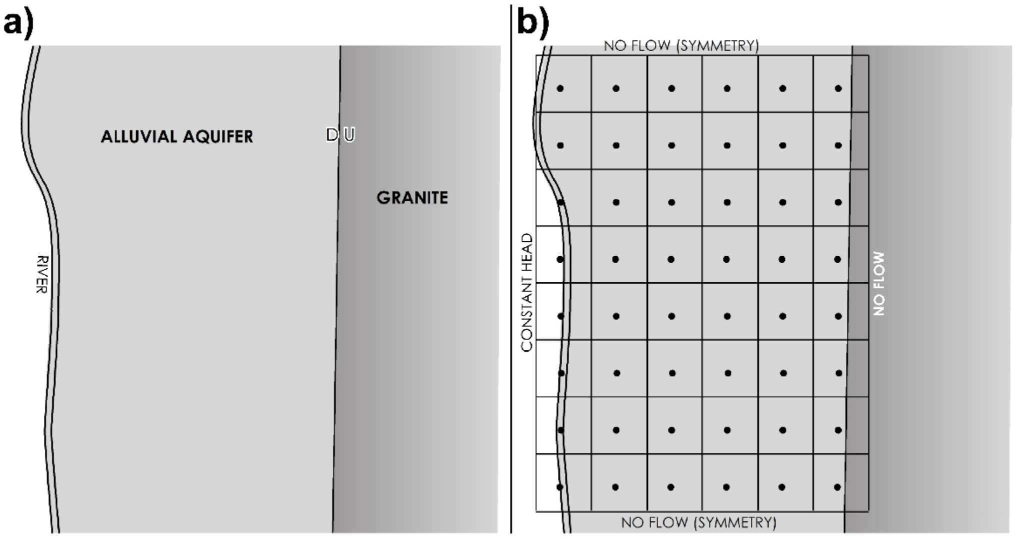


Figure 73 – a) Map of an alluvial aquifer, bounded by a river to the west and a fault-block granite to the east; and b) discretization of the alluvial aquifer with boundary conditions and model nodes.

Once the model grid has been discretized and the equations are added to each node, an initial value of head is assigned to each node and the set of equations are solved. However, the value of head at each node is dependent upon the value at each of the four surrounding nodes (Figure 74). If the head at a given node is calculated based on the heads at the surrounding nodes, then when the head is calculated at the next adjacent node, its head changes so the head at the first node needs to be recalculated.

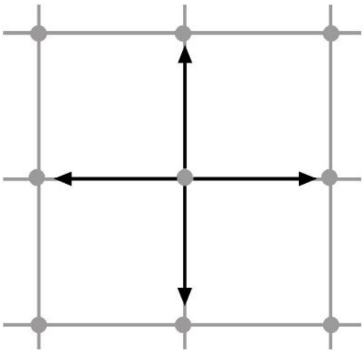


Figure 74 - The value at any node is dependent upon the values at the surrounding nodes.

This is where the numerical part of the method is applied. This is done by assigning initial heads to all nodes, then using matrix solutions—which can be thought of in a simplified sense as starting at one corner of the model, calculating a head at that node, then systematically moving across the row to calculate the value at each node. The same process is then applied to the next row, then the next, and so on, until all nodes have been calculated and the value at each node has been recorded. The process is then repeated, and the values from the second set of calculations—that is, the second iteration—are compared to the first set of calculations and the difference between each set is calculated. This process is then repeated multiple times until the change in head from one iteration to the next is so small that it can be considered negligible. At this point, the model has *converged* on an approximate solution. Simple models with well-chosen initial heads can converge in only a few iterations, while complex models can take hundreds or even thousands of iterations to converge. Complex models can also experience mathematical instability, which in turn results in models that never converge.

This iterative process is where the importance of modern computers comes in. A person using a pencil, paper, and a hand-held calculator could take years to calculate and solve a simple model with only a few nodes. Developing numerical models was not feasible until the development of digital computers that can do lots of calculations in incredibly short time. Computing power is so great now that inexpensive desktop computers can solve sophisticated models in a relatively short time. However, the processing time can become significant if there is a lot of detail in the model, or if an automated calibration or uncertainty analysis is applied to a large model.

Discretizing a system involves decisions concerning how much detail needs to be built into the model, and these decisions are based in many ways on the model objectives and the amount of detail required for the results. The more nodes that are included in the model grid, the more computationally intensive the model. Also, more complexity increases the possibility of numerical instability and a model that won’t converge. In many cases, simpler models are better because there is less estimation and uncertainty, and frequently they are sufficient to provide the information that we need.

10.4 The Method of Finite Differences

The equation applied to each model node is a finite version of the groundwater flow equation, Equation (91). We can re-state that in the following way. Equation (91) represents the flow at an infinitesimal point, a point with no dimensions. When a finite number of nodes is added to a numerical model, the equation must be transformed so that it accounts for the distance from one node to the next and/or the overall volume of aquifer which is represented by the node. There are several mathematical methods that we can use to do this; the one used in most groundwater models is the method of *finite differences*.

The exact derivation of the finite difference equations is not presented here, nor a comprehensive overview of the method. Instead, the goal of this section is to provide a very simple and general overview of the procedure. To illustrate the approach in the simplest manner, we can assume that the equations calculate the value of head at each node as an *average* of the heads at the surrounding nodes. If the aquifer is modeled as heterogeneous, or isotropic, or if the node spacing is variable, it gets a little more complicated than that, but for now we just need to consider that the value at each node is dependent upon the four surrounding nodes.

Each node in a finite difference model grid requires a hydraulic conductivity and a storage parameter—specific yield if unconfined, specific storage if confined. Recharge can be added to each node. This is accomplished mathematically by adding a volume of water per unit time to the node, and the storage parameter controls the resulting head change. Pumping wells can also be added at any node. Pumping wells are the inverse of recharge—that is, a volume of water per unit time is removed. Recharge and wells are *specified flux* boundaries.

Boundary conditions must be applied to the grid at the edges of the model domain. A no-flow boundary is treated like an image well problem, with an imaginary node on the opposite side of the boundary with head equal to the active node inside the boundary (Figure 75a). For a constant head boundary, the constant head value is assigned to the node and is not recalculated based on head at surrounding nodes (Figure 75b).

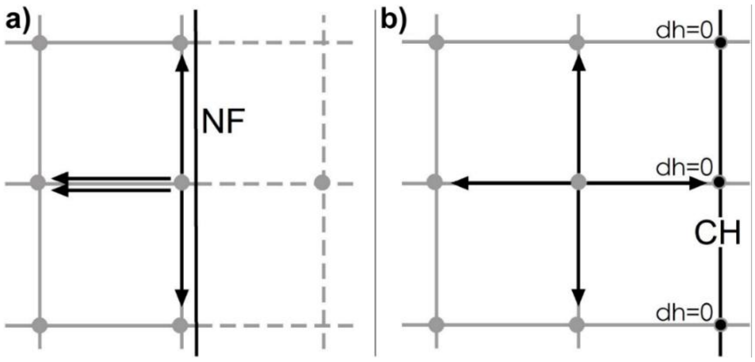


Figure 75 - Implementation of boundary conditions in finite-difference grids for: a) no-flow boundary with dashed lines representing the ‘image’ nodes; and b) constant head boundary with black nodes representing locations where head is assigned so it influences calculations as surrounding nodes but is not recalculated during iteration.

Finite difference model grids must follow certain rules. The simplest approach to discretizing a model is to use a consistent row and column spacing throughout the model grid (Figure 76). If there is an area within the center of the overall model domain that requires high resolution—for example, a proposed well field for a new factory—then variable row and column spacing can be applied to the grid (Figure 77). The advantage of this approach is that it uses fewer nodes overall, which makes the model less computationally intensive. A potential disadvantage is that the long rectangular model cells at the edges can create model instability and result in models that do not converge. There are recently developed versions of publicly available groundwater modeling software that utilize *unstructured* or *nested grids*, which allow for finer grid resolution within larger grid cells thus avoiding the elongated rectangles at model edges (Figure 78)—for example, MODFLOW-USG, MODFLOW 6↗. These models allow for more efficient grid spacing with high resolution in some areas and larger grid cells in others.

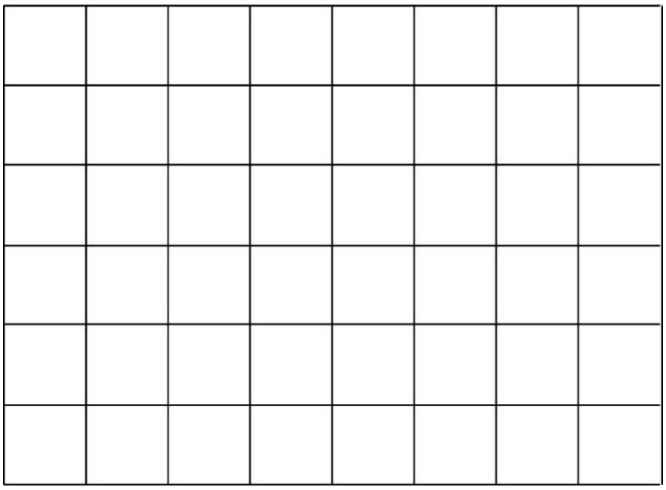


Figure 76 - Example of a finite difference grid with constant row and column spacing.

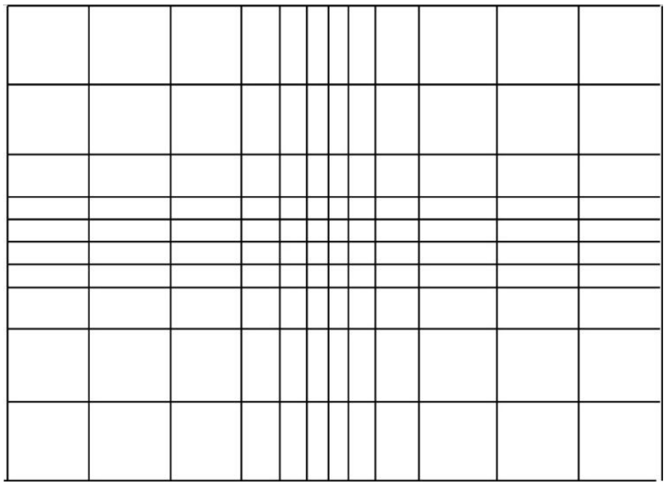


Figure 77 - Example of a finite difference grid with variable grid spacing. This can be applied to a model which has an area in the middle of the model domain that requires high resolution.

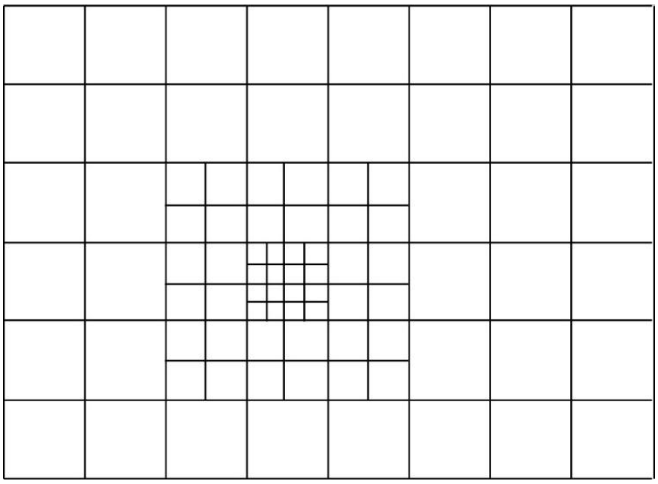


Figure 78 - Example of a finite difference grid with nested grids. This is a more stable and efficient way of simulating high-resolution areas within the model grid as compared to Figure 78.

Figure 79 is an example of an unacceptable finite difference model grid with grid cells that are not rectilinear. There is an alternate model approach called the method of *finite elements* that allows for variable model elements such as those in Figure 79. The finite element method will not be discussed further in this book.

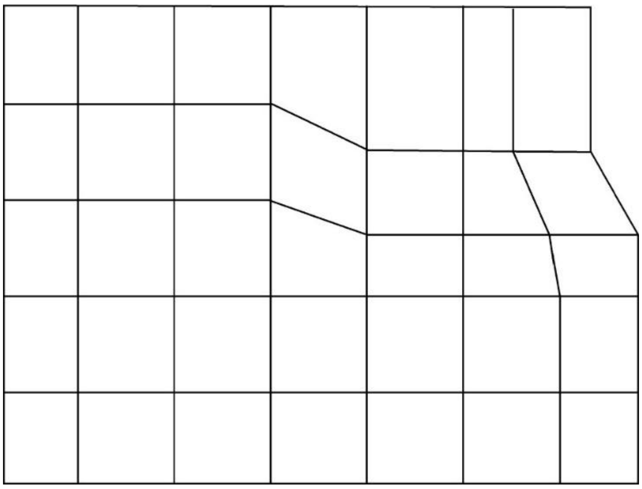


Figure 79 - Example of an unacceptable finite difference grid with non-rectilinear grid cells.

10.5 An Introduction to MODFLOW

A rigorous finite difference model consists of computer code that reads the model grid and other input values, performs the iterative calculations, and records the output in some usable format. A finite difference model code requires the following items.

- A means of inputting grid node locations, boundary conditions, and hydraulic parameters (K, S_s) at each node.
- An algorithm that calculates heads at each value and controls the iterative process—that is, compares calculated heads from each successive iteration, to determine if the convergence criteria are met.

- A means of writing the final head values to an output file for processing.

Constructing simple finite difference models using FORTRAN, python, or some other computer language to do the calculations is not too difficult. However, models can get very complex in a very short period, and the time spent writing the code can increase dramatically with increasing complexity.

In the early days of groundwater modeling the modeler needed to write the computer code to build the model for any given specific problem. In the late 1970s, hydrogeologists and programmers started writing model codes that were flexible and could be applied to a variety of situations. Then in the early 1980s, two hydrogeologists at the USGS named McDonald and Harbaugh (1984) developed and published the code for a finite-difference-based software package called MODFLOW, which quickly became the industry standard for developing groundwater flow models.

What made their model so useful and universally acceptable was that, rather than set up a single computer program with all the steps outlined above, they set up their software as a series of subroutines, or *modules*, with each module associated with a specific part of a finite difference model. For example, they have a pumping well module, a recharge module, a river boundary module, etc. The user is still responsible for putting together the input files and setting up the grid, but then they can choose the modules that are appropriate for their model and construct it accordingly. This, along with the fact that their software was free, resulted in widespread distribution of the model and an almost universal acceptance of the program.

Over the years since the original release of MODFLOW there have been numerous subsequent versions with various improvements, and several non-USGS affiliated companies started developing and selling *pre-processor* software that use graphical user interfaces (GUI) and special programs to aid in grid construction and parameter distribution. These are then linked directly to MODFLOW, so the model grid and parameters go right into the MODFLOW algorithm. They also developed *post-processor* programs that take the output and make nice images and animations illustrating model results. These programs are in widespread use today, and they give any individual with a standard PC computer the ability to generate and run fairly complex groundwater models.

This section provides a limited overview of groundwater modeling and the MODFLOW modeling environment. Students and practicing hydrogeologist who want more information on groundwater modeling are encouraged to download and read *Getting Started with MODFLOW*↗ by Winston (2023) and *Geologic Frameworks for Groundwater Flow Models*↗ by Brandenburg (2020); both books are available as free downloads from The Groundwater Project.

11 Groundwater Chemistry

11.1 Introduction

In this section, we use the term "mass" to mean anything in the water that isn't water. Mass exists in several phases within a groundwater system. Mass exists in a solid phase, consisting of either the minerals that make up the geologic units or suspended particles moving through the aquifer. Mass also exists as a separate liquid phase; for example, many organic contaminants do not mix with water very well and stay in a liquid state in the subsurface. Mass also occurs as a gas phase; most soils contain $\mathrm{CO_2}$ in their pores, and bubbles of gas can occur in the saturated zone of aquifers. Finally, mass exists as a dissolved phase. All these phases are significant parts of aqueous geochemical systems; however, hydrogeologists are mostly concerned with the dissolved phase, and they generally lump the other three phases together as sources for dissolved species in the water.

There are two aspects of water chemistry that interest hydrogeologists. The first aspect deals with the natural chemistry of groundwater systems and the interaction of groundwater with solid mineral phases and organic matter. Water is known as the *universal solvent* because it reacts with and/or dissolves nearly anything it encounters. The rocks and sediments that make up an aquifer—primarily *inorganic* compounds—impart a chemical signature to the water, in the form of a specific set of dissolved ions. This is useful to us because we can use the groundwater chemistry to infer where the water is coming from and to link the groundwater to specific geologic units.

The second is the problem of groundwater contamination and the spread of contamination in the subsurface. This is discussed in Section 12.

In this section, we focus on mass as dissolved phases in groundwater, and consider any solid, liquid, or gas phases together as a separate phase that reacts with the groundwater. Groundwater is constantly reacting with the other masses in the subsurface. These reactions are responsible for the redistribution of mass in the subsurface and are what control the chemistry of the groundwater.

11.2 First, a Little Background Chemistry

This will be review for most people, but it is provided to ensure everyone understands the terminology and concepts.

11.2.1 Basic Chemistry Concepts and Definitions

Atoms and *elements* are the basic building blocks of nature. They are described in the periodic table. An interactive periodic table is available at https://ptable.com/#Properties↗. Each element has a symbol associated with it—for example, carbon is C, potassium is K, neodymium is Nd. Atoms are made of the following smaller particles (Figure 80):

- *Protons* are positively charged particles that reside in the nucleus of the atom. The number of protons in an atom is called the *atomic number*. The atomic number is the large number written above the element symbol on a periodic chart, and it determines the element—for example, carbon has 6 protons, argon has 18, uranium has 92. Protons have mass and make up part of the mass of the atom. Each proton has a mass equal to 1 *amu (atomic mass unit).*
- *Electrons* are negatively charged particles that orbit the nucleus of the atom. The number of electrons in a stable atom equals the number of protons—so there is no net charge. Electrons have a mass that is so small it is negligible, so we will consider their mass is zero.
- *Neutrons* are particles with mass, but no charge, which reside in the nucleus with the protons. The number of neutrons usually equals the number of protons; however, the number can vary—usually a few more than the protons. The number of neutrons does not affect the properties of the element, but it does affect the mass of the element.

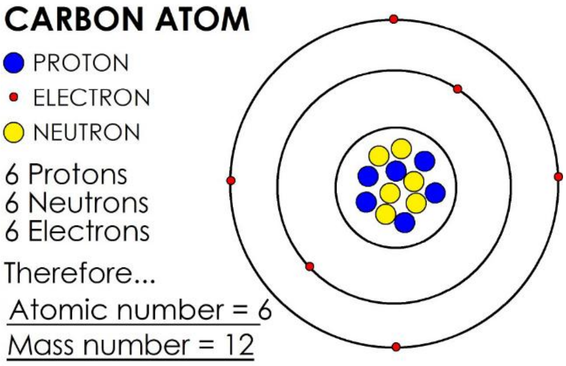


Figure 80 - General structure of a carbon atom.

Elements with the same number of protons, but different numbers of neutrons, are called *isotopes* of the element.

- For example, a stable carbon atom has 6 protons and 6 electrons. Carbon atoms in nature usually have 6 neutrons (6 neutrons + 6 protons = carbon 12, or ^{12}C). Some carbon atoms have 7 neutrons (^{13}C) or 8 neutrons (^{14}C). These are all isotopes of carbon.
- Each of these isotopes is still carbon; they are all the same size and react with other elements in the same way. The main difference is that ^{13}C and ^{14}C are heavier, and ^{14}C is radioactive—that is, it is unstable and decays into ^{14}N.

The mass of the protons and neutrons add up to make the *atomic mass* of the element—the smaller number written below the element symbol on the periodic table.

Molecules contain two or more atoms bonded together into a single particle. Molecules can be made up of a single type of element—for example, nitrogen in the atmosphere is N_2—or it could be several elements—for example, my favorite molecule, $C_8H_{10}N_4O_2$.

Ions are elements that have gained or lost one or more electrons after they dissolve into water, and so have a positive or negative charge. For example, when table salt ($NaCl$) dissolves in water, it dissociates into sodium ions (Na^+) and chloride ions (Cl^-). Molecules that dissociate into ions in solution are called *electrolytes*.

- Positively charged ions are called *cations*. Examples include:
  - calcium - Ca^{2+}
  - sodium - Na^+
- Negatively charged ions are called *anions*. Examples include:
  - chloride – Cl^-
  - bicarbonate – HCO_3^-
  - sulfate – SO_4^{2-}

Some ions, like bicarbonate and sulfate, consist of two or more elements.

Some compounds do not dissociate into ions in water; rather they stay in molecular, or nonionic, form. Most organic molecules are nonionic, although some will dissociate into organic cations and anions.

The basic types of chemical bonds—the forces that hold atoms together in a molecule—are as follows:

- *ionic bonds*: positive ions stick to negative ions—for example, table salt;
- *covalent bonds*: two elements share an electron—for example, many organic molecules;
- *metallic bonds:* are arranged in crystals within a "soup" of electrons that are freely shared by all nuclei; and
- *Van der Waals bonds*: are held by a weak intermolecular force that temporarily bonds molecules together.

11.2.2 Atomic Mass and the Mole

As previously stated, the atomic mass is related to the number of protons and neutrons in an element. The mass changes when different isotopes are present, but the average mass of all the isotopes is given on the periodic chart and is sufficiently accurate for use in our calculations.

In any sort of chemistry, we are interested in knowing the number of atoms, ions, or molecules that are being exchanged when reactions occur. For example, consider the reaction of calcite dissolution and precipitation:

$$CaCO_3 + CO_2 + H_2O \leftrightarrows Ca^{2+} + 2HCO_3^-$$

Each molecule of calcite reacts with one carbon dioxide molecule and one water molecule to produce one calcium ion and two bicarbonate ions. So, if we know how many calcite molecules we have, we can figure out how much water and carbon dioxide we need to dissolve all of it, and how much calcium and bicarbonate will be produced.

However, we can't directly measure the number of atoms or ions in a solution. What we can easily measure is the *mass* of the elements in the solution. This is a problem because each element has a unique atomic mass, so a given mass of one element will not have the same number of atoms as a similar mass of another element. Fortunately, we can easily relate the mass of an element to the number of atoms using the unit called the *mole*.

A mole is the number of atoms (defined by *Avogadro's Number,* $6.02\text{x}10^{23}$) in a mass of element equal to its atomic mass in grams. For example:

- Oxygen has an atomic mass of 16—actually, it's 15.9994, but 16 is close enough for our purposes here. Therefore, 16 grams of oxygen contains $6.02\text{x}10^{23}$ oxygen atoms, and 1 mole of oxygen weighs 16 grams.

We can also express compounds as moles by determining the *formula weight* of the molecule. This is done by summing up the masses of the individual atoms in the molecule. For example, the formula mass of caffeine ($C_8H_{10}N_4O_2$) is the sum of:

- atomic mass of C is 12 amu in each of 8 atoms = 96 amu;
- atomic mass of H is 1 amu in each of 10 atoms = 10 amu;
- atomic mass of N is 14 amu in each of 4 atoms = 56 amu;
- atomic mass of O is 16 amu in each of 2 atoms = 32 amu;
- total atomic mass = 194 amu;

therefore, one mole of $C_8H_{10}N_4O_2$ has a mass of 194 grams.

In any given chemical reaction, a certain number of atoms of one element will react with a certain number of atoms of another element. We can use the atomic mass to convert the masses of our elements to moles, then calculate how many moles of each element react with the other elements, then convert back to mass.

11.2.3 Concentrations of Solutions

An element that dissolves into water—or some other liquid—is called a *solute*. Frequently, the individual elements or molecules that are dissolved will be called *dissolved species* or *dissolved constituents*. The liquid it dissolves into is called a *solvent*. We can measure and express the concentration of the solute in the solvent in terms of the mass of the solute or the number of moles of the solute.

- *Mass solute per unit volume of solution* is most often expressed as milligrams per liter (mg/L) or micrograms per liter (µg/L).
- *Mass of solute per unit mass of solution* used to be expressed as parts per million (ppm) or parts per billion (ppb); now they are usually reported as mg/kg or µg/kg.
- *Molarity, or molar concentration* (M) refers to the number of moles of a species per liter of solution. *Molality*, or *molal concentration*, (m) refers to the number of

moles of a species per kilogram of solution. These two values are generally the same for aqueous solutions since a liter of fresh water has a mass of about 1 kilogram. However, the mass of a liter of solution increases as the concentration increases, so for a saline solution like seawater, molarity and molality are not the same.

One can use the formula weight of an element to convert between mass concentrations and mole concentrations. To do so, express the concentration as *grams per volume*, then divide by the formula weight to get *moles per volume*.

The reason we use mole concentrations is because chemical reactions involve *numbers of atoms* rather than *masses of atoms*. However, in dealing with ionic reactions—like the precipitation or dissolution of salts—the number of ions that will react with each other depends on the charge of the ions. An ion with a +2 charge—for example, the calcium ion, or Ca^{2+}—will react with two ions with a –1 charge—for example, the chloride ion, or Cl^{-}. Therefore, it is sometimes useful to express concentrations in terms of *equivalent charge concentrations*. The equivalent charge concentration—in *equivalents* (eq) per liter—is equal to the molarity times the charge of the ion. The following provide a couple of examples.

- Sodium (Na^{+}) has a charge of plus one, ∴1 M sodium solution = 1 eq/L
- Calcium (Ca^{2+}) has a charge of plus two, ∴1 M calcium solution = 2 eq/L

Equivalent concentrations are usually expressed in milliequivalents per liter (meq/L) or per kilogram (meq/kg). We can convert from mg/L to meq/L by dividing by the formula weight and then multiplying by the charge of the ion. Exercise 20↴ provides an opportunity to make these conversions and use them in later exercises.

11.3 Water Analysis and Naturally Occurring Dissolved Species

Groundwater taken from any given place contains a wide variety of dissolved constituents. We have many methods for analyzing water, both in the field and in the laboratory, which characterize the concentrations of dissolved solutes in the water. Here we discuss the parameters that are typically measured.

11.3.1 Basic Chemical Parameters

It is common practice to measure temperature, pH, alkalinity, total dissolved solids (TDS), and electrical conductance when analyzing groundwater.

- *pH* is defined as the inverse log of hydrogen ion activity in the water (*activity* is very similar to concentration). For example, a pH of 7 means that the activity (or concentration) of hydrogen in the water is equal to 1×10^{-7} mol/L. A pH of 5.6 means that hydrogen activity is equal to $1 \times 10^{-5.6}$ mol/L.
- *Alkalinity* refers to the ability of the water to neutralize an acid. This is directly related to specific dissolved species in the water; for most groundwater systems the predominant acid-neutralizing species is the bicarbonate ion. Alkalinity is

therefore often used as a measurement of the concentration of bicarbonate in groundwater. Alkalinity is usually measured in the field, when the samples are taken, because exposure to air can cause some of the bicarbonate in solution to form carbon dioxide and leave the solution.

- *Total dissolved solids* represent the total concentration of dissolved constituents in the water; usually measured by evaporating a liter of the sample and measuring the weight of the remaining solids. TDS is measured in mg/L. Shallow groundwater (within a few hundred meters of the surface) generally ranges from 100 mg/L to 10,000 mg/L TDS (seawater is 35,000 mg/L). The standard limit for drinking water is generally considered to be less than 1,000 mg/L, with less than 500 preferable—although people can tolerate up to 2000 mg/L for short periods of time.
- *Electrical conductance (or electrical conductivity)* is a measure of the ability of the sample to conduct electricity. The units of electrical conductance are microsiemens per centimeter (μS/cm). Conductance is a proxy for TDS, since the electrical conductivity of water is a function of the amount of dissolved material in the water. Conductance is also a function of water temperature, so it is occasionally reported as *specific conductance*, which is the equivalent conductance at 25 °C.

In addition to these basic chemical parameters, we also analyze for the concentration of individual constituents in the water. Most of the dissolved constituents in groundwater are inorganic cations and anions derived from the dissolution of minerals. In any natural groundwater sample about 95 to 99 percent of the dissolved constituents (by weight) will consist of the following constituents.

- Bicarbonate
- Calcium
- Chloride
- Magnesium
- Silicon
- Sodium
- Sulfate
- Carbonic acid

These are usually referred to as the *major constituents* or *primary constituents,* and generally have concentrations greater than 5 mg/L. These constituents come directly from the dissolution of mineral phases in the subsurface, and a set of laboratory analyses will generally test for them.

Minor constituents, which generally occur in concentrations ranging from 0.1 to 10 mg/L, include the following constituents.

- Boron
- Carbonate
- Fluoride
- Iron
- Nitrate
- Potassium
- Strontium

In addition to these, there are a host of *trace constituents* that occur in concentrations less than 0.1 mg/L. Some of these are heavy metals that are toxic in small quantities. These constituents are generally not analyzed for except when looking for contamination or for investigating some specific geochemical process.

If a water sample is analyzed for the major and minor constituents, the analytical results will identify almost all the mass dissolved in the sample. There are two ways that we can check the analytical results, to ensure the quality of the analyses.

- The sum of the concentrations of the individual species, as determined by various lab procedures, can be compared to the total dissolved solids concentration as determined by evaporation in the lab. If the two are significantly different, then most likely there is a problem with the lab analyses.
- Water is electrically neutral; for every positive ion in solution, there is an equal number of negative charges to offset the charge. A *charge balance* (also called *electroneutrality*) can be calculated to see if the sum of the cations equals the sum of the anions. The charge balance (C.B.) is calculated by dividing the difference between the sum of the cation concentrations (in meq/L) and the sum of the anion concentrations (in meq/L) by the sum of the concentrations of all ions as shown by Equation (92).

$$\text{C. B.} = \frac{(\Sigma_{cations} - \Sigma_{anions})}{(\Sigma_{cations} + \Sigma_{anions})}(100) \tag{92}$$

Ideally, the charge balance should be within ±5 percent—allowing for analytical uncertainty and the trace elements that were missed in the analysis. Exercise 21↴ provides an opportunity to assess the charge balance of the sample analysis evaluated in Exercise 20. Exercise 22↴ provides an opportunity to assess the charge balance of a different sample.

11.4 A Little about Chemical Reactions

The following is a basic overview of the types of reactions that can occur in groundwater systems. We will not get into the specific details of any of the reactions.

However, a basic understanding of the nature of these reactions, and how we study them, is important.

Most of the reactions that affect groundwater chemistry involve reversible reactions, like precipitation and dissolution of mineral phases and certain oxidation-reduction reactions. Reversible reactions are reactions in which the system reaches a *dynamic equilibrium*. For example, consider a small piece of the mineral calcite—the primary mineral in limestone—completely submerged in a glass of water. The water will dissolve the calcite until it becomes saturated with the ions in the calcite mineral. At that point, the system is at equilibrium; however, the reactions don't stop; what happens is that calcite is precipitating and dissolving at the same rate, so the concentration in the water doesn't change. This is the dynamic part of the equilibrium, the reactions (i.e., mineral precipitation and dissolution) still occur but are proceeding in both directions at an equal rate. The characteristics of this equilibrium—for example, how much is in solution and how much is in the mineral, are based on the *thermodynamics* of the system. In other words, these reactions are all about changes in energy in the system, and they are all about the system trying to reach a state of minimum energy. Several widely used mathematical models that represent these systems are based on the thermodynamics of equilibrium. These models and approaches to groundwater chemistry ignore *reaction rates*—in other words, if we assume that a system is at equilibrium, we know nothing about how long it took to get there or how fast the reactions are proceeding.

When we strive to understand the dynamics of geochemical systems, we often assume that the system is at equilibrium. This is like a lot of other assumptions that we make—it simplifies things for us and turns a complicated system into something that we can understand, while in reality, it is wrong. Natural aqueous systems are rarely at equilibrium; instead, they are in a state of moving towards equilibrium at a rate that is related to the reaction rates. In some cases, it may be more appropriate to use a *kinetic* approach to modeling the chemistry. Kinetic models are appropriate for reactions that proceed slowly.

Some reactions in groundwater systems are *irreversible*; they go in only one direction. Examples include radioactive decay, some oxidation-reduction reactions, and degradation of organic contaminants. When dealing with irreversible reactions, the concept of equilibrium does not apply, and we are required to use a kinetic approach.

For more details and information about geochemical reactions, students are encouraged to review *The Geochemistry of Natural Waters*↗ by Drever (1997), which is available as a free download from The Groundwater Project.

11.5 Inferences about Groundwater Flow Systems

When we collect groundwater samples and analyze them, we are compiling information about the dissolved species in the water. The dissolved constituents were derived from the rocks and sediments through which the water flowed, which makes them useful to hydrogeologists in their quest to understand flow systems. If we have chemical analyses on groundwater samples and know something about the type of rocks or sediments in the subsurface, we can make inferences about where the water flowed and what rocks it encountered along the way. Here are some examples of how chemical analyses of groundwater can be used.

- If a groundwater sample taken from a well is relatively low TDS but contains a lot of dissolved calcium and sulfate with relatively equal molar concentrations, it is likely that the water chemistry is controlled by interactions with the minerals gypsum and anhydrite— $\mathrm{CaSO_4 \cdot 2\,H_2O}$ and $\mathrm{CaSO_4}$, respectively. If we know that there are beds of sedimentary rocks containing gypsum and anhydrite dipping into the subsurface from nearby mountains, we may infer that the groundwater is relatively recent recharge that has flowed to the study area through the gypsum and anhydrite beds.
- Suppose two groundwater samples, one taken from downgradient of the other, have very similar chemistry with the exception that the upgradient sample has more calcium and magnesium in it while the downgradient sample has less $\mathrm{Ca^{2+}}$ and $\mathrm{Mg^{2+}}$ and much more sodium $(\mathrm{Na^+})$. If we know that there are a lot of clays in the area, we may infer that *cation exchange* is occurring—that is, the clays are grabbing the divalent cations and releasing monovalent cations. Cation exchange is also discussed in Section 12.3.

11.6 Graphical Representations of Geochemical Data

Dealing with large amounts of geochemical data can be facilitated by using some standard graphical methods for displaying and analyzing the data. The following sections contain general descriptions of methods that are commonly used for presenting and interpreting groundwater chemistry data.

11.6.1 Piper Diagrams

Piper diagrams are a specific type of ternary diagram used to plot the concentrations of the major cations and anions (Figure 81). The Piper diagram consists of two ternary diagrams—one for cations and one for anions—and a central diamond-shaped field into which the cation and anion values are projected. The relative percentages of the concentrations (in meq/L) of the three major cations (Ca, Mg, and Na) are plotted on the ternary diagram to the left. Usually, potassium and sodium are added together since they are similar in size and readily substitute for each other in minerals. The same is done with the three major anions (Cl, $\mathrm{HCO_3}$, and $\mathrm{SO_4}$). The location of each point is then projected up into the central area, and the intersection of the projections defines a point on the central field.

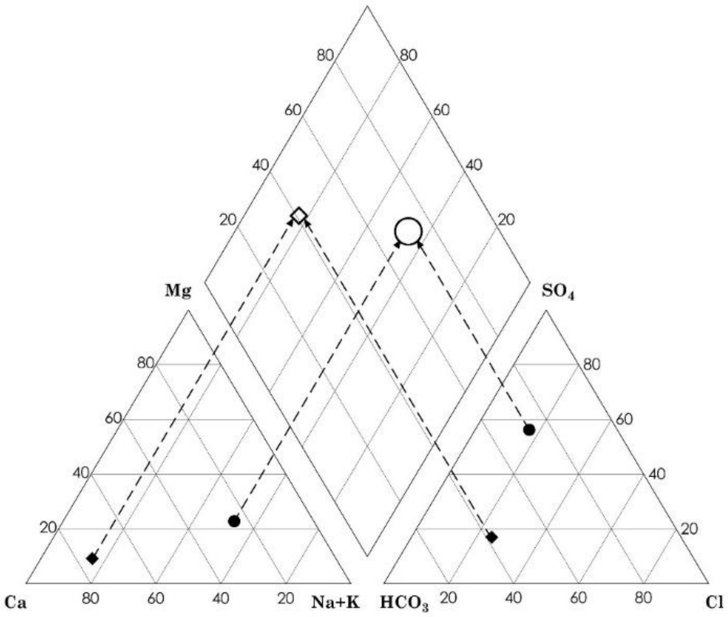


Figure 81 - Piper diagram: the upper diamond integrates the ions from the bottom left triangle (cations) and lower right triangle (anions)

Piper diagrams are good for looking at lots of data, because they can show distinct groups of water types (Figure 82) as well as the relationships between those groups. Exercise 23↓ provides an opportunity to plot the water analysis of Exercise 20 and 21 on a piper diagram.

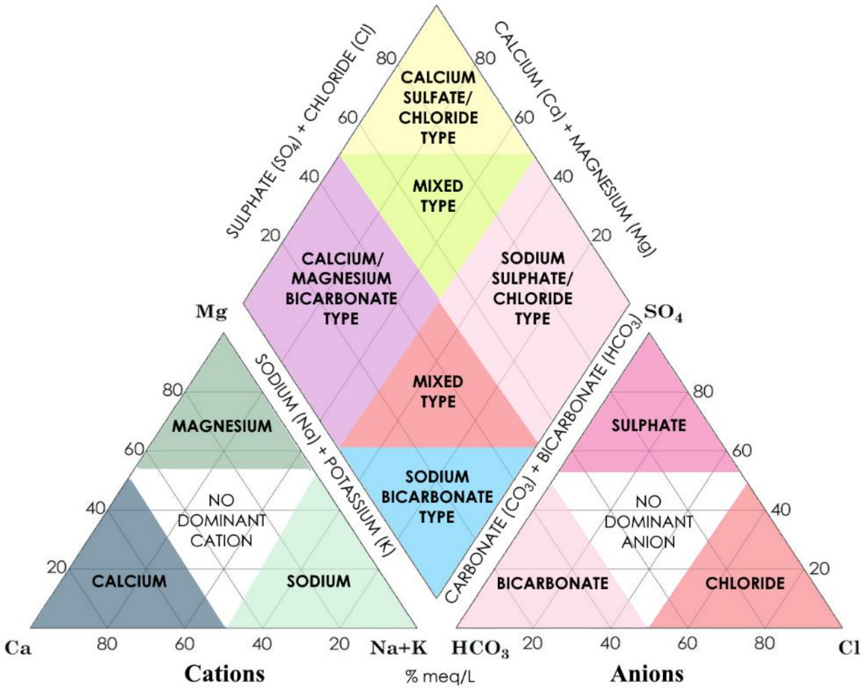


Figure 82 - Water types as delineated by the location of a water analysis on a Piper diagram (modified from Hatari Labs).

An example is given in Figure 83. The data are from groundwater samples taken from wells in Presidio County, Texas. Samples from wells in the Rio Grande Alluvium and from those in the upgradient volcanic formations fall into distinct fields on the diagram. The samples in the Presidio Basin sediments, located between the igneous uplands and the Rio Grande Alluvium, fall into a broad area that overlaps both fields with some samples clearly falling in the Igneous/Volcanic field, some in the Rio Grande Alluvium field, and some along the boundary. The locations of the Presidio Basin samples on the plot can therefore provide clues about the origins of the water in those wells.

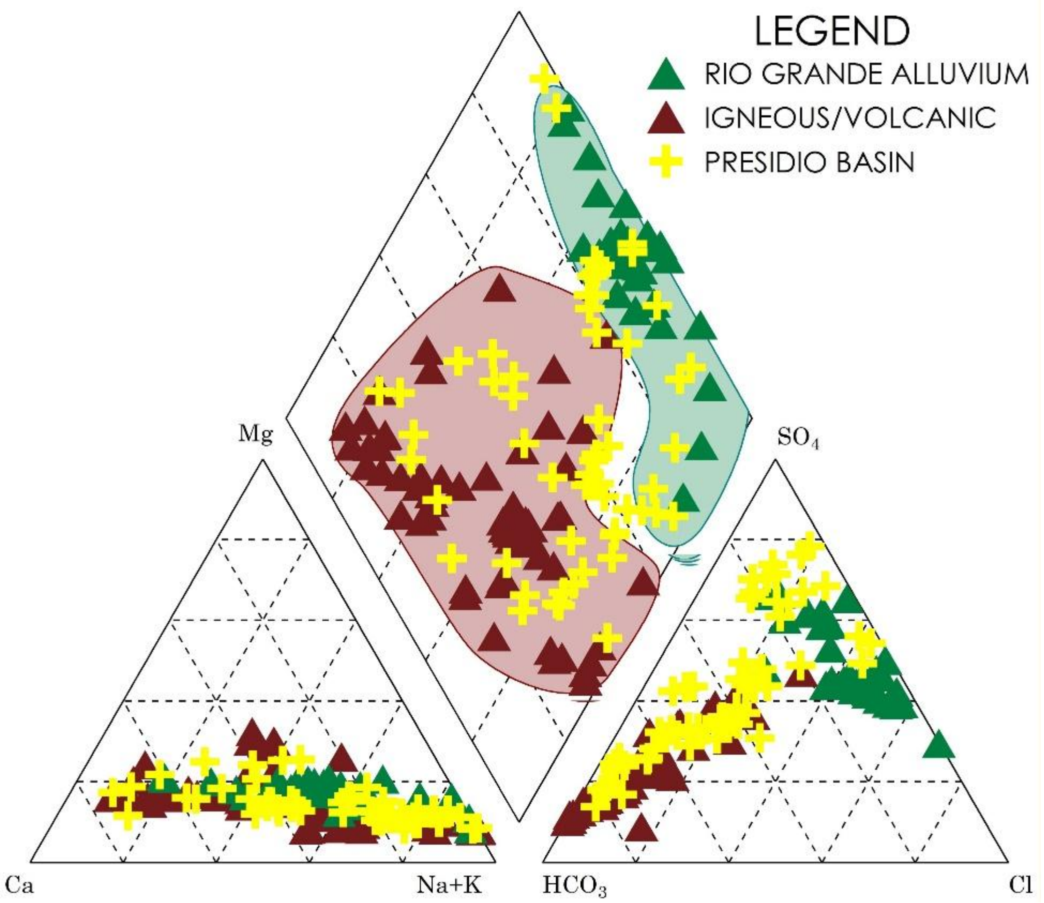


Figure 83 - Piper diagram of analytical data from water samples taken in Presidio County, Texas (McKiernan and Uliana, 2005; all data acquired from the Texas Water Development Board Water Well Database in February 2005).

11.6.2 Stiff Diagrams

Stiff diagrams are a way of representing analyses from an individual water sample. A Stiff diagram is basically a graph that has concentration (in meq/L) on the horizontal axis and the major cations and anions on the vertical axis (Figure 84). The central vertical line represents zero concentration, and the cation concentrations are to the left while the anion concentrations are to the right. The concentrations are plotted, and the points are joined with a solid line to create a polygon. The utility of these diagrams is that their shape gives the viewer an instantaneous picture of the water chemistry. For example, Figure 84a is from a water sample that contains predominantly calcium and sulfate, while Figure 84b is from a water sample that is mostly sodium and chloride. We can use these diagrams to compare samples and to identify groups of samples.

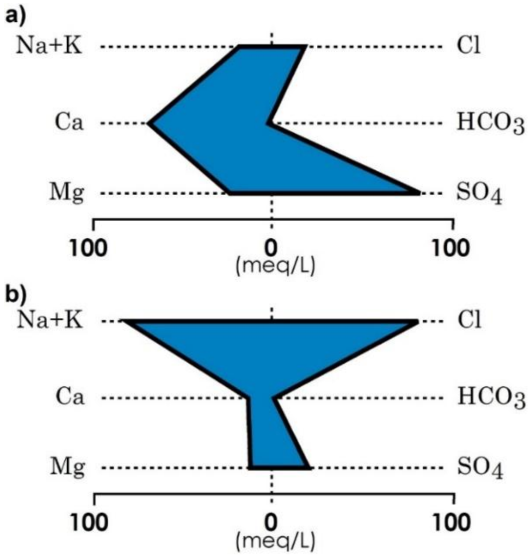


Figure 84 - Stiff diagrams.

11.6.3 Case study: Using Geochemical Diagrams to Identify a Contamination Source

The following is an example of an application of Piper and Stiff diagrams in an area that was allegedly impacted by leaking deep salt-water disposal (SWD) injection wells (Uliana, 2005). Several shallow wells located on a ranch in West Texas (north of Orla, TX, USA) had suddenly become too salty for the cattle to tolerate. The well owners suspected that SWD injection wells located on the ranch property were responsible for the contamination. The SWD injection wells are injecting brines that contain high concentrations of sodium (Na^+) and chloride (Cl^-). The SWD wells are extremely deep and the head in the injection intervals is much higher than ground surface so any breach in the casing or cement of the injection wells will result in brine moving into the shallow aquifer. Samples were taken from several of the shallow ranch wells in the vicinity of the brine injection wells and analyzed for the major cations and anions. Figure 85 shows the results of those samples, plotted as Stiff diagrams.

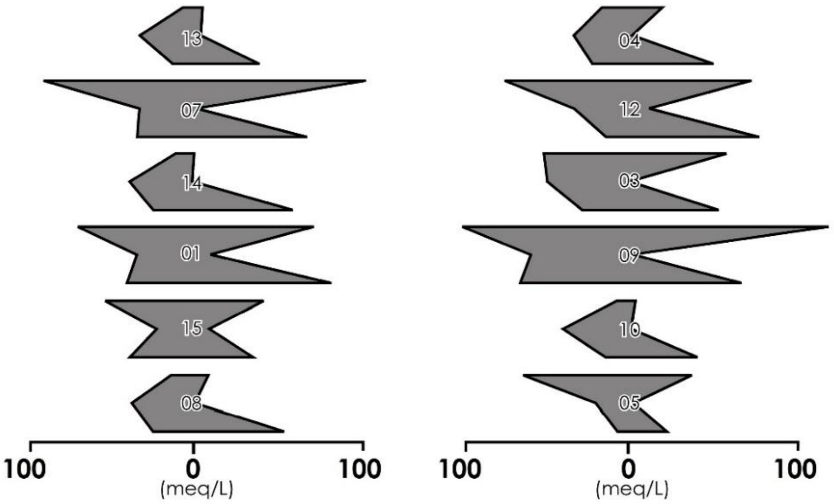


Figure 85 - Stiff diagrams from shallow wells near brine disposal wells (modified from Uliana, 2005).

The TDS, sodium concentrations, chloride concentrations, and the shapes of the Stiff diagrams suggested that the samples fell into two distinct groups; a lower TDS group with a chemical signature that had more calcium and sulfate, and a higher TDS group that had more sodium and chloride (Figure 86). Groundwater samples taken from wells located away from the ranch indicate that the natural groundwater in this region is dominated by calcium and sulfate with very little sodium and chloride. It was therefore hypothesized, based on these data, that the Group 1 samples are more like the natural water chemistry with minimal impact from the brine injection wells, while the Group 2 samples have been impacted by saline water.

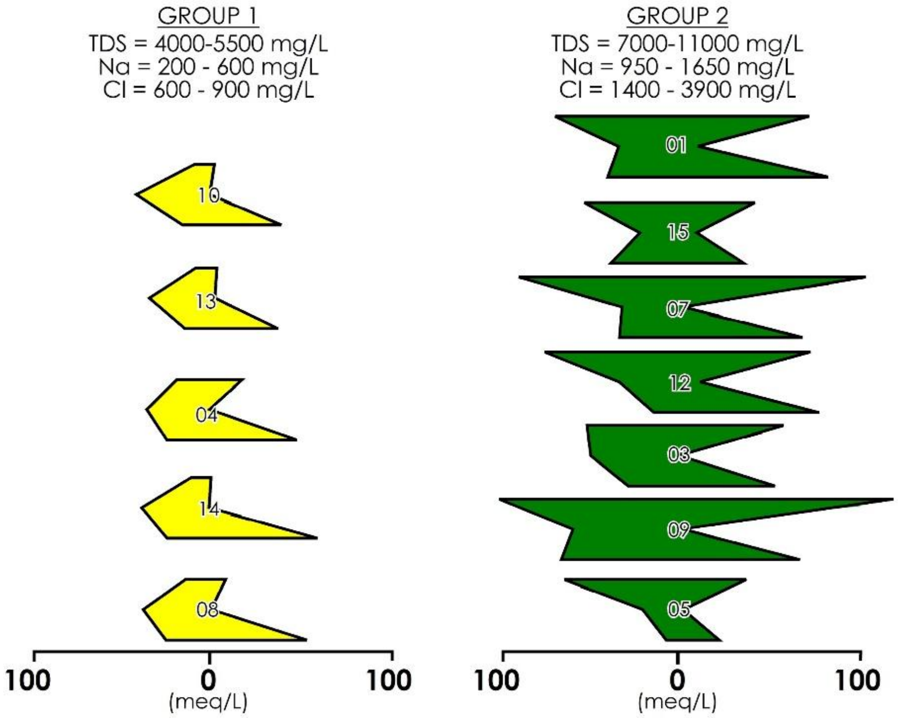


Figure 86 - Samples are placed into groups based on concentration and shapes of the Stiff diagrams (modified from Uliana, 2005).

Plotting the same data on a piper diagram (Figure 87) supports the hypothesis that the brine injection wells could be responsible for the contamination. The background samples from off-site wells plot in a restricted region that is dominated by calcium and sulfate, which is consistent with the abundance of gypsum and anhydrite in the local aquifer. The brine injection wells are high sodium and sulfate as expected, and they plot in a similarly restricted area. The wells in Group 1 plot very close to the background wells, while the Group 2 wells plot between the brine injection wells and the Group 1 wells. This is also consistent with the hypothesis.

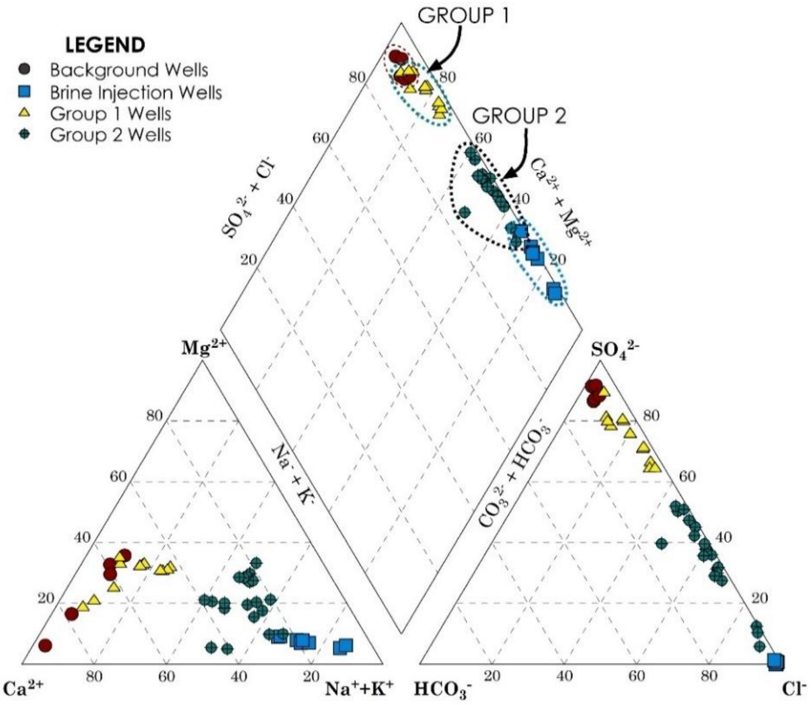


Figure 87 - Piper diagram for water samples taken in Culberson County, Texas (modified from Uliana, 2005).

Plotting the Stiff diagrams on a map with their well locations (Figure 88) indicates a clear pattern with the Group 2 samples closest to the brine injection wells, while the others are farther away. This suggests that the Group 2 samples have been contaminated by the brine injection wells, and that the Group 1 samples reflect the natural water chemistry.

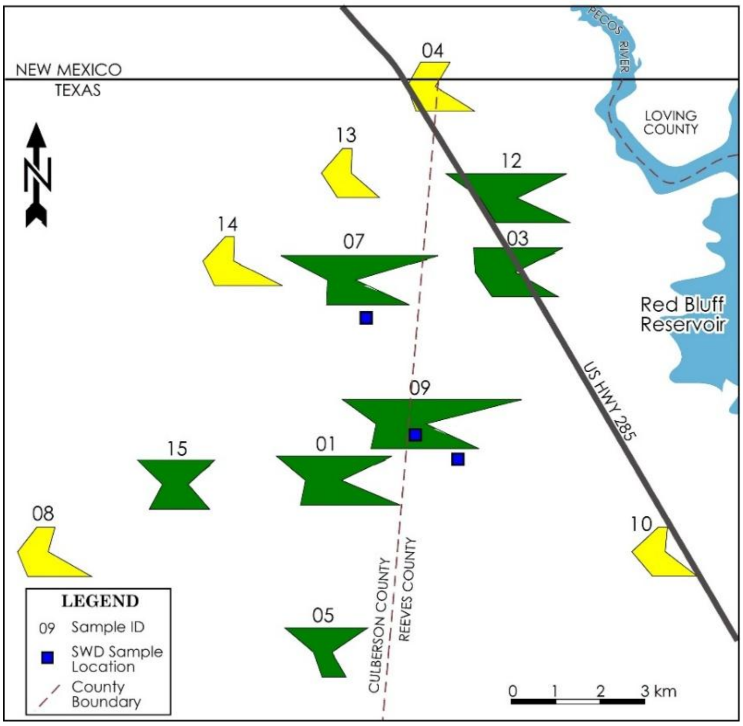


Figure 88 - Map of sample locations with Stiff diagrams and location of brine injection wells (modified from Uliana, 2005).

11.6.4 Schoeller Diagrams

Schoeller diagrams are another graphical way of expressing the chemistry of a single sample (Figure 89). In these diagrams, the horizontal axis is each of the major cations and anions, while the vertical axis is the concentration of each in meq/L on a log scale. When the data are plotted, they are connected with straight-line segments to create a crooked line that, like in the Stiff diagrams, gives an immediate picture of the chemistry of the samples that can then be compared to other samples.

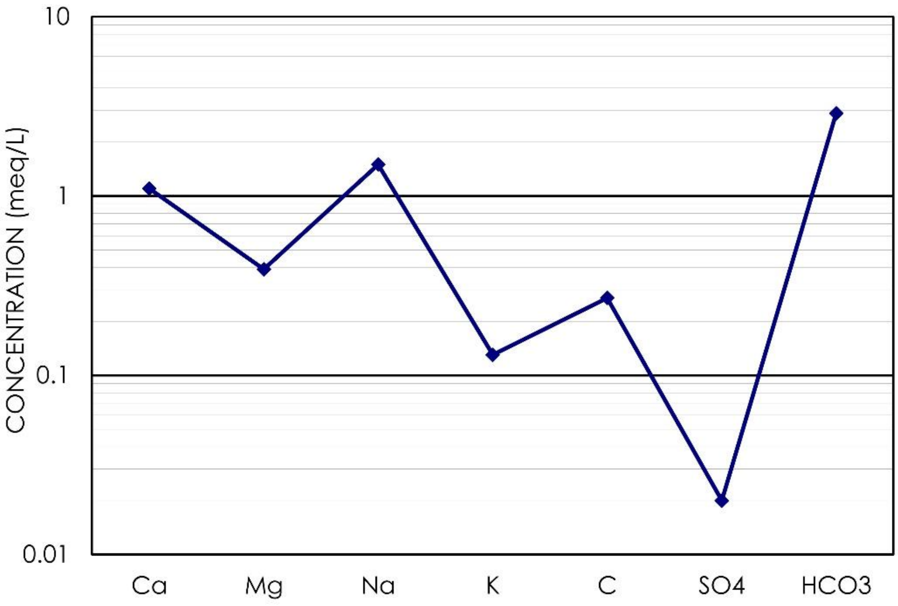


Figure 89 - Schoeller diagram.

11.7 Evaluation of Water Quality Data

Much of the focus of Section 11 is on presenting the important chemical parameters of, and dissolved constituents in, groundwater; the ways that those parameters and constituents are measured; basic calculations for and graphical representations of water quality data; as well as the processes that link the water chemistry to geologic settings. A common reason why groundwater samples are collected and analyzed for chemical constituents is to evaluate whether the water quality is acceptable for the intended use of the water. In most areas, groundwater designated for a municipal water supply (i.e., for drinking and household use) must meet specific water quality standards (e.g., the National Primary Drinking Water Regulations promulgated by the U.S. Environmental Protection Agency[5]) or be treated to comply with those standards before water is distributed to consumers. Groundwater developed for other uses may require much different standards. For example, water for livestock or irrigation can contain higher concentrations of many constituents, as livestock and plants can typically tolerate higher concentrations of various ions. Water is also required for most industrial processes (e.g., power generation, manufacturing, mining), and different industrial uses require different minimum water quality – in some cases the use may require very pure water with low salinity, while others can use high salinity water with no problem.

When performing water quality evaluations, it is very important to understand the requirements of the anticipated use before developing sampling and analysis program. All relevant regulations should be determined so that the correct laboratory analyses can be chosen. Proper sampling protocols must be established, and the laboratory should be contacted in advance so that proper sample containers are used, acceptable hold times (i.e., the time between sample collection and delivery to the laboratory) are not exceeded, and laboratory transmission protocols (e.g., samples kept on ice) are followed. All laboratory results should be checked for charge balance.

11.8 Additional References

Readers who wish to learn more about groundwater geochemistry are encouraged to download and review *The Geochemistry of Natural Waters*↗ by Drever (1997), which is available at no charge from The Groundwater Project. An additional excellent resource is *Study and Interpretation of the Chemical Characteristics of Natural Water* by Hem (1985), this book is available as a free download from the US Geological Survey on the following website: https://pubs.usgs.gov/wsp/wsp2254/pdf/wsp2254a.pdf↗.

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[5] https://www.epa.gov/ground-water-and-drinking-water/national-primary-drinking-water-regulations↗

12 Mass Transport

12.1 Introduction

Beyond the natural geochemistry of groundwater discussed in Section 11, the other aspect of water chemistry that interests hydrogeologists is groundwater contamination and the spread of contamination in the subsurface. These contaminants include road salt; agricultural herbicides, pesticides, and fertilizers; manufacturing byproducts; petroleum products; wastes from mines and industrial processes; and pathogens (i.e., viruses, bacteria, fungi, protozoa, and worms). These materials may be introduced over a large area such as agricultural fields (non-point source) or in a discrete area like a salt storage facility. Many of the contaminants are organic compounds—chemical compounds that contain carbon—that are associated with petroleum-based fuels, solvents, and other industrial chemicals. Organic compounds often reach groundwater in liquid form, as lighter than water liquids (*LNAPLs* – light non-aqueous phase liquids) or denser than water liquids (*DNAPLs* – dense non-aqueous phase liquids). These liquids are often spilled on or near the surface then migrate downward to the groundwater where the LNAPLs float on the water table while the DNAPLs sink through the flow system. The pool of NAPL dissolves slowly into the groundwater and thus becomes a source of contamination. Sites that have experienced releases of contamination to the groundwater generally have a zone of dissolved contaminants around and down-gradient of the source that is called a contaminant *plume*. Groundwater contamination is a problem that our society has been aware of for much of the last century, and a lot of effort has been done to understand the processes involved, characterize and delineate contaminant plumes, and clean up contaminated sites. It is an extremely complicated problem because there are numerous processes that control the movement of contaminants and many possible reactions that create, change, and break down contaminants as they move in groundwater flow systems.

In Section 12.2, we develop a general overview of the processes that move mass (i.e., suspended particles and dissolved chemicals) through groundwater systems. In Section 12.3, we discuss reactions that affect the concentration of both contaminants and naturally occurring dissolved material. We do not discuss specific contaminants nor the reactions that those contaminants experience in groundwater systems.

The Groundwater Project offers many books in the contamination category↗ that can be downloaded free of charge. Information on organic contaminants in groundwater can be found in *Properties of Organic Contaminants*↗ by Mackay and others (2024), *Petroleum Hydrocarbons in Groundwater*↗ by Brown and others (2017), *Dissolved Organic Carbon in Groundwater Systems*↗ by Chapelle (2022), and *Septic System Impacts on Groundwater Quality*↗ by Robertson (2021). Each of these books is available as a free download from The Groundwater Project. Additional books related to mass transport and groundwater contamination are currently in development for the Groundwater Project as of the publication of this book, so readers are encouraged to regularly review the offerings at the Groundwater Project for related publications↗.

12.2 Mass Transport in Groundwater

In this section, we use the term "mass" to mean anything in the water that isn't water. Mass occurs in water as dissolved ions, molecules, colloidal particles, or other suspended particles. *Mass transport* refers to the movement of these dissolved species and particles in groundwater systems.

If we could measure the velocity of groundwater flow at every point in the subsurface and, if we had the computational power to calculate the movement of mass by advection and molecular diffusion at that level of detail, then we could track the movement of mass and predict how its movement would proceed if the groundwater flow field were to change in response to stresses such as recharge events and variable well pumping rates. Given that we cannot fully capture the velocity field in measurements, the hydrogeologic profession has resorted to using a 'fudge factor[6]' that is commonly known as *dispersion* to represent the complex velocity field that we are not able to measure. Dispersion represents molecular diffusion enhanced by a dispersivity term that accounts for all the spatial and temporal variations of the subsurface velocities that we are not are able to measure and simulate. This difficulty in measuring the details of the velocity field is not unique to groundwater. Another example is our inability to measure the details of velocity in the atmosphere, although in the atmosphere is it easier to see or at least measure the migration of mass. The problem with measuring the details of velocity in a moving fluid is more readily observed in the atmosphere by, for example, watching the discharge of smoke from a variety of sources, particularly on a windy day.

Thus, instead of using the variable velocity field and molecular diffusion to describe and predict mass movement, we use an average velocity and then include a dispersion component to account for the velocity variations that disperse mass. In most situations diffusion is a trivial contributor to mass movement so the diffusion process is 'lumped' into the dispersion term that represents the variable velocity field that we are not able to measure. Ultimately, for the purpose of using mathematics to describe and predict mass transport in groundwater, two processes are used: *advection* and *dispersion*.

12.2.1 Advection

Groundwater flowing through a porous medium carries dissolved species and particles along with it by a process called *advection*. Figure 90a shows purely advective mass transport from a *continuous source of contamination*. The cross hatched area signifies the source zone of the contamination that is maintained at a constant value of concentration—

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[6] A "fudge factor" is a variable included in an equation or calculation to account for some unquantified but significant phenomenon or to ensure a desired result.

for example, a settling pond, a leaking underground storage tank—and the gray area represents the contamination in the groundwater at the time (t) required for the contaminant to move 50 meters. Figure 90 displays conditions at 500 days for a groundwater velocity of 0.1 m/d. The concentration of contamination is constant throughout the gray area.

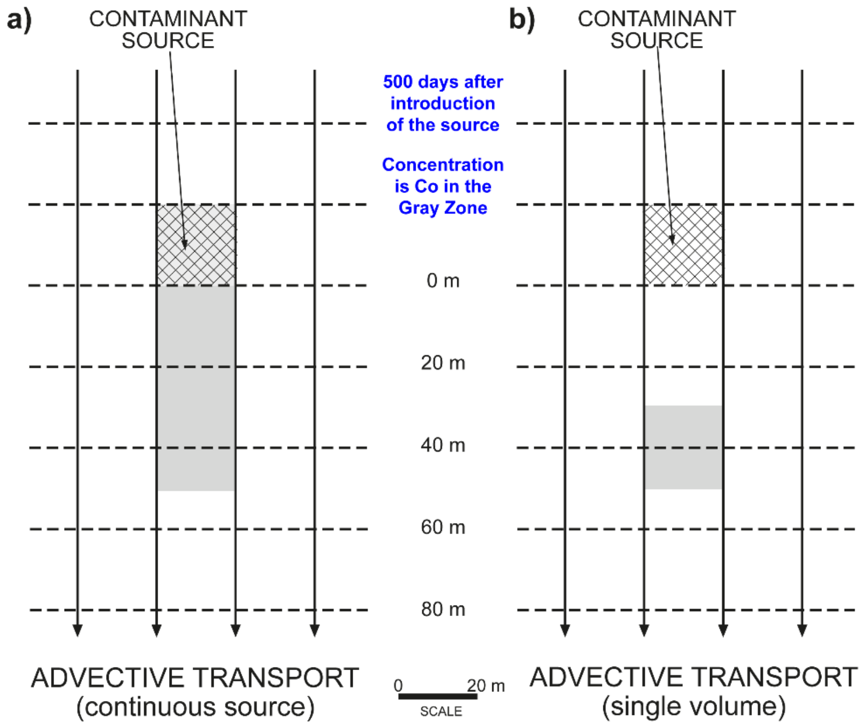


Figure 90 - Position of a plume (light gray) by advective transport 500 days after the source is introduced given a groundwater velocity of 0.1 m/d. a) For a continuous source that is 20 m by 20 m and is maintained at a concentration of C_o starting at time zero. b) For a single volume containing contaminant at a concentration of C_o that occupies a 20 m by 20 m block of the aquifer at time zero.

Figure 90b shows purely advective mass transport from a *single volume of contamination,* also called a *pulse* or a *slug,* released from a rectangular area at a point in time. The cross hatched area represents the location of the contamination release—for example, a spill from an oil tanker—and the gray area represents the contamination in the groundwater at the time (t) required for the contaminant to move 50 meters. The concentration of the contaminant in the gray area is the same as it was at the initial release.

Advective transport is a function of the velocity of the groundwater. If we assume that advective transport is the only process moving the contaminant, and we know the hydraulic conductivity and effective porosity of our aquifer, we can calculate a *travel time,* which is the time necessary for the contaminant to move a specific distance (L), using the following procedure.

- Calculate the average linear velocity using Darcy's law and the effective porosity, as shown in Equation (93):

$$v_{avg} = -\frac{K}{\varphi_{eff}}\frac{\partial h}{\partial x} \tag{93}$$

where (parameter dimensions are dark green font with mass as M, length as L, time as T):

v_{avg} = average linear velocity (LT^{-1})

K = hydraulic conductivity (LT^{-1})

φ_{eff} = effective porosity (:)

$\partial h/\partial x$ = hydraulic gradient (:)

- Divide the distance (L) by the average linear velocity to get travel time.

Thought Question 11*: This assumes that the mass itself does not affect or influence the pattern of flow. Is there a possibility that the contaminant itself may affect flow? (Hint: review Section 3.3.3).*

Breakthrough Curves

A good way of visualizing and understanding mass transport processes is using *breakthrough curves*. Breakthrough curves are graphs that show the either the *spatial distribution of the contaminant at a point in time after release* or *the temporal distribution of the contaminant at one location from the point of release*. Breakthrough curves for the situations in Figure 90a and Figure 90b are given in Figure 91 and Figure 92, respectively.

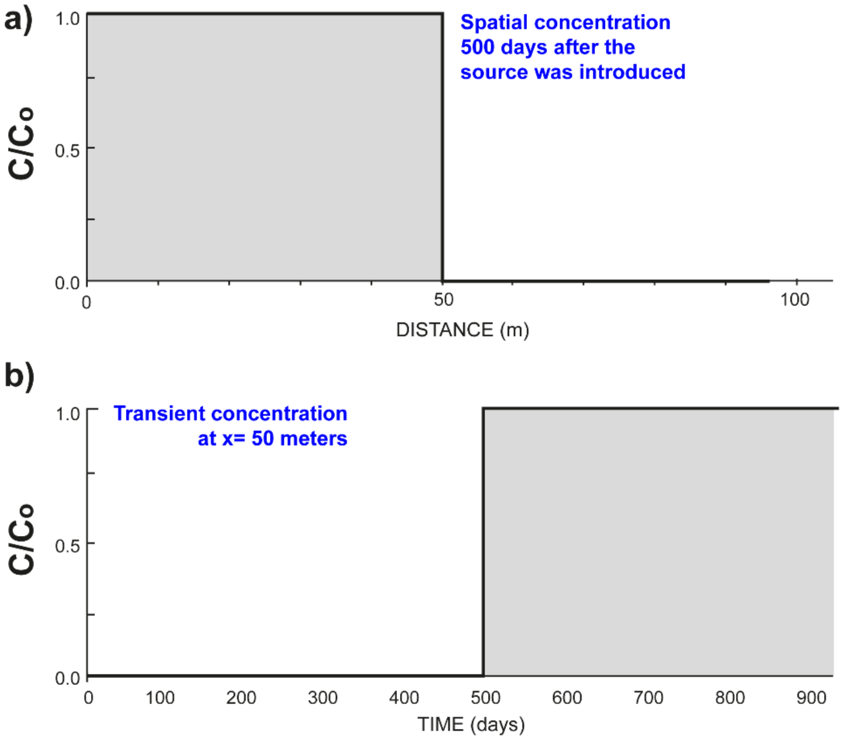


Figure 91 - Concentration, *C*, relative to initial concentration, C_o, for purely advective transport at a groundwater velocity of 0.1 m/d from a continuous source as shown in Figure 90a. a) C/C_o as a function of distance from the source 500 days after the source is introduced. b) C/C_o at a distance 50 meters from the source as a function of time since the source began releasing contaminants. C/C_o changes from zero to one at 500 days, then remains at one until the continuous source is terminated.

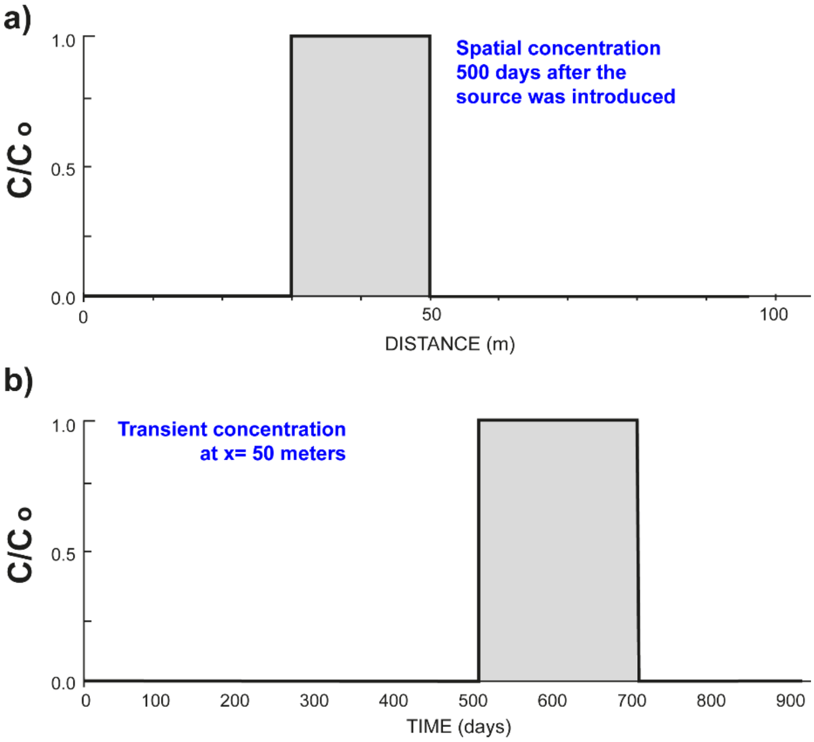


Figure 92 - Concentration, *C*, relative to initial concentration, C_o, for purely advective transport at a groundwater velocity of 0.1 m/d from a single volume source as shown in Figure 90b. a) C/C_o as a function of distance from the source 500 days after the source is introduced , and b) C/C_o at a distance 50 meters from the source as a function of time since the source was released. C/C_o, changes from zero to one at 500 days after the source is introduced, then remains at one until the tail end of the 20-meter-long volume of contaminants moves beyond 50 meters, which occurs 200 days later at 700 days.

The vertical axis on the breakthrough curves represents the concentration of the contaminant (C) divided by the original concentration (C_o). This quantity represents the fraction of contamination in the water; where C is the concentration in the water at a given location and time, C_o is the source concentration, and C/C_o is their ratio. Where $\frac{C}{C_o} = 1,$ the contamination is equal to the source contamination. The ratio, C/C_o, can be multiplied by the source concentration to determine the concertation at that location and time.

Concentration of contamination is usually expressed as a mass per volume of water (e.g., mg/L) or as a dimensionless mass per mass (e.g., parts per million). Using C/C_o is a dimensionless way of expressing the concentration as a fraction or percentage of the original total concentration, and we use it here to illustrate the basic concepts of contaminant transport.

12.2.2 Dispersion

In groundwater systems, advection is usually the process responsible for moving the largest quantity of mass. However, as the contaminant moves through the subsurface, several processes act on it and cause contamination to spread, resulting in variation of concentration throughout the contaminated area. This spreading of contaminants in the subsurface is called *dispersion*, or sometimes *hydrodynamic dispersion*. Dispersion parallel to the flow direction is called *longitudinal dispersion*, while dispersion perpendicular to the flow direction is called *transverse dispersion*. Longitudinal dispersion spreads out the contaminant in the direction of flow, while transverse dispersion spreads it out laterally—perpendicular to the direction of flow—as the contaminant moves with flowing groundwater. Although dispersion acts in all directions it is often simplified into longitudinal, transverse lateral, and transverse vertical components.

Dispersion is caused by two processes. The first is *molecular diffusion*, which is the net movement of dissolved solutes under the influence of a concentration gradient—that is, from areas of high concentration to areas of low concentration. This process is driven by the random motions of the dissolved ions and is a function of the temperature of the water; that is, higher temperatures = more energetic and lively ions = more diffusion. It is possible to measure a *diffusion coefficient*—a parameter that relates concentration to the rate of diffusion—for various chemicals under laboratory conditions.

All spreading beyond that caused by diffusion is by a second process called *hydrodynamic (or mechanical) dispersion*. Hydrodynamic dispersion is caused by variations in groundwater velocity (i.e., local variations in magnitude and direction of flow) along the tortuous paths taken by groundwater as it moves through pore spaces and regions of different hydraulic conductivity from point to point in a groundwater system—this is also discussed in sub-section 3.3.7 Fluid Velocity. Dispersion occurs at the pore scale and at larger scales. In nearly every situation, spreading is greater in the direction of flow, which is described as *longitudinal spreading*, while spreading in other directions is called *transverse spreading*. The variations become more complex as transient stresses on the groundwater flow field such as recharge events and well pumping, or other human impacts, shift flow paths over time. Another example of human impact on groundwater flow fields is the presence of open well bores where groundwater moves vertically to enter flow in different depth zones.

Again, as mentioned in Section 12.2 and repeated here for emphasis, it is not possible to measure all the parameters that control velocity variations in a groundwater system, and even if we could measure them, our computers would not be able to simulate those variations. Some advances have been made in more rigorously measuring the subsurface and in modeling transport in groundwater systems using alternative methods such as fractional calculus, and perhaps someday the mathematical calculation of mass transport in groundwater will be a trivial task. However, until that time, for practical reasons, we use hydrodynamic dispersion as a 'fudge factor' to account for the unquantified but significant phenomena of spreading mass in a groundwater flow system. Extremely simplified portrayals of the influence of dispersion on contaminant migration is shown in Figure 93.

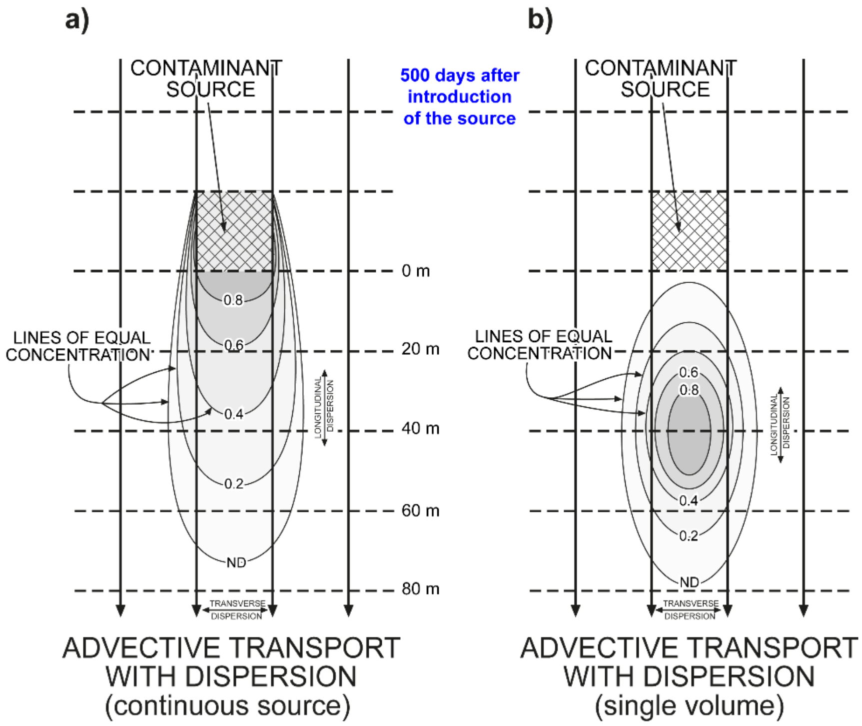


Figure 93 - Advective transport at groundwater velocity of 0.1 m/d with dispersion, showing concentrations relative to initial concentration in the groundwater 500 days after the release of contaminants for the case of a) a continuous source and b) a single volume pulse.

Figure 93a and Figure 93b show the effects of dispersion on contaminant transport from a continuous source and from a single volume pulse of contamination. These figures show the same situation as Figure 90 with the additional process of dispersion. Figure 93a shows advective mass transport with dispersion from a continuous point source of contamination. The cross hatched area signifies the source of the contamination—for example, a settling pond, a leaking under-ground storage tank—and the lines of equal concentration represent the fraction of the initial contaminant concentration at some time (t) after the release of contamination.

Figure 93b shows advective mass transport with dispersion from a single volume of contamination released from a rectangular source at a specific point in time. The cross hatched area represents the original location of the contamination release—for example, a spill from an oil tanker—and the lines of equal concentration represent the fraction of the initial contaminant concentration at time (t) after the release of contamination.

The breakthrough curves for Figure 93a and Figure 93b are shown in Figure 94 and Figure 95, respectively.

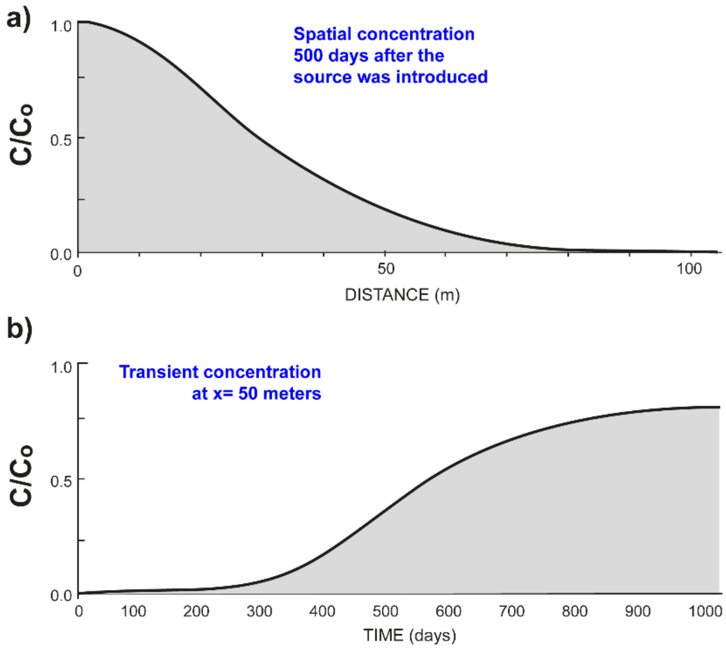


Figure 94 - Concentration, *C*, relative to initial concentration, C_o for advective transport at a groundwater velocity of 0.1 m/d with dispersion from a continuous source as shown in Figure 93a. a) C/C_o as a function of distance from the source 500 days after the source is introduced. C/C_o never quite reaches a value of one and approaches 1 later than that shown in Figure 91 because dispersion spreads out the contaminants in the aquifer. b) C/C_o at a distance 50 meters from the source as a function of time since the source began releasing contaminants. C/C_o begins to exceed a value of zero early as a few molecules of contaminants move rapidly ahead, then and rises slowly, never reaching one because the contaminants spread laterally.

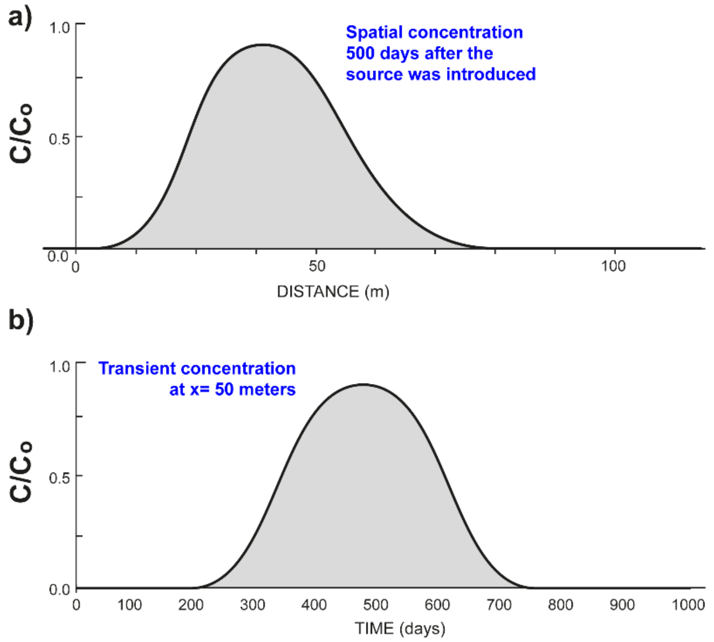


Figure 95 - Concentration, *C*, relative to initial concentration, C_o, for advective transport at a groundwater velocity of 0.1 m/d with dispersion from a single volume source as shown in Figure 93b. a) C/C_o as a function of distance from the source 500 days after the source is introduced. b) C/C_o at a distance 50 meters from the source as a function of time since the source was released. C/C_o rises slowly earlier than for the purely advective transport because some contaminants move rapidly out ahead of the rest. C/C_o never reaches one because the contaminants spread laterally.

***Thought Question 12**: How does the dispersivity affect the shape of the breakthrough curves—that is, what would a curve with a large dispersivity look like versus one with a small dispersivity?*

12.3 Reactions and Retardation

Some constituents in groundwater are relatively stable and do not undergo significant reactions in the subsurface. These are called *conservative* constituents. An example is the chloride ion—chloride doesn't stick to anything, it doesn't undergo biological reactions involving micro-organisms in the subsurface, and it is generally one of the last ions to precipitate out of groundwater into a mineral phase. Not all solutes and contaminants are like that; in fact, most solutes and contaminants are *reactive* in the subsurface. This complicates our understanding of mass transport because, in addition to dilution of the contaminant from dispersion, the mass of the solute or contaminant in the system changes with time.

Reactions in the groundwater usually involve degradation or decay of the contaminant. For radioactive contaminants, decay of the contaminant will occur based on the half-life of the radioactive constituent. For non-radioactive contaminants, decay and degradation is typically related to some sort of biologically controlled process, or *biodegradation*, that involves microorganisms in the groundwater breaking down contaminants as part of their metabolism. Biologically controlled degradation is much more difficult to understand and quantify, as there are often multiple types of organisms that are "consuming" the contaminant and they are all connected within complex subterranean ecosystems. In addition, biological degradation involves changing the contaminant to a different molecule or type of molecule, which reduces the mass of the contaminant in the system. This can be problematic when the *daughter products*, or the molecules that result from degradation, are also considered contaminants; for example, the degradation of chlorinated solvents can result in the production of additional chlorinated compounds that are just as toxic or dangerous as the original compound.

More information on microbial ecosystems in groundwater and on biodegradation can be found in *Groundwater Microbiology*↗ by Ferris and others (2021), *Biotic Transformations of Organic Contaminants*↗ by Rittmann (2023), and *Processes Controlling the Natural Attenuation of Fuel Hydrocarbons and MTBE in the UK Chalk Aquifer*↗ by Thornton and others (2006); each of these books is available as a free download from The Groundwater Project. More information on chlorinated solvents in groundwater can be found in the classic text *Dense Chlorinated Solvents and other DNAPLs in Groundwater*↗ by Pankow and Cherry (1996), which is also available as a free download from The Groundwater Project.

Some of the reactions do not involve degradation of the contaminant; rather they slow down the overall movement of the contaminant through the groundwater. This process is called *retardation*. Most retardation reactions involve *sorption*, which is a general term that describes how one entity sticks to another. Sorption is usually reversible, so there is no loss in mass, just a delay in travel time (hence the term retardation). Sorption works in two ways.

- *Absorption*: A contaminant particle is absorbed into the volume of something else (Figure 96). For example, most organic compounds have high solubility in other organic compounds but have low solubility in water; organic contaminants in groundwater can basically *dissolve* into organic matter in soil or subsurface sediments.
- *Adsorption*: A contaminant particle sticks onto the surface of something else (Figure 96). This is basically the same as absorption, but it is a surface phenomenon. This happens with ions and clays; clays have charges on their surfaces, and this electrostatic attraction can grab ions out of solution.

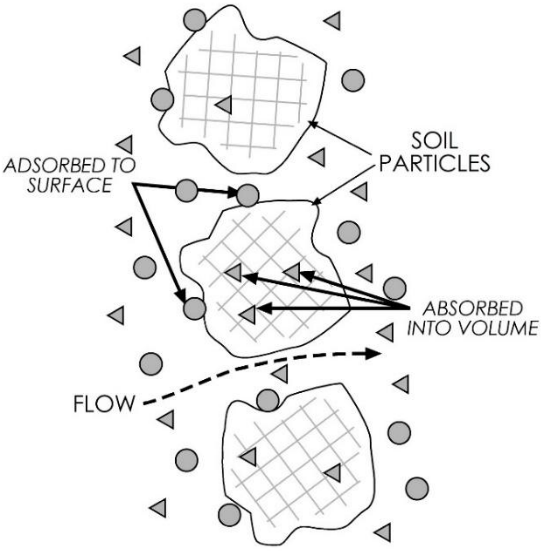


Figure 96 - Adsorption and absorption of different types of contaminant particles in soil.

Clays have very large surface area per volume and tend to have significant electrostatic charge on the surface. In clays, the negatively charged sites are more abundant than positive, as the positive charges tend to occur on the edges of the minerals while negative charges cover the surface. Divalent cations—cations with a +2 charge, like calcium and magnesium—are more strongly attracted to clays than monovalent cations—cations with a +1 charge, like sodium. In some groundwater systems, divalent cations in solution will be pulled out by the clays and replaced with sodium ions. This process is called *cation exchange*.

More information on the movement of particles and sorption processes for *colloids* (i.e., nano- and micro-particles) can be found in *Colloid (Nano- and Micro-Particle) Transport and Surface Interaction in Groundwater*↗ by Johnson and Pazmiño (2023), which is available as a free download from The Groundwater Project.

12.4 Mass Transport Equations

We represent mass transport in groundwater mathematically as a combination of:

- advective transport,
- molecular diffusion,
- mechanical dispersion,

along with retardation processes such as

- solutes exchanging places with ions on the surface of minerals that make up the subsurface material,
- sorption of mass to surface material,
- chemical reaction including dissolution of subsurface material, precipitation of mass to form solids, and
- decay (both radioactive and biological) of mass which can decrease concentration and introduce additional mass that are daughter products of a decay process.

Similar to the discussions in Sections 4, 5, and 6 of this book, it is possible to develop governing equations for transport of solutes and particles in groundwater systems by considering an elemental volume and deriving a general three-dimensional differential equation that can be solved for various practical applications. A detailed derivation of this equation is beyond the scope of this publication; however, a basic introduction to the concepts and mathematics is presented here.

Development of the mass transport equation in porous media starts with Fick's Law, which describes the diffusive flux of solutes in response to a concentration gradient. Fick's Law is presented in Equation (94).

$$J = -D_{mol}\frac{\partial \varphi}{\partial x} \tag{94}$$

where (parameter dimensions are dark green font with mass as M, length as L, time as T):

J = diffusion flux, which is the amount of substance diffusing per unit area per unit time ($\text{molesL}^{-3}\text{T}^{-1}$)

D_{mol} = molecular diffusion coefficient ($\text{L}^{2}\text{T}^{-1}$)

$\frac{\partial \varphi}{\partial x}$ = concentration gradient, which has units of concentration per length (:)

Like hydraulic conductivity, molecular diffusion can be measured in the laboratory.

Fick's Law is then combined with the equation for advective transport, Equation (90), and a mechanical dispersion term that represents the dispersion described in Section 12.2.2 to develop Equation (95). Unlike the groundwater flow equation in which all the terms are second partial derivatives, this equation is a hyperbolic equation because the last term is not a second derivative. This variation makes it more difficult to solve the transport equation as compared with the flow equation.

$$\frac{\partial C}{\partial t} = D_l \frac{\partial^2 C}{\partial {l_l}^2} + D_t \frac{\partial^2 C}{\partial {l_t}^2} + D_v \frac{\partial^2 C}{\partial {l_v}^2} - \overline{v_l} \frac{\partial C}{\partial l} \tag{95}$$

where (parameter dimensions are dark green font with mass as M, length as L, time as T):

C = concentration in fluid (ML^{-3})

t = time (T)

l = concentration gradient (ML^{-4})

D = hydrodynamic dispersion coefficient (L^2T^{-1})

$\bar{v}$ = interstitial velocity (LT^{-1})

l_l = in the direction of flow (:)

l_t = in the direction laterally transverse to flow (:)

l_v = in the direction vertically transverse to flow (:)

Unlike the diffusion coefficient, the hydrodynamic dispersion coefficient cannot be measured in a laboratory experiment. Rather, the hydrodynamic dispersion coefficient is determined by adjusting its value until the distribution of calculated concentrations in space and time—as determined by Equation (95)—match the observed concentrations in a laboratory or field flow system. The hydrodynamic dispersion coefficient is a bulk parameter that combines the molecular diffusion coefficient of Equation (94), flow velocity, and a term called *dispersivity*, which is intended to represent the heterogeneous tortuosity of the medium through which the groundwater flows. Like hydraulic conductivity, dispersivity is viewed as anisotropic, that is, it has a different value in the longitudinal direction (i.e., in the direction of groundwater flow) and in the transverse lateral and vertical directions (i.e., perpendicular to the direction of fluid flow) thus there is a dispersion coefficient for each of the major directions of flow. In fact, the dispersion coefficient may vary in non-orthogonal directions, but using three directions as presented in Equation (96) simplifies the mathematics and captures concentration variations in the direction of flow as well as to the sides which is generally less and then above and below which is typically even more restricted due to the typical layering of geologic materials.

$$D_{l,t,v} = \alpha_{l,t,v} \overline{v_l} + D_{mol} \tag{96}$$

where (parameter dimensions are dark green font with mass as M, length as L, time as T):

$D_{l,t,v}$ = dispersion coefficient in the longitudinal, transverse lateral, or transverse vertical direction of flow (L^2T^{-1})

$\alpha_{l,t,v}$ = dispersivity in the longitudinal, transverse lateral, or transverse vertical direction of flow (L)

$\overline{v_l}$ = average linear velocity in the longitudinal direction of flow (LT^{-1})

D_{mol} = molecular diffusion coefficient in porous medium (L^2T^{-1}), typically insignificant, being on the order of 1x10^{-9} m^2/s

Dispersivity, α, is a function of heterogeneity at the pore-, unit-, and formational-scale. It is a very difficult term to characterize. Previous studies (e.g., Schulze-Makuch, 2005) have shown that dispersivity is a function of scale of measurement such that studies which incorporate larger volumes of aquifer (e.g., larger contaminant plume sizes) will likely experience behavior consistent with greater dispersivity coefficients.

Equation (95) is often reduced to its one-dimensional form as shown in Equation (97) and used for predicting movement and distribution of contaminants along a groundwater flow path.

$$\frac{\partial C}{\partial t} = D_x \frac{\partial^2 C}{\partial x^2} - \overline{v_x} \frac{\partial C}{\partial x} \tag{97}$$

There are many analytical solutions for the mass transport equation that can be used to calculate concentration at a specified location and time. These can be used to predict the spatial distribution of contaminant at a given time, or the shape and timing of contaminant breakthrough curves at a given location. Analytical expressions are available for one-, two-, and three- dimensional spreading from both continuous and slug sources. Some are enhanced to reflect retardation and/or decay of contaminants. The equations may look daunting, but a solution can be obtained by systematically entering the values and following through each step of the calculation. A couple of example solutions are provided here to offer insight into the character of the equations. One is for one-dimensional spreading of a continuous source in a uniform, one-dimensional flow field. It is representative of contaminant moving through a column full of sand in a laboratory, Equation (98). This equation is discussed by Fetter (1999) in section 2.8.4 of his book. This is similar to the plume shown in Figure 93a but with no lateral spreading. The other equation is for three-dimensional spreading of a specified amount of mass introduced at a point in a uniform, one-dimensional flow field, Equation (99). This is similar to the plume shown in Figure 93b but with the source defined as an amount of mass at a point location intend of a block of fluid with a given concentration. lateral spreading.

$$C = \frac{C_o}{2}\left[\operatorname{erfc}\left(\frac{x - \overline{v_x}t}{2\sqrt{D_x t}}\right) + \exp\left(\frac{x\overline{v_x}}{D_x}\right)\operatorname{erfc}\left(\frac{x + \overline{v_x}t}{2\sqrt{D_x t}}\right)\right] \tag{98}$$

$$C = \frac{M}{8(\pi t)^{\frac{3}{2}}\sqrt{D_x D_y D_z}}\exp\left(-\frac{X^2}{4D_x t} - \frac{Y^2}{4D_y t} - \frac{Z^2}{4D_z t}\right) \tag{99}$$

where (parameter dimensions are dark green font with mass as M, length as L, time as T):

C = concentration in fluid (ML^{-3})

C_o = concentration of the source (ML^{-3})

erfc = complimentary error function, which is the same as 1-erf, if the value within the erf is negative then the erf is minus the erf of its absolute value, so erfc(x)=1-erf(x) and if x is negative then erfc(x)=1-(-erf(x)); erf can be looked up in a mathematical table or obtained using an online calculator such as https://www.medcalc.org/manual/erf-function.php↗

x = distance from the source in the x direction (L)

t = time since source began releasing contaminant (T)

$\overline{v_x}$ = interstitial velocity in the x direction (LT^{-1})

D = hydrodynamic dispersion coefficient (L^2T^{-1}), subscript indicates the directionality of the coefficient

X = distance from the center of mass of the plume in the x direction at time t (L), the center of mass is at x=$\bar{v}t$

Y = distance from the center of mass of the plume in the y direction (L)

Z = distance from the center of mass of the plume in the z direction (L)

It is essential to determine the dimensionality of the setting and the character of the source when selecting an equation to calculate concentration. Exercise 24⤵ provides an opportunity to calculate a concentration from one-dimensional spreading of a continuous contaminant source using Equation (98). Exercise 25⤵ provides an opportunity to calculate three-dimensional spreading of a slug contaminant source Equation (99).

12.5 Management of Groundwater Contamination

There are a variety of actions that can be taken to manage groundwater contamination including:

- source removal (e.g., digging up the source);
- containment:
  - an infiltration barrier, which is a very low permeability cap over the source,
  - cut-off walls around the source, and sometimes including the plume, which are created by digging trenches around the source and filling them with low permeability material such as bentonite clay,
  - hydrodynamic isolation which involves creating a hydraulic barrier by injection and usually also withdrawal of groundwater to isolate the contaminated zone from the regional flow field, and

- contaminant removal, including;
    - pumping and treating the contaminated water,
    - funnel and gate systems involving cutoff walls that direct flow through an emplaced material that reduces contamination by changing the Eh or pH, providing material for chemical reaction, or providing biota that can degrade the contaminant,
    - skimmer pumps to remove floating liquid contaminants,
    - soil vapor extraction for volatile contaminants,
    - natural attenuation in which natural processes break down the contaminant into innocuous material, and
    - in-situ bioremediation (natural attenuation expedited by introduction of something that breaks down the contaminants (e.g., microbes) or something that enhances the natural microbial population (e.g., nutrients, oxygen sources, carbon substrates, pH buffers, and specific electron donors or acceptors).

The most common activity in contaminant remediation is pumping groundwater to capture contaminants. To implement this, one must estimate the number, locations and rate of pumping wells that will capture the contaminants. This is a useful concept to consider when striving to understand groundwater systems because it is applicable to more than remediation as it can define areas of recharge that should be protected to keep water supply wells safe, or to determine the extent of dewatering to allow excavation below the natural water table. Thus, capture zone analysis is discussed in Section 12.5.1.

12.5.1 Capture Zone Analysis

One way of reducing contamination in the subsurface is through *pump-and-treat* remediation systems. These involve pumping water out through a well, treating it, and discharging the treated water back into the subsurface. When a well is pumped, there is a zone around the pumping well called a *zone of capture*. This zone corresponds to the cone of depression, and all the water within the zone of capture flows towards the well. In an aquifer with a perfectly horizontal potentiometric surface, the zone of capture is a circle—or ellipse, if the aquifer is anisotropic. In the field, potentiometric surfaces generally slope, so the capture zone is an elongated area that extends slightly down-gradient and mostly up-gradient of the well. Figure 97 shows a cross-section and a map of a capture zone in a homogeneous, isotropic aquifer. These images can be generated for different aquifer properties and pumping rates by superposing drawdowns calculated using the Theis equation on the regional hydraulic head surface. Superposition is described in Section 8.9.1.

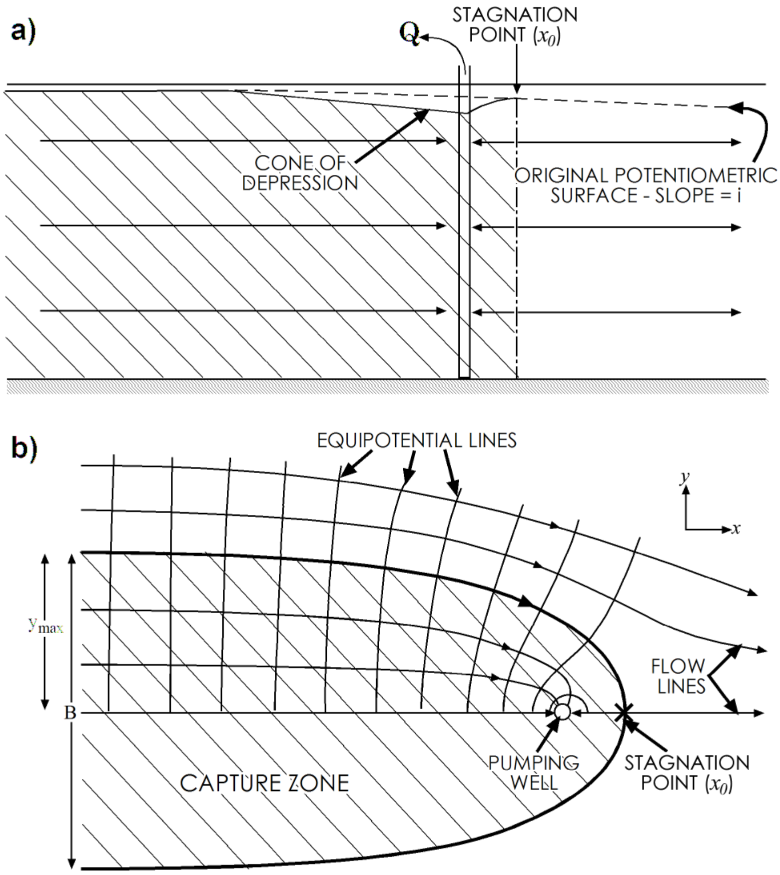


Figure 97 - a) Cross-section of a capture zone (hatched area), showing the original potentiometric surface and the stagnation point; and b) map view of capture zone showing the flow divide (heavy black line) that delineates the zone.

The pumping of groundwater reverses the gradient for a short distance down gradient of the well. The capture zone is surrounded by a flow divide with flow on one side of the divide flowing towards the pumping well and flow on the other side continuing down the natural gradient. Downgradient of the well is a *stagnation point*, which marks the downgradient limit of the capture zone and a point in the aquifer where flow away from the pumping well is balanced by flow towards the pumping well. The stagnation point is defined as an infinitesimally small point within the flow system at which the gradient is zero and the flow velocity is, therefore, also zero.

The x coordinate of the stagnation zone, x_o, can be calculated using Equation (100), the maximum width of the capture zone, B, can be calculated using Equation (101), and the x coordinate for a given half-width of the capture zone, y, can be determined using Equation (102), such that the outline of the flow divide surrounding the capture zone can be mapped. The full theoretical foundation for the equations can be found in a publication by Grubb (1993).

$$x_o = \frac{Q}{2\pi K i b} \tag{100}$$

$$B = 2y_{max} = \frac{Q}{Kib} \tag{101}$$

$$x = \frac{-y}{tan\left(\frac{2\pi Kiby}{Q}\right)} \tag{102}$$

where (parameter dimensions are dark green font with mass as M, length as L, time as T):

x_o = distance downgradient from the well where flow is stagnant (L)

Q = pumping rate (L^3T^{-1})

K = hydraulic conductivity (LT^{-1})

i = regional gradient (:)

b = aquifer thickness (L)

B = width of the capture zone at its maximum width (L)

y_{max} = half width of the capture zone at its maximum width (L)

x = x coordinate where the width of the capture zone equals y (L)

y = half width of the capture zone at its maximum width (L)

The Groundwater Project provides an online interactive tool for exploring the nature of well capture zones↗. Exercise 26⤵ offers an opportunity to consider how to limit pumping rate to avoid a spilled contaminant reaching a water supply well.

Although pump-and-treat is a common remediation method, it is also typically subject to failure. Causes of failure include inadequate capture zone analysis, sorbed contaminants released too slowly, contaminants in low permeability zones not captured, and vertical spreading of floating contaminants within the cone of depression. These difficulties can greatly increase the cost of remediation and ultimately reduce the effectiveness of site cleanup and remediation.

13 Surface Water

13.1 Introduction

This book is about groundwater, and to this point the material has dealt with the physics, mathematics, and chemistry of groundwater flow systems. However, groundwater is directly linked to the surface water component of the hydrologic cycle. The freshwater cycle—including precipitation, evaporation, infiltration, runoff, and accumulation in surface reservoirs—is the primary source of recharge to, and discharge from, the groundwater. In addition to sustaining groundwater, the water cycle (Figure 98) is the source of water for people, agriculture, and industry, thus it is important to have a basic understanding of the system. This section discusses surface water and the interactions between surface water and groundwater.

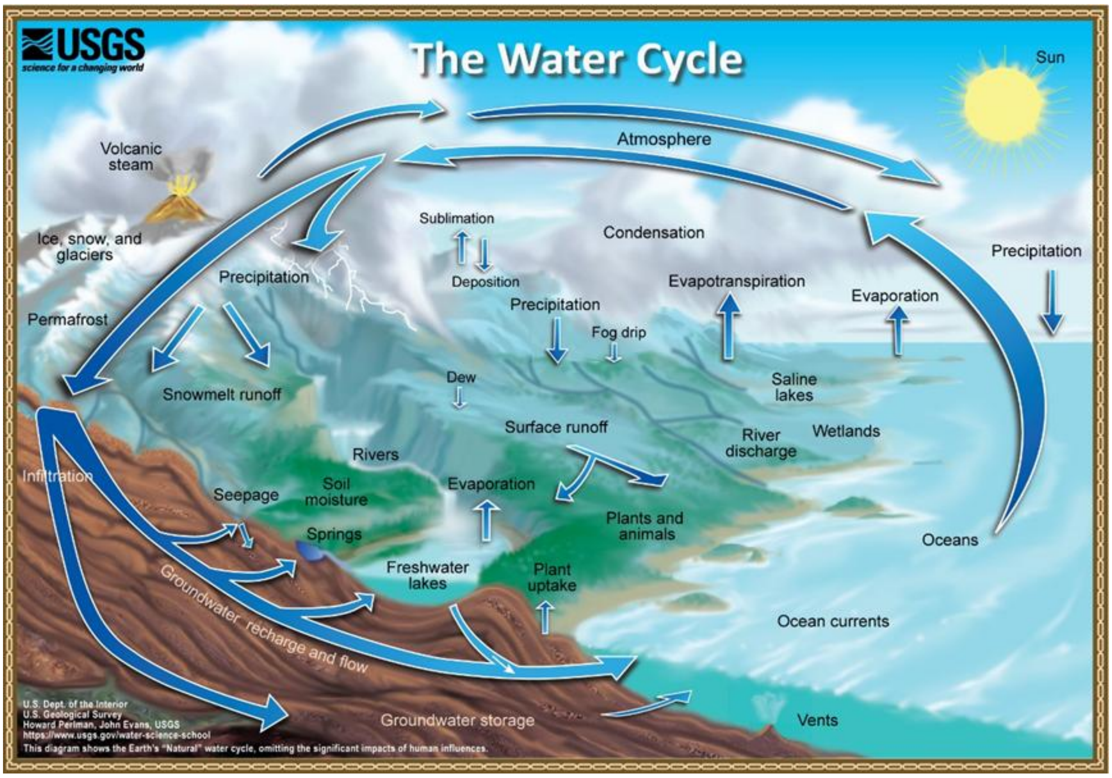


Figure 98 - The hydrologic cycle (reproduced from US Geological Survey website at https://www.usgs.gov/media/images/natural-water-cycle-jpg↗).

13.2 Basics of Surface Water

When rain falls on the Earth, some of it soaks into the ground, some is taken up by plants and animals, and some evaporates into the air. The rest of the water—the part that drains to the nearest creek or river and flows towards the ocean—makes up the part of the hydrologic cycle that we call *surface water*.

A *stream* is the general term used to describe a body of flowing water on the Earth's surface that is contained within a channel. Streams receive their water from a surrounding area called a *drainage basin, watershed,* or *catchment*. Watersheds are defined by the topography surrounding the stream, and watersheds are determined by arbitrarily selecting a point on a stream and delineating the area that drains to that point (Figure 99 and Figure 100).

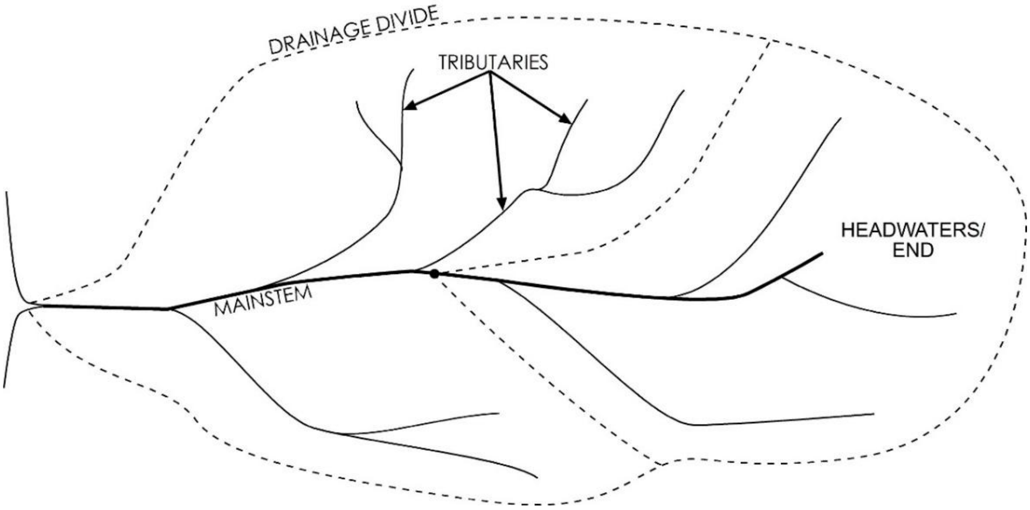


Figure 99 - Schematic diagram of two watersheds delineated by dashed lines with the watershed on the right that includes the headwaters being a portion of the entire water shed.

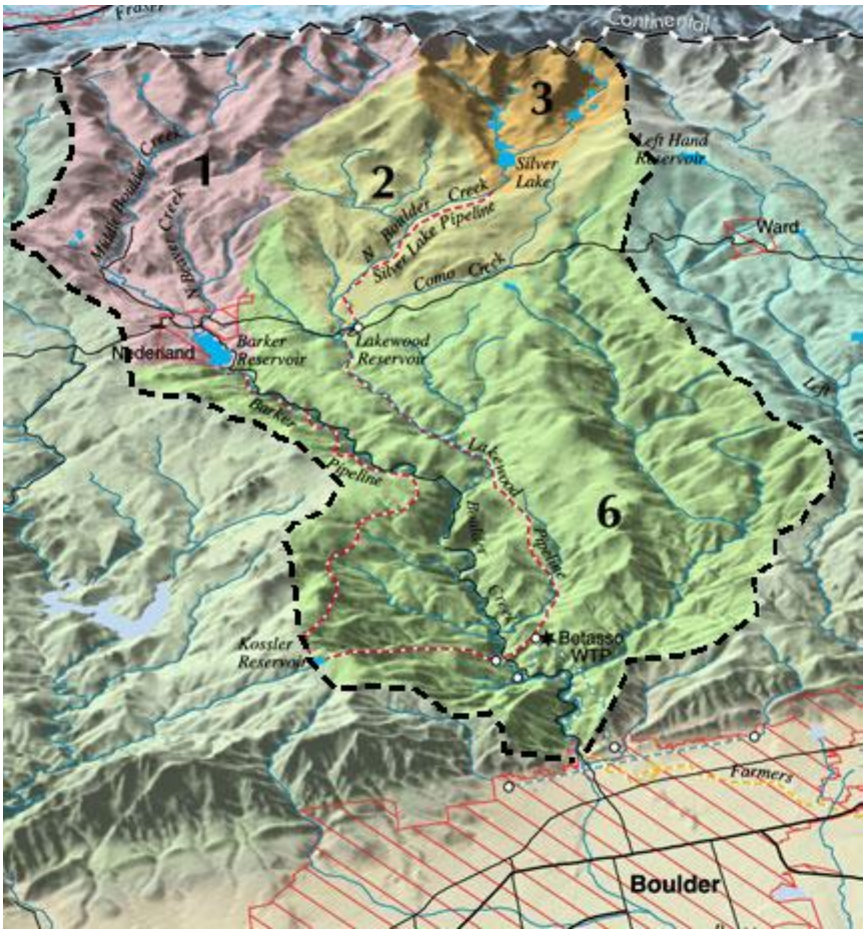


Figure 100 - Shaded-relief image of a watershed (reproduced from https://bcn.boulder.co.us/basin/waterworks/watersheds.html↗).

Some definitions relevant to the discussion of surface water include the following.

- The topographic high that separates one watershed from the next is called a *drainage divide*—the black dashed line in Figure 100.
- The place where the stream starts flowing at the highest point in the watershed is called the *headwaters* or the *end* of the stream.
- The point where the stream finally flows into another body of water—a river, lake, reservoir, or the ocean—is called the *mouth* of the stream.
- The primary channel in the stream is called the *mainstem*; smaller streams that flow into the mainstem are called *tributaries*.

13.3 Evaporation, Transpiration, and Precipitation

13.3.1 Evaporation

Where liquid water exists at the Earth's surface, water molecules are continually exchanged between the liquid phase and the atmosphere. *Evaporation* occurs when the number of water molecules passing to the vapor state in the atmosphere exceeds the number joining the liquid phase. The rate of evaporation is based on the energy in the system, which is related to the water temperature as well as the temperature, pressure, and humidity of the air above the water. *Humidity* refers to the amount of moisture in the air; ways of expressing humidity include the following.

- *Absolute humidity*: total mass of water per unit volume of air.
- *Saturation humidity*: maximum mass of water a unit volume of air can hold at a given temperature and pressure.
- *Relative humidity*: absolute humidity divided by the saturation humidity (i.e., the percent ratio of the amount of moisture in the air to the maximum it could hold).

Evaporation from lakes and rivers, and even directly from the groundwater, is a significant flux in the water cycle and must be considered in water-budget studies. Evaporation rates from a lake or a reservoir can be determined indirectly by measuring the inflows, outflows, and changes in storage in the lake, and using the continuity equation to calculate the evaporation portion. This can be challenging because it is difficult to reliably measure rates and volumes of water that are entering or leaving a groundwater system that is hydraulically connected to a lake.

Evaporation can be measured directly using shallow pans of water that are exposed to the atmosphere. Evaporation pans are maintained throughout the US by the National Weather Service (Arkansas–Red Basin River Forecast Center, 2023) for constant monitoring of evaporation rates. Water is maintained in these pans at a fairly constant depth, and the amount of water added to the pan by the operators, as well as by precipitation, is used to calculate evaporation rates.

One of the complications of using pans to estimate the amount of water that would evaporate from a nearby reservoir is that the pan absorbs heat from the sun much more readily than the reservoir. Therefore, pan measurements overestimate the rate of evaporation. Pan evaporation rates must be multiplied by a pan coefficient, which is a number less than 1 (usually somewhere between 0.5 and 0.8) to develop more meaningful evaporation rates that can be used in water balance studies.

13.3.2 Transpiration

Plants are constantly pumping water from the ground into the atmosphere through a process called *transpiration*. Plants take up water for growth—that is, for building plant tissue—but only a small percentage of what they take up gets used, while the rest is released to the atmosphere through leaves and needles. Transpiration is a difficult thing to quantify; it varies with the time of the day—mostly, transpiration takes place during daylight hours, when photosynthesis is occurring—and time of year. Different plant species take up water at different rates.

Transpiration is significant wherever plants exist. In some cases, transpiration rates can be so high that they drastically reduce the amount of water in streams and serve as a major discharge flux in shallow groundwater systems and wetlands. Plants called *phreatophytes* extend their roots down into the saturated zone, and some of these can remove water at extremely high rates, stressing the groundwater system in a manner similar to pumping wells. Other types of plants are *xerophytes*, which are shallow rooted plants that live in desert areas and require little water, and *hydrophytes*, which are aquatic plants that live directly in water.

13.3.3 Evapotranspiration

In practice it is very difficult to separate water lost to evaporation from transpiration losses; therefore, for groundwater models and water balance studies we usually lump them together as *evapotranspiration* (ET). To understand this concept, we need to distinguish between *potential evapotranspiration* and *actual evapotranspiration*.

Potential evapotranspiration is the water loss that would occur if an unlimited supply of water were available for transpiration and evaporation. In reality, the amount of water that transpires or evaporates is limited by the amount of water that is available. If the amount of water available is less than the potential evapotranspiration, then the *actual evapotranspiration* will be lower than the potential.

Figure 101 shows an example relationship between precipitation and potential/actual ET for the northern hemisphere with a warm, dry summer and a cool, wet fall/winter/spring. In the summer months, when precipitation is low, there is not enough water to satisfy the potential; therefore, the actual ET is less than the potential.

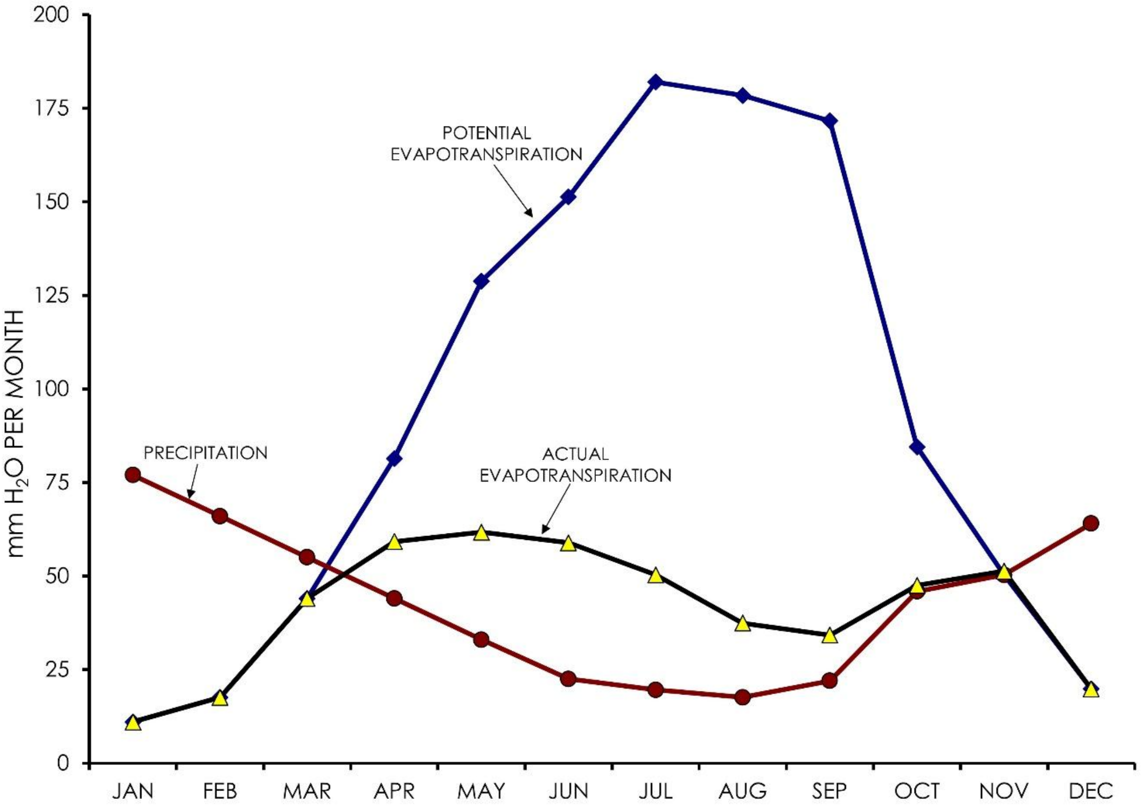


Figure 101 - Graph of hypothetical data showing an example relationship between potential evapotranspiration, actual evapotranspiration, and precipitation in the northern hemisphere with a warm, dry summer and a cool, wet fall/winter/spring.

Actual ET cannot exceed the potential, but if precipitation and the capacity of the soil to store water are both low, then actual can be much less than potential. In areas where precipitation is more evenly distributed throughout the year, actual ET will be close to potential ET. This is important because we can measure potential evaporation—that is, pan evaporation—and determine potential transpiration for specific plants, but we must acknowledge that the potential will not be reached if there is not enough water available.

Precipitation

As masses of air cool, the saturation humidity decreases and the relative humidity increases. When the relative humidity approaches 100 percent condensation begins to occur. Condensation is initiated on particles of dust or ice crystals suspended in the air and can form clouds. When air masses rise, they expand; as they expand, they cool, and water condenses. If enough water condenses, it will exceed the air's capacity to hold it and water falls out as precipitation.

The amount of rain that falls during a storm is usually measured in terms of a depth. This depth refers to the depth of water that is left in a rain gauge after the storm. In theory, if we know the depth at any given point, and the area of the watershed upon which rain fell, we can estimate the volume of water that fell in the drainage basin during the storm. The rate and the total depth of precipitation, however, will vary from place to place in an area during a storm. When attempting to determine the volume of rain that actually falls in a watershed, we need to calculate an average precipitation, or *effective uniform depth* (EUD) of precipitation for the watershed.

If the rain gauge stations are distributed evenly throughout the area of interest, each gauge will represent an equal area of the watershed. In that case, we can take the arithmetic average of all the rainfall values at each station to estimate an effective uniform depth. Unfortunately, rain gauges are never evenly distributed throughout a watershed, and we need to adjust the values so that an EUD can be determined. There are several ways of doing this; two of the more popular methods are presented below.

1. *Isohyet method*: Isohyetal lines are lines of equal rainfall. These are drawn on a map of the watershed just like any contour map. The area bounded by adjacent isohyets is measured, and the average depth of the two isohyets is applied to that area. The areas are then weighted and averaged based on the relative size of each isohyetal area. Figure 102 shows an isohyetal map. One advantage to using this method is that it is easy and does not require specialized software or other equipment. A disadvantage of this method is that, since the distribution of rain is never the same, a new map must be redrawn with each rain event.

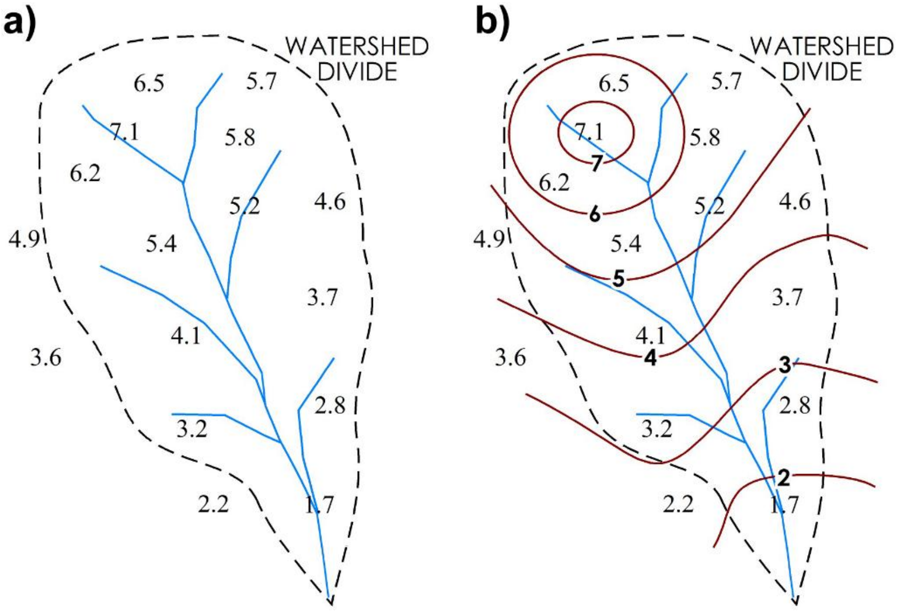
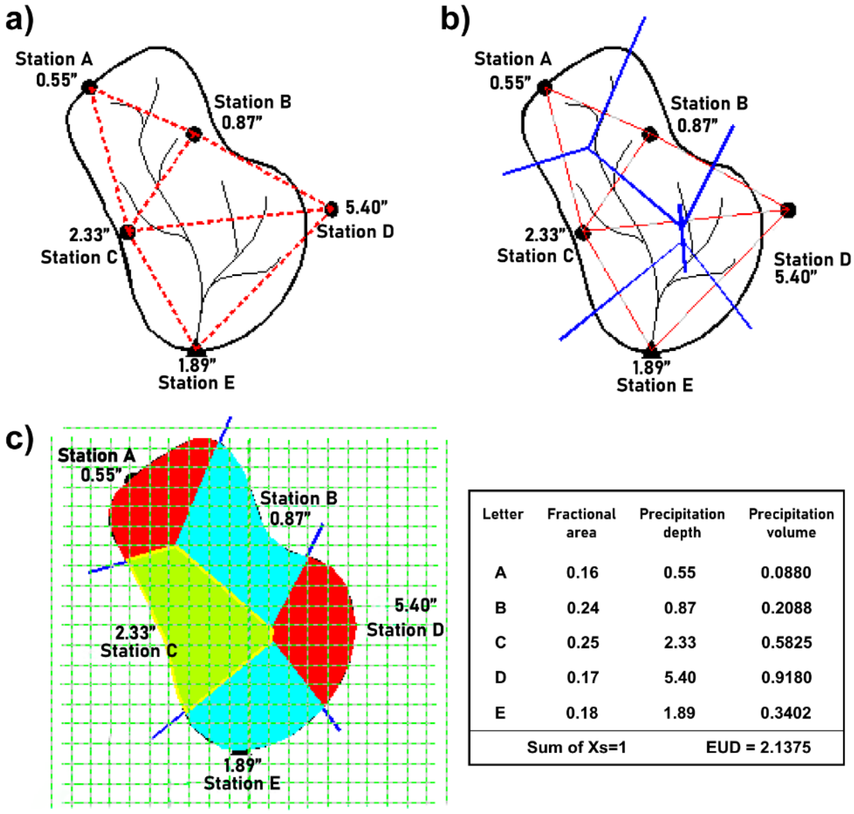


Figure 102 - a) Watershed map with rainfall data from several gauges. b) Isohyetal lines drawn on the map.

2. *Thiessen method*: This is similar to the previous method, except that it weights the areas around each gauge rather than areas between rainfall contours. The Thiessen method delineates areas that include all points closer to each gauge than to any other gauge in the basin. To determine the zone containing all points closest to each station, first adjacent stations in the watershed are connected with a network of lines. Then a perpendicular line is drawn at the midpoint of each of the network

lines. These perpendicular-bisectors are extended until they intersect with the nearest perpendicular-bisector, thus creating a set of polygons covering the area. The area of each polygon is measured, and its fraction of the total area is determined. The rainfall depth at each station is multiplied by the fractional area represented by that rain gauge, and these are summed to determine the EUD. Figure 103 shows the development of a Thiessen polygon map. The advantage of the Thiessen polygon method is that you only need to calculate the weighting factors for the gauge stations once, then they can be used to calculate EUD for any storm or for each year.



Letter	Fractional area	Precipitation depth	Precipitation volume
A	0.16	0.55	0.0880
B	0.24	0.87	0.2088
C	0.25	2.33	0.5825
D	0.17	5.40	0.9180
E	0.18	1.89	0.3402
	Sum of Xs=1		EUD = 2.1375

Figure 103 - Development of a Thiessen polygon map for a rain gauge network. a) Lines (red dashed) are drawn connecting the gauge locations. b) A perpendicular line (solid blue) is drawn at the midpoint of each line. c) The perpendicular lines are extended until they connect to form polygons and the fractional area of each polygon is determined then multiplied by the precipitation recorded at the gauge within that polygon (reproduced from Arkansas–Red Basin River Forecast Center [ARBRFC], 2023).

These methods have limitations in mountainous areas, where orographic effects can create vastly different microclimates over short distances. In such areas, detailed studies of vegetation or more detailed rain gauge coverage is needed to properly determine the distribution of rainfall.

13.4 What Happens When It Rains?

When precipitation falls from the sky, several things can happen to it. Some will be intercepted by plants before it reaches the ground; this water can evaporate or can eventually fall to the ground or move down the stems of plants or trees to the ground. Some of the rainfall that reaches the ground can infiltrate into the soil.

The *infiltration capacity* of the soil refers to the rate at which the soil can absorb water. For dry soils, the infiltration capacity is high due to capillary action pulling the water into the soil. As the soil becomes wetter, the infiltration capacity diminishes, and less water can infiltrate.

Infiltration capacity changes as a function of time throughout a rain event and follows an exponential decay curve that approaches a minimum called the *equilibrium infiltration capacity*, which represents the lowest rate that the infiltration capacity will reach.

If the precipitation rate does not exceed the equilibrium infiltration capacity, all the precipitation reaching the land surface will infiltrate and there will be no runoff. If the precipitation rate is greater than equilibrium infiltration capacity, but less than the initial capacity, initial infiltration will accept all the precipitation followed by runoff or formation of puddles of water on the surface—called *depression storage*. If the precipitation rate exceeds the equilibrium infiltration capacity then infiltration will occur at the rate of the initial infiltration capacity, while runoff and depression storage commence immediately.

Water that infiltrates into the subsurface will percolate vertically unless it encounters the water table or some variation in permeability that causes it to move laterally. Layers of low-permeability material in the subsurface can slow down the vertical percolation of groundwater and cause it to move horizontally towards a stream. This flow of water is called *interflow*.

13.5 Streams

Streams are generally connected hydraulically to the underlying groundwater. When groundwater is discharging into the stream, we call the stream a *gaining* stream. The amount of water that flows into the stream from the groundwater is called *baseflow*. If infiltration causes the water table to rise, the hydraulic gradient between the groundwater and the stream stage will increase and the amount of baseflow will also increase. For many streams, baseflow is the source of water to the stream except during storms, when precipitation in the watershed exceeds the infiltration capacity and the depression storage is filled. In this case, runoff (also called overland flow) will occur, and this runoff will flow into the stream. Natural runoff varies widely in its contribution to the total volume of water flowing through a watershed. In arid regions where the streams are *losing* streams and are not receiving baseflow, natural runoff is a significant contributor to stream flow. In heavily urbanized areas, with a lot of impervious cover—parking lots, roads, buildings—runoff is a much more significant contribution to the surface water system.

13.5.1 Discharge

Discharge is defined as the volume of water moving past a point on a stream in a given period of time. In the simplest terms, stream discharge (Q) is equal to the velocity of the water (v) times the cross-sectional area of the stream (A), or $Q = vA$. The complication is that water in the stream is flowing fastest in the middle of the stream and slowest at the edges and along the bed of the stream. Figure 104a shows a cross section of a stream flowing into (or out of) the page. Figure 104b shows contours indicating the changes in velocity LT^{-1}) throughout the stream. The line a–a′ shows the velocity profile; with arrows on the left side representing the magnitude of velocity with direction into, or out of the page.

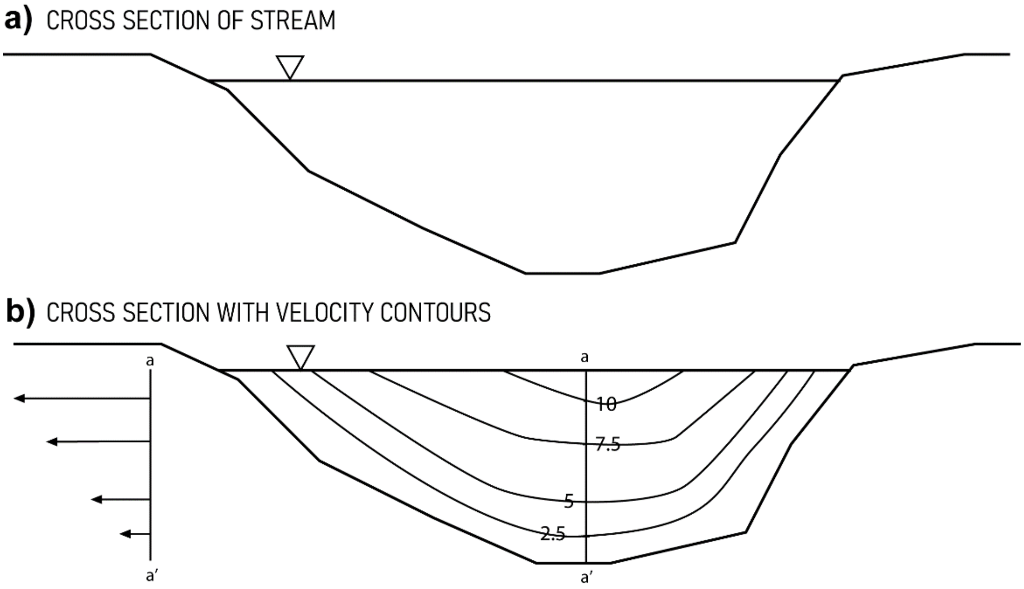


Figure 104 - Cross-section of stream showing stream profile and velocity contours. The contour values might represent feet per second.

To measure discharge, the average velocity of the water in the stream must be estimated. Stream velocity can be measured at points in the channel using a *current meter*. Experience has shown that, in water greater than about 2.5 feet (0.76 m) deep, averaging velocity measured at 0.2 and 0.8 of the total depth provides a good average water velocity for the segment. If the stream is less than 2.5 feet (0.76 m) deep, one measurement at 0.6 of total depth is representative of the average velocity (Figure 105). Measurements are made at regular intervals (e.g., 0.5 feet or 0.15 m) across the stream. The velocity is then multiplied by the interval and the depth of the stream in that interval to get a discharge for each vertical strip of the stream. The sum of the individual discharge measurements is the overall discharge of the stream.

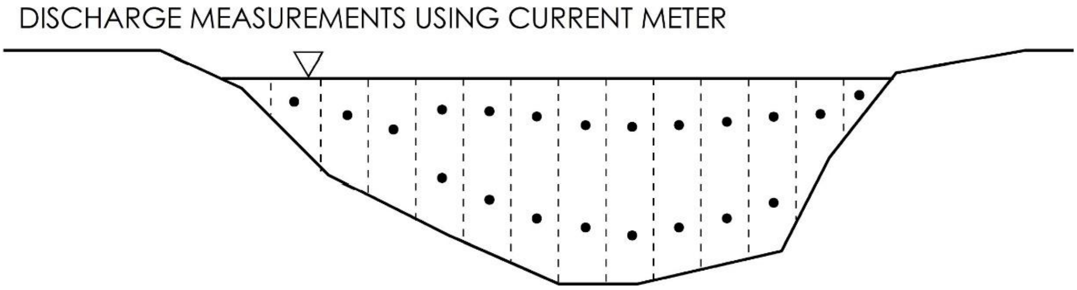


Figure 105 - Cross section of stream showing stream gaging measurement locations.

If a lot of stream gage measurements are made at different times of the year under different flow conditions—that is, low, moderate and high flows—a relationship between stream *stage*—that is, water surface elevation—and stream discharge can be developed. This is done by repeatedly measuring the stage height each time the discharge is measured, then plotting the values on a graph and connecting them with a curve. This curve is called a *stage-discharge rating curve* (Figure 106). The utility of this is that it is much easier to measure stream stage, because once a stage-discharge curve is developed, only the stage of the stream needs to be measured to estimate discharge.

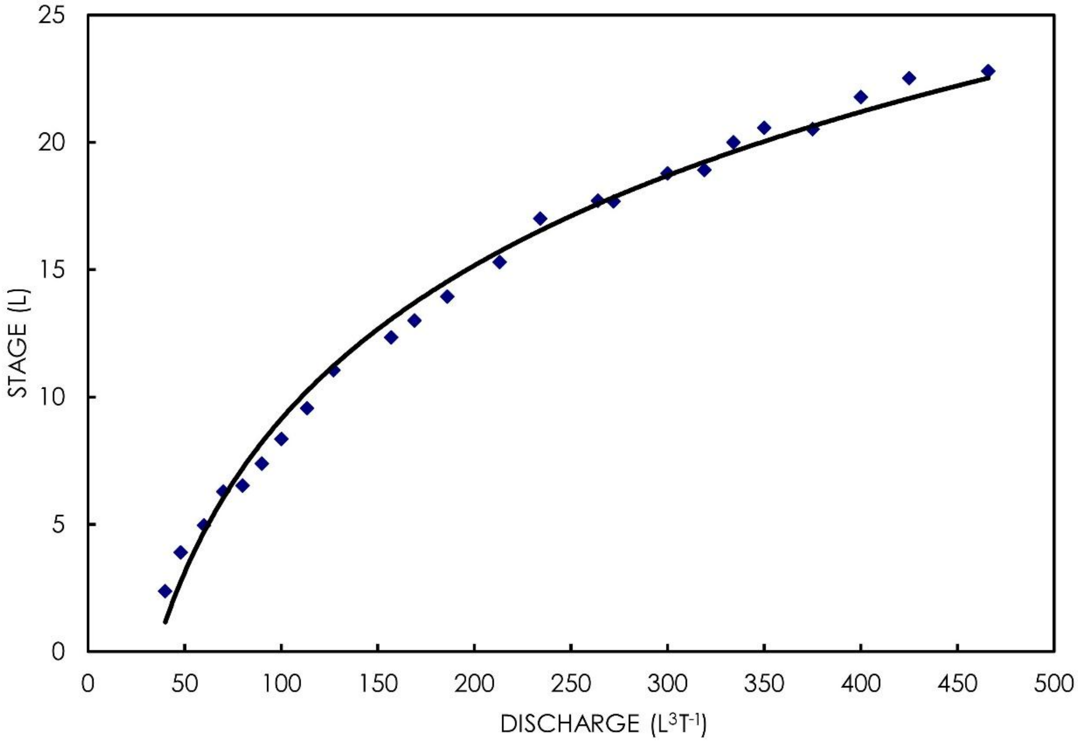


Figure 106 - Stage-discharge rating curve.

Stage-discharge rating curves are not linear. This is because the relationship between cross-sectional area and elevation of the stream water surface is not linear. More specifically, as the river stage increases, the overall area available for water to flow through increases much more dramatically and the discharge increases more rapidly than the stage increases.

***Thought Question 13**: What might cause the stage-discharge rating curve for a specific stream to change?*

One can also measure stream discharge in small streams using a *weir*. A weir is a dam with a small opening of a pre-determined size and geometry that allows water to flow through. The most common weirs are the *rectangular weir*, the *trapezoidal* (also called *Cipolletti) weir*, and the *90° V-notch weir* (Figure 107). The water level just behind the weir blade can easily be measured and used to estimate discharge through the weir opening based on equations that are specific to each weir shape.

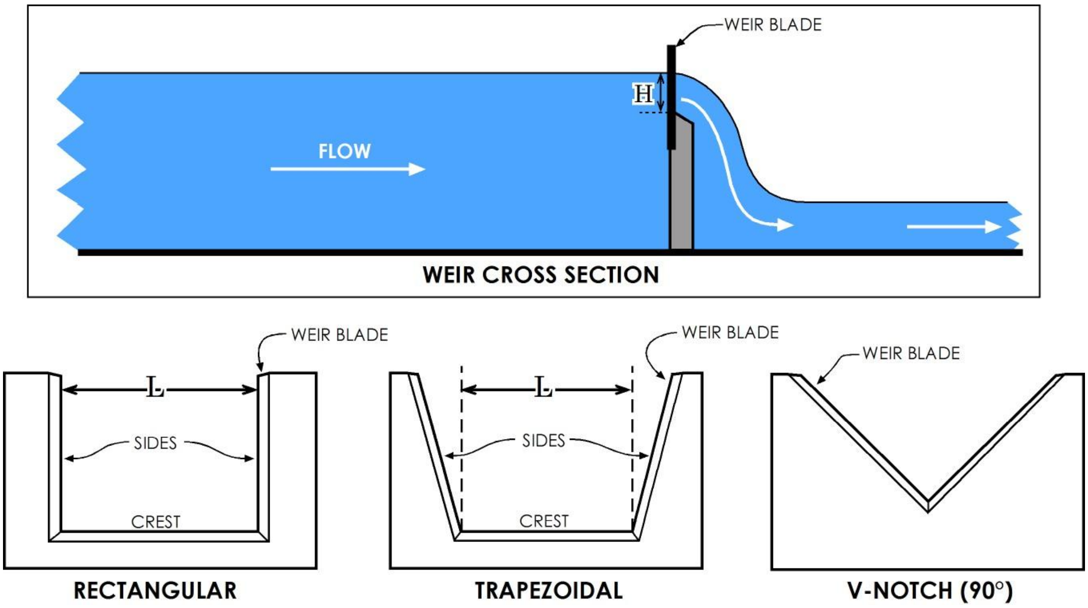


Figure 107 - Different types of weirs.

Equation (103) and Equation (104) are equations for calculating discharge using rectangular and V-notch weirs respectively.

Rectangular Weir

$$Q = 3.33(L - 0.2H)H^{\frac{3}{2}} \tag{103}$$

90° V-notch Weir

$$Q = 2.5H^{\frac{5}{2}} \tag{104}$$

where these empirical equations need to be used with specific units:

Q = discharge in cubic feet per second (cfs)

L = length of the weir crest in ft

H = head of the backwater above the weir crest in ft

These equations are empirically derived, so the coefficients in the equations don't have any real meaning—they empirically relate water elevation to discharge. The coefficients used here are tied to units of feet, so different coefficients are required if different units are used.

13.5.2 Manning Equation

The average velocity of flow in an open channel can be calculated using the *Manning equation* as shown in Equation (105) for units of feet and seconds. To use the formula with meters and second, the 1/n is replaced with 1.49/n.

$$v = \frac{1}{n} R^{2/3} S^{1/2} \qquad (105)$$

where:

v = velocity (meters per second)

R = hydraulic radius, or the ratio of the cross-sectional area of flow to the wetted perimeter (meters)

S = energy gradient (or the slope of the water surface)

n = Manning roughness coefficient - values are accessible via internet search (e.g., the engineering toolbox↗) – some examples of its magnitude for streams are provided here.

0.04-00.05 mountain stream with rocky bed

0.035 winding natural stream with weeds

0.025 natural stream with little vegetation

0.02 straight unlined earth canal

0.012 smooth concrete

The wetted perimeter refers to the length of the channel that is in contact with the water. The Manning roughness coefficient is an empirically derived number that basically describes how the roughness of the channel creates friction that slows down the water in the channel. As with the weir equations, the Manning equation has empirically derived coefficients that require specific units of feet and seconds.

This equation is a way of determining flow velocity or, indirectly, discharge. It can be used to model streams that don't exist yet, like a proposed drainage channel for a new housing development. It can also be used to estimate flow in an existing channel under various storm events, provided that the slope and dimensions of the channel are known.

13.5.3 Stream Hydrographs

A *stream hydrograph* is a graph that represents river discharge at a single point on a river as a function of time. A stream hydrograph shows how discharge in the stream changes with time and, if plotted with precipitation, also shows how discharge responds to storms. A specific hydrograph for a storm event is called a *storm hydrograph.* Figure 108 shows a typical storm hydrograph.

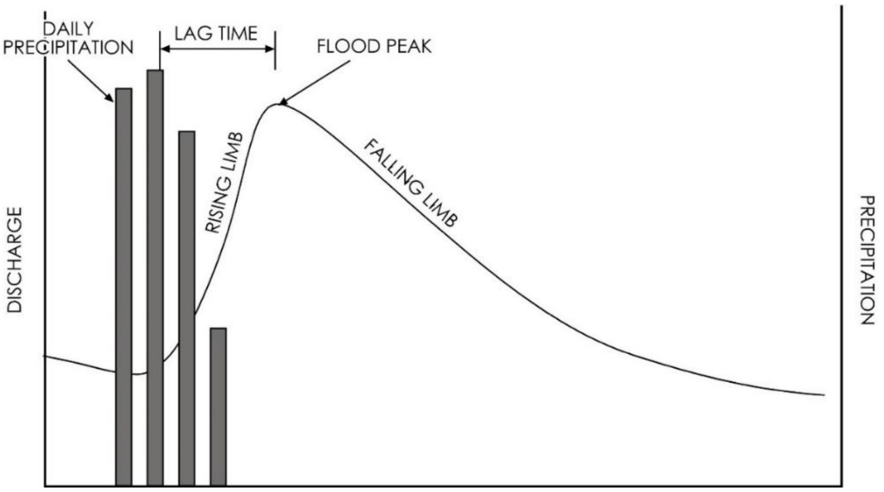


Figure 108 - Hypothetical storm hydrograph showing response to a 4-day storm event.

The columns represent daily precipitation with the scale on the right axis. The *flood peak*, or point where flow is highest, occurs at some time after the precipitation peak. This difference in time is called the *lag time*. The lag occurs because it takes time for water to flow overland or through the subsurface as interflow. The lag time is different for each watershed and is a function of the size and geometry of the watershed as well as the surface conditions and geologic materials in the watershed. The part of the curve that is increasing is the *rising limb* of the curve; the part that is decreasing as storm water drains out of the watershed is the *falling limb*.

The hydrograph can be broken up into specific components (Figure 109). The flow in the stream is a combination of baseflow from groundwater and surface runoff from the storm event. The baseflow increases throughout the storm event in response to rising groundwater water levels around the stream. At the point where the runoff from the watershed is exhausted and the stream is fed only by baseflow, the stream is in *recession*.

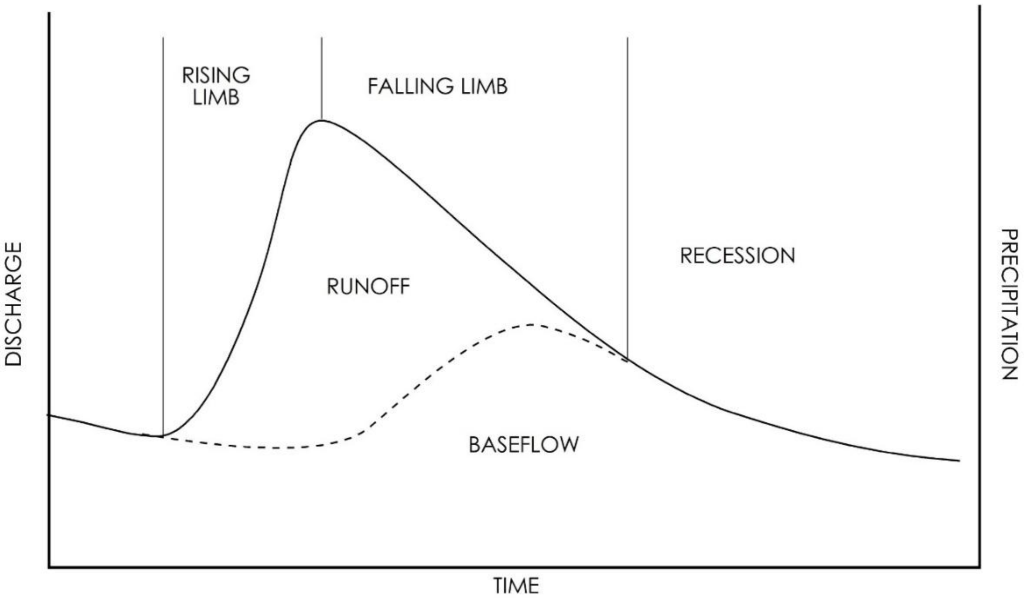


Figure 109 - Storm hydrograph showing the specific parts of the curve.

Thought Question 14: *Why are the lag times between the runoff peak and the baseflow peak different?*

In the absence of precipitation, the recession part of the hydrograph continues to decrease. Eventually, as the groundwater continues to drain and groundwater levels decline further, the stream stops receiving baseflow from the groundwater, and stream discharge is zero. The shape and slope of the *baseflow recession hydrograph* is a function of the geometry of the basin as well as the geology and soil characteristics and is unique to each stream.

Thought Question 15: *What factors control the shape of the runoff and baseflow parts of the curve?*

14 Unsaturated Zone

14.1 Introduction

We've looked at groundwater and we've looked at surface water…now let's look at what is between the two. The layer between the ground surface and the water table—commonly referred to as the *unsaturated zone*—contains a combination of water and air. This part of the subsurface is important for several reasons: The soils and moisture in this zone support plants, chemical and organic processes filter contaminants and affect the chemistry of groundwater, and the unsaturated zone is an avenue of recharge to unconfined aquifers. The same basic laws of physics that govern the flow of fluids in saturated units are at work in the unsaturated zone, however, the processes involved are much more complicated in the unsaturated zone than they are in saturated rock and sediment. In this section, we discuss some of the processes at work in the unsaturated zone.

14.2 Distribution of Water in the Shallow Subsurface

14.2.1 Saturation

At the surface of the Earth, the soil, sediment, or rock have some porosity, and some fraction of that porosity contains water. Thes fraction of porosity that contains water is called the *degree of saturation* (S_d) and is defined in Equation (106). This quantity can be expressed as a fraction or, if multiplied by 100, as a percentage.

$$S_d = \frac{volume_{water}}{volume_{voids}} \tag{106}$$

The degree of saturation tends to increase with depth, and the point at which the degree of saturation reaches 100 percent marks the boundary between the *saturated zone* below and the *unsaturated zone* above (Figure 110).

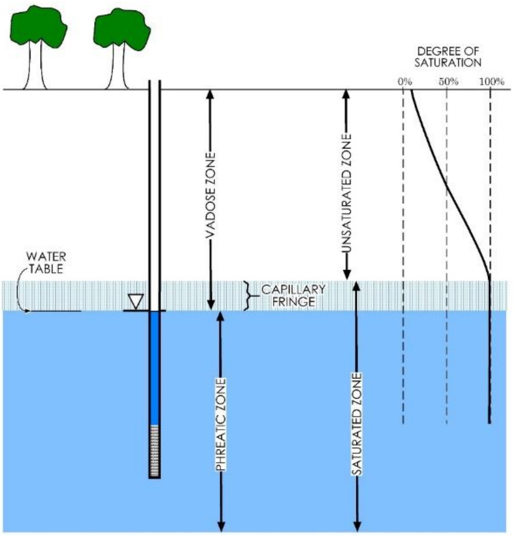


Figure 110 - Cross-section showing the distribution of water in the subsurface.

This boundary is often considered to be the same as the *water table,* which we previously defined as:

- the point in an unconfined aquifer in the zone of saturation where the *pore-water pressure* and the atmospheric pressure are equal, and
- the level to which water will rise in a well completed in an unconfined aquifer.

It may seem like we are making this unnecessarily complicated, and we should just define the water table as the boundary between the saturated and unsaturated zones, but the truth is that the two are not exactly the same because of *capillarity*.

14.2.2 Capillary Forces

To understand the role of capillary forces in groundwater, we first need to understand a little about pore fluid pressures. Let us consider a coffee cup that is half full of water—or half empty depending on your emotional outlook at the moment. Atmospheric pressure is pushing down on the water in the cup (Figure 111), and at the surface of the water the fluid pressure in the water is equal to the atmospheric pressure—for every force there is an equal and opposite force, as you may recall from your high-school physics class. Then, below the surface, fluid pressures increase with depth in the cup as the column of water above it increases.

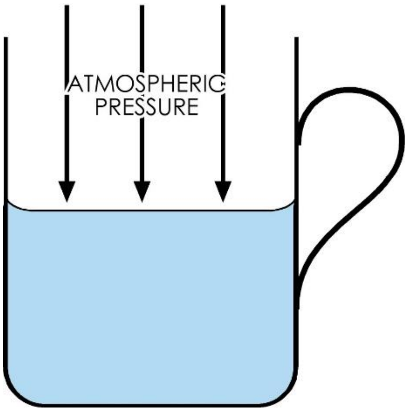


Figure 111 - Coffee cup half-full of water.

Now let us consider that the coffee cup is full of dry sand. If we pour some water into the cup so that the bottom half of the cup is saturated, we will have a surface in the middle of the cup that separates the saturated zone from the unsaturated zone. Atmospheric pressure is pushing down on the water in the pore spaces in the sand just like in the cup in Figure 111, and at the surface of the water, the *pore fluid pressure* is equal to the atmospheric pressure (Figure 112a). At any point below the surface, the pore fluid pressure is equal to the atmospheric pressure plus the pressure associated with the column of water above that point.

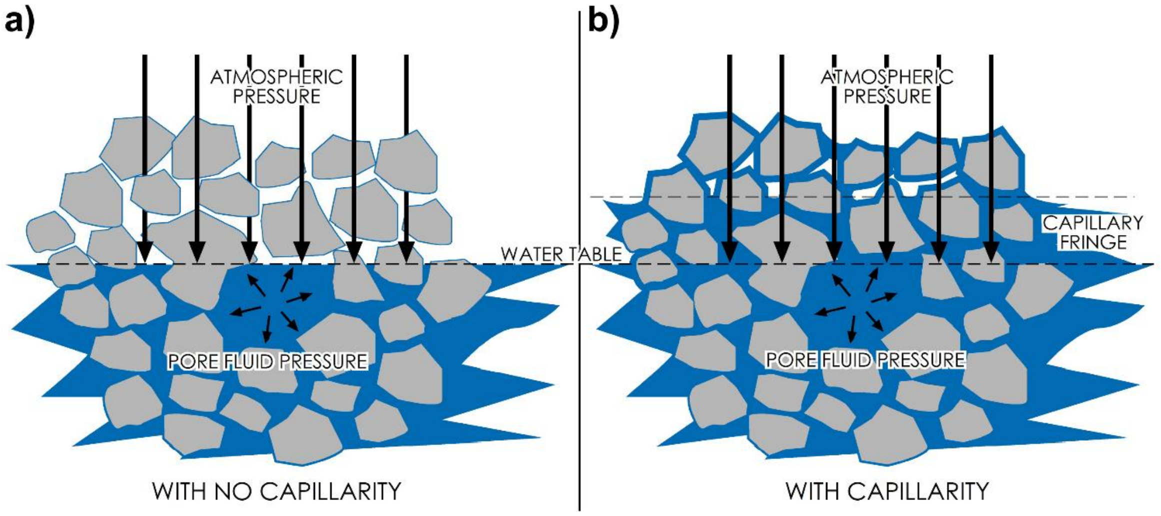


Figure 112 - Schematic diagram showing the interface between the saturated zone and the unsaturated zone a) with no capillarity, and b) with capillarity.

Up to this point, we have ignored atmospheric pressure in our calculations of hydraulic head in groundwater. This is because we use atmospheric pressure as the reference pressure so we define it as zero pressure. When dealing with unsaturated zone problems, pore fluid pressures are less than atmospheric, and given that we define atmospheric pressure as zero, pressures in the unsaturated zone are negative.

The polarity of water molecules (Figure 1) gives water a high surface tension and makes it 'stick' to things. Near the water table, the force associated with surface tension is enough to counteract the force of gravity and pull water upwards along mineral surfaces. This can be demonstrated by dipping the edge of a paper towel into water and watching the water move up into the towel against the pull of gravity. Porous rock or sediments are just like the paper towel—there is a lot of surface area in the pore spaces, surface tension causes water to stick to those surfaces, and water gets pulled upward from the water table. This force pulling water up into the pore spaces above the water table is called *capillarity*, or the *capillary force*. The capillary forces at work in the unsaturated zone pull enough water up above the water table to create a thin saturated zone above the water table called a *capillary fringe* (Figure 110 and Figure 112b). Capillary forces are greater when the openings are small so the capillary fringe is thicker in fine-grained materials and when the grains such as clay carry charge that attracts water molecules.

The capillary forces that are pulling water up into the capillary fringe are offsetting the forces of gravity that are pushing water down towards the water table. As we move from the water up into the capillary fringe, pore fluid pressures decrease. We can then define the capillary fringe as *the part of the saturated zone in which pore fluid pressures are less than atmospheric pressure.*

Figure 110 shows that the subsurface is separated into the saturated zone and the unsaturated zone, and the boundary between the two is the top of the capillary fringe. We also separate the subsurface into the *vadose zone*—the zone where pore fluid pressures are less than atmospheric pressure—and the *phreatic zone*—where pore fluid pressures are *greater* than atmospheric pressure. The surface that separates these two zones is the water table.

Often you will hear *vadose zone* and *unsaturated zone* used interchangeably—similarly the *phreatic* and *saturated zone* terms are used interchangeably. This is usually ok, since the capillary fringe is generally thin and probably not that important in most groundwater flow problems. However, it is useful to be aware of what vadose and phreatic really mean, and that in a strict sense, vadose and unsaturated are not the same.

Thought Question 16*: The thickness of the capillary fringe is not constant; it varies from aquifer to aquifer and from place to place in a single aquifer. What factors affect the thickness of the capillary fringe?*

14.3 Movement of Water in the Vadose Zone

The ease with which water moves in the vadose zone is directly associated with the moisture content of the rock or sediments. Before we talk about the movement of water in the vadose zone, let's have a quick review of the ways we express moisture content.

14.3.1 Moisture Content

We have already reviewed degree of saturation, which is the fraction of the pore space that is filled with water. Moisture content can also be expressed as a gravimetric moisture content—that is, moisture content *by weight*—or as a *volumetric moisture content*—that is, moisture content by volume. The *gravimetric moisture content* (ω) is the weight of water in the sample divided by the weight of the *solids* in the sample as expressed in Equation (107).

$$\omega = \frac{weight_{water}}{weight_{solids}} \tag{107}$$

Volumetric moisture content (θ) is the volume of water in the sample divided by the total volume of the sample as shown by Equation (108).

$$\theta = \frac{volume_{water}}{volume_{total}} \tag{108}$$

Note that volumetric moisture content is always some value less than the porosity of the rock.

14.3.2 Pore–Water Tension

In the vadose zone above the capillary fringe, water exists as a film or coating around the sediment or soil grains that is held to the grains by surface tension. This tension can be thought of as a *negative pressure head* or *matric potential*, and we commonly use ψ to denote this quantity. The terminology is such that a higher matric potential is a lower (i.e., more negative) pressure.

Negative pore pressures in the unsaturated zone can be measured using an instrument called a *tensiometer* (Figure 113). A tensiometer consists of a sealed tube with a porous ceramic cup at one end and a pressure gauge on the other end. The tube is filled with water and sealed with the pressure gage set to zero so the pressure in the tube is the same as atmospheric pressure outside of the tube.

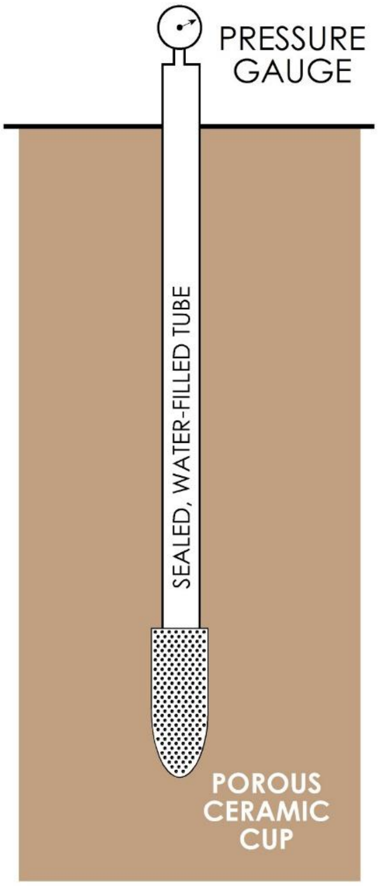


Figure 113 – Typical tensiometer design.

The tensiometer is inserted into the ground so that the ceramic cup is in contact with the soil. The negative pore pressures in the soil create a pressure gradient through the cup on the water in the tensiometer; these pressures are then transmitted to the pressure gauge. The amount of tension, or negative pressure head, is inversely related to the moisture content of the sediments (i.e., more-negative pressure values indicate lower moisture content).

With low moisture contents (Figure 114a), the film of water on the sediments is very thin, the volume of water in the film is small compared to the surface area of the water, and the surface tension is very high. When surface tension is high, the negative pressure head is high, and water movement is impeded by the surface tension. When moisture content is higher (Figure 114b), surface tension is lower, and water moves more easily.

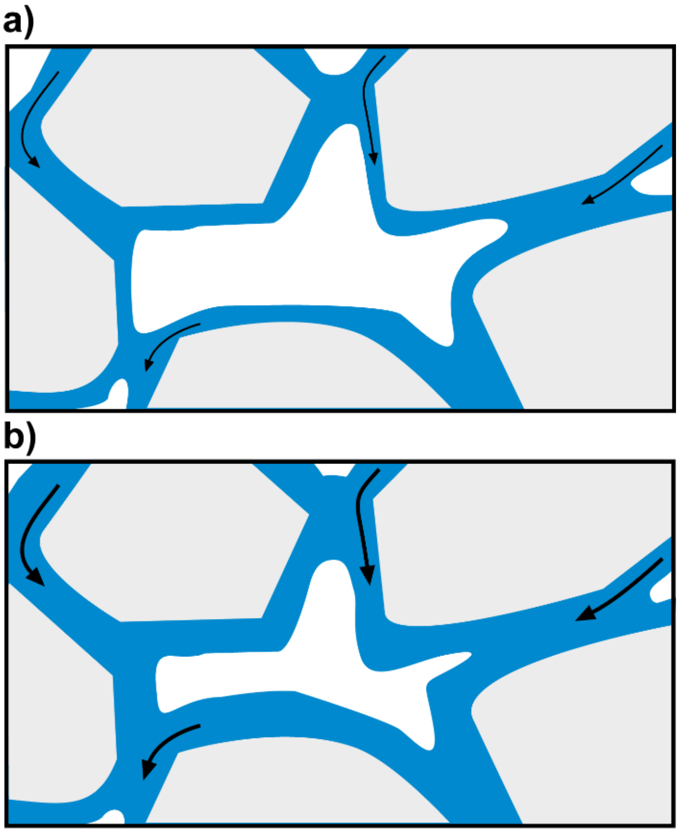


Figure 114 - Pore fluid in the vadose zone with a) low moisture content and slow fluid movement indicated by the thin black arrows; and b) high moisture content with moisture moving more readily.

We can deduce that the permeability of sediments in the vadose zone is directly related to the moisture content. This is in fact true—increasing moisture content increases the permeability and decreasing moisture content decreases the permeability. The primary reason for this is that higher moisture content leads to more interconnection between pockets of fluid in the pores and the increased thickness of water on the pore walls provides a larger flow area. Unfortunately, the relationship is not linear—in fact, there is an extremely complicated relationship between moisture content and flow, and the theory of flow in the unsaturated zone is not as simple as in the saturated zone.

The general relationship between moisture content and these other parameters—matric potential and permeability—is shown in Figure 115. Permeability—that is, unsaturated hydraulic conductivity—increases as moisture content increases, while matric potential decreases with increasing moisture content. Matric potential is a negative number, and *higher matric potential* is a *more negative pressure* so decreasing matric potential (e.g., a value increasing from say -10^7 to -10^2) is related to a decrease in the force that pulls water to solid surfaces in the unsaturated zone. A decrease of matric potential also reflects an increase in moisture content. These combine to result in increased flow downward through the vadose zone.

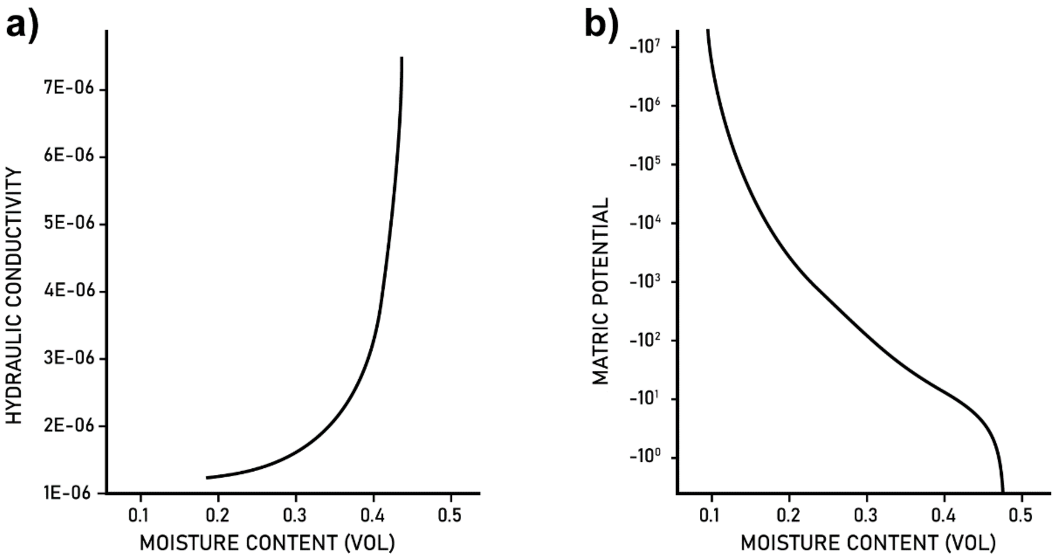


Figure 115 - Graphs of a) hydraulic conductivity versus volumetric (vol) moisture content; and b) matric potential versus volumetric moisture content. It is useful to note that the y-axis of the graph in (b) is reversed such that the smaller value (-10^7) is higher on the graph than the larger value (-10^2) because a smaller negative number is described as a higher matric potential. The pressure is orders of magnitude lower at the top of the graph than the bottom of the graph.

14.4 Theory of Flow in an Unsaturated Porous Medium

In the unsaturated zone water is pulled down under the force of gravity and flow is, for the most part, vertical and downward. In saturated groundwater, flow is driven by the hydraulic gradient, which refers to the distribution of hydraulic heads in the flow system. Flow in the vadose zone is also driven by differences in head, and the head at any point in the unsaturated zone is like the saturated zone head in that it composed of an elevation component and a pressure component. More specifically, unsaturated zone head (hu) is defined in Equation (109).

$$hu = \psi + z \tag{109}$$

where (parameter dimensions are dark green font with mass as M, length as L, time as T):

ψ = matric potential (or negative pressure head, thus a negative length) (L)

z = elevation head (L)

Darcy's law is also applicable in the unsaturated zone, since we are dealing with laminar flow, so we can describe the permeability in terms of a hydraulic conductivity. However, as previously stated, the matric potential and the hydraulic conductivity are both a function of the volumetric moisture content (θ). The implication is that, unlike in saturated sediments, hydraulic conductivity is no longer a constant, rather, it is a variable dependent upon θ. When you think about it, the nonlinear nature of unsaturated flow becomes apparent—water infiltrates into the unsaturated zone at a rate that is dependent upon the moisture content, however, as it infiltrates, the moisture content increases, permeability increases, and water infiltrates faster, changing the permeability more, and so on. At the same time, the matric potential is also changing, so as water infiltrates, matric potential decreases, the gradient decreases, and the water infiltrates more slowly, and so on. The problem in modeling the unsaturated zone, therefore, is that both primary parameters (i.e., ψ and K) change as conditions in the model change, and the changes are not linear—that is, doubling θ does not necessarily double K. Not only do they change, but they change independent of each other—the magnitude of change of K will not be the same as the change in ψ for the same change in θ.

The nonlinearity is complicated by the fact that the magnitude of the change in K or ψ depends on whether water is being *added* to the soil (*wetting*) or water is being *removed* from the soil (*drying*). In other words, the shape of the curves in Figure 115 will change depending on how the conditions are changing. This process is called *hysteresis*—that is, the dependence of the state of a system is based on its history. Figure 116 shows idealized curves of moisture content versus matric potential and hydraulic conductivity versus matric potential with one curve for draining (i.e., drying) a material and a different curve for rewetting the material. If the same sample were repeatedly drained and wetted, the drying and wetting curves would be different each time. The implication of this is that the prior moisture history of the soil will affect and control the relationship between ψ and K.

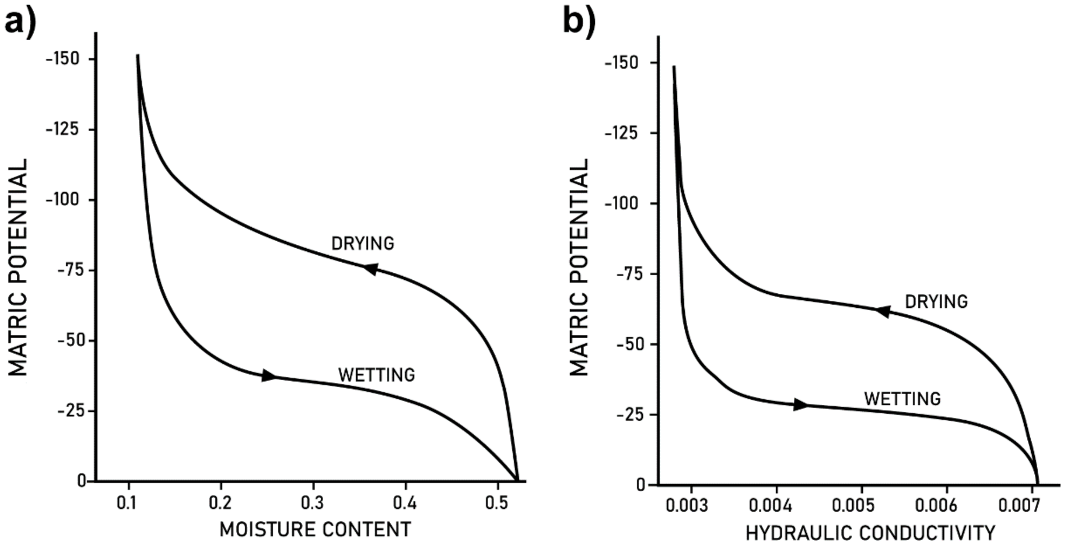


Figure 116 - Idealized plots of a) water content and b) hydraulic conductivity versus matric potential showing separate curves for the drying and wetting paths.

14.5 Studying the Unsaturated Zone: Instruments and Techniques

This section contains basic descriptions of a few instruments and techniques used to investigate hydrologic processes in the unsaturated zone. This is by no means a comprehensive list.

Tensiometers are used to measure *matric head*, or soil suction (Figure 113). These can be installed at varying depths to infer moisture profiles. Usually these are installed between 15 cm and 1 meter below the surface. Unlike the saturated zone, it is not possible to measure a water level in a piezometer in the unsaturated zone because water will not flow into the well. Instead, tension in the soil is measured and then added to elevation to get total head (Equation (7)).

A *lysimeter* is like a tensiometer, but instead of measuring matric potential it is used for sampling the moisture in the pore spaces of the soil so the water can be analyzed for its chemical content. A lysimeter has a tube and a porous cup like the tensiometer; however, the tube is left empty, and a partial vacuum is generated in the tube through a valve at the top. The ceramic cup is placed in the soil, and the vacuum in the tube draws water in from the soil, where it can be sampled via the sampling tube (Figure 117).

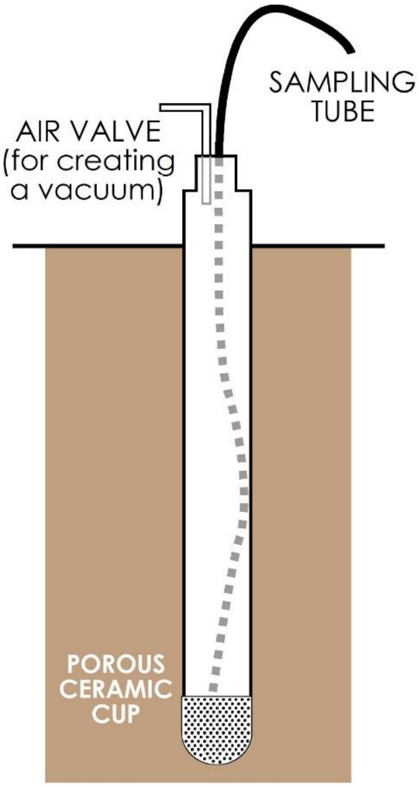


Figure 117 - Lysimeter for vadose zone fluid sampling.

Gypsum blocks are small (1 to 2 cm) blocks of gypsum with embedded wires that can be hooked up to a box that measures electrical resistance. These are buried in the soil with the wires exposed. They can be used to indirectly measure soil moisture because gypsum is not very conductive and so, changes in moisture within the block are inversely proportional to changes in electrical resistance. These need to be replaced frequently (≈1 to 3 years).

Electrical permittivity sensors provide longer term monitoring using metal probes that are inserted into the soil. They are connected to a logger which sends an electrical signal and records a response relating the electrical permittivity to soil moisture. The logging interval is set by the user.

A common method for using tensiometers, lysimeters, and gypsum blocks is to set up a field experiment called an *infiltration plot*. In an infiltration plot, a plot of ground is isolated using wooden boards or a ring of sheet metal to create an enclosed area that can be filled with water to a certain depth. Instruments like those described above are installed within the infiltration plot at various depths and monitored for a period of time—for example, days, weeks—to record the initial conditions. The plot is then filled with water, and the instruments are monitored over the next few days as the water infiltrates into the ground. From this data one can develop a series of *moisture profiles* that illustrate movement of the *wetting front* down through the subsurface (Figure 118).

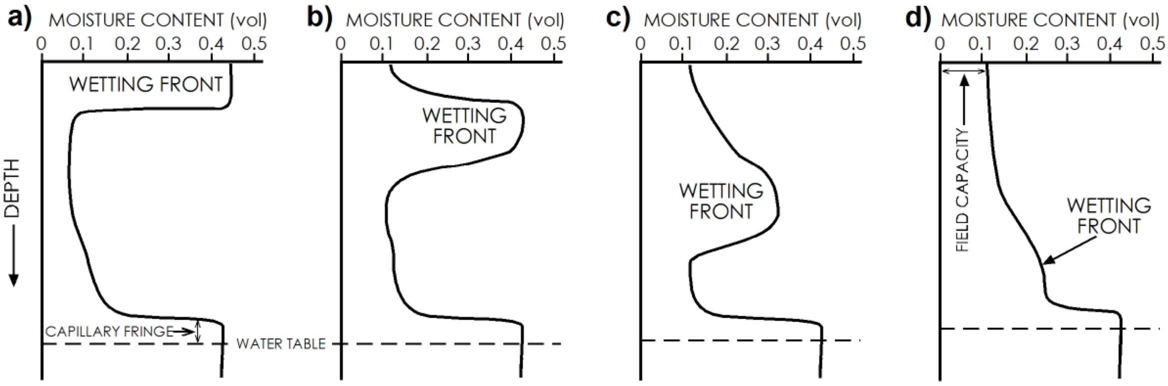


Figure 118 - Moisture profiles showing the movement of a wetting front from an initial slug of water in an infiltration plot: a) initial infiltration saturates the shallow soil; b) with time the water percolates downward at varying rates such that the volume of water is spread vertically and the moisture content is less than full saturation; c) continued downward percolation increases the vertical spread of moisture and decreases the moisture contents; and d) some of the water introduced to the plot reaches, thus recharges, the water table causing a slight rise.

15 Consolidation and Subsidence

15.1 Introduction

In Section 5 we discussed the response of a confined aquifer to pumping from a well. To review, when a well is pumped in a confined aquifer, the pore spaces stay filled with water, and the water comes from the *elastic response* of the aquifer. More specifically, as pressure is reduced in the pores the pores get a little bit smaller and the overall pore volume decreases slightly. When we discussed this earlier, we did not consider the fact that changing the volume of the individual pores might change the overall volume of the aquifer.

Well…, it does change the overall volume of the aquifer. As the fluid pressure decreases, the grains expand, the pores get smaller, and the overall volume of the aquifer decreases. This is called *consolidation*. Just as different rocks react to tectonic stresses in different ways, different aquifers and hydrostratigraphic units react to changes in fluid pressure in different ways. Change in aquifer volume in response to change in pore fluid pressure is the subject of this section.

15.2 Review of Elastic Response in a Confined Aquifer

Consider a confined aquifer that consists of a layer of sand overlain by a layer of clay that acts as a confining layer. If a well is installed in this confined aquifer and water is removed from it via a pump, the heads around the well will lower in such a way that the well creates a *cone of depression* in the potentiometric surface (Figure 119). The more water that is removed from the well, the larger the cone of depression. The example in Figure 119 shows that, even though pumping has removed a volume of water and lowered the water level in the well and in the surrounding aquifer, the saturated thickness of the aquifer has not changed—that is, the aquifer is still saturated up to the base of the confining layer.

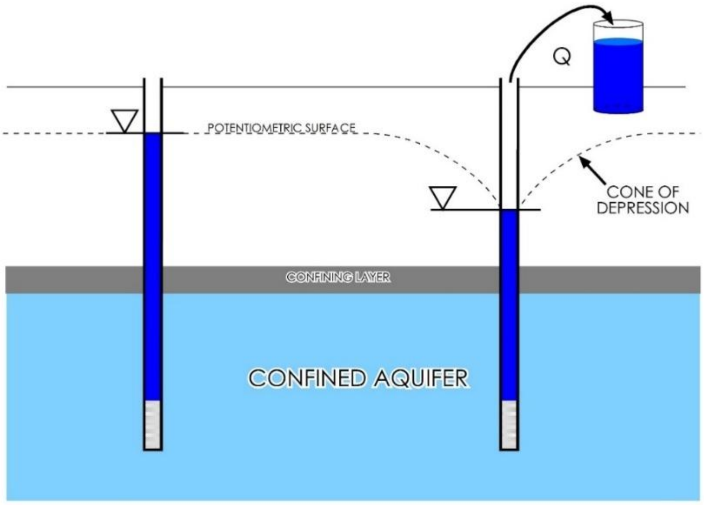


Figure 119 - Pumping well in a confined aquifer.

What is happening has to do with the additional fluid pressure, or *excess fluid pressure*, in the confined aquifer (Figure 120). The pressure head in the aquifer right next to the well screen is related to the column of water above that point in the well. In an unconfined aquifer, the pressure head is equal to the thickness of the saturated zone above the well screen. In a confined aquifer, it is equal to the thickness of the saturated zone plus the additional pressure that is reflected in the elevation of the potentiometric surface above the base of the confining layer.

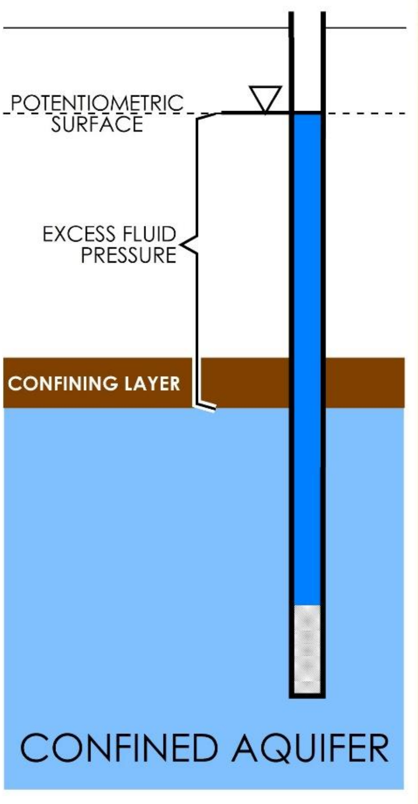


Figure 120 - Excess fluid pressure in a confined aquifer.

To put it into other words, in a confined aquifer, there is some additional pressure in the pressure head component, which is represented by the length of the column of water in the well above the top of the aquifer. If there was no additional pressure—as in an unconfined aquifer—the water level in the well would rise to the water table. It is the extra pressure in the confined aquifer that is pushing the aquifer matrix apart and creating extra space for the water in the aquifer, and when that pressure is relieved, the grains relax back to their original positions (Figure 121). Decreasing the fluid pressure in the aquifer causes the potentiometric surface to decline and the individual pore spaces to decrease. The change in volume of each individual pore is miniscule but can add up to a significant change in volume of the aquifer.

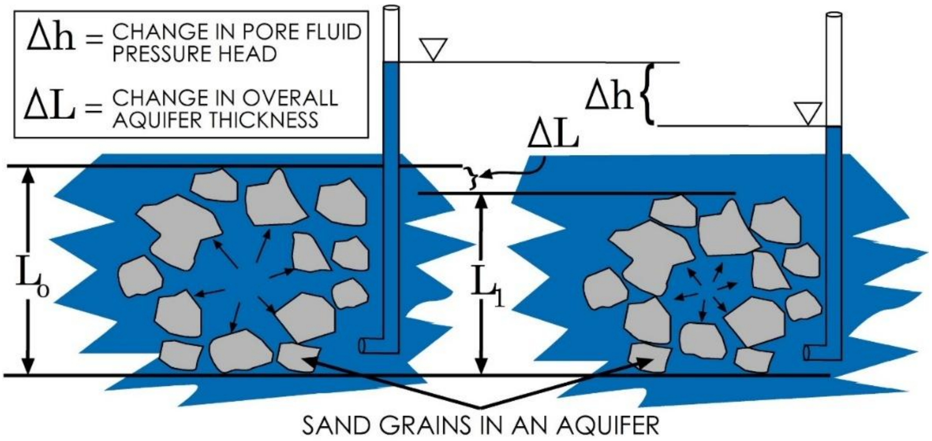


Figure 121 – Reduction of pore fluid pressure causes grains to move closer together such that the confined aquifer thickness decreases.

What we are talking about here is a *strain*—that is, change in volume or change in length—resulting from a *stress*—some force or pressure applied in three dimensions. The stresses in an aquifer are due to the weight of the column of rock and water above a given point, and the strains that we observe are consolidation of aquifer materials, which can often result in subsidence of the ground surface. Before we talk about strain—that is, consolidation—let's talk about the states of stress in the subsurface.

15.3 States of Stress in the Earth's Crust

At all points in the subsurface, the rocks and sediments are under stress. Stresses can be *normal* or *shear* relative to the point of observation, and normal stresses can be either compressional or tensional (Figure 122).

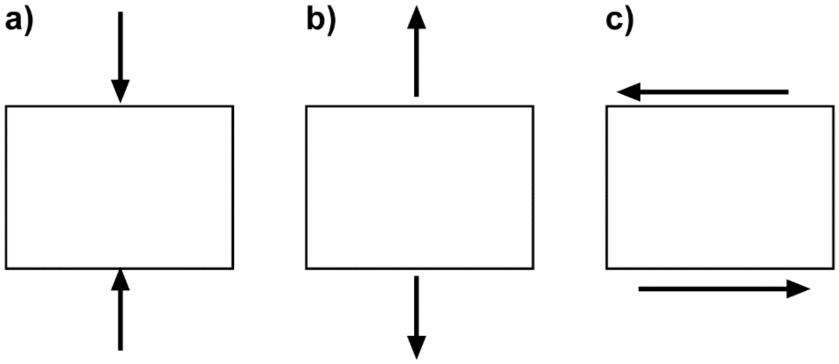


Figure 122 - Schematic diagrams showing a) compressional stress, b) tensional stress, and c) shear stress.

Stress is a tensor quantity, meaning that it exists in three dimensions in the subsurface and the magnitude of stress varies with respect to location and direction. If we are dealing with an area that is at a tectonically active margin, or at a location with lots of topography and vertical relief—for example, next to a cliff face—the horizontal stresses could be significant. Fortunately, in most hydrogeologic settings with fluid-related consolidation problems, we can generally ignore the horizontal stresses and only deal with vertical stress, so for our purposes we will ignore horizontal stress and only consider vertical stress—and resulting vertical strain.

The vertical stress at a given point in the subsurface is simply a function of the pressure exerted on that point by the weight of the overlying material. This pressure is equal to the unit weight of the material times the thickness of the material. We can express that relationship in Equation (110).

$$\sigma_v = p_b g z \tag{110}$$

where (parameter dimensions are dark green font with mass as M, length as L, time as T):

σ_v = vertical stress ($ML^{-1}T^{-2}$)

p_b = bulk density of the overlying material (ML^{-3})

g = gravitational acceleration (LT^{-2})

z = thickness of layer material (L)

This equation calculates the total stress at any given depth in the subsurface, based on the thickness of the overlying material. The total stress, also called the lithostatic stress, is the sum of two different stresses: the *effective stress*, which is the stress from the column of rock above the point (Figure 123), and the *pore fluid pressure*, which is the stress related to the fluid in the pores (Figure 121). This relationship is summed up by Equation (111), also called *Terzaghi's Law*. We are therefore considering two stresses—the effective stress, which is directed downward, and the pore fluid pressure, which is acting in all directions but can be considered to be acting opposite to the effective stress.

$$\sigma = \sigma' + p_f \tag{111}$$

where (parameter dimensions are dark green font with mass as M, length as L, time as T):

σ = total vertical stress; also called the normal stress ($ML^{-1}T^{-2}$)

σ' = effective stress, or "grain to grain" stress in the rock ($ML^{-1}T^{-2}$)

p_f = pore fluid pressure ($ML^{-1}T^{-2}$)

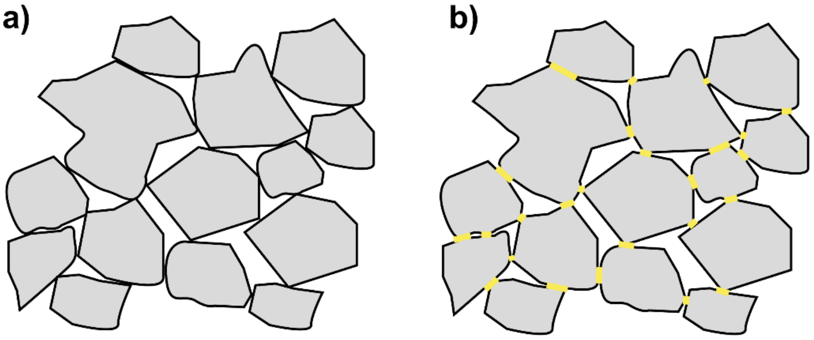


Figure 123 - a) Close-up of sand grains in the subsurface; b) same grains with grain contacts highlighted. The effective stress is the stress related to the weight of the sediment grains and is supported by the grain contacts.

In an unconfined aquifer, the pore fluid pressure is simply the pressure from the column of water above the point in the aquifer—just like the total stress is the pressure associated with all the material above that point. This is called the *hydrostatic pressure*. We can use Equation (112) to express the hydrostatic pressure.

$$p_h = p_w g z \tag{112}$$

where (parameter dimensions are dark green font with mass as M, length as L, time as T):

p_h = hydrostatic pressure ($ML^{-1}T^{-2}$)

p_w = density of water (ML^{-3})

z = thickness of the overlying layer of water (L)

In a confined aquifer, the pore fluid pressure is equal to the hydrostatic pressure, plus the additional pressure that causes water to rise above the top of the aquifer in a well—that is, the excess fluid pressure (Figure 120). We can therefore combine Equation (110) and Equation (111) and rewrite the Terzaghi equation as Equation (113).

$$\sigma = \sigma' + p_w g z + u \tag{113}$$

where (parameter dimensions are dark green font with mass as M, length as L, time as T):

u = excess fluid pressure ($ML^{-1}T^{-2}$)

As this equation indicates, the stresses are a function of depth, and the lack of exponents in the equations tells us that they are linearly proportional to depth. We can therefore draw a diagram (Figure 124) that graphically represents the distribution of each component of stress—lithostatic, hydrostatic, and excess fluid pressure—in the subsurface. Hydrostatic pressure, p_h, is zero at the water table and increases linearly with depth. Lithostatic pressure, σ', is zero at the surface and increases linearly with depth. A confined aquifer has an additional component of pressure, u. The effective stress, σ, is the sum of p_h, σ', and u.

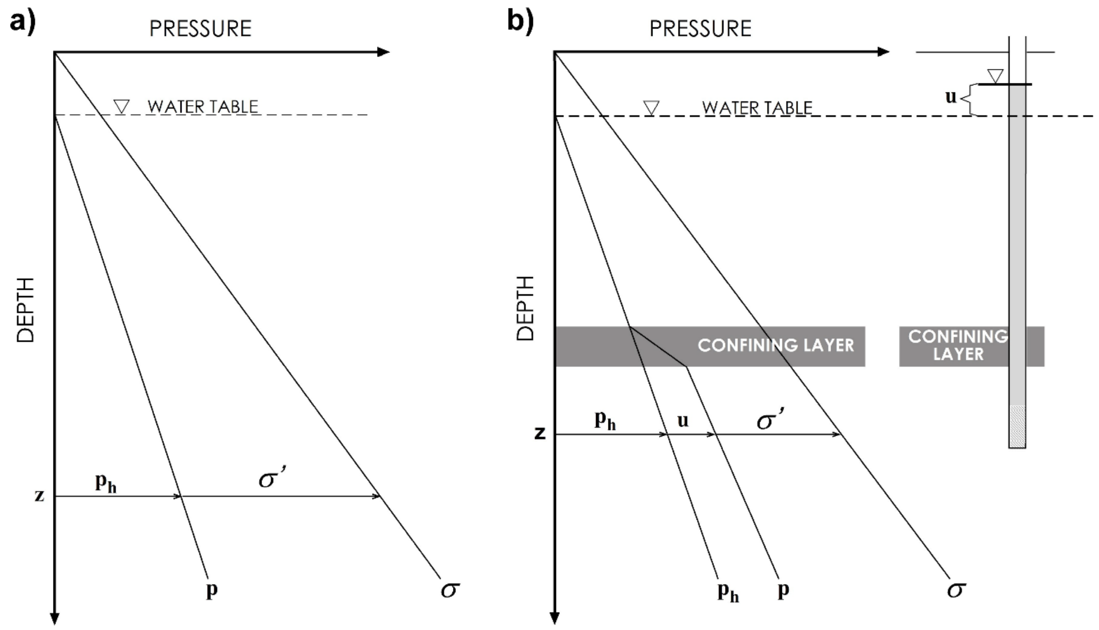


Figure 124 - Changes in hydrostatic (p_h) and lithostatic (σ) stress with depth in a) an unconfined aquifer, and b) a confined aquifer with excess fluid pressure (u). The well on the right in (b) illustrates the definition of u relative to water levels in a well.

Figure 124a shows the relationship between stress and depth in an unconfined aquifer. Both the lithostatic and the pore fluid pressures increase linearly with depth, and the pore fluid pressure is generally a little less than half of the lithostatic. Figure 124b shows the relationship between stress and depth in a confined aquifer with an overlying unconfined aquifer. In the water table above the confining layer, the pore fluid pressure increases linearly; below the confining layer there is an additional component of pore fluid pressure (u). The additional pore fluid pressure decreases the effective stress accordingly—in other words, the pore fluid pressure pushes against the grains and reduces the stress at the grain contacts. The additional stress, u, represents the additional pressure head associated with the elevation of the potentiometric surface above the water table.

Thought Question 17*: As pore fluid pressure increases, it decreases the effective stress—in other words, it pushes against the stress of the weight of the overlying rock. What would happen if the fluid pressure increased so much that it equaled the lithostatic stress—that is, if the effective stress were zero?*

15.4 Compressibility and Storage Parameters

We have discussed stress; now let's talk about strain and the relationship between strain and stress. In any given material, a certain amount of stress will result in a certain amount of strain. The ratio of strain to stress is called the compressibility. In terms more specific to what we are discussing here, the compressibility is the amount of shortening or extension—that is, the change in thickness of a layer—that occurs due to a change in stress—in this case, the change in stress associated with a change in pore fluid pressure. We can express this as Equation (114).

$$\alpha = \frac{\frac{\Delta L}{L}}{\Delta \sigma'} \tag{114}$$

where (parameter dimensions are dark green font with mass as M, length as L, time as T):

α = compressibility (LT^2M^{-1})

$\frac{\Delta L}{L}$ = the change in length (thickness) over the total length (:)

$\Delta \sigma'$ = the change in effective stress ($ML^{-1}T^{-2}$)

Compressibility is just a simple coefficient relating strain to stress, and it can be applied in several ways. If we know the compressibility of a material—for example, a clay confining layer—we can calculate how much the layer will consolidate for a given change in head. To do that, we convert the change in head to a change in pressure, then multiply by the compressibility to calculate the change in length over the total length.

The specific storage is directly related to the compressibility of the aquifer by the following relationship shown in Equation (115).

$$S_s = \rho_w g \, \phi \, (\alpha + \beta) \tag{115}$$

where (parameter dimensions are dark green font with mass as M, length as L, time as T):

S_s = specific storage (L^{-1})

ρ_w = density of water (ML^{-3})

g = gravitational acceleration (LT^{-2})

α = compressibility of the aquifer framework ($M^{-1}LT^2$)

ϕ = porosity (:)

β = compressibility of water ($M^{-1}LT^2$)

15.5 Land Subsidence

As shown in Section 15, decreased pore fluid pressure in aquifers (i.e., decreased stress) results in strain that manifests as a reduction in thickness of the formation. In certain types of geologic environments, the changes in thickness can be significant, resulting in a reduction in the elevation of the ground surface called *land subsidence*. A primary cause of land subsidence throughout the world is extensive removal of groundwater through pumping from water supply wells, and many metropolitan areas – including Jakarta, Bangkok, Houston, and Mexico City – have experienced land surface subsidence of up to tens of meters in many places. Numerous hazards and negative impacts are associated with subsidence. On a local scale, subsidence results in instability that can damage building foundations, utility lines, and pavements. On a regional scale, subsidence changes surface drainage patterns and makes flooding events more intense and damaging. Coastal areas experiencing subsidence are especially vulnerable to storm surges and other issues associated with rising sea levels. For many of these areas, the dependence upon groundwater for municipal, and other, water supplies makes it extremely difficult to mitigate subsidence. Much of the subsidence that occurred to date is non-recoverable, which means that the ground surface elevation lost to subsidence will never be reversed or restored.

More information on land subsidence can be found in *Land Subsidence and its Mitigation*↗ by Gambolati and Teatini (2021), which is available from The Groundwater Project as a free download or to be read online.

16 Flow in Non-Porous Media

16.1 Introduction

To date, our treatment of groundwater flow has some implicit assumptions concerning flow in a porous medium. Basically, when we talk of flow in *porous media*, we are referring to water moving through small, evenly distributed pores, with laminar flow driven by a hydraulic gradient where the flow is linearly and directly proportional to the gradient.

However, this is often not the case—aquifers are not always geologic layers with small, evenly-distributed pore spaces. Groundwater flow in many geologic units is controlled by *discontinuities* in the rock or sediment—cracks, bedding planes, breccia zones along faults, and conduits formed by dissolution. The physics of groundwater flow in these discontinuities is different from flow in a porous media system. Fractures or conduits often control the direction and magnitude of flow in a groundwater system. Flow in these discontinuities, or in *non-porous media*, is the subject of this section, which focuses on fracture flow. Many of the concepts apply to conduit flow as well. More specific information related to conduit flow is provided by Kuniansky and others (2022) in *Introduction to Karst Aquifers*↗ and by Stevanović and others (2024) in *Karst: Environment and Management of Aquifers*↗, both are available as free downloads from The Groundwater Project.

For our purposes, a *fracture* is defined as any relatively planar discontinuity in a rock or sediment. We tend to think of fractures as a break in a rock, but for hydrogeologists, a bedding plane acts the same as a stress-release joint in bedrock. When we look at rocks in nature, we find that all rocks exposed at Earth's surface are fractured to some degree. In many rocks, the fractures are not a significant part of the flow system, and it is often a valid assumption to ignore them or to assume that flow in them is acting the same as flow in a porous medium. Fractures and planar discontinuities, however, are frequently a significant part of an aquifer flow system.

16.2 Conceptualizing Fractured Media

The term *fractured media* refers to an aquifer or formation with fractures that are a significant part of the flow system. Fractured media can be classified into four categories based on hydraulic properties of the fractures relative to the *matrix*—that is, the blocks of rock between the fractures (Figure 125). The hydraulic properties of interest are the permeability, or hydraulic conductivity, and the storage characteristics of each component (i.e., fractures and matrix) of the system. These are idealized conceptualizations of natural materials, the differences between them are gradational, and natural systems may be a combination of two or more classifications shown in (Figure 125).

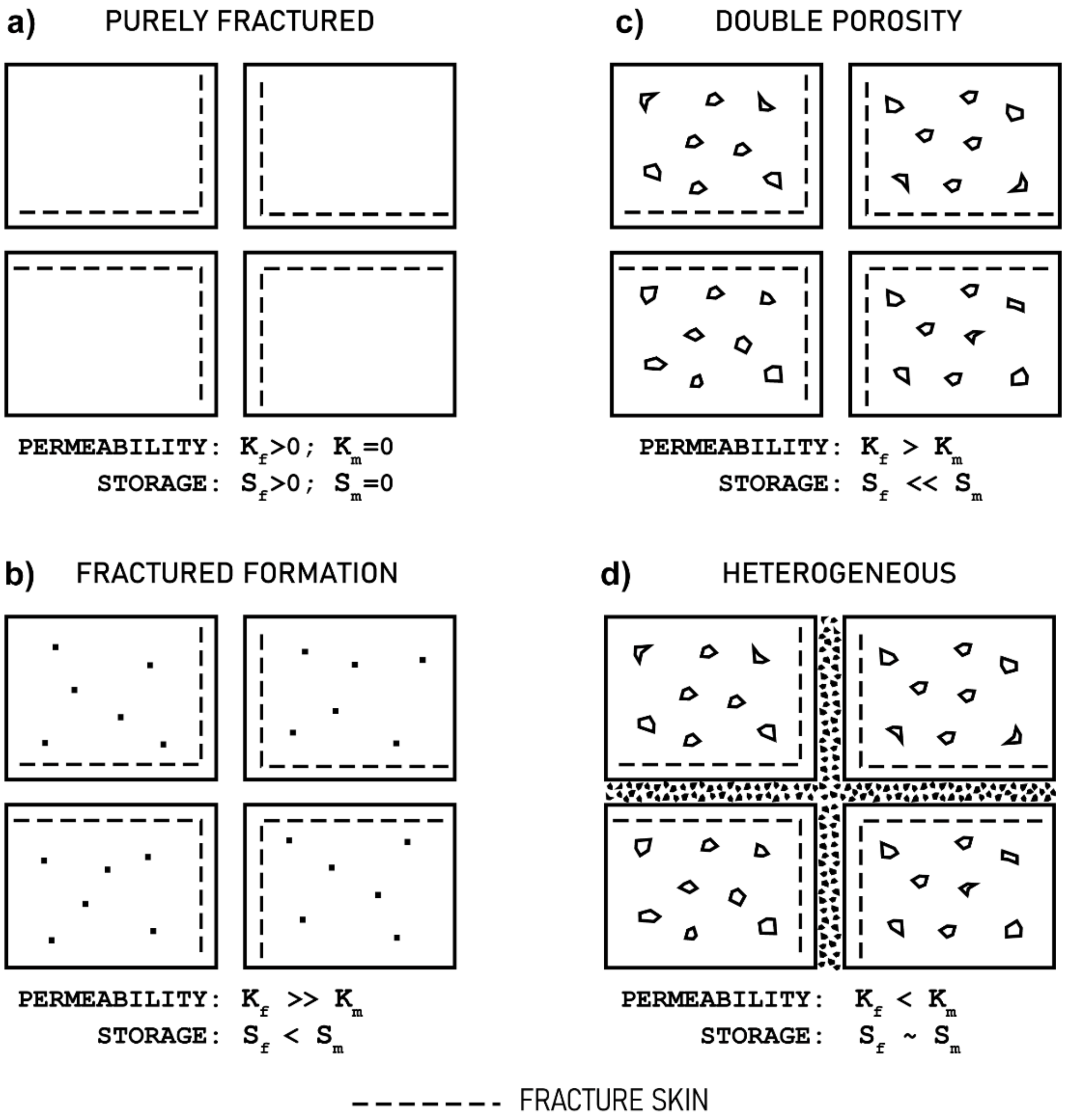


Figure 125 - Hydrogeologic classification of fractured media: K_f is permeability of the fractures; K_m is permeability of the matrix; S_f is storage coefficient of the fractures; S_m is storage coefficient of the matrix (modified from Sharp, 1993).

In a purely fractured system (Figure 125a), the permeability and porosity of the matrix is so low that it is essentially impermeable; such that all fluid flow in the system occurs in the fractures and all fluid stored in the system is stored in the fractures.

In a fractured formation (Figure 125b), the permeability of the matrix is relatively low, but the porosity is significant. In these systems, the permeability of the fractures is so much higher than the matrix that all of the fluid flow is in the fractures, but most of the water is stored in the matrix. As we show later, it does not take a very large fracture to have a high permeability. Fractured formations are very significant in the petroleum industry; in many petroleum reservoirs, most of the oil is stored in the matrix, but it has to flow through the fractures to get to the oil wells.

In the double porosity system (Figure 125c), permeability of the matrix is high enough to allow for significant flow in the matrix as well as in the fractures. As with the fractured formation, most of the fluid is stored in the matrix. In this type of system there can be a great deal of interaction between the two (water moving from the matrix into the fracture and vice versa), which can complicate modeling of contaminant transport. This type of system, as well as the previous two, are difficult to model and quantify because the geometry of the fractures is poorly known and because it is difficult to characterize the matrix-fracture interactions.

The final system is called a heterogeneous system (Figure 125d). In this system, the fractures have been filled in with sediment that has a different permeability than the matrix—frequently lower. In this case, the fractures may act as barriers to flow, rather than conduits.

Figure 125 also introduces the concept of a *fracture skin* (dashed line). The fracture skin refers to a zone along and immediately beneath the surface of the fracture in which the matrix has been altered. This alteration could be from precipitation of minerals (e.g., iron or manganese oxides, calcite), dissolution of minerals from the matrix, or deposition of clay particles in pores (from transport through the aquifer or from alteration of feldspars and micas in the matrix). Fracture skins are significant because they affect the hydraulic relationship between the matrix and the fractures. They can also change the sorption and exchange properties of fracture surfaces, which can in turn affect contaminant transport.

In most cases, fracture skins have a lower permeability than the matrix, and they act to impede the exchange of water between the matrix and the fracture. In some cases, especially where the fracture skin has experienced dissolution, the skin can have a higher permeability than the matrix.

Thought Question 18: *Give a geologic example of each type of fractured media.*

16.3 Fracture Properties

16.3.1 Orientation

Fractures are three-dimensional structures that are generally assumed to be planar or close to planar. Therefore, just like any planar structure we can characterize the orientation with strike and dip, where the *strike* is the line created by the intersection of the fracture with the horizontal plane, and the *dip* is the angle of the fracture plane below the horizontal plane. We can measure these orientations in the field or on aerial photographs, and then use rose diagrams (Figure 126), radial plots, or stereonets to present the distribution of these orientations.

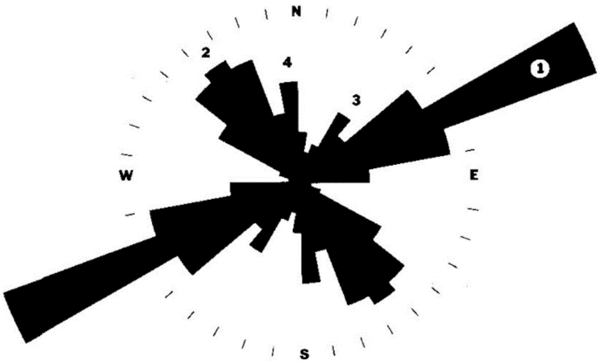


Figure 126 - Sample bi-directional rose diagram showing the strike direction of fracture sets in a rock formation with the length of the rose petal representing one of a variety of features such as the frequency of fractures in that set relative to all fractures in the rock, the length of the fractures in that set, the average magnitude of their dip. The numbered labels represent the 4 main fracture set directions.

Rose diagrams can be plotted on maps to show fracture trends and changes in orientation throughout a region. These diagrams can identify sets of fractures, which can then be related to groundwater flow directions using several methods, such as geochemical tracers.

16.3.2 Aperture

Fracture aperture is the distance between the fracture walls (Figure 127).

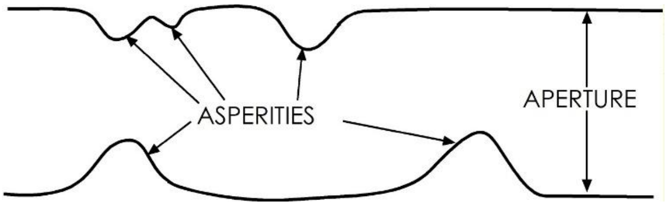


Figure 127 - Fracture aperture and asperities on fracture wall.

In an idealized fracture with smooth walls and a constant aperture—that is, the fracture walls are parallel to each other—the hydraulic conductivity and the discharge are proportional to the aperture. However, rock fractures always have *asperities* that stick out from the wall of the fracture. These affect the properties of the fracture in the following ways.

- Asperities create roughness on the fracture surface. This roughness has several effects.
    - *Channelization* of fluid along the surface of the fracture—most of the water flow only within channels on the fracture surface and not uniformly over its entire surface (Figure 128).
    - *Tortuosity* (τ)—defined as the distance water travels divided by the straight-line distance between two points, the more numerous the asperities and the larger their amplitude, the more tortuous the flow path, thus water must travel farther —this is sometimes defined as the reciprocal (Figure 129).

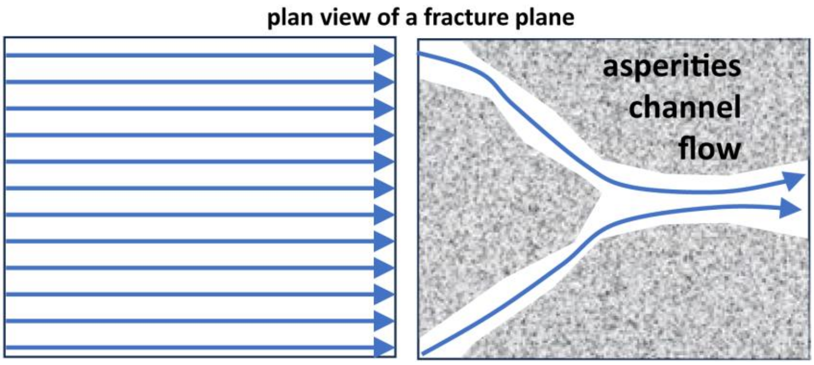


Figure 128 - Plan view of a fracture surface: a) without asperities; and b) with channeling of flow along the surface of a fracture around asperities.

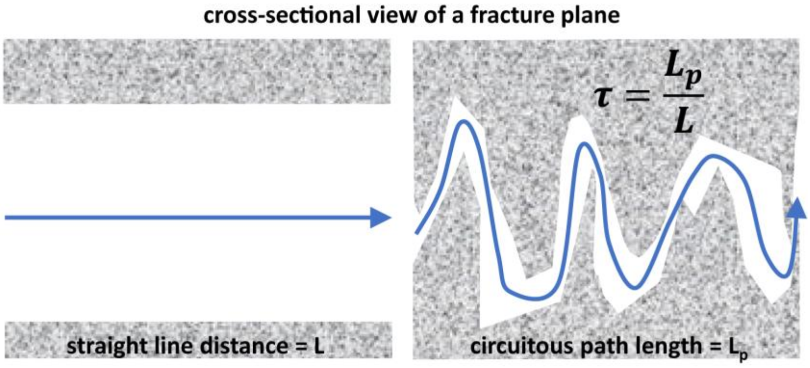


Figure 129 - Cross sectional view of a fracture plane a) without asperities; and b) with asperities causing tortuous flow along the fracture.

- Friction associated with surface roughness impedes the flow of fluids. Friction also affects the geotechnical properties of the fracture—higher roughness means that the fracture is less likely to fail or activate in response to changes in stress.
- In the subsurface, where the fracture is under compressive stress, the asperities hold the fracture open.

16.3.3 Length, Width, and Depth

Fractures are three-dimensional structures that, in addition to aperture, have a length and a width. An idealized fracture in the subsurface, which does not intersect the surface, will generally have an irregular elliptical shape with an aperture that is maximum near the middle and tapers off to the edges (Figure 130).

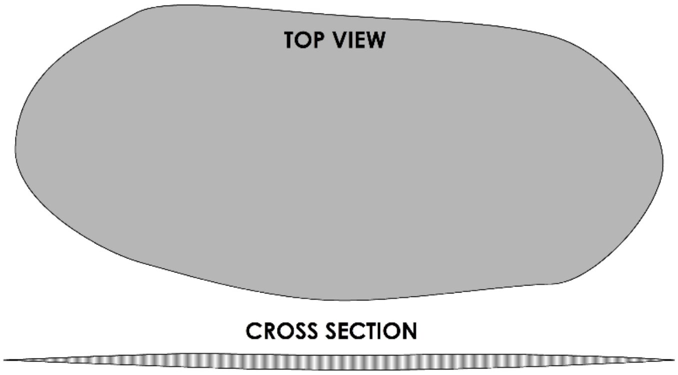


Figure 130 - General fracture geometry.

When we talk about length and width, we are generally talking about the dimension that is exposed at the Earth's surface—either at an outcrop or identified from an aerial photo. In outcrop, we frequently cannot see the ends of the fracture—it either terminates at the top of the outcrop or it terminates somewhere underground, or both. Length is generally used to describe the exposed length of a vertical fracture in an outcrop; width is often used to describe horizontal fractures. Depth refers to the distance that the fracture goes into the outcrop. This can sometimes be measured or observed using geophysical methods—ground penetrating radar, shallow seismic, electromagnetic—or using boring logs. However, geophysical methods are limited because they do not quantify whether the fracture is open (i.e., able to support flow) or filled and sealed.

16.3.4 Spacing and Density

A given volume of rock will generally not have only a single fracture in it; rather, there will be several fractures, and some of those fractures will have similar orientations due to the fact that they were formed by the same stresses. The *spacing* of the fractures refers to the distance from one fracture to the next; the *density* of fractures refers to the number of fractures in a unit volume of rock.

- Spacing is measured by running a scanline or traverse across an exposed rock face and measuring the distance between the points where the fractures intersect the scanline.
- Density is measured by adding up the lengths of all the fractures in a volume or area of rock and dividing the total length by the volume, or area. We frequently use area because these studies are often two-dimensional outcrop studies—that is, on the surface—rather than three-dimensional subsurface studies.

Spacing and density seem to be related to the aperture. Studies have indicated that, in specific rocks, there are direct relationships between fracture spacing and aperture—that is, smaller apertures tend to be more closely spaced, and vice versa. This relationship seems to hold across several scales—from thin section to outcrop to air photo—and if it holds true it could have a major impact on the way we model fractured systems.

16.3.5 Connectivity

Fractures are only effective as conduits for fluid flow if they are connected to each other. This is especially true in purely fractured or fractured formation systems. Fracture connectivity can be evaluated by observing fracture terminations and plotting the results on ternary diagrams. There are four basic types of fracture terminations (Figure 131a).

- *Abutting* (A)—the fracture ends in the middle of another fracture.
- *Blind* (B)—the fracture ends in the middle of the rock.
- *Crossing* (C)—the fracture crosses another fracture.
- *Diffuse* (D)—the fracture ends in a diffuse set of smaller fractures.

The number of each type of fracture termination can be counted in outcrop and plotted on a ternary diagram. Since there are four categories, and only three endmembers on a ternary diagram, two of the categories must be grouped. There are two different ways of plotting; the Barton and Hsieh (1989) method (Figure 131b) and the Laubach (1992) method (Figure 131c). In the Barton and Hsieh method, blind and diffuse terminations are lumped together. In the Laubach method, the abutting and crossing terminations are lumped together as *connected*. Exercise 27↓ demonstrates one way that ternary diagrams of fracture termination can be applied.

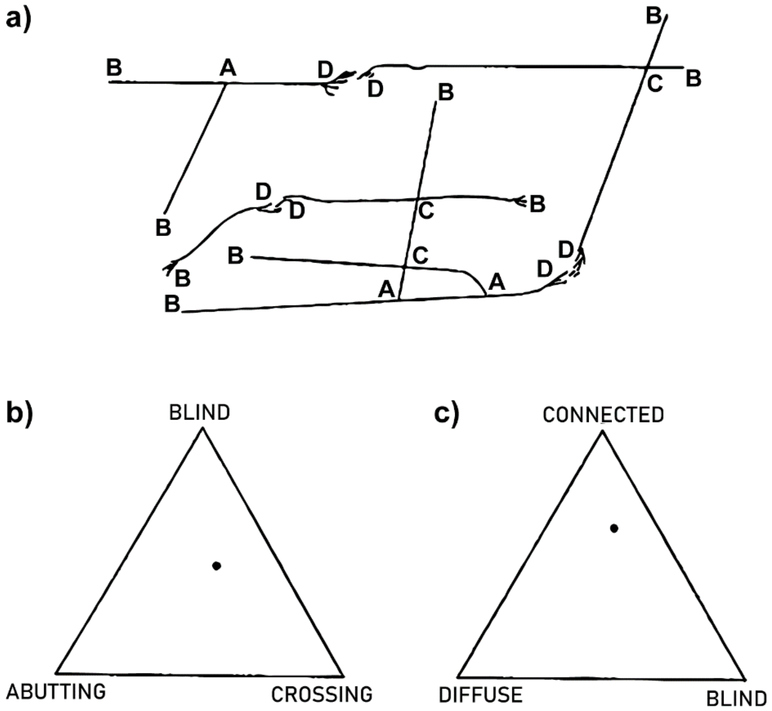


Figure 131 - Fracture terminations. a) Type of termination: A=abutting; B=blind; C=crossing; D=diffuse (from Laubach, 1992). Ternary diagrams for displaying the number of different fracture types: b) as proposed by Barton and Hsieh (1989); and c) as proposed by Laubach (1992).

16.4 Formation of Fractures

There are generally three modes of fracture loading, which involve different fracture surface displacements (Figure 132). The three modes are listed here.

- Mode 1: opening or tensile mode—the fracture faces are pulled apart (Figure 132a).
- Mode 2: sliding or in-plane shear—the fracture surfaces slide over each other (Figure 132b).
- Mode 3: tearing or anti-plane shear—the fracture surfaces move parallel to the leading edge of the fracture and relative to each other (Figure 132c).

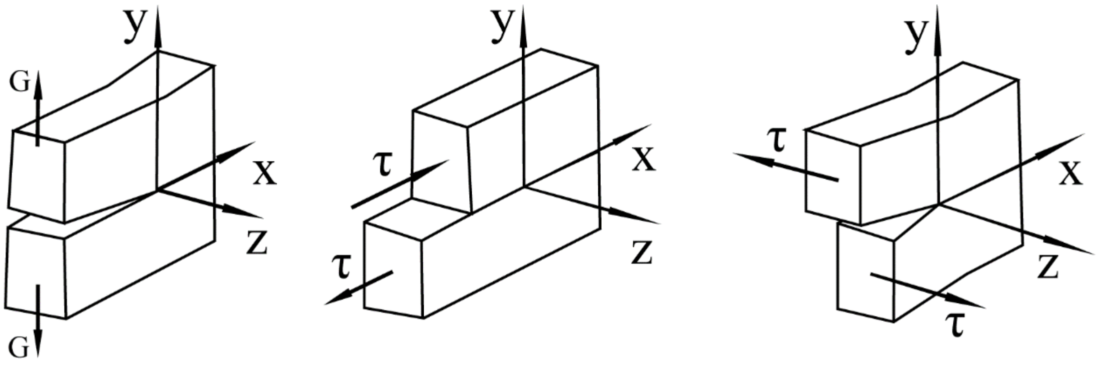


Figure 132 - Fracture modes based on the stress and the relative displacement (reproduced courtesy of John M. Sharp, University of Texas, Austin).

Different fracture modes will result in different fracture properties. Modes 2 and 3 have shear stresses that will affect the walls of the fracture. As a result, asperities will be sheared off and minerals may form on the surface, creating surface features called *slickensides*.

As a fracture forms and propagates through a rock, the stresses that are breaking the rock are distributed throughout the rock so that high compressive stresses occur just ahead of the tip of the fracture. Because of this, when two fractures propagate towards each other, the stresses of each will deflect the direction of propagation for the other fracture (Figure 133).

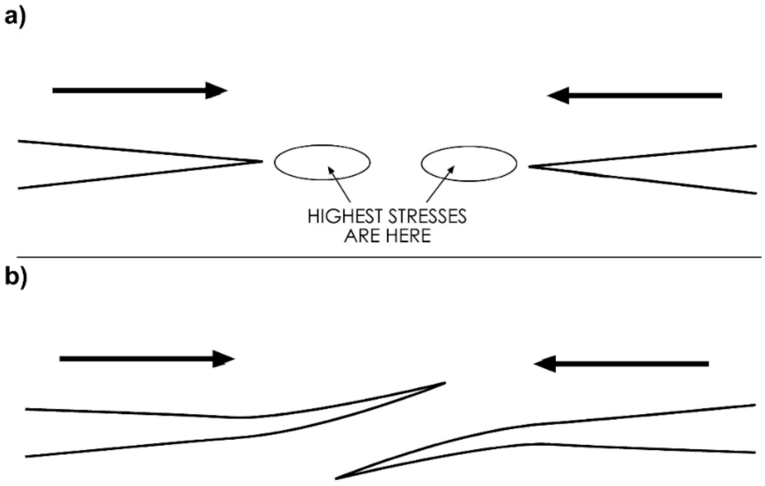


Figure 133 - a) As fractures propagate, high stresses occur just ahead of the fracture tip. b) When the fracture tips get close, the high stress region deflects the direction of propagation.

Stresses that create fractures include the following.

- Tectonic movements.
- Unloading and exfoliation—when a rock is buried, it is under compressive stress; when the overburden is released, the release of stress causes the rock to fracture.

- Cooling—cooling lava will often contract, creating joint sets that are usually vertical and often result in sets of polygonal columns called *columnar joints*.
- Gravity and freeze-thaw—can cause existing fractures to propagate further.

16.5 Hydraulic Properties of Fractures

16.5.1 Porosity

In most natural formations, the porosity associated with fractures is very low and is generally much lower than the matrix porosity, unless it is a purely fractured system with very little porosity in the matrix. Permeability, however, is a different story, as a fracture with even a small aperture can potentially move a lot of water.

16.5.2 Permeability

Discharge in a single idealized fracture with smooth walls and a constant aperture is governed by Equation (116), called the *cubic law*.

$$Q = \frac{\rho_w g b^3 i}{12\mu} \tag{116}$$

where (parameter dimensions are dark green font with mass as M, length as L, time as T):

Q = volumetric discharge per unit width of fracture plane (L^2T^{-1})

ρ_w = density of water (ML^{-3})

g = gravitational acceleration (LT^{-2})

b = aperture (distance between walls of fracture) (L)

i = hydraulic gradient (dh/dl) (:)

μ = viscosity of water ($ML^{-1}T^{-1}$)

The hydraulic conductivity of a fracture is expressed by Equation (117).

$$K = \frac{\rho_w g b^2}{12\mu} \tag{117}$$

This equation shows that hydraulic conductivity is proportional to the *square* of the aperture, and discharge is proportional to the *cube* of the aperture—hence the name *cubic law*.

This is the basis for a conceptual and mathematical understanding of fluid flow in fractures. In an idealized fracture, the hydraulic conductivity and discharge are a function of, and directly proportional to, the size of the aperture—that is, an increase of the aperture results in an increase in the hydraulic conductivity and an increase in the discharge, assuming the same gradient. In addition, the relationship between aperture and conductivity/discharge is *not* linear—a small increase in aperture results in a large increase in permeability and an even larger increase in discharge.

One can use the cubic law to calculate discharge and permeability of fractures. When that is done, it shows that a fracture with an aperture as small as 1 mm has permeability close to 1 m/s, which means that it can transmit a lot of water. Fractures therefore don't have to be very large to have a big influence on a flow system.

16.6 Modeling Fluid Flow in Fractures

There are several ways we can conceptualize fluid flow in fractures. We won't get into the mathematics associated with each, but it is important to conceptually understand the methods.

16.6.1 Parallel Plate Model

A purely fractured system can be modeled using the cubic law and the assumptions that 1) all fluid flow is in the fractures, and 2) the fractures can be represented as a series of parallel plates—in other words, the fractures are all smooth-walled and have constant apertures. The spacing and density of fractures in a volume of rock can be used to integrate the cubic law over the whole volume, so that a hydraulic conductivity tensor for the whole volume is calculated. There are also ways to factor in roughness and variations in aperture; usually these involve multiplying the cubic law by some factor that reduces the flow accordingly.

This method has some limitations. It has an implicit assumption that the fractures follow a constant spacing, or some spacing that varies in a predictable way. Also, it ignores flow in the matrix, which may or may not be a good assumption.

16.6.2 Discrete Fracture Model

If the exact locations, extents, orientations, and hydraulic properties of all the fractures in a system are known with some degree of confidence, the system can be simulated using a model that contains and calculates flow in each fracture. This is used frequently in the petroleum industry for reservoirs that have been extensively studied through drilling and geophysics such that locations of the major fractures are known with some certainty. The obvious limitation with this is that, to get a meaningful model, you need to have lots of data and high confidence in those data. For many groundwater problems, this is not within the budget, thus those data are not available.

16.6.3 Stochastic Models

Stochastic means random or statistical. In stochastic models, the probability of occurrence of a fracture is input to the model rather than actual orientations. The model generates many possible fracture configurations and calculates flow for all of them to provide a range of flow results. These can be used where data is sparse and are most effective when it can be shown that the fractures follow some sort of predictable distribution.

16.6.4 Double-Porosity Models

Some numerical models incorporate a parallel-plate or discrete fracture approach with a standard porous media model approach to simulate flow in a double-porosity system. A difficult part of these models is characterizing the relationship between the fractures and the matrix blocks—a mathematical function must be specified that controls the flow of water from fractures to the matrix and vice-versa. This function is most likely non-linear and represents a process that is not well understood.

16.6.5 Equivalent Porous Media

A simple way of modeling double-porosity systems is by ignoring the fractures and developing equivalent hydraulic parameters that characterize the aquifer as a single porous medium. The idea is that, even though the system is fractured, the model contains a representative elemental volume (REV) of the formation or aquifer. This approach is used frequently, most likely because modelers don't have enough data to meaningfully apply an alternative approach. This method works if the fractures are so numerous and well-connected that the whole aquifer acts as an equivalent porous medium, but it does not work well if a few large fractures are controlling the flow system. Sometimes a few of the large fractures will be represented by extremely high permeability porous zones.

16.6.6 Equivalent Porous Media with Fracture Domains

A more appropriate way of using the equivalent porous media approach is by establishing *fracture domains*, or areas in which the fractures have common characteristics—orientation, connectivity, and so on (Figure 134). Investigations (air photo, outcrop) can be used to establish regions with similar fracture characteristics. The parallel plate approach can be used to establish a hydraulic conductivity tensor for each region. These can then be input into a heterogeneous porous media model with variable anisotropy.

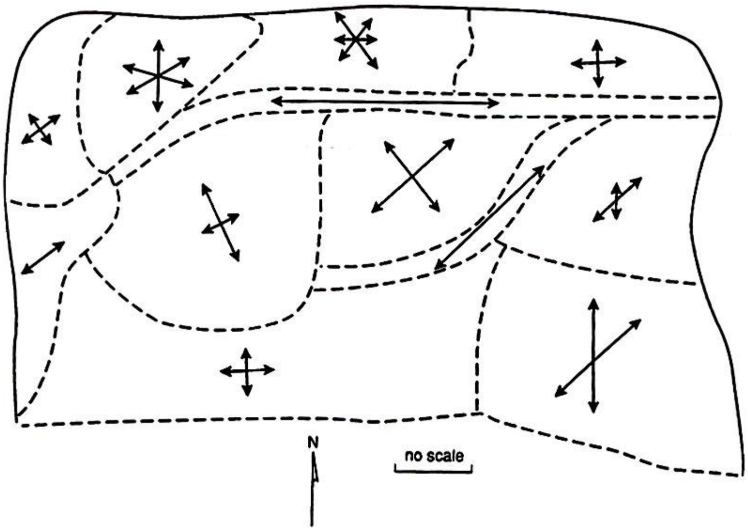


Figure 134 - Hypothetical fracture domains in map view with arrows representing hypothetical fracture set directions within the delineated zones (reproduced courtesy of John M. Sharp, University of Texas, Austin).

16.7 Influence of Fractures on Fluid Flow and Mass Transport

Fractures affect fluid flow in several ways. The obvious way is that open fractures provide a conduit for water to move in the subsurface. Due to their planar nature, fractures can impart a significant anisotropy in an aquifer, which in turn affects flow paths in the aquifer. The preferred flow paths created by fractures can significantly affect the paths that contamination takes in the subsurface, and the high permeability associated with those features can drastically affect travel time for contaminants. In addition, fracture skins can influence transport of contaminants by impeding diffusion into the matrix or even facilitating retardation processes—for example, sorption of contaminants in the fracture skins. As a result of these factors, it is not uncommon for teams of hydrogeologists to invest a great deal of time and money characterizing a contaminated aquifer and developing predictions about contaminant transport directions and travel times, only to observe contamination showing up very quickly in a completely unexpected location. The observation of contaminants in fractured aquifers moving much faster than expected and appearing in places we do not expect is so common that it is known as “Black’s Law” (Black, 1993).

It should also be noted that fractures can act as barriers to flow in an aquifer as well as conduits. Fractures often contain precipitates of crystalline minerals or *fault gouge—that is,* ground-up or broken rock and/or sediment—that can drastically reduce permeability. Faults can also juxtapose permeable beds against units with very low permeability, which in turn reduces the lateral extent of an aquifer.

17 Groundwater in Various Geologic Settings

17.1 Introduction

We’ve discussed various aspects of groundwater flow, ranging from the properties of porous media to the response of aquifers to a pumping well to the study of regional flow systems to the chemistry of groundwater systems. Most of our discussions dealt with idealized conceptual systems, like a perfectly confined aquifer, or a homogeneous layer of sand, or a set of evenly spaced fractures with a constant aperture. But the natural world is not ideal like those conceptualizations. Groundwater occurs in a variety of different geologic settings, from unconsolidated alluvium to karstified limestone to fractured granite, and each geologic setting has specific geologic and hydrogeologic characteristics. To study and understand hydrogeologic systems, one needs to first understand the geology. In this section, we will look at the occurrence of groundwater in a variety of geologic systems and examine the relationship between geology and the water that is flowing through it.

17.2 Basic Classification of Hydrogeologic Environments

Groundwater occurs everywhere in the subsurface, and almost any geologic setting can host an aquifer even if it only provides small volumes of water. However, there are certain geologic environments that tend to have sufficient porosity and permeability to support a substantial aquifer. Keep in mind that both parameters are necessary—a clay or vuggy basalt unit may have lots of porosity and hold a lot of water, but if the pores are too small or not connected, permeability will be low and well yields will not be adequate. Conversely, a set of fractures in a granite may have an extremely high permeability—because it only takes a small aperture to get a very permeable fracture—but if the matrix porosity is low, a well will quickly drain the fractures and will not yield much water over time.

To understand these systems, we classify five basic geologic settings—with variations within each setting—plus a few special cases.

17.3 Alluvium

Alluvium refers to sediments deposited by running water. These include gravel in a stream channel, sand deposited on a riverbank, and the mud that is deposited outside of the river channel during a flood. Alluvial systems are often excellent aquifers. The natural sorting of sediments by flowing water often results in continuous deposits of coarse sediments that have high permeability and porosity. There are three basic types of alluvial deposits that we will consider here: alluvial fans, deltas, and floodplain/river valley deposits.

17.3.1 Alluvial Fans

Sediment transport in a stream is controlled by the energy of the stream—highly energetic streams in mountainous areas can carry lots of large grains, while slower streams with flatter stream gradients carry finer sediments. Whenever the gradient—and therefore, the energy—of a stream suddenly decreases, much of the sediments carried by the stream drop out of the water and settle on the surface. This occurs when a stream flows out of a mountainous area on to a flat plain, leaving behind a large pile of sediment called an *alluvial fan* (Figure 135 and Figure 136).

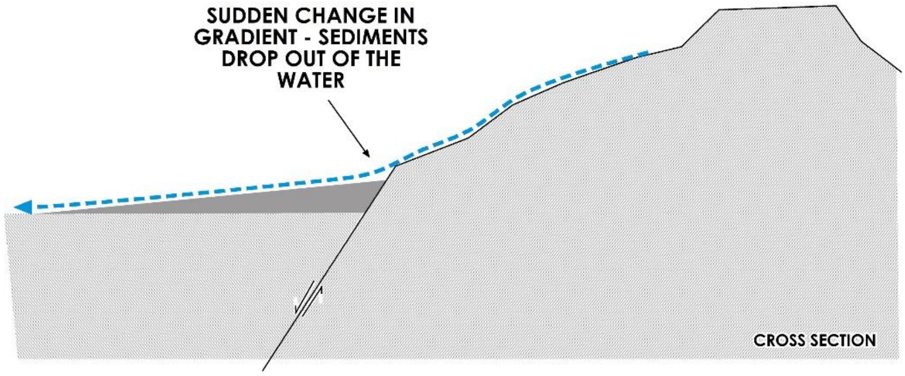


Figure 135 - Cross section of an alluvial fan (dark gray) developing at the foot of a fault-block mountain range where an abrupt change in the stream (dashed blue line) gradient causes rapid deposition of coarse sediment.

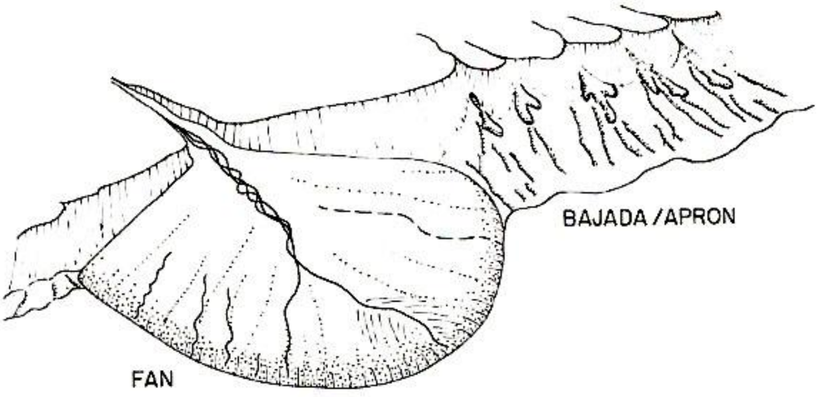


Figure 136 - Aerial view of a single alluvial fan adjacent to a bajada or alluvial apron (reproduced from Galloway & Hobday, 1996).

Streams flowing out onto alluvial fans tend to carry a wide variety of sediments that range from mud and silt to sand to coarse gravel. The changes in stream energy across the fan results in sorting of sediments so that coarser sediments are located near the *proximal* end—that is, near the mountain—and finer sediments are near the *distal* end. Because of this, the most productive wells will generally be found near the break in slope at the foot of the mountains. The sediments in an alluvial fan are generally unconsolidated, although in arid regions, extensive evaporation can cause mineral precipitation that partially cements the sediments. Also, evaporation can leave behind *caliche*, which is a layer of fairly hard calcium carbonate deposited in the soil zone that can inhibit infiltration of water into the sediments.

Sometimes a series of alluvial fans will coalesce into a large package of sediments called a *bajada*, or *alluvial apron* (Figure 136). In fault-block mountain areas like New Mexico and Nevada in the US, entire valleys can be filled with alluvial fan sediments to depths of as much as 1,000 meters.

Groundwater systems in alluvial fans are usually unconfined and receive much of their recharge from infiltration of precipitation and inflow of water from losing streams. Sometimes interbedded sands and clays result in locally confined units. Groundwater flow is usually controlled by the topography. Streams near the proximal end are usually losing flow to the aquifers, while the distal end of the fans often contain springs and gaining streams. These systems usually have very little fine sediments in them, because the finer sediments tend to get carried away by streams, so permeabilities are often high.

Alluvial-filled, fault-block valleys often contain significant regional-scale groundwater flow systems (Figure 137).

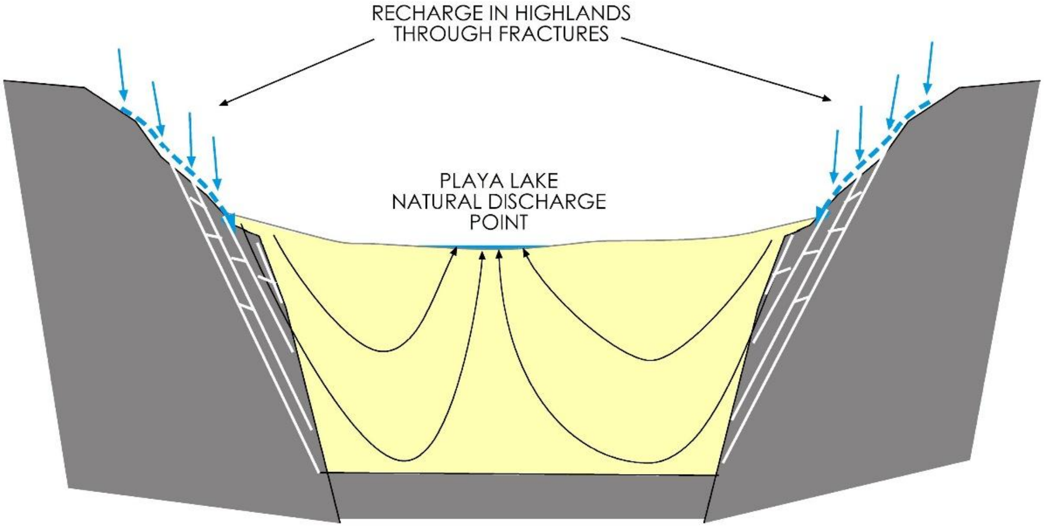


Figure 137 - Cross section of alluvial-filled, fault-block valley and groundwater flow system. Solid blue arrows represent recharge, dashed blue arrows represent streams that deposit and recharge alluvial fans, curved black arrows represent groundwater flow paths.

In arid regions, much of the alluvium includes a shallow layer of caliche that inhibits infiltration of precipitation. Also, high evapotranspiration rates cause most of the precipitation on the valley floor to be utilized before it reaches the water table. Usually, recharge in these arid systems is primarily through fractures in the bedrock exposed in the highlands, which then convey water to the unconsolidated alluvium. The groundwater system flows toward the middle, with eventual discharge to shallow lakes in the center of the valley called *playa lakes*. In some places, groundwater discharges directly into the lake, and then flows out of the valley on the surface. In many arid and semi-arid regions, the lakes are usually dry, and the primary mode of discharge is through evaporation directly from the water table below the playa, which results in highly saline groundwater just below the playa and extensive deposits of salt at the surface called *salt flats*.

Alluvial fans are often important sources of water in arid and semi-arid regions because the lack of thick vegetation in semi-arid climates tends to result in much greater erosion and sedimentation. As a result, alluvial fans tend to be much larger in area and thicker. Less vegetation also results in less evapotranspiration, which in turn can increase infiltration and recharge. The lack of surface water in these areas generally means that groundwater is more crucial.

17.3.2 Deltas

Deltas are in many ways like alluvial fans, but they form where a stream enters a larger body of water, such as a lake, inland sea, gulf, or ocean. As the stream enters the larger body of water, water velocity—and related fluid energy—abruptly decreases drastically, which results in the stream depositing much of its sediment load. A major difference between alluvial fans and deltas is that deltas contain much more fine-grained material. This is because, in alluvial fans, the water energy of the stream leaving the depositional area of the fan is still sufficient to carry away most of the fine-grained material, while the typical delta has low fluid energy with coarse-grained sediments deposited only in localized areas.

Deltas can be classified based on their shape, which is directly related to the forces that control deposition and ultimately geometry. Deltas are classified as *fluvial-dominated, wave-dominated,* or *tide-dominated,* or usually some combination of these.

Fluvial-, or river-, dominated deltas (Figure 138) are deltas whose shape is controlled by the input of sediment. These form in locations where rivers flow into relatively low energy environments, typically an inland sea, or gulf. These are often called *birds-foot* deltas because the streams tend to form relatively narrow linear piles of sediment that extend out into the bay or ocean that are similar to the shape of a bird's foot. The Mississippi River delta on the Gulf Coast of Louisiana in the USA is an example of a fluvially-dominated delta.

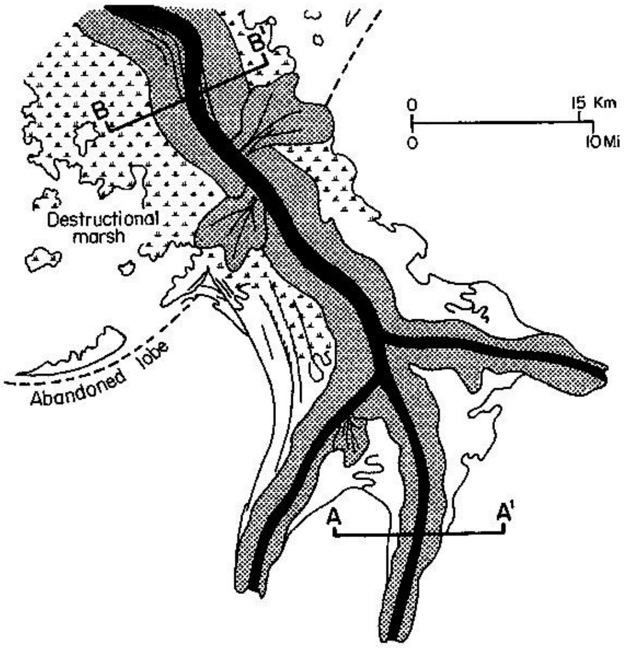


Figure 138 - Map showing a hypothetical example of a fluvial-dominated delta (reproduced from Galloway & Hobday, 1996).

Wave-dominated deltas (Figure 139) form where rivers drain into bodies of water with relatively high wave action. The shape of the delta is affected by the waves so that the delta front is smoothed out. Wave action also results in a lot of re-working of the sediments in the delta front, which in turn results in removal of fine sediment and sorting of the sand fractions. An example of a wave-dominated delta is the Nile River delta in Egypt.

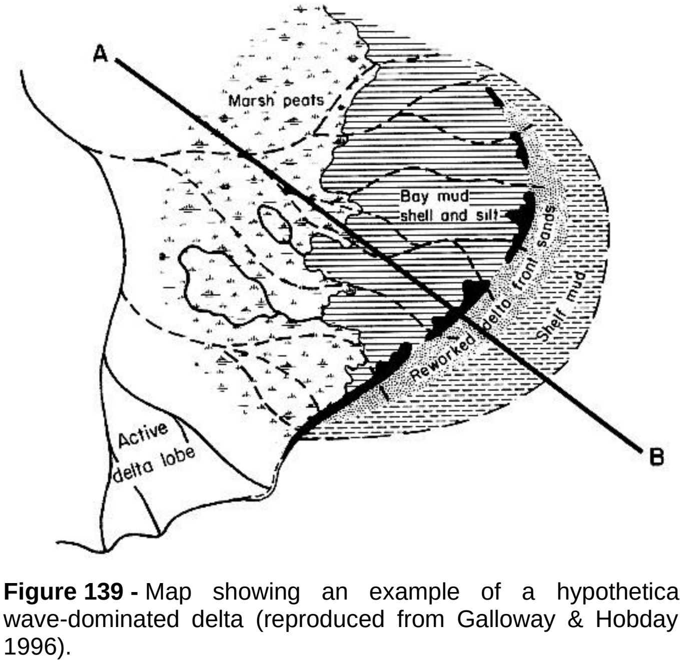


Figure 139 - Map showing an example of a hypothetical wave-dominated delta (reproduced from Galloway & Hobday, 1996).

Tide-dominated deltas (Figure 140) are like wave-dominated deltas in that tidal fluctuations are extensive reworking the sediments and the shape of the delta front. In contrast to wave-dominated deltas, in tide-dominated deltas; the energy is usually perpendicular to the wave front. Whereas wave action creates a longshore drift that moves sediment parallel to the front, the daily movement of tides perpendicular to the coastline reworks the sediments to create lobes of sediment perpendicular to the delta front. An example of a tide-dominated delta is along the coast of Bangladesh where numerous river systems drain into the Bay of Bengal.

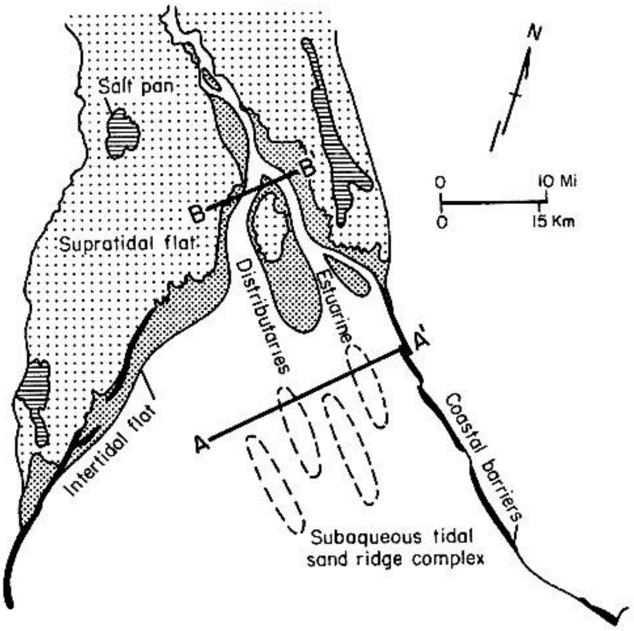


Figure 140 - Map showing an example of a hypothetical tide-dominated delta (reproduced from Galloway & Hobday, 1996).

In active fluvial-dominated deltas, coarse-grained deposits are generally in abandoned stream channels. These channels are usually surrounded by mud and are isolated from each other. Therefore, the success of wells drilled into delta sediments depends upon penetrating a channel deposit of sufficient size. In wave or tide-dominated deltas, the delta fronts will be very sandy and will contain lots of water, but that water is often non-potable sea water.

In general, active deltas are usually not very good sources of potable water. Permeable units are too few and far between, and the groundwater is usually too salty to drink or contains too much organic material from swamps and tidal flat areas. Exceptions do occur; the Nile River Delta is generally very permeable, and the groundwater is clean enough to drink. However, high concentrations of manganese and iron causes steel well screens in the Nile Delta to clog with rust and manganese oxides; usually these wells only last a few years before they must be replaced.

17.3.3 Floodplain/river Valley Deposits

The area around rivers can contain very productive aquifers, and these areas have historically been extensively used as a source of water for irrigation and drinking. River valleys usually contain unconsolidated, high-permeability, coarse-grained, channel deposits interspersed with finer grained floodplain deposits. They are usually shallow and easy to drill. They are connected to surface streams, have a fairly constant source of recharge, and usually have fairly good water quality. In larger rivers, these deposits can be up to 50 meters thick with a significant saturated thickness.

In addition to current river valley deposits, floodplain and channel deposits from ancient rivers can often be found in glaciated areas or in deserts. In the Sahara Desert of Africa, for example, ground-penetrating radar has identified an extensive system of river valleys, buried beneath the sand, that contain vast groundwater reserves. Another example is the Ogallala aquifer in the High Plains of the US, which is an extensive fluvial system that formed as the Rocky Mountains were uplifted in the west. This network of streams left behind extensive deposits of sands and silt that make up one of the largest and most prolific aquifers in the world.

17.4 Glacial Deposits

During the most recent ice age—that is, the period of multiple glacial cycles that occurred between about 2.6 million and 12 thousand years ago—much of the North American and Eurasian continents were covered by glaciers. Expansion of the glaciers ripped up a considerable amount of rock and sediment, which was then carried over the continents. After the ice retreated, the glaciers left behind extensive deposits of glacial sediments. These deposits have a large range of grain sizes, from mud and silt to boulders the size of a building, which result in deposits that have a broad range of permeability. Modern glacial sediments can also be found worldwide in high mountainous areas. We group all glacial deposits—called drift—into two classes—unstratified and stratified, or layered.

Unstratified drift is called *diamicton* or *till*. Diamicton is a general term describing any large deposit of geological material that contains clasts of many different sizes that are not well sorted. This includes glacial deposits as well as mass wasting deposits—for example, mudflows, avalanches. Till is a specific diamicton of known glacial origin. It consists of mixed sediments containing sediment grains ranging from boulders to very fine clays. Till may be hard and compacted by the weight of glaciers that flowed over it. It is generally poorly sorted, although tills deposited upon or beside glaciers may be graded in glacial outwash channels.

Most well yields in till are very small. The poor sorting leads to low porosity and permeability. Productive wells must tap sandy layers or fractures in the till or else be in a very sandy till.

Stratified drift is comprised of glacial sediments that are deposited in layers. In contrast to till, these units can be very productive. Stratified glacial drift is generally formed by one of the processes as listed here.

- **Outwash deposits** have characteristics like alluvial systems because they are both formed by running water. Outwash can be found in vast *aprons*, or plains that spread out in front of a glacier. These are formed when melt water from the glacier runs out from beneath the ice and carries sediments from the glacier. Localized outwash deposits are found in *pitted* plains and as ice-contact stratified drift. These tend to be smaller deposits than outwash plains and may be more poorly sorted. Outwash can also be intercalated—*sandwiched* or interbedded—between low permeability tills.
- **Productive river valley (floodplain) type deposits** are common in deglaciated areas. Some may be buried by glacial drift (0 to 150 m thick), while some form and are exposed at the surface in valleys downstream from glacier termini.
- **Eolian deposits** are sediments that were transported by wind. We identify two basic types of eolian deposits: dunes and loess. Sand dunes, if saturated, are excellent aquifers—they usually have well-sorted fine sand sized sediment grains with little silt and clay. Loess is silt-sized grains that are deposited in extensive layers and are generally not good aquifers, as they tend to have low porosities and permeabilities.
- **Lacustrine deposits** are also common in deglaciated areas. These form from small lakes that dry out, leaving behind deposits of very fine-grained material. The deposits are thinly layered, very low-permeability silts and clays that do not form productive aquifers. These can, however, form non-continuous low permeability units that can support perched aquifers in overlying sandy units.

17.5 Clastic Sedimentary Rocks—Sandstone/Shale

Like unconsolidated sediments, the porosity and permeability of sandstones is affected by sorting. grain size, and cementation. Well-cemented sandstones can have very low permeabilities and hydraulic properties that are more like granite than sandstone. As a rule, older sedimentary rocks tend to be more cemented than younger, although this is not always the case.

Other factors control the suitability of sandstones as aquifers. The depositional system determines the grain size, grain distribution, and overall geometry of the deposits. A beach or wave-dominated delta complex will usually result in continuous layers of well-sorted sand, while a fluvial or submarine channel system will tend to have the sandy parts in channel deposits that may not be well-connected. Some eolian sandstones can cover vast areas—like the Nubian Sandstone in Africa—and contain very productive aquifers.

Cross bedding, clay drapes, and other sedimentary structures can create anisotropies that may influence and control flow directions.

Structures related to depositional and tectonic stresses can also affect flow in sandstones. Growth faults associated with prograding delta sediments can create linear trends of high-sand (i.e., sands containing dense minerals) deposits that are not well connected to each other. Folds and faults from tectonic stresses can create anisotropies and affect the overall geometry of the aquifer. Fractures can play a major role in transmitting precipitation from the surface to the aquifer, which facilitates recharge. Fractures can greatly increase permeability in the subsurface; however, they can create barriers to flow when filled-in with post-diagenetic minerals or where they juxtapose permeable sedimentary rocks against low-permeability units.

Shales are generally very poor aquifers, and usually act as confining zones when interbedded with sandstones. If the shales are fractured and sufficiently thick, however, they can produce enough water to supply low-yield water wells. In some places, like the Clare Valley of South Australia, the fractured shales are the most permeable units in the area and are the predominant source of water.

17.6 Chemical Sedimentary Rocks—Carbonates and Evaporites

Limestones, dolomites, and, to a lesser extent, evaporites like gypsum and halite, are widespread over the surface of the Earth. These rocks have an extremely wide range of hydrogeologic characteristics and are responsible for both some of the most productive aquifers in the world as well as extensive low permeability units with very low water quantity and quality.

Limestones can have a wide range of primary porosities and permeabilities. Limestones created from lime muds usually have low permeability and low porosity, while limestones created by the accumulation of invertebrate shells or in reef environments have high primary porosity and permeability. Since calcite—one of the primary carbonate minerals—is fairly soluble, these rocks can easily become well-cemented, and some limestones have very little open pore space.

The high solubility of limestones means that they are also subject to extensive *karstification*. Karstification refers to the formation of caves and other voids associated with dissolution of soluble minerals. Karst development usually starts in fractures and tends to occur at the water table where CO_2 outgasses from the system and groundwater becomes acidic by mixing with the CO_2 to form carbonic acid. The acidic groundwater flows through fractures, small pores, and other voids, dissolving the soluble carbonate minerals and enlarging the interconnected pores to create open spaces with very high permeabilities. The resultant cavities may be filled with reddish silt, re-precipitated calcite, or they may remain open for fluid flow. Extensive karstification results in extremely transmissive and productive aquifers as discussed in *Introduction to Karst Aquifers*↗ by Kuniansky and others

(2022) and *Karst: Environment and Management of Aquifers*↗ by Stevanović and others (2024), both are available as free downloads from The Groundwater Project. Some well-known and extensively studied karst aquifers include the Floridian aquifer and *The Edwards Aquifer*↗ as presented by Sharp and Green (2022); their book is available as a free download from The Groundwater Project. These aquifers respond quickly to recharge from storms, and flow rates are very high. They are also extremely susceptible to contamination.

Evaporite units rarely form aquifers. They tend to have low primary porosity and are so ductile that fractures don't stay open for long. Some aquifers occur in gypsum—for example, the Blaine aquifer of Texas and southwest Oklahoma, US—but these aquifers typically yield low-quality water and are limited in extent.

17.7 "Hard" Rocks

Granites, basalts, and metamorphic rocks are usually not very good aquifers. These rocks tend to have very low primary porosities, and in the ones that do have significant porosity—for example, vuggy basalts—the porosity is often not well connected. There are exceptions to this; some granites will have a small amount of porosity along the grain boundaries that are significant in a large enough thickness of the material. However, in general, productivity of these aquifers is almost always dependent on fractures and on weathered zones at the base of the overlying soils.

Fractures in igneous rocks can come from cooling stresses, from exfoliation and release of overburden stress, as well as from tectonic stresses. The key to understanding these systems is determining the distribution of the fractures. In regions dominated by regional tectonic processes, the fractures will often follow distinct orientations based on the structural history of the area. Cooling joints will tend to be vertical and fairly evenly spaced, although variations in spacing do occur.

Volcanic deposits can be stratified like sedimentary deposits, and the various layers will have different properties. Often basalt flows are interbedded with ash-flow tuffs and other ash deposits. These will have different hydraulic properties and will respond to fracturing differently, and as a result cooling joints in the more brittle basalts will generally not propagate into the tuffs.

In some active volcanic areas, the basalt bedrock can have extremely high permeability. The basalt bedrock of the Hawai'ian islands in the US was created by flows of low-viscosity lavas that tend to leave behind lava tubes and large open spaces. These lava tubes create conduits that produce very high permeability. The aquifers of Hawai'i, however, tend to contain mostly seawater with only a thin layer of fresh water in the uppermost part of the saturated zone. The Columbia River Basalts in the northwestern USA are interbedded with sediments that accumulated between lava flows resulting in very productive zones near the top of each of many basalt layers. Wells in the area produce high-quality water at rates up to 5000 gpm (~19,000 liters per minute) as documented by Whitehead (1994).

More information about aquifer systems in general and an overview of the largest and most significant aquifer systems can be found in *Large Aquifer Systems Around the World*↗ by van der Gun (2022), which is available as a free download from The Groundwater Project.

18 Wrap-up

This book is not intended to represent a comprehensive treatment of hydrogeology. Rather, it contains a selection of key concepts and topics designed to support a university-level course in groundwater hydrology for students planning to either pursue full-time employment in hydrogeology or additional graduate-level study in groundwater science. It is assumed that students taking a course appropriate for this book have had at least a one class in basic Earth Science, some introductory physics and chemistry, and mathematics that at least include basic geometry and algebra.

The concepts in this book are therefore presented in an order and within an overall context that is intended to facilitate teaching a hydrogeology course. The book is arranged in two parts. In the first part, the fundamentals of groundwater science are presented in a context that initially focuses on the energy driving the movement of groundwater, then the properties of the media through which it flows, and finally presents a quantitative overview focusing on increasing scales of measurement ranging from a small volume of aquifer to a zone around a water well to the entire aquifer. The second part of the book presents a set of supporting hydrogeology topics related to water chemistry, parts of the hydrologic cycle that are directly connected to the groundwater systems, stress and strain in groundwater systems, the influence of fractures and discontinuities in groundwater systems, and an overview of various geologic settings which typically host aquifers. The key concepts presented in each section are summarized below.

18.1 Part 1: Fundamentals

Section 1 contains an overview of the hydrologic cycle and definitions of some of the terms used in the book, including the term aquifer (a subsurface formation that is capable of yielding economically viable volumes of water to a well). This section also discusses systems and natural cycles and helps set the stage for how groundwater systems are studied and understood.

Section 2 discusses the distribution of energy in the subsurface and how variations in fluid energy drive the movement of water. The key concepts presented in Section 2 are as follows.

- All the components of fluid energy at a point in a groundwater flow system can be collectively quantified by a length – more specifically, a water level elevation (for a water of uniform density) above a datum – called head.
- Head varies from point to point within an aquifer system. The variation in head with respect to distance is called the hydraulic gradient.
- Fluid flow in the subsurface is driven by the hydraulic gradient; that is, fluid moves from areas of higher fluid energy (or head) to areas of lower energy.

- Hydraulic gradient exists in three dimensions and does not necessarily mimic or reflect the slope or orientation of the ground surface – in other words, groundwater does not necessarily flow "down hill".

Section 3 describes the properties of the media (i.e., rock and sediments) through which groundwater flows, including quantitative definitions of the basic parameters that affect the occurrence and movement of fluid in groundwater systems. The key concepts in this section are as follows.

- Geologic materials contain some fraction of empty (i.e., pore) space. The fraction of pore space by volume is called the porosity.
- The flow of a fluid in a porous media – either as a velocity in one dimension or a volumetric rate through a cross-sectional area of aquifer – is linearly proportional to the hydraulic gradient. The constant of proportionality is called permeability.
- Permeability is a function of both the properties of the media (the sizes and shapes of pores as well as the degree of pore connectivity) and properties of the fluid (density and viscosity).
- If the fluid is assumed to be water, then the properties of water are incorporated into the permeability term and it is called hydraulic conductivity.

Section 4 brings together concepts from Sections 2 and 3 to develop a quantitative framework for understanding groundwater systems based on a few general equations. The key concepts are as follows.

- Groundwater flow is governed by an empirically derived equation called Darcy's law, which states that the volumetric discharge of groundwater though a cross-sectional area of aquifer is linearly proportional to the hydraulic gradient with the hydraulic conductivity representing the constant of proportionality.
- For three-dimensional aquifers, groundwater flow is governed by a three-dimensional second-order partial differential equation that describes the divergence of flux and transient changes in storage at each point in a groundwater flow system.
- The divergence of flux is controlled by Darcy's law, thus it is dependent upon hydraulic gradient and hydraulic conductivity.

Section 5 builds on concepts presented in Section 4 to provide a quantitative overview of the parameters that control storage of water within an aquifer. The key concepts are as follows.

- Storage is only relevant to transient conditions when water levels change.
- Storage parameters are quantitative values that relate the change in volume of water in the subsurface to the change in water level and fluid energy. The overall storage parameter for an aquifer is the storativity.

- Confined aquifers contain an additional component of fluid pressure that causes the water level in a well to rise above the top of the aquifer.
- For confined aquifers, removal of water from the aquifer is associated with changes in fluid pressure that in turn cause change in pore volume. The storage parameter that describes elastic changes in the aquifer is the specific storage, and storativity is the product of the specific storage and the aquifer thickness.
- Unconfined aquifers do not have an additional component of fluid pressure; therefore, water level in a well rises to roughly the same elevation as the water table.
- For unconfined aquifers, removal of water is associated with drainage of pores in the aquifer. The storativity of an unconfined aquifer is thus controlled by the effective porosity of the aquifer.

Section 6 continues to build on the quantitative treatment of groundwater flow systems by introducing the difference between steady-state and transient conditions and presenting basic equations of groundwater flow and methods for solving them. The key concepts are as follows.

- Steady state means that the flux of groundwater into a given volume of a groundwater system is equal to flux out of that volume. For this condition, storage does not change, and storage parameters are not relevant to evaluation of the system.
- In transient conditions the flux into a volume of the system is not equal to flux out of the volume. For this condition, storage changes with time and storage parameters are relevant to evaluation of the system.

The concepts covered in the first six sections deal with general hydrogeologic situations. Section 7 then introduces a particularly important situation in hydrogeology; specifically, the flow of subsurface fluids to a well. Key concepts are as follows.

- A well is a hole constructed for the purpose of accessing fluid in the subsurface.
- Pumping fluid from, or injecting fluid into, a well results in a radial flow field around the well with flow lines either converging on or diverging from the well.
- Changes in head in a confined aquifer due to pumping from, or injecting into, a well are governed by the Theis equation.
- The Theis equation can be used to make predictions about how heads will vary in an aquifer in response to pumping or injection.

Section 8 builds on Section 7 by presenting the fundamentals of aquifer testing. Key concepts are as follows.

- The Theis equation, and variations of it, can be used to estimate aquifer parameters by pumping a well at a known rate and solving for the values of permeability and storage needed to produce the observed changes in water levels around the well.

- Aquifer responses to pumping occur in predictable ways that can be used to identify aquifer boundaries and leaky confining units.
- Aquifer testing can also be used to evaluate the future performance of the well and to estimate the sustainability long-term pumping.

Sections 9 and 10 expand on the scale of consideration to include study of groundwater flow systems at local, intermediate, and regional scale. Key concepts are as follows.

- Groundwater flow systems have physical boundaries that affect groundwater flow at various scales.
- Groundwater flow systems are dynamic systems that have areas and mechanisms that allow water to enter the system (i.e., recharge) and natural areas and mechanisms that allow water to leave the system (i.e., discharge).
- Variation of aquifer parameters influence the rate and direction of groundwater flow in aquifers.
- Aquifer studies can be used to develop a conceptual hydrogeological model, which is a summary of the relevant aspects of an aquifer. The conceptual hydrogeological model can be the basis for a quantitative computer-based model.
- Quantitative computer-based models are commonly used to make predictions about aquifer behavior (e.g., changes in water levels or reductions in spring flows in response to pumping from wells). These models are also useful for developing a better understanding of aquifers and for evaluating uncertainty in associated with aquifer characterization.

18.2 Part 2: Related Topics

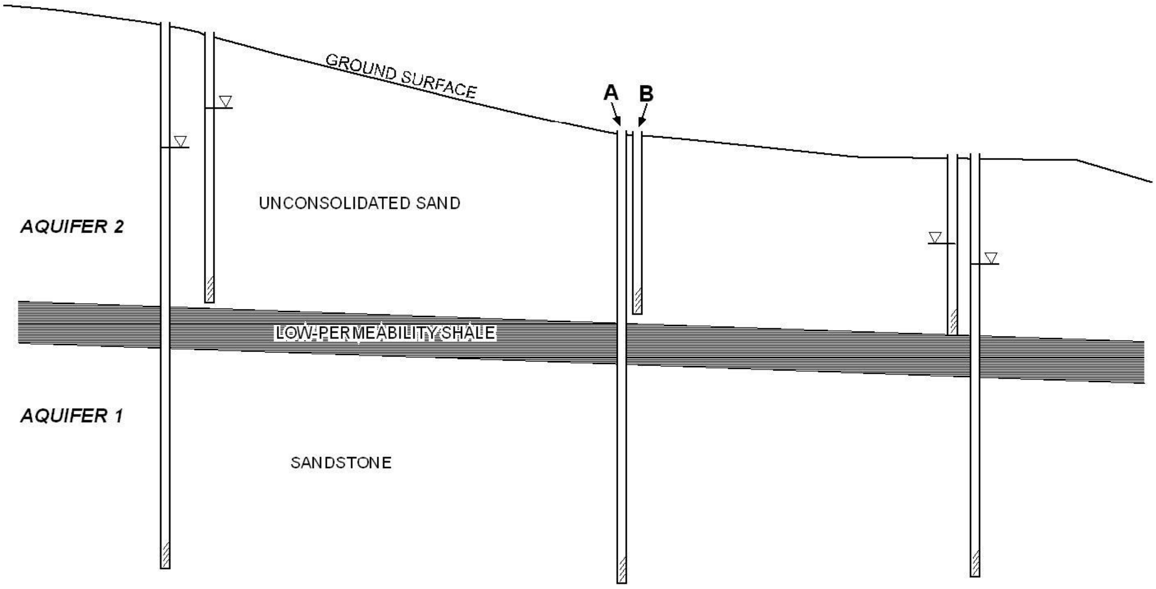
Sections 11 through 17 address associated topics that are of interest to practicing hydrogeologists. Section 11 covers the basics of groundwater chemistry and focuses on the chemical relationship between subsurface fluids and the minerals that make up the formations through which the fluids flow. Section 12 introduces the fate and transport of contaminants in groundwater. Sections 13 and 14 deal with parts of the hydrologic cycle that are "adjacent" to the groundwater systems discussed in the main part of the book. More specifically, Section 13 deals with water flowing on the Earth's surface and includes a basic introduction to open-channel flow and drainage networks. Section 14 introduces concepts associated with the unsaturated part of the subsurface between the ground surface and the top of the saturated zone. Section 15 addresses stress and strain in the subsurface and discusses the difference between fluid pressure and lithostatic pressure associated with the overlying column of rock or sediment at given depth in the subsurface. Section 16 discusses groundwater flow in fractures and discrete channels to illustrate that many groundwater systems do not fit the simple models of fluid flow discussed in previous sections. Finally, Section 17 discusses specific geologic environments that are favorable to the development of aquifers and groundwater flow systems.

Readers are encouraged to review the other free books provided by the Groundwater Project↗ for more specific information on various aspects of groundwater science. Appropriate links to specific books are provided throughout this book. Also, the author of this book is developing a companion practicum book with additional problem sets and a semester-long laboratory curriculum to support students with additional practical problems and exercises.

19 Exercises

Exercise 1

The following image is a cross section showing several wells completed at various levels. The two aquifers are separated by a confining layer; some wells are completed in the upper aquifer (aquifer 1) and some are completed in the lower aquifer (aquifer 2). The symbol with the upside-down triangle next to a well indicates the water level in that well. The water levels in wells A and B have not yet been measured.



Based on the water levels shown in the diagram, answer the following questions:

a) Which aquifer is confined?

b) In which well would you expect to see a higher water level: well A or well B?

c) Use the upside-down triangle symbol to indicate where you would predict the water levels are located in wells A and B.

d) Which direction is groundwater flowing in each aquifer? Draw two arrows on the cross section to indicate the direction of flow in each aquifer.

Solution for Exercise 1⤵

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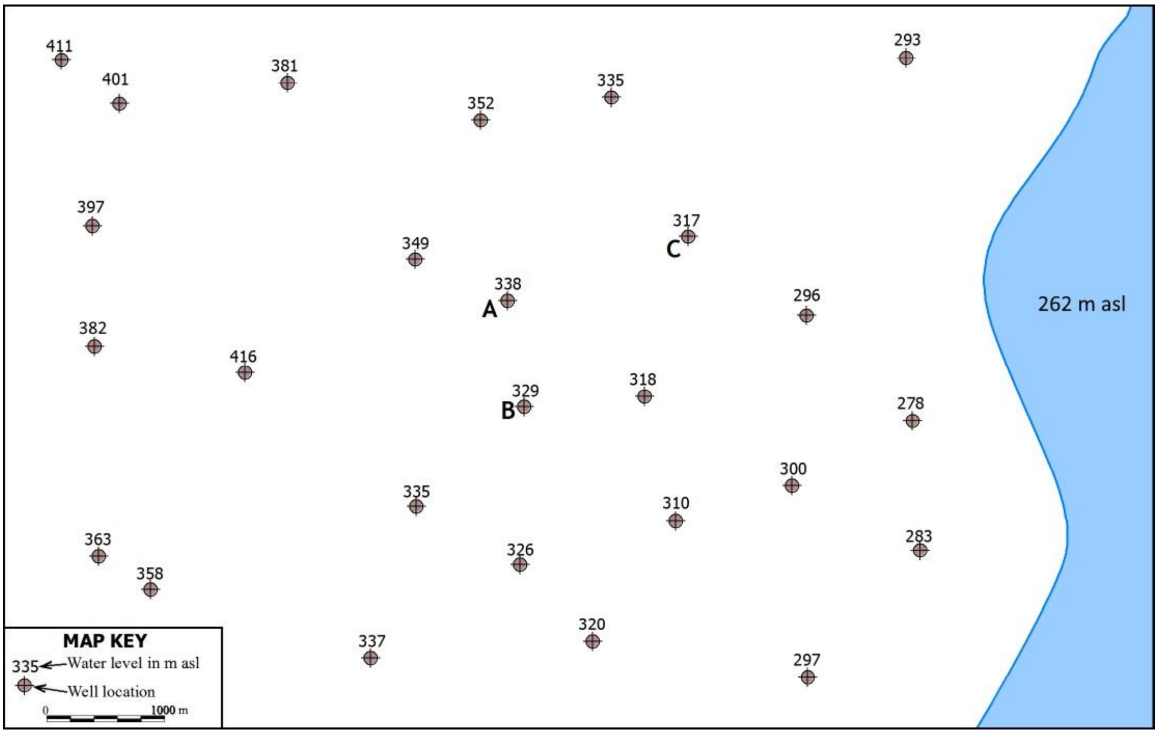
Exercise 2

It's your first day at your new job as a hydrogeologist and you are given the following map.



Given the data shown on the map:

a) Calculate the gradient between wells A and B and wells A and C.

b) Contour the water levels in the wells. Use 20-meter intervals starting at 280 m ASL. Do any values not fit the overall pattern? If so, identify and state why you excluded them (e.g., water level too low).

c) Using the contours, calculate the hydraulic gradient in the vicinity of the wells labeled A, B, and C. How does this gradient compare to the gradients you measured in part a)?

d) Give three possible reasons explaining the anomalous water level identified in part b). For each reason, describe what you would need to do or need to know to identify that reason as the culprit.

Solution for Exercise 2⤵

Return to where text linked to Exercise 2⤴

Exercise 3

Use the vertical and horizontal scales in the lower left corner of Figure 9 to calculate the vertical hydraulic gradient between wells a and b and the horizontal hydraulic gradient between wells b and c.

Be sure to indicate the direction of the gradient and express your final answer in the correct units.

Solution for Exercise 3↓

Return to where text linked to Exercise 3↑

Exercise 4

Part 1: A completely saturated sample of sandstone has a bulk density of 2.61 g/cm^3, an effective porosity of 10 percent, and a total porosity of 12 percent.

Determine the density of the solid phase (i.e., the density of the mineral).

Part 2: One cubic meter of quartz sand has a dry mass of 2,320 kg.

a) Calculate the porosity of the sand.
b) Calculate the mass (in kg) of the sand for the following degrees of saturation: 25 percent, 75 percent, 100 percent.

Part 3: A sample of quartz sand has a dry mass of 753.7 kg and a volume of 0.36 m^3.

a) Calculate the porosity of the sand.
b) Calculate the mass (in kg) of the sand assuming a degree of saturation equal to 62 percent.

Solution for Exercise 4↓

Return to where text linked to Exercise 4↑

Exercise 5

a) Use the hydraulic gradients determined for Figure 9 in Exercise 3 to calculate an average linear velocity (in m/s and in m/d) for groundwater in the aquifer between wells b and c. Assume that the hydraulic conductivity is 0.050 m/s and the effective porosity is 0.15.
b) A volume of an unknown contaminant is poured into well b and allowed to flow away from the well in the aquifer. Assuming that the contaminant began flowing away from well b just after it was poured into the well, use the average linear velocity calculated in part (a) to calculate the average time (in days) required for the contaminant to reach well c.

Solution for Exercise 5↓

Return to where text linked to Exercise 5↑

Exercise 6

Apply the procedure described in Section 4.3 to Darcy's law with intrinsic permeability rather than hydraulic conductivity; refer to the following Equation.

$$Q = k\frac{\rho_{fluid}g}{\mu_{fluid}}A\frac{dh}{dl}$$

where (parameter dimensions are dark green font with mass as M, length as L, time as T):

Q = volumetric discharge (L^3T^{-1})

k = intrinsic permeability (L^2)

ρ_{fluid} = density of the fluid (ML^{-3})

g = gravitational acceleration (LT^{-2})

A = cross-sectional area perpendicular to flow (L^2)

dh/dl = hydraulic gradient (:)

μ_{fluid} = viscosity of the fluid ($ML^{-1}T^{-1}$)

Solution for Exercise 6⤵

Return to where text linked to Exercise 6⤴

Exercise 7

Print out the page containing Figure 33 and draw a flow net in the dam according to the procedure described in Section 6.3.3. Calculate the total volumetric discharge per unit width of the dam assuming a hydraulic conductivity of 0.2 meters per day.

Solution for Exercise 7⤵

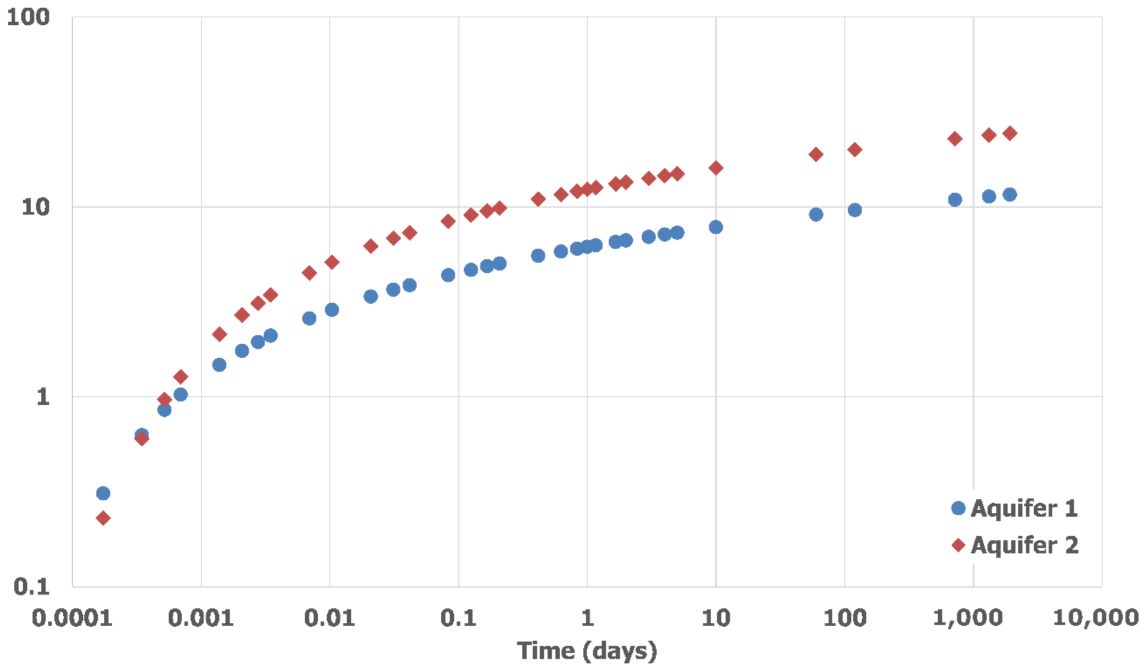
Return to where text linked to Exercise 7⤴

Exercise 8

Part 1: Two wells are completed in two different confined aquifers. All things are equal (e.g., same saturated thickness, storage parameters, well diameter, screened intervals, well efficiency, and so on) except that the transmissivity of aquifer A is 1.2 times greater than the transmissivity in aquifer B (i.e., $T_a = 1.2T_b$). If each well is pumped at the same rate for the same amount of time, which well will experience the greater drawdowns? Circle one of the answers:

a) The well in aquifer A will have greater drawdown.
b) The well in aquifer B will have greater drawdown.

Part 2: The following graph shows aquifer test results—specifically, time-drawdown data from observation wells—from two different aquifer tests in two different confined aquifers. For each test, the pump rate is the same and the distance to the observation well from the pumping well is the same. Assume that each aquifer has the same storativity.



Which aquifer has the greater transmissivity? Circle one:

a) Aquifer 1 has a greater transmissivity.
b) Aquifer 2 has a greater transmissivity.

Solution for Exercise 8⤵

Return to where text linked to Exercise 8⤴

Exercise 9

An observation well located 75 m from a pumping well has a static water level of 175 m ASL. The pumping well is pumped for 36 hours at a rate of 545 m^3/d. Calculate the drawdown and final head in the observation well at the end of the 36-hour pumping period. Assume the following:

$K = 0.61$ m/d (7.06×10^{-6} m/s)

$b = 75$ m

$S = 0.0004$

Solution for Exercise 9⤵

Return to where text linked to Exercise 9⤴

Exercise 10

Use the curve match example from Figure 46 to calculate T (in m^2/s) and S. Assume the data were taken from an observation well located 150 m from the pumping well and the well was pumped continuously at 300 m^3/d throughout the test. Also assume the time on the graph is in minutes and the drawdown on the graph is in meters. It is important to take care to use consistent units.

Solution for Exercise 10⤵

Return to where text linked to Exercise 10⤴

Exercise 11

This image shows time-drawdown data from a multi-well aquifer test that involved one pumping well and one observation well. The data are from the observation well located 20 meters from the pumping well. The pumping well was pumped at 100 m^3/d for 72 hours.

Assume the aquifer is a fully confined sandstone with a constant saturated thickness of 20 m.

a) Manually fit the data to the Theis type curve provided below it.

b) Then, for $t = 1$ and $s = 1$, select match points for $1/u$ and $W(u)$ and calculate T, S, and K for the test.

c) Provide the values of t, s, 1/u, W(u) for the selected match point and show the calculations to determine T, S, and K.

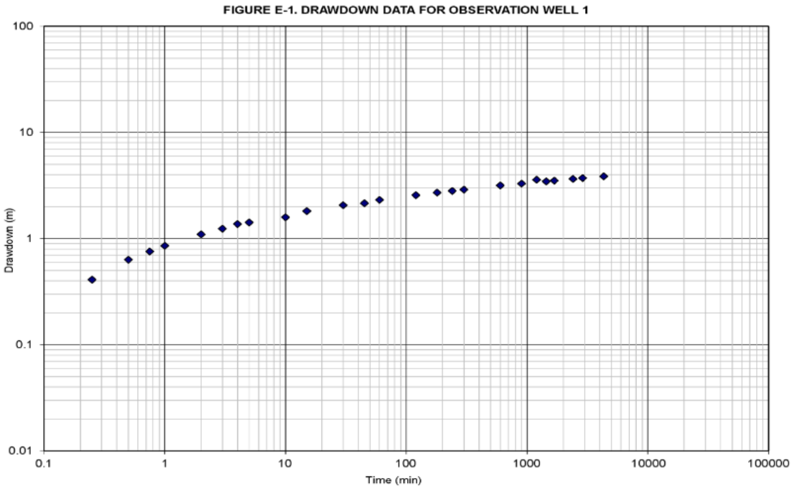
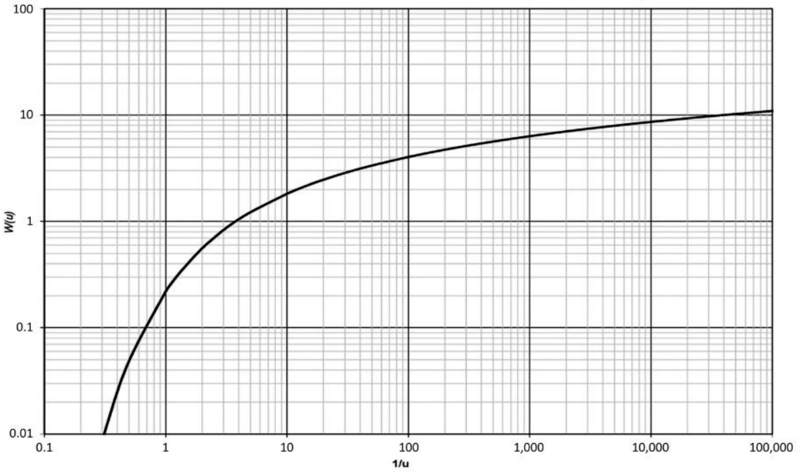


FIGURE E-1. DRAWDOWN DATA FOR OBSERVATION WELL 1

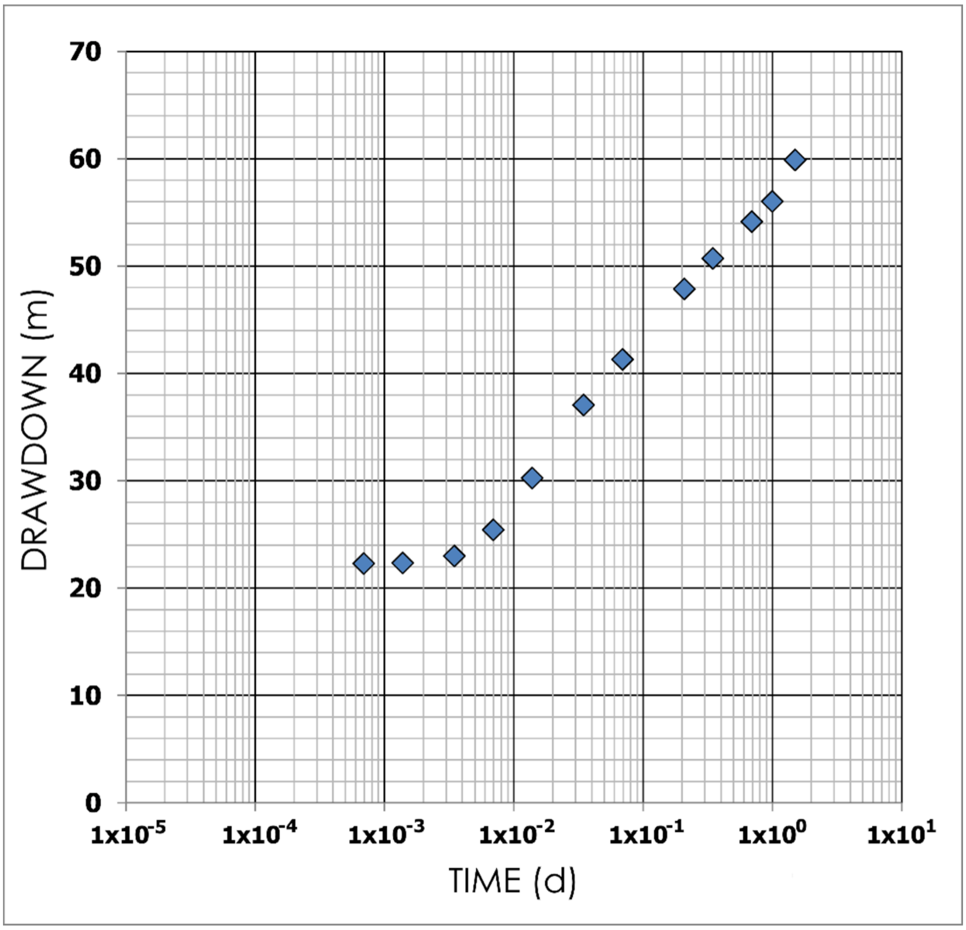


Solution for Exercise 11↴

Return to where text linked to Exercise 11↥

Exercise 12

A well is pumped at 300 m³/d for 36 hours. Drawdowns are measured in an observation well located 2 meters from the pumping well. The following image shows the drawdown data versus time on a semi-log plot. Use the Cooper–Jacob simplification of the Theis equation to determine T and S for the formation.



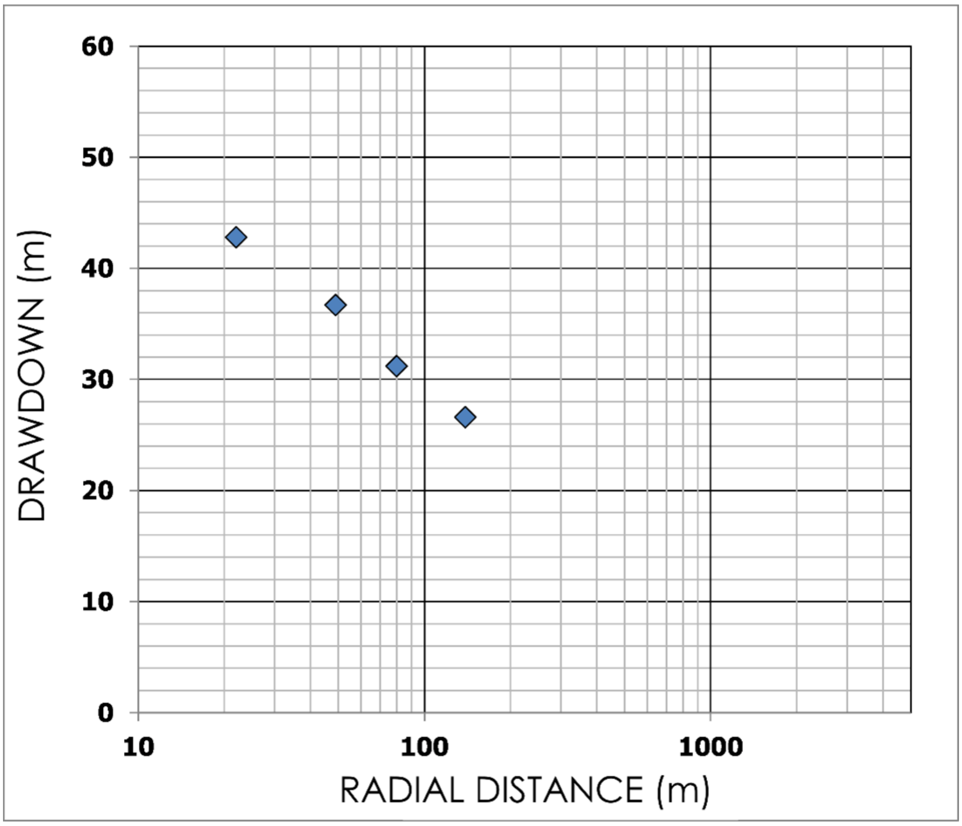
Click for solution to Exercise 12⤵

Return to where text linked to Exercise 12⤴

Exercise 13

A well is pumped at 200 m^3/d for 24 hours. Drawdowns are measured in four observation wells located at distances ranging from about 20 to about 140 meters from the pumping well. The table shows the distance to each well and drawdown in that well 24 hours after pumping began. The graph shows the drawdown data on a linear scale as a function of distance on a semi-log scale. Use the Cooper–Jacob simplification of the Theis equation to determine T and S for the formation.

Distance from PW m	Drawdown m
22	42.8
49	36.7
80	31.2
139	26.6



Solution for Exercise 13⤵

Return to where text linked to Exercise 13⤴

Exercise 14

Part 1: A pumping well has a known efficiency of 90 percent at a pump rate of 500 m^3/d. Would the measured drawdown in the well be greater than the expected drawdown or less than the expected drawdown?

Part 2: At a pumping rate of 600 m^3/d, would you expect the efficiency to be higher—that is, greater than 90 percent—or lower—that is, less than 90 percent? Justify your answer.

Part 3: A pumping well is installed in a confined aquifer with a known transmissivity (0.0003 m^2/s) and storativity (0.0004). The diameter of the borehole used to install the well is 0.30 m.

a) Using the radius of the borehole (0.15 m) as r, calculate the drawdown in the well after 24 hours of pumping at 500 m^3/d.

b) The well is pumped for 24 hours, and the measured drawdown in the well is 17 meters. Calculate the well efficiency for that pumping rate and determine the amount of well loss.

Part 4: An aquifer test is performed in a pumping well with no observation wells (i.e., a single-well aquifer test). The resulting transmissivity is 1,000 ft^2/d. If we assume that the well has an efficiency of 75 percent, which of the following three statements is the most appropriate? Circle one and justify your answer.

a) The test provides a good estimate of the actual formation transmissivity.

b) The test results overestimate the actual formation transmissivity.

c) The test results underestimate the formation transmissivity.

Solution for Exercise 14↴

Return to where text linked to Exercise 14↥

Exercise 15

A newly constructed well has a static water level of 645 m ASL (above sea level). The well is pumped for 24 hours at a rate of 500 m^3/d. At the end of the pumping period, the water level in the well is 619 m ASL.

a) What is the drawdown in the well after 24 hours? Show your calculations.

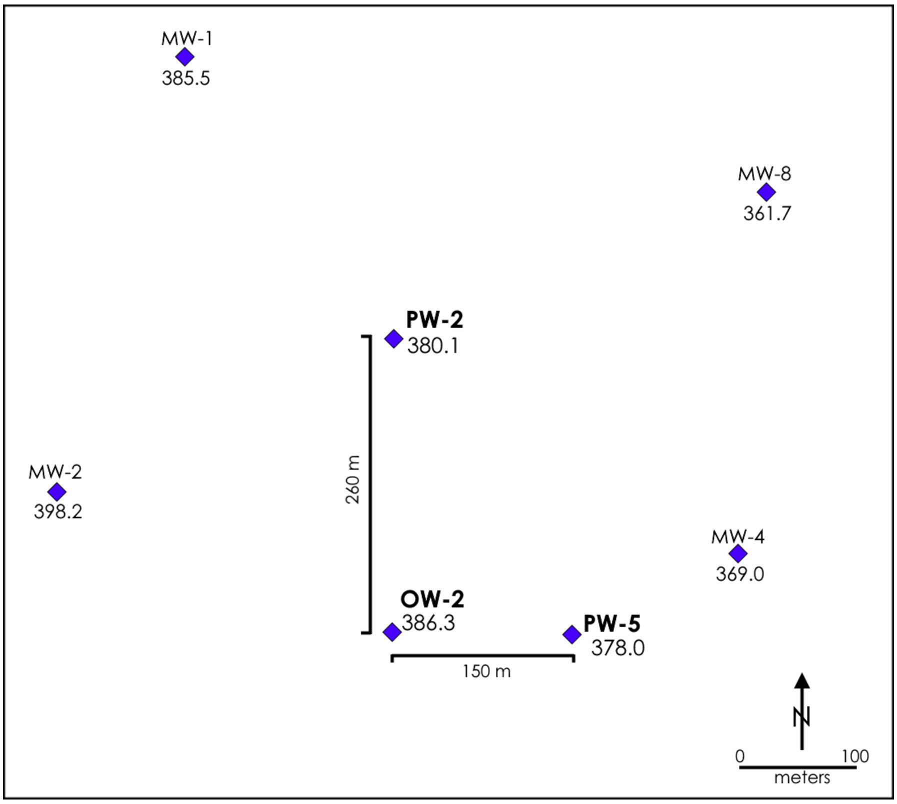
b) What is the specific capacity of the well? Show your calculations.

Solution for Exercise 15↴

Return to where text linked to Exercise 15↥

Exercise 16

The image shown here is a map of the locations of several wells. The number next to each well is a measured hydraulic head in that well (in m ASL). Wells PW-2 and PW-5 are pumping wells. As shown on the map, observation well OW-2 is 150 meters from pumping well PW-5 and 260 meters from PW-2.



Assume the following:

- All wells are completed into the same confined aquifer.
- The elevation of the top of the aquifer is 55 m ASL.
- The elevation of the base of the aquifer is 25 m ASL.
- $K = 7.6 \text{ x } 10^{-6}$ m/s
- $S_s = 0.000001/\text{m}$

Using the information provided:

a) Calculate the drawdown and hydraulic head in OW-2 assuming that PW-2 is pumped at a constant rate of 500 m^3/d for 20 years.

b) During the final 10 years of the pumping period for PW-2, well PW-5 is also pumped at a constant rate of 200 m^3/d. Calculate the drawdown in OW-2 associated with pumping PW-5 for 10 years, then determine the final hydraulic head in OW-2 assuming 20 years of pumping at 500 m^3/d in PW-2 and 10 years at 200 m^3/d in PW-5.

Solution for Exercise 16↴

Return to where text linked to Exercise 16↥

Exercise 17

Calculate the drawdown at the observation well in Figure 55 and Figure 57 given the following assumptions:

- the system behaves in a nearly linear manner even after pumping, i.e., drawdown is small relative to aquifer thickness
- $r_{boundary} = 250$ m
- $r_{pump\ well\ to\ observation\ well} = 200$ m
- $Q = 1{,}000$ m^3/d
- $K = 2.0\text{x}10^{-5}$ m/s
- $b = 20$ m
- $S = 0.0001$
- $t = 10$ years

[Solution for Exercise 17](#)↴

[Return to where text linked to Exercise 17](#)↥

Exercise 18

Before answering the questions at the end of this exercise, a sequence of instructions are provided to expedite your use of the TopoDrive tool. Create a representation of a topographically driven flow system by using the online TopoDrive educational tool↗ (Hsieh, 2001) to observe what controls local, intermediate, and regional flow systems; and to track groundwater flowlines and particles through the system you create. After connecting to that link and reading the brief introduction to the tool, click on 'Go To Interactive Tool'. The menu on the black bar at the top of the page has a 'How to Use' tab that provides a concise quick-start guide.

Launch the Topodrive software, then choose start and enter a basin length of 5000 m and a vertical exaggeration of 1 and select OK. Click on 'Water Table' in the menu and when directed to draw a water table, click OK, then click high up on the left side of the left axis of the drawing panel, then move slightly down and to the right and click again, continue adding points as you move to the right, some lower and some higher, until reaching the right side of the drawing area, then double click on the last point. Select "Mesh" from the menu and select 50 rows and 50 columns, then click on properties and choose the green option of 1e-4 m/s and 20 percent porosity, click OK and use the mouse to draw a shape around the entire shape that you just created, double click on the last point and the shape will turn green to indicate those properties. You can make patterns of many different hydraulic conductivity and porosity values and you can change the value in the table where you select the properties, but for now just create a homogeneous basin.

Next, click the menu item 'Head" and select 50 contours. The head contours will be drawn. The head at the top is determined by where you created the surface and the other boundaries are impermeable so the software calculates the contours from that information. Then select "flow" from the menu and leave the default of flow path tracking and both forward and backward, then click OK. Now click anywhere on the image and the associated flow line will be drawn – click many times to get a feel for the flow field. Next click 'Animation' on the menu and chose 100 days of travel per 1 second of animation with a smoothness of 10 frames per second, then click OK, then click on the image and the flow line you just drew will be animated. You can also return to 'Flow' and instead select 'Particle Movement' with and initial particle spacing of 50 m. After clicking OK, use the mouse to draw a rectangle in the upper left corner of your image, double clicking on the final point and a group of dots will appear spaced 50 m apart. Now, select "Animation' and keep the previous settings of 100 and 10, then click OK followed by a click on the image to watch the particles move.

Did the system you created exhibit local, intermediate, and regional flow systems? If not, how might you change the water table you drew to create three system levels? If so, how might you change the water table you drew to eliminate the local and intermediate systems? Try redrawing the water table to create the conditions that your original drawing did not produce. Were you correct about what you needed to change?

Return to the 'Properties' menu item and add substantial heterogeneity. How does that change the flow patterns?

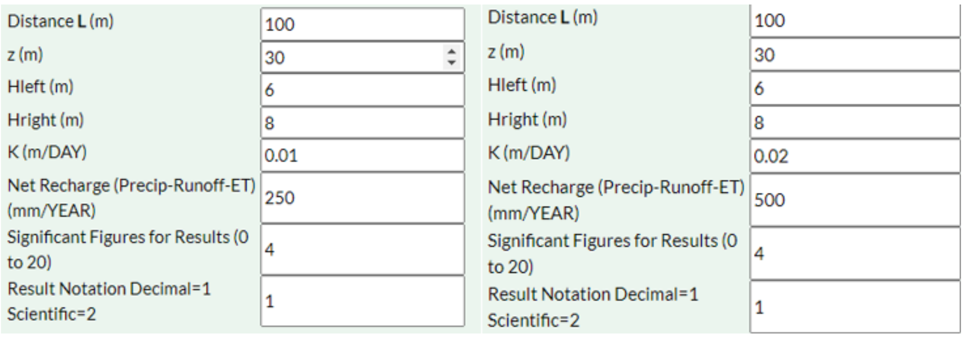
Solution for Exercise 18↴

Return to where text linked to Exercise 18↥

Exercise 19

Groundwater model non-uniqueness is the phenomenon in which different sets of model parameters produce the same groundwater system conditions. It is common practice to use groundwater models to determine system properties by matching model results to field observations such as water levels and groundwater discharge. This process is called calibrating a model. After a match is obtained, the model is typically used to make predictions. However, non-uniqueness can cause the calibration to produce parameters that are not representative of the system, resulting in inaccurate model predictions.

Use the Groundwater Project interactive educational tool called WTR↗ that simulates flow in a land mass between two water bodies with recharge at the surface. This was described briefly in Section 6.3.2. Use the two input data sets shown below which are identical except the second data set has a value of hydraulic conductivity and net recharge that are both twice the values in the first data set.



Distance **L** (m)	100
z (m)	30
Hleft (m)	6
Hright (m)	8
K (m/DAY)	0.01
Net Recharge (Precip-Runoff-ET) (mm/YEAR)	250
Significant Figures for Results (0 to 20)	4
Result Notation Decimal=1 Scientific=2	1

Distance **L** (m)	100
z (m)	30
Hleft (m)	6
Hright (m)	8
K (m/DAY)	0.02
Net Recharge (Precip-Runoff-ET) (mm/YEAR)	500
Significant Figures for Results (0 to 20)	4
Result Notation Decimal=1 Scientific=2	1

a) How many differences can you see in the output results?
b) How do the water levels differ?
c) How do the flow rates differ?
d) If a hydrogeologist had only head measurements for the system and adjusted parameter inputs until the calculated heads matched the measured heads, would they obtain accurate values of hydraulic conductivity and recharge?
e) If not, what could be measured to provide accurate estimates?
f) Would you trust predictions of a model that matches the water levels, but does not consider the flow balance? Why, or why not?

Solution for Exercise 19⤵
Return to where text linked to Exercise 19⤴

Exercise 20

The following table contains the results of an analysis of a water sample with the concentrations of various ions (column A) given in milligrams per liter (mg/L) in column B.

a) Use the periodic table at https://ptable.com/#Properties↗ to determine the atomic weight of each ion and enter the atomic weights in column C.
b) Use the atomic weight of each ion to convert the concentration from mg/L to molarity (mmol/L) and enter the answers in column D.
c) Enter the valence charge for each ion in column E.
d) Use the valence charge and values from column D to calculate the concentration in milliequivalents per liter (meq/L)

A	B	C	D	E	F
Solute	**Measured Concentration (mg/L)**	**Atomic weight(g)**	**Molarity (mmol/L)**	**Valence (charge)**	**meq/L**
Ca^{2+}	92				
Mg^{2+}	34				
Na^{+}	8.2				
K^{+}	1.4				
Fe(III)	0.1				
HCO_3^-	325				
SO_4^{2-}	84				
Cl^-	9.6				
NO_3^-	13				

Solution for Exercise 20↴
Return to where text linked to Exercise 20⮥

Exercise 21

Use the data and calculated values in Exercise 20 to calculate a charge balance for that sample. State if the charge balance is acceptable or not.

Solution for Exercise 21↴
Return to where text linked to Exercise 21⮥

Exercise 22

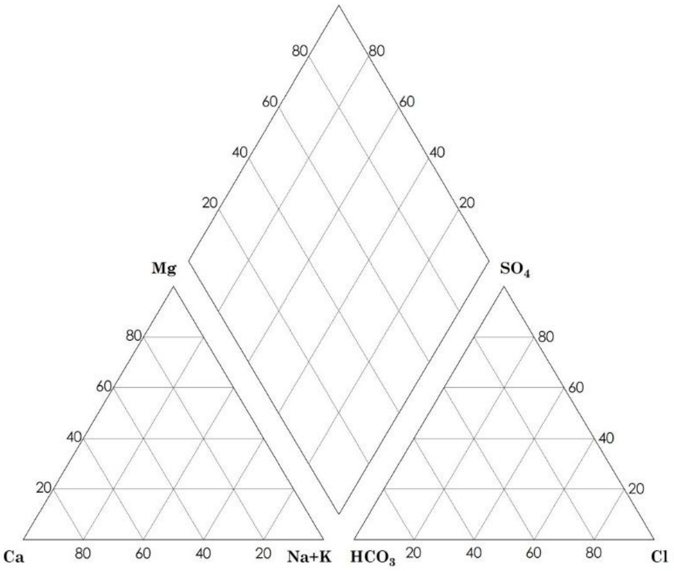
Calculate molarity and milliequivalents, then determine the charge balance for the following sample. The atomic weight and valance charge for each ion are provided in the table. State if the charge balance is acceptable or not. If the charge balance is not acceptable, state if the cations or the anions are in excess of the balance.

A	B	C	D	E	F
Solute	**Measured Concentration (mg/L)**	**Atomic weight(g)**	**Molarity (mmol/L)**	**Valence (charge)**	**meq/L**
Ca^{2+}	75	40.08		2	
Mg^{2+}	25	24.31		2	
Na^{+}	5.6	23		1	
K^{+}	2.1	39.1		1	
Fe(III)	0.2	55.8		3	
HCO_3^-	109	61		1	
SO_4^{2-}	242	96		2	
Cl^-	9.3	35.5		1	
NO_3^-	15	62		1	

Solution for Exercise 22↴
Return to where text linked to Exercise 22↥

Exercise 23

Plot the data provided and evaluated in Exercise 20 and 21 on a Piper Diagram and determine the water type.



Solution for Exercise 23↓
Return to where text linked to Exercise 23↑

Exercise 24

Suppose that a source of contaminant is introduced to the up-gradient end of a column filled with sand in a hydrogeology laboratory. If the source is maintained at a continuous concentration of 1000 mg/L, the hydraulic conductivity is 0.1 cm/sec, effective porosity is 0.2, the dispersivity in the x direction is 5 cm, dispersivity in the y direction is 1/5 of the dispersivity in the x direction, dispersivity in the z direction is 1/10 of the dispersivity in the x direction, and the head drop across the 100 cm long cylinder is 10 cm.

What will the concentration be at:

(a) 25 cm after 1000sec?
(b) 50 cm after 1000sec?
(c) 75 cm after 1000sec?

d) Do the answers you obtained make sense? Why, or why not?

e) Would you expect measurements of concentrations in plumes in a field setting to produce the values you calculated? Why, or why not?

Solution for Exercise 24↓
Return to where text linked to Exercise 24↑

Exercise 25

Suppose a mass of 1000 mg is introduce in the center of a large tank of sand in a hydrogeology laboratory with uniform, one-dimensional, flow moving through the tank. The hydraulic conductivity is 0.1 cm/sec, effective porosity is 0.2, the dispersivity in the x direction is 5 cm, dispersivity in the y direction is 1/5 of the dispersivity in the x direction, dispersivity in the z direction is 1/10 of the dispersivity in the x direction, and the head drop across the 100 cm long tank is 10 cm.

What will the concentration be at:

(a) 25 cm after 1000sec?

(b) the center of mass of the plume after 1000sec?

(c) a point that is 10 cm to the left and 5 cm below the center of mass of the plume after 1000sec?

d) Why is the concentration at 25 cm from the source so different for Exercise 24 part (a) and this exercise part (a)?

e) Would you expect measurements of concentrations in plumes in a field setting to produce the values you calculated? Why, or why not?

Solution for Exercise 25↴
Return to where text linked to Exercise 25↥

Exercise 26

a) How large is the capture zone of a well that pumps 0.005 m^3/s in a 20-meter-thick aquifer with a hydraulic conductivity of 5.5×10^{-3} m/s, and a gradient of 2.5×10^{-4} directed due east?

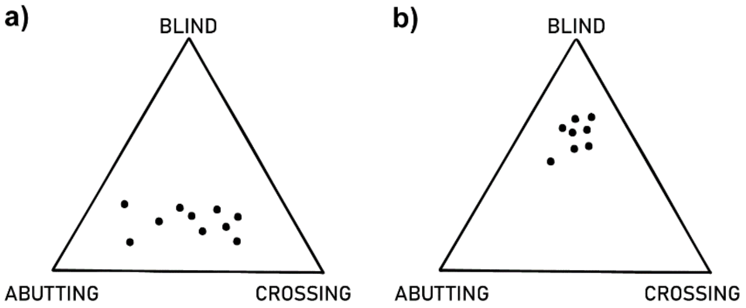
b) How far down gradient it the stagnation point?

c) If a contaminant spill occurs 200 meters northwest of the well, how much does the pumping rate need to be reduced to avoid drawing in contaminated water?

Solution for Exercise 26↴
Return to where text linked to Exercise 26↥

Exercise 27

This figure shows ternary diagrams from two different fractured aquifers. Each dot represents an outcrop; for each aquifer about 9 outcrops were studied. Based on these results, and assuming that fracture permeability is much higher than the matrix permeability of the rock, in which aquifer would you expect to see the higher permeability?



Solution for Exercise 27⤵
Return to where text linked to Exercise 27⤴

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21 Boxes

Box 1 - Derivation of the Radial Flow Equation

In Section 8, we stated the two-dimensional groundwater flow equation with respect to Cartesian coordinate system x, y is as shown in Equation Box (1-1).

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = \frac{S}{T}\frac{\partial h}{\partial t} \qquad \text{Equation Box (1-1)}$$

where (parameter dimensions are dark green font with mass as M, length as L, time as T):

h = hydraulic head (L)

S = storativity (:)

T = transmissivity (L^2T^{-1})

t = time (T)

Flow to a well is radial; in other words, flow lines are converging on the well from all sides (Figure Box 1-1).

Because the flow lines converge, the equipotential lines that define the cone of depression around the well get closer together near the well. In this case, we cannot use the Cartesian form of the equation, and therefore need to express the equation in terms of a polar coordinate system.

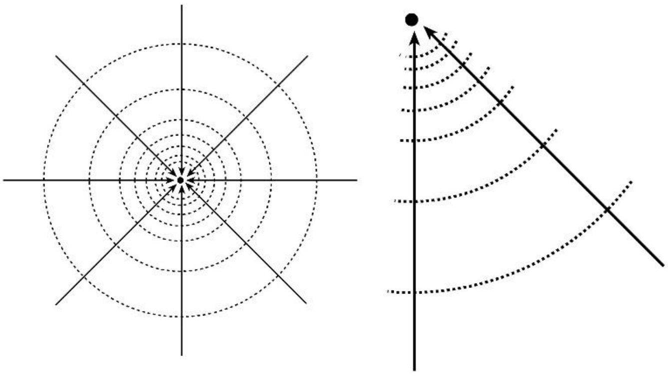


Figure Box 1-1 - Radial flow to a well.

In a polar coordinate system, a point is defined by its location with respect to a fixed central pole (Figure Box 1-2). The location of a point is defined by its radial distance from the pole (r) and the angle (θ) between the radial line and some fixed polar axis (e.g., north).

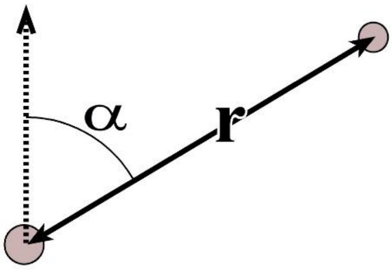


Figure Box 1-2 - Polar coordinate system.

The derivation of the transformation of the equation in Cartesian coordinates to polar coordinates follows below.

The Pythagorean theorem as shown by Equation Box (1-2).

$$r = (x^2 + y^2)^{1/2} \qquad \text{Equation Box (1-2)}$$

where (parameter dimensions are dark green font with mass as M, length as L, time as T):

r = radial distance from the central pole (L)

x, y = Cartesian coordinates (L)

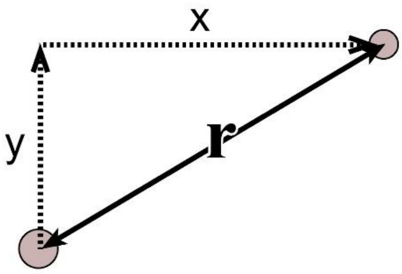


Figure Box 1-3 - Definition of terms in Equation Box 1-2.

If we assume that $h(x, y) = h(r)$, then

$$\frac{\partial h}{\partial x} = \frac{\partial h}{\partial r}\frac{\partial r}{\partial x} \qquad \text{Equation Box (1-3)}$$

and

$$\frac{\partial h}{\partial y} = \frac{\partial h}{\partial r}\frac{\partial r}{\partial y} \qquad \text{Equation Box (1-4)}$$

The second partial of h with respect to x is

$$\frac{\partial}{\partial x}\left(\frac{\partial h}{\partial x}\right) = \frac{\partial}{\partial x}\left[\frac{\partial h}{\partial r}\frac{\partial r}{\partial x}\right] \qquad \text{Equation Box (1-5)}$$

Using the product rule, we can remove the $\partial h/\partial r$ and take the second derivative of r with respect to x to get $\left(\frac{\partial h}{\partial r}\right)\left(\frac{\partial^2 r}{dx^2}\right)$. This product rule also gives us $\left(\frac{\partial r}{\partial x}\right)$ $\left(\frac{\partial}{\partial x}\right)$ of $\left(\frac{\partial h}{\partial r}\right)$, or

$$\frac{\partial}{\partial x}\left(\frac{\partial h}{\partial x}\right)=\frac{\partial h}{\partial r}\left[\frac{\partial^2 r}{\partial x^2}\right]+\frac{\partial r}{\partial x}\left[\frac{\partial}{\partial x}\frac{\partial h}{\partial r}\right] \qquad \text{Equation Box (1-6)}$$

Now we use the chain rule on the terms in the brackets at the end of the equation. Recognizing that h is a function of r, which itself is a function of x and y—that is, $h[r(x,y)]$—we get $\partial^2 r/dx^2$times an additional $\partial r/\partial x$ out of those brackets, or

$$\frac{\partial}{\partial x}\left(\frac{\partial h}{\partial x}\right)=\frac{\partial h}{\partial r}\left[\frac{\partial^2 r}{\partial x^2}\right]+\frac{\partial r}{\partial x}\left[\frac{\partial^2 h}{\partial r^2}\frac{\partial r}{\partial x}\right] \qquad \text{Equation Box (1-7)}$$

Therefore, the second partial derivative of h with respect to x is:

$$\frac{\partial\left(\frac{\partial h}{\partial x}\right)}{\partial x}=\frac{\partial^2 h}{\partial x^2}=\frac{\partial h}{\partial r}\frac{\partial^2 r}{\partial x^2}+\frac{\partial r}{\partial x}\frac{\partial^2 h}{\partial r^2}\frac{\partial r}{\partial x}=\frac{\partial h}{\partial r}\frac{\partial^2 r}{\partial x^2}+\frac{\partial^2 h}{dr^2}\left(\frac{\partial r}{\partial x}\right)^2 \qquad \text{Equation Box (1-8)}$$

and the second partial derivative of h with respect to y is:

$$\frac{\partial\left(\frac{\partial h}{\partial y}\right)}{\partial y}=\frac{\partial^2 h}{\partial y^2}=\frac{\partial h}{\partial r}\frac{\partial^2 r}{\partial y^2}+\frac{\partial r}{\partial y}\frac{\partial^2 h}{\partial r^2}\frac{\partial r}{\partial y}=\frac{\partial h}{\partial r}\frac{\partial^2 r}{\partial y^2}+\frac{\partial^2 h}{dr^2}\left(\frac{\partial r}{\partial y}\right)^2 \qquad \text{Equation Box (1-9)}$$

Now, the partial derivative of r with respect to x is:

$$\frac{\partial r}{\partial x}=\frac{1}{2}(x^2+y^2)^{\frac{-1}{2}}2x=\frac{1}{2}\frac{1}{r}2x=\frac{x}{r} \qquad \text{Equation Box (1-10)}$$

Next, we use the quotient rule to get the second derivative of r with respect to x. The quotient rule says that:

$$\left(\frac{f}{g}\right)=\frac{f'g-fg'}{g} \qquad \text{Equation Box (1-11)}$$

so, if $\partial r/\partial x \ = \ x/r$, then $\partial^2 r/\partial x^2$ is as below (because $f' \ = \ \partial x/\partial x \ = \ 1$).

$$\frac{\partial^2 r}{\partial x^2}=\frac{r-x\left(\frac{\partial r}{\partial x}\right)}{r^2}=\frac{r(r)-r\left(x\left(\frac{x}{r}\right)\right)}{r(r^2)}=\frac{r^2-x^2}{r^3} \qquad \text{Equation Box (1-12)}$$

but since

$$r=(x^2+y^2)^{\frac{1}{2}}\text{ and }r^2=x^2+y^2 \qquad \text{(Equation Box 1-13)}$$

and

$$y^2=r^2-x^2 \qquad \text{(Equation Box 1-14)}$$

The second partial with respect to x is:

$$\frac{\partial^2 r}{\partial x^2}=\frac{y^2}{r^3} \qquad \text{(Equation Box 1-15)}$$

And, similarly, the partials of r with respect to y are:

$$\frac{\partial r}{\partial y} = \frac{y}{r} \qquad \text{(Equation Box 1-16)}$$

and

$$\frac{\partial^2 r}{\partial y^2} = \frac{x^2}{r^3} \qquad \text{(Equation Box 1-17)}$$

We can substitute into the expression for the second partial of h with respect to x and y to obtain:

$$\frac{\partial^2 h}{\partial x^2} = \frac{\partial h}{\partial r}\frac{y^2}{r^3} + \frac{\partial^2 h}{\partial r^2}\frac{x^2}{r^2} \qquad \text{(Equation Box 1-18)}$$

$$\frac{\partial^2 h}{\partial y^2} = \frac{\partial h}{\partial r}\frac{x^2}{r^3} + \frac{\partial^2 h}{\partial r^2}\frac{y^2}{r^2} \qquad \text{(Equation Box 1-19)}$$

Therefore, we have:

$$\begin{aligned}\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} &= \frac{\partial h}{\partial r}\frac{y^2}{r^3} + \frac{\partial^2 h}{\partial r^2}\frac{x^2}{r^2} + \frac{\partial h}{\partial r}\frac{x^2}{r^3} + \frac{\partial^2 h}{\partial r^2}\frac{y^2}{r^2} \\ &= \frac{\partial h}{\partial r}\frac{(x^2+y^2)}{r^3} + \frac{\partial^2 h}{\partial r^2}\frac{(x^2+y^2)}{r^2} \\ &= \frac{\partial h}{\partial r}\frac{r^2}{r^3} + \frac{\partial^2 h}{\partial r^2}\frac{r^2}{r^2} \\ &= \frac{1}{r}\frac{\partial h}{\partial r} + \frac{\partial^2 h}{\partial r^2}\end{aligned} \qquad \text{(Equation Box 1-20)}$$

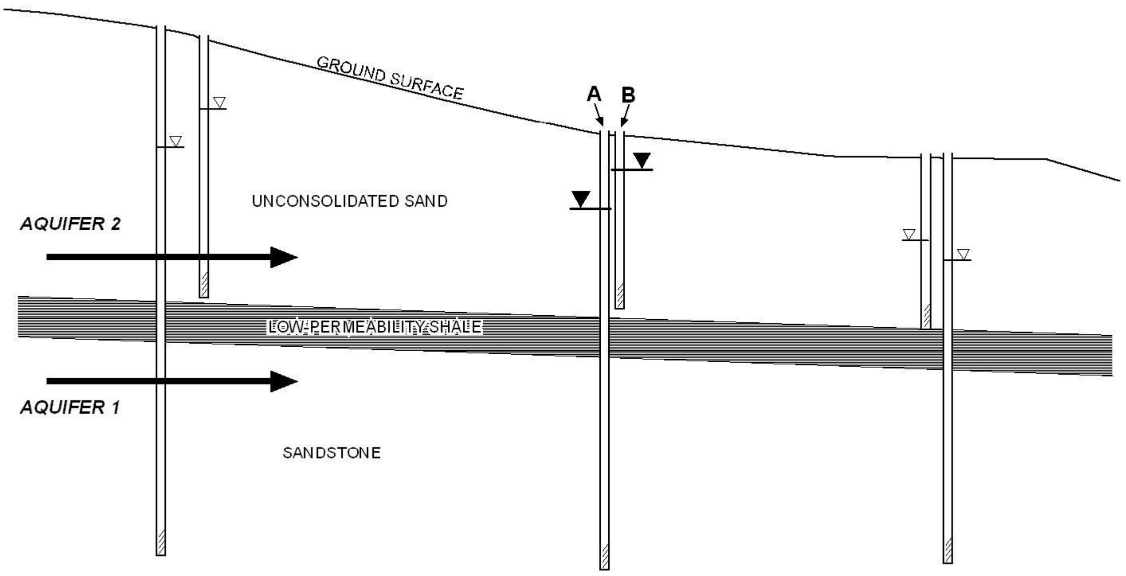
and substituting for the left side of Equation Box (1-1):

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} = \frac{S}{T}\frac{\partial h}{\partial t} \qquad \text{(Equation Box 1-21)}$$

Return to where text linked to Box 1↥

22 Exercise Solutions

Solution Exercise 1



a) Aquifer 1 is confined.

b) Well B would have a higher water level.

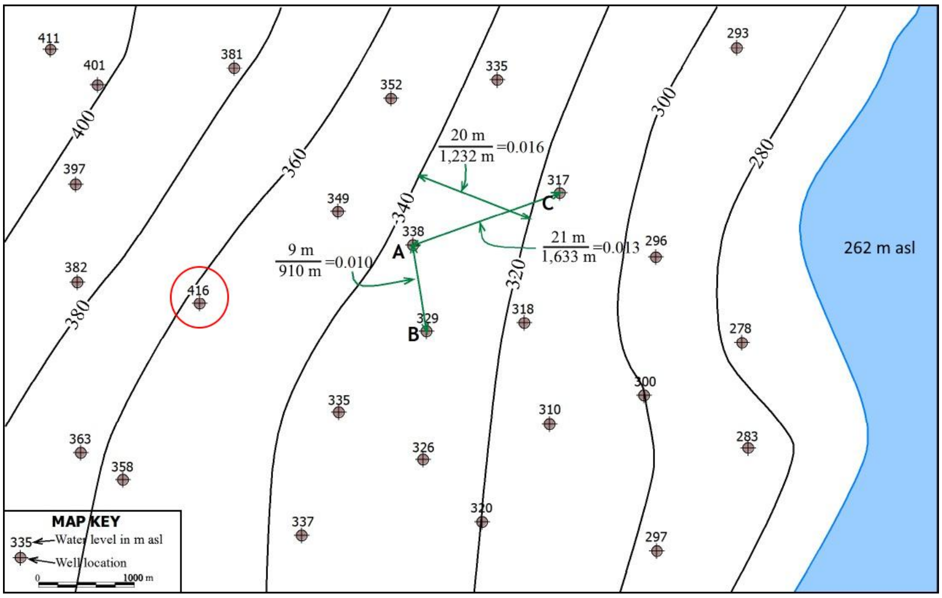
c) See image.

d) See image.

Return to Exercise 1⮥

Return to where text linked to Exercise 1⮥

Solution Exercise 2



a) AB: $\frac{9\text{ m}}{910\text{ m}} = 0.010$

AC: $\frac{21\text{ m}}{1{,}633\text{ m}} = 0.013$

b) See map. Circled value appears to be too high compared to the other values.

c) See map. 0.016 This value is greater than the values of gradient between wells because it is calculated as the maximum gradient, which in an isotropic material is the direction of flow.

d) Reason and How to identify.

Reason: Erroneous data point (incorrectly measured or recorded, bad survey data).
How to identify whether this is the reason: Re-measure or resurvey; check previous data values.

Reason: Problem with well (screen is clogged, water level is standing surface water).
How to identify whether this is the reason: Pump well and check recovery; downhole camera survey.

Reason: Surface recharge feature in area (pond or other standing water; leaking underground pipes).
How to identify whether this is the reason whether this is the reason: Perform field survey; check underground lines in area.

Reason: The well is completed in a different formation.
How to identify: Check well depth against depths of other wells; review driller's logs and other stratigraphic information at site; look for other wells in upper or lower formations and compare with their water levels.

Reason: Nearby injection well (possibly for remediation or mitigation).

How to identify whether this is the reason: Field survey; review state and federal permits for injection wells to determine if any are nearby.

Additional option:

Reason: May not be anomalous; may not be enough data to show that there is a good reason for high water levels at that point.

How to identify whether this is the reason: Look for additional data to the west.

Return to Exercise 2↥

Return to where text linked to Exercise 2↥

Solution Exercise 3

$\Delta h_{ab} = 2\ m$

$\Delta l_{ab} = 5$ m

Vertical gradient (a-b) = $\frac{2\text{ m}}{5\text{ m}}$ = 0.364

The gradient is downward.

$\Delta h_{bc} = 1$ m

$\Delta l_{bc} = 1{,}500$ m

Horizontal gradient (b-c) = 1 m/1,500 m = 0.00067

The gradient is from left to right in Figure 9.

Return to Exercise 3↥

Return to where text linked to Exercise 3↥

Solution Exercise 4

Part 1:

$$p\text{Bulk} = (p\text{min}\ [1-n]) + (p\text{H}_2\text{O}\ n);$$

$$p\text{min} = p\text{Bulk} - \frac{p\text{H}_2\text{O}\,n}{1-n}$$

$$\left(2.61\frac{\text{g}}{\text{cm}^3}\right) - \frac{1\,(0.12)}{0.88} = 2.83\ \frac{\text{g}}{\text{cm}^3}$$

Part 2:

a) $p\text{Bulk} = (p\text{min}\ [1-n])$

$$n = 1 - \left(\frac{p\text{Bulk}}{p\text{min}}\right)$$

$$1 - \frac{2{,}320\frac{\text{kg}}{\text{m}^3}}{2{,}650\frac{\text{kg}}{\text{m}^3}} = 0.125$$

b) 25 percent: $0.125\ \left(1{,}000\ \frac{\text{kg}}{\text{m}^3}1\ \text{m}^3\ (0.25)\right) = 31.1\ \text{kg} + 2{,}320\ \text{kg} = 2{,}351\ \text{kg}$

75 percent: $0.125\ \left(1{,}000\ \frac{\text{kg}}{\text{m}^3}1\ \text{m}^3\ (0.75)\right) = 93.4\ \text{kg} + 2{,}320\ \text{kg} = 2{,}413\ \text{kg}$

100 percent: $0.125\ \left(1000\ \frac{\text{kg}}{\text{m}^3}1\ \text{m}^3\ (1.00)\right) = 124.5\ \text{kg} + 2{,}320\ \text{kg} = 2{,}445\ \text{kg}$

Part 3:

a) $p\text{Bulk} = \frac{\text{mass}}{\text{volume}} = \frac{753.7\ \text{kg}}{0.36\ \text{m}^3} = 2093.5\ \frac{\text{kg}}{\text{m}^3}$

$$p\text{Bulk} = p\text{min}\ [1-n]$$

$$n = 1 - \frac{p\text{Bulk}}{p\text{min}}$$

$$1 - \frac{2093.5\frac{\text{kg}}{\text{m}^3}}{2650\frac{\text{kg}}{\text{m}^3}} = 0.21$$

b) $\left((0.21)\ 1000\ \frac{\text{kg}}{\text{m}^3}\ 0.36\ \text{m}^3\ (0.62)\right) + 753.7\ \text{kg} = 46.87\ \text{kg}\ 753.7\ \text{kg} = 800.6\ \text{kg}$

Return to Exercise 4↥

Return to where text linked to Exercise 4↥

Solution Exercise 5

a) The horizontal gradient, i, determined in the solution to Exercise 3 (0.00067) is used with the hydraulic conductivity of 0.05 m/s and effective porosity of 0.15 to calculate the average linear velocity between wells b and c.

$$v = \frac{q}{\varphi_{\text{eff}}} = \frac{K\,i}{\varphi_{\text{eff}}} = \frac{0.05\frac{\text{m}}{\text{s}}\,0.00067}{0.15}\,\frac{86{,}400\ \text{s}}{1\ \text{day}} = 19.3\,\frac{\text{m}}{\text{d}}\ \text{from left to right in Figure 9}$$

b) Travel time is determined by dividing the distance to be traveled by the velocity just as is done to figure out how long it will take to drive somewhere at a set speed.

$$time = \frac{distance}{v} = \frac{1500\ \text{m}}{19.3\,\frac{\text{m}}{\text{d}}} = 78.125\ \text{days}$$

Return to Exercise 5↥

Return to where text linked to Exercise 5↥

Solution Exercise 6

1. The items and their dimensions are defined in the table of Exercise 5. Their SI units are as follows.

$$Q = k\frac{\rho_{fluid}g}{\mu_{fluid}}A\frac{dh}{dl}$$

Q = volumetric discharge (m^3 sec^{-1})

k = intrinsic permeability (m^2)

ρ_{fluid} = density of the fluid (kg m^{-3})

g = gravitational acceleration (m sec^{-2})

A = cross-sectional area perpendicular to flow (m^2)

dh/dl = hydraulic gradient (:)

μ_{fluid} = viscosity of the fluid (kg m^{-1} sec^{-1})

2. Variables are $Q, A, \frac{dh}{dl}, k, \rho_{fluid}, \mu_{fluid}$. Constants are g.

3. Discharge is directly proportional to permeability, fluid density, gravitational acceleration, area, and hydraulic gradient. Discharge is inversely proportional to viscosity.

4. There are no exponents or logarithms in the equation; therefore, all proportionality relationships are linear.

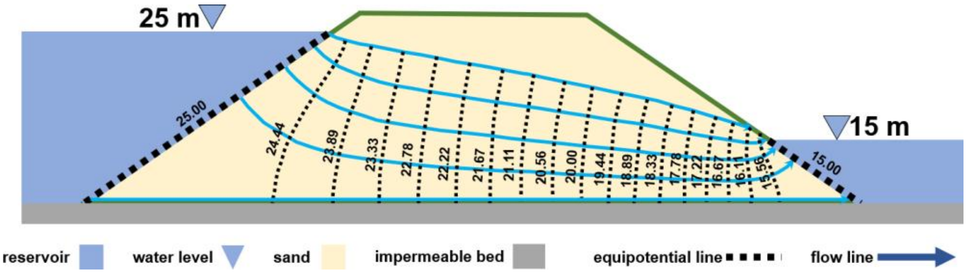
Thought Question 19*: Explain why discharge is directly proportional to density but inversely proportional to viscosity.*

Return to Exercise 6↑

Return to where text linked to Exercise 6↑

Solution Exercise 7

Details of the solution are not presented here; readers are instead encouraged to review Box 4↗ of *Graphical Construction of Groundwater Flow Nets*↗ by Poeter and Hsieh (2020) The book is available as a free download at that link. The book provides further instruction and examples of using flow nets to solve for groundwater flow. A figure from Poeter and Hsieh (2020) for a similar problem is shown here.



(from Poeter & Hsieh, 2020)

Return to Exercise 7↥

Return to where text linked to Exercise 7↥

Solution Exercise 8

Part 1: (b) because lower transmissivity results in more drawdown near the well.

Part 2: (a) because greater transmissivity results in less drawdown near the well.

Return to Exercise 8↥

Return to where text linked to Exercise 8↥

Solution Exercise 9

Variables:

$T\ = (\mathrm{k})(\mathrm{b}) = (75\ \mathrm{m})(7.06\ \mathrm{x}\ 10^{-6}\ \mathrm{m}/_{\mathrm{s}}) =\ 0.00053\ \mathrm{m}^2/s$

$S\ =\ 0.0004$

$$Q\ = \frac{545\,\frac{\mathrm{m}^3}{\mathrm{d}}}{86400\ \frac{s}{d}} = 0.00631\ \frac{m^3}{s}$$

$r\ =\ 75\ m$

$t = (36\ hrs)\left(3600\ \frac{s}{hr}\right) = 129{,}600\ seconds$

Calculations:

$$u\ = \frac{r^2S}{4Tt} = \frac{[(5625\ \ \mathrm{m}^2)0.0004]}{\left[4\left(0.00053\,\frac{\mathrm{m}^2}{\mathrm{s}}\right)129{,}600\ s\right]} =\ 0.0082$$

reading from Table 4, $W(u)\ =\ 4.26$

Drawdown at 36 hours $s = \frac{Q}{4\pi T}\mathrm{W}(u) = \frac{\left(0.00631\frac{\mathrm{m}^3}{\mathrm{s}}\right)4.23}{4\ \pi\ 0.00053\frac{\mathrm{m}^2}{\mathrm{s}}}\ =\ 4.02\ m$

Final head at 36 hours = $175\ -\ 4.02\ =\ 171\ \mathrm{m}$

Return to Exercise 9↥

Return to where text linked to Exercise 9↥

Solution Exercise 10

Variables:

r = 150 m

$$Q = \frac{300\,\frac{m^3}{d}}{86400\,\frac{s}{d}} = 0.00347\,\frac{m^3}{s}$$

Match points:

t = 1 minute (60 seconds)

s = 1 meter

u = 0.31

W(u) = 7

Solve for T:

$$s = \frac{Q}{4\pi T}\mathrm{W}(u) \quad \text{rearrange for } T = \frac{Q}{4\pi s}\mathrm{W}(u)$$

$$u = \frac{r^2 S}{4Tt} \quad \text{rearrange for } S = \frac{u\,4Tt}{r^2}$$

$$T = \frac{0.00347\,\frac{\mathrm{m}^3}{\mathrm{s}}}{4\ \pi\ 1\ \mathrm{m}}\,(7) = 0.0019\,\frac{\mathrm{m}^2}{\mathrm{s}}$$

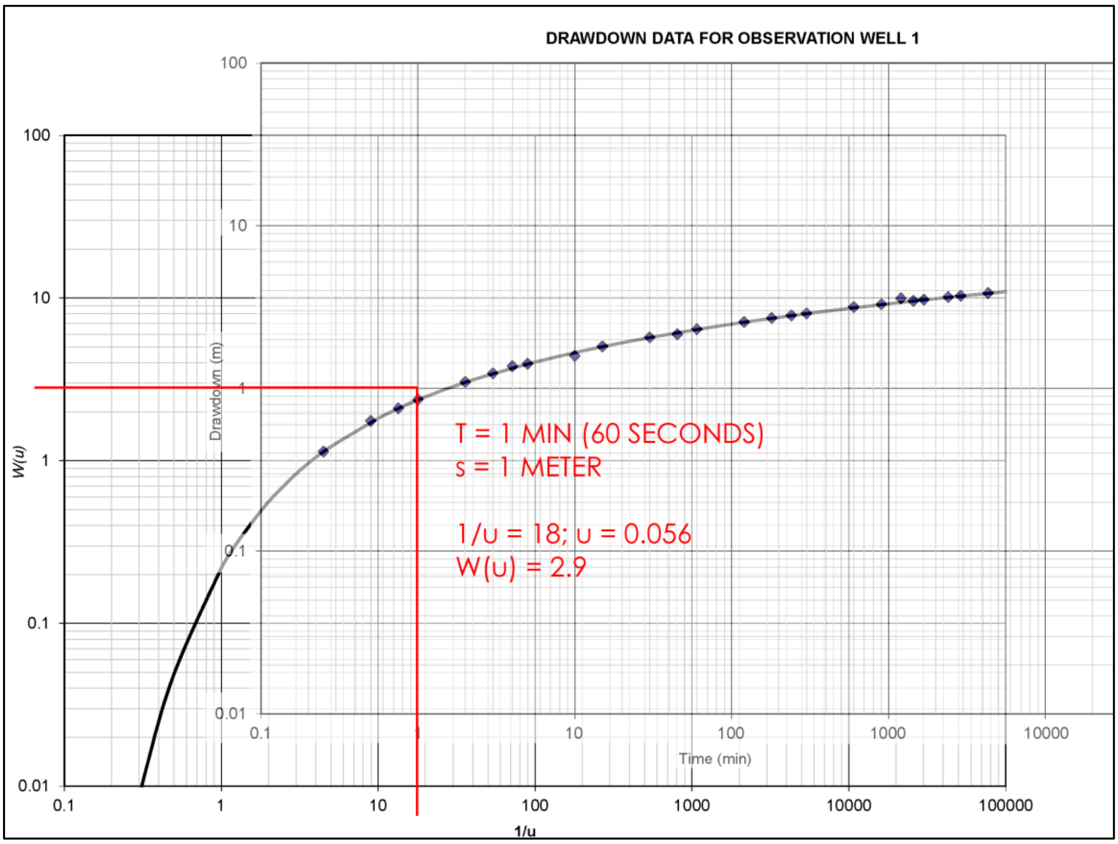
$$S = \frac{0.31\ (4)\ 0.0019\,\frac{\mathrm{m}^2}{\mathrm{s}}\ 1\ \mathrm{min}\,\frac{60s}{min}}{(150m)^2} = 6.4 \times 10^{-6}$$

Return to Exercise 10↥

Return to where text linked to Exercise 10↥

Solution Exercise 11

Curve match:



Variables:

$Q: 100\ \frac{\text{m}^3}{d}; 0.00116\ \frac{\text{m}^3}{s}$

$r: 20\ m$

$b: 20\ m$

$t: 1\text{ min (60 seconds)}$

$s:\ 1\text{ m}$

$\frac{1}{u}: 18; \quad u = 0.056$

$W(u): 2.9$

$$T = \frac{Q\ \text{W}(u)}{4\ \pi\ s} = \frac{0.00116\ \frac{\text{m}^3}{\text{s}}\ (2.9)}{4\ \pi\ 1\text{ m}} = 0.000267\frac{\text{m}^2}{\text{s}}$$

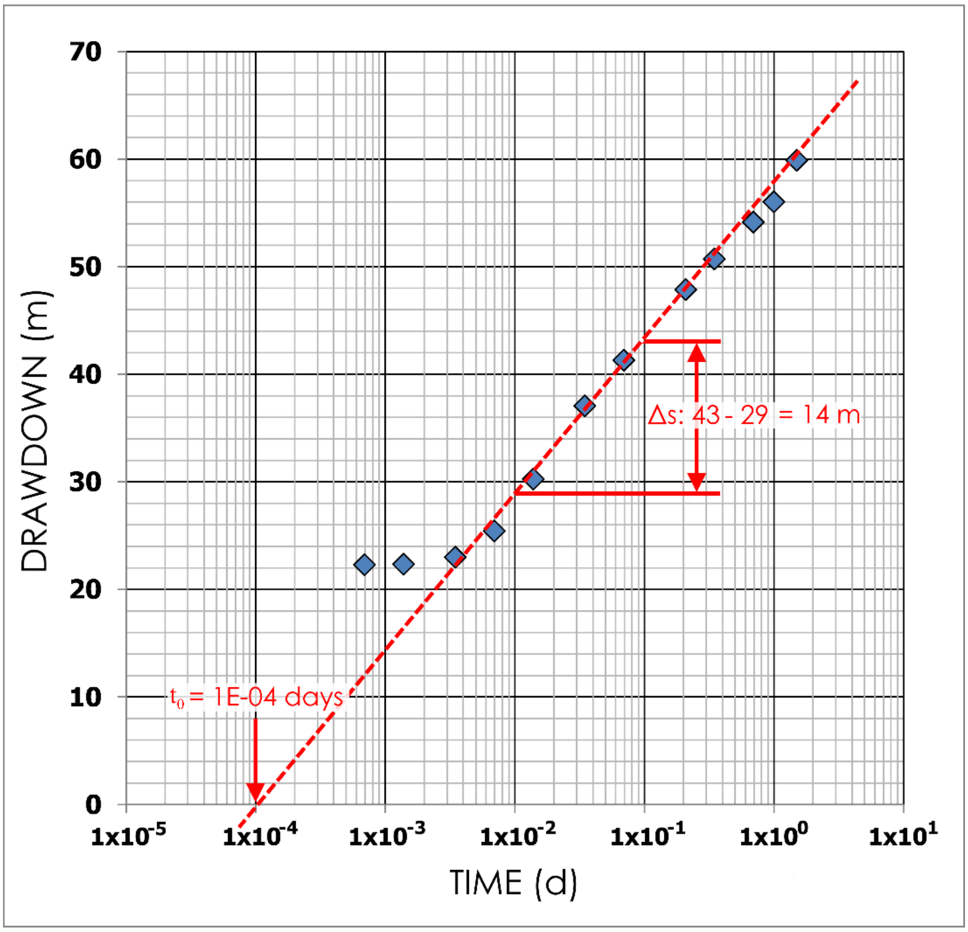
$$S = \frac{4\ T\ u\ t}{r^2} = \frac{(4)\ 0.000267\ \frac{\text{mt}^2}{\text{s}}\ (0.056)\ 60s}{(20\text{ m})^2} = 9.0x10^{-6}$$

$$K = \frac{T}{b} = \frac{0.000267\ \frac{\text{m}^2}{\text{s}}}{20\text{ m}} = 1.34\ x\ 10^{-5}\ \frac{\text{m}}{\text{s}}$$

Return to Exercise 11⮥

Return to where text linked to Exercise 11⮥

Solution Exercise 12



$$t_0 = 1x10^{-4} \text{ days; } 8.64 \text{ seconds}$$

$$\Delta s = 14 \text{ m}$$

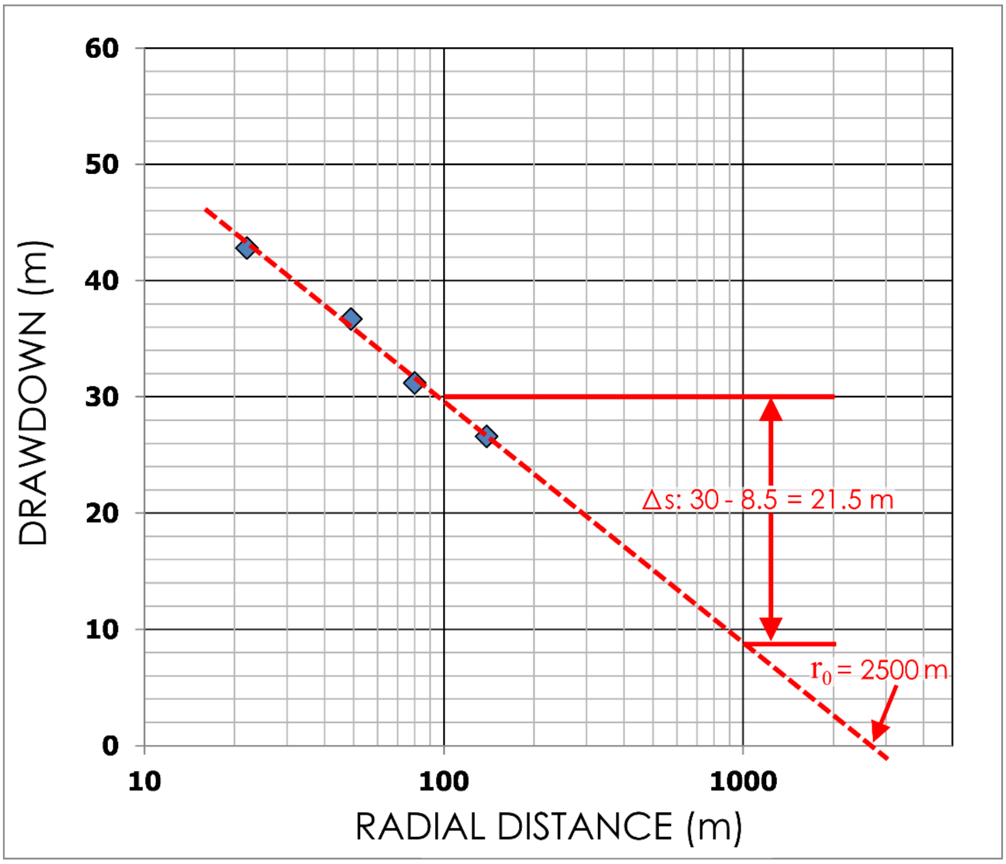
$$T = \frac{2.3\ Q}{4\ \pi\ \Delta s} = \frac{2.3\left(0.00347\frac{\mathrm{m}^3}{\mathrm{s}}\right)}{4\ \pi\ 14} = 4.54\ x\ 10^{-5}\ \frac{\mathrm{m}^2}{\mathrm{s}}$$

$$S = \frac{2.25\ T\ t_0}{r^2} = \frac{2.25\left(4.54\ x\ 10^{-5}\ \frac{\mathrm{m}^2}{\mathrm{s}}\right)\ 8.64\ seconds}{(2\ m)^2} = 2.2x10^{-4}$$

Return to Exercise 12↥

Return to where text linked to Exercise 12↥

Solution Exercise 13



$$r_0 = 2{,}500 \text{ m}$$

$$\Delta s = 21.5 \text{ m}$$

$$T = \frac{2.3\ Q}{4\ \pi\ \Delta s} = \frac{2.3\ \left(0.00231\frac{\text{m}^3}{\text{s}}\right)}{4\ \pi\ 21.5 \text{ m}} = 2.0\ x\ 10^{-5}\,\frac{\text{m}^2}{\text{s}}$$

$$S = \frac{2.25\ T\ t}{{r_0}^2} = \frac{2.25\ \left(2.0\ x\ 10^{-5}\,\frac{\text{m}^2}{\text{s}}\right)\ 86{,}400 \text{ s}}{(2{,}500 \text{ m})^2} = 6.1\text{x}10^{-7}$$

Return to Exercise 13↥

Return to where text linked to Exercise 13↥

Solution Exercise 14

Part 1: Drawdowns would be greater than expected due to the well loss associated with the lower efficiency.

Part 2: Well loss is directly proportional to a power (greater than 1) of the pumping rate with greater well loss at higher pumping rates. The well efficiency at a higher pumping rate (i.e., at 600 m^3/d) would therefore be lower than that at the previous pumping rate (500 m^3/d).

Part 3:

a) The drawdown at 24 hours is 14.44 m, this is calculated from the Theis Equation with $u = 8.68\text{x}10^{-8}$ and $\mathrm{W}(u) = 15.68$.

b) The well efficiency is 85 percent (calculated as 14.44 m / 17 m). The well loss is 2.56 m (calculated as (17 m – 14.44 m).

Part 4:

The answer is (c) test results underestimate the formation transmissivity.

The transmissivity estimated from the test is based upon the measured drawdowns in the well. Lower efficiency means greater drawdown than expected from a well with 100 percent efficiency. Since the observed drawdowns are greater than expected, the calculated transmissivity would be less than expected, and therefore the test results would underestimate the actual transmissivity.

Return to Exercise 14⮥

Return to where text linked to Exercise 14⮥

Solution Exercise 15

$$\text{Drawdown} = 645\ \text{m} - 619\ \text{m} = 26\ \text{m}$$

$$\text{Specific capacity} = \frac{500\ \text{m}^3/d}{26\ \text{m}} = 19.2\,\frac{\text{m}^3/d}{\text{m}}$$

Return to Exercise 15⮥

Return to where text linked to Exercise 15⮥

Solution Exercise 16

Variables:

$$T = 2.28\,x\,10^{-4}\,\frac{\mathrm{m}^2}{\mathrm{s}}$$

$$S = 0.00003$$

$$Q_{PW-2} = 500\frac{\mathrm{m}^3}{\mathrm{d}} = 0.0058\,\frac{m^2}{s}$$

$$r = 260\ \mathrm{m}$$

$$t = 631{,}152{,}000\ \text{seconds}\quad (20\ \text{years with 1 year} = 365.25\ \text{days})$$

Calculations:

Part 1 a:

$$u = \frac{r^2S}{4Tt} = \frac{(260\ \mathrm{m})^2\ \ 0.00003}{4\left(2.28\ \mathrm{x}\ 10^{-4}\,\frac{\mathrm{m}^2}{\mathrm{s}}\right)\ 631{,}152{,}000\ \text{seconds}} = 3.5\mathrm{x}10^{-6}$$

$$W(u) = 12$$

$$\text{Drawdown at 20 years: } s = \frac{Q}{4\pi T}\mathrm{W}(u) = \frac{0.0058\,\frac{\mathrm{m}^3}{\mathrm{s}}(12)}{4\,\pi\ 2.28\mathrm{x}10^{-4}\,\frac{\mathrm{m}^2}{\mathrm{s}}} = 24.3\ \mathrm{m}$$

Part 1 b:

Changed Variables:

$$Q_{PW-5} = 200\frac{\mathrm{m}^3}{\mathrm{d}} = 0.0023\,\frac{\mathrm{m}^2}{\mathrm{s}}$$

$$r = 150\ \mathrm{m}$$

$$t = 315{,}576{,}000\ \text{seconds}\quad (10\ \text{years with 1 year} = 365.25\ \text{days})$$

$$u = \frac{r^2S}{4Tt} = \frac{(150\ \mathrm{m})^2\ \ 0.00003}{4\left(2.28\mathrm{x}10^{-4}\ \frac{\mathrm{m}^2}{\mathrm{s}}\right)\ 315{,}576{,}000\ \text{seconds}} = 2.3\mathrm{x}10^{-6}$$

$$W(u) = 12.4$$

$$\text{Drawdown at 10 years: } s = \frac{Q}{4\pi T}\mathrm{W}(u) = \frac{0.0058\,\frac{\mathrm{m}^3}{\mathrm{s}}(12.4)}{4\,\pi\ 2.28\ x\ 10^{-4}\,\frac{\mathrm{m}^2}{\mathrm{s}}} = 10\ \mathrm{m}$$

Drawdown at 20 years due to both wells pumping: 24.3 m + 10 m = 34.2 m

Head is 386.3 n ASL – 34.2 m = 352.1 m ASL which is well above the top of the aquifer at 55 m ASL.

Return to Exercise 16↥

Return to where text linked to Exercise 16↥

Solution Exercise 17

Drawdown at Observation Well in Figure 55 with no-flow boundary

Variables:

$$Q\ =\ 1000\frac{\mathrm{m}^3}{\mathrm{d}}\ or\ 0.0116\ \frac{m^3}{s}$$

$t = 315{,}576{,}000$ *seconds* (10 *years assuming* 365.25 days per year

$$T\ =\ Kb\ =\ 0.00002\frac{\mathrm{m}}{\mathrm{s}}\ 20\ \mathrm{m}\ =\ 0.0004\ \frac{\mathrm{m}^2}{\mathrm{s}}$$

$$S\ =\ 0.0001$$

$$r_{pumping}\ =\ 200\ \mathrm{m}$$

$$r_{image}\ = 300\ \mathrm{m}$$

Drawdown at OW from the pumping well:

$$u\ = \frac{r^2S}{4Tt} = \frac{(200\ \mathrm{m})^2\ 0.0001}{\left[4\left(0.0004\frac{\mathrm{m}^2}{\mathrm{s}}\right)315{,}576{,}000\ \mathrm{seconds}\right]} =\ 7.92\mathrm{x}10^{-6}$$

$$W(u)\ =\ 11.17$$

$$s = \frac{Q}{4\pi T}\mathrm{W}(u) = \frac{0.0116\frac{\mathrm{m}^3}{\mathrm{s}}}{4\ \pi\ 0.0004\frac{\mathrm{m}^2}{\mathrm{s}}}(11.17) =\ 25.78\ \mathrm{m}$$

Drawdown at OW from image well:

$$u\ = \frac{r^2S}{4Tt} = \frac{(300\ \mathrm{m})^2\ \ 0.0001}{4\ \ (0.0004\ \frac{\mathrm{m}^2}{\mathrm{s}})\ 315{,}576{,}000\ \mathrm{seconds}} = 1.78\mathrm{x}10^{-5}$$

$$W(u)\ =\ 10.39$$

$$s\ = \frac{0.0116\frac{\mathrm{m}^3}{\mathrm{s}}}{4\ \pi\ 0.0004\frac{\mathrm{m}^2}{\mathrm{s}}}(10.39) =\ 23.98\ \mathrm{m}$$

Combined Drawdown at OW:

$$s =\ 25.78\ \mathrm{m} + 23.98\ \mathrm{m} =\ 49.76\ \mathrm{m}$$

Drawdown at Observation Well in Figure 55

Calculations are identical except Q of the image well is injection $Q = -0.0116\frac{\mathrm{m}^3}{\mathrm{s}}$. For the total drawdown, the drawdown associated with the image well is subtracted from the real well drawdown rather than added. The solution is:

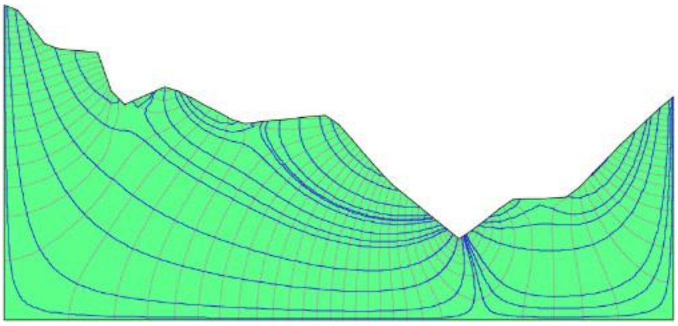
$$s =\ 25.78\ \mathrm{m} + (-23.98\ \mathrm{m}) =\ 1.8\ \mathrm{m}$$

Return to Exercise 17↑

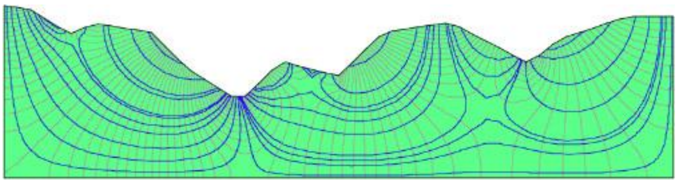
Return to where text linked to Exercise 17↑

Solution Exercise 18

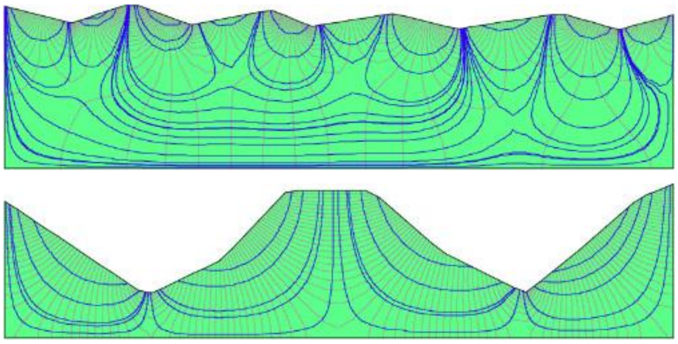
Everyone’s response will be different. You can return to this software to explore many conceptual alternatives. First, I created a basin shown below. There were some small local and intermediate systems.



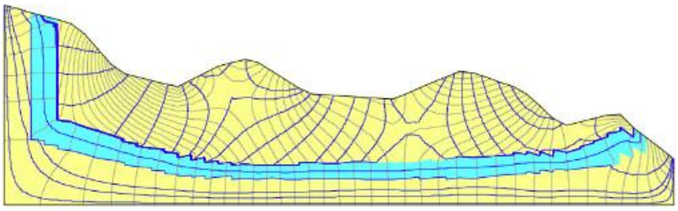
I decided I could create more significant local and intermediate systems by using less relief and that was the case as shown below.



I decided the hierarchy of flow systems would also be significantly impacted by the number and the amplitude of the variations of the water table, as shown below.



For heterogeneity I added a high K blue layer embedded in a low K yellow material and noted that this had a major impact on the flow paths, with flow being channeled through the high K material and diverting up to the surface along the length of the basin, with a few local systems in the shallow portion of the basin.



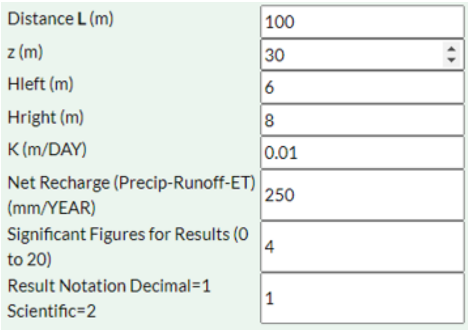
Return to Exercise 18⮥

Return to where text linked to Exercise 18⮥

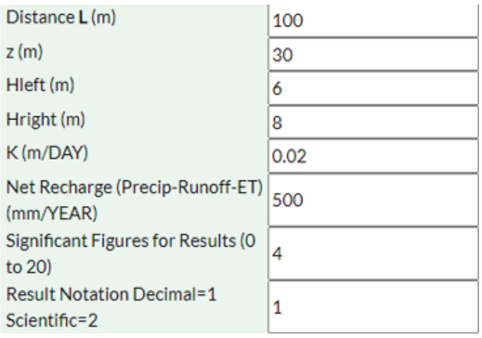
Solution Exercise 19

I visited https://interactive-education.gw-project.org/wtr/wtr.html↗ and input the following data sets.

Data Set 1 Date Set 2



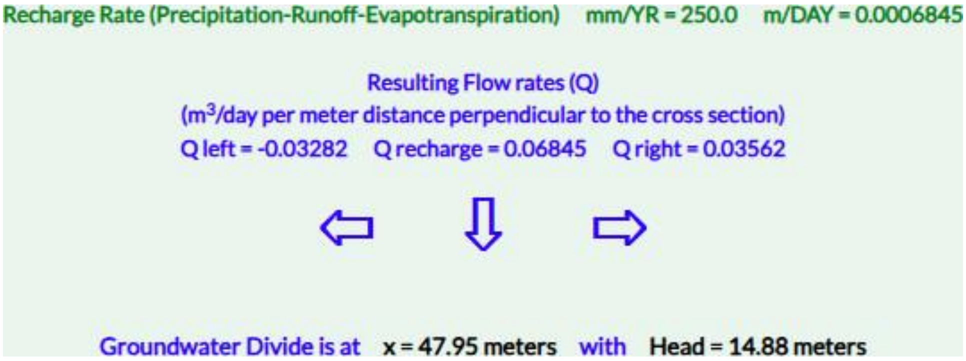
Distance **L** (m)	100
z (m)	30
Hleft (m)	6
Hright (m)	8
K (m/DAY)	0.01
Net Recharge (Precip-Runoff-ET) (mm/YEAR)	250
Significant Figures for Results (0 to 20)	4
Result Notation Decimal=1 Scientific=2	1



Distance **L** (m)	100
z (m)	30
Hleft (m)	6
Hright (m)	8
K (m/DAY)	0.02
Net Recharge (Precip-Runoff-ET) (mm/YEAR)	500
Significant Figures for Results (0 to 20)	4
Result Notation Decimal=1 Scientific=2	1

The output from Data Set 1 and Data Set 2 have identical graphs of head, while the flows are twice as large for Data Set 2.

Data Set 1



Recharge Rate (Precipitation-Runoff-Evapotranspiration) mm/YR = 250.0 m/DAY = 0.0006845

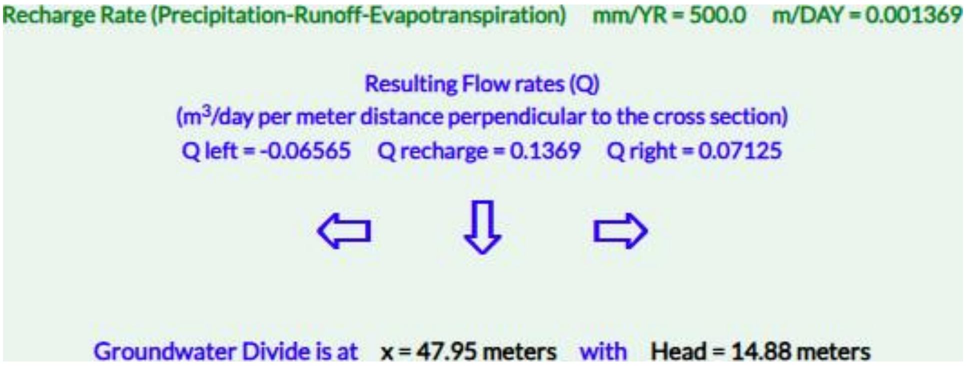
Resulting Flow rates (Q)

(m^3/day per meter distance perpendicular to the cross section)

Q left = -0.03282 Q recharge = 0.06845 Q right = 0.03562

Groundwater Divide is at x = 47.95 meters with Head = 14.88 meters

Data Set 2



Recharge Rate (Precipitation-Runoff-Evapotranspiration) mm/YR = 500.0 m/DAY = 0.001369

Resulting Flow rates (Q)

(m^3/day per meter distance perpendicular to the cross section)

Q left = -0.06565 Q recharge = 0.1369 Q right = 0.07125

Groundwater Divide is at x = 47.95 meters with Head = 14.88 meters

a) The only difference in the two outputs are the flow rates.
b) The water levels are identical.
c) The flows are a factor of 2 higher for Data Set 2.
d) If a hydrogeologist had only head measurements for the system and adjusted parameter inputs until the calculated heads matched the measured heads, there would be no way of knowing if the K and recharge rate were correct because if

both are adjusted by the same factor the head distribution stays the same, so only the ratio of recharge to K can be determined, not the absolute values. This is known as model non-uniqueness.

e) If a measurement of flow were obtained and considered along with the head measurements, then the magnitude of recharge could be determined and knowing the ratio of K to recharge the K could also be independently estimated. Such a measurement could be obtained by making a measurement of seepage to the water body on either side of the system. This is sometimes done by measuring stream discharge at two locations and taking the difference to determine how much groundwater flowed into the stream over that reach.

f) I would not trust predictions of a model that matches the water levels, but does not consider the flow balance because the values of K and recharge would not be reliable so any predictions made with the model would not be reliable. For example, if the values of Data Set1 were representative of the field system but the values of Data Set 2 were used to make predictions, hydrogeologists would predict that the system could support twice the well pumping rate than could be sustained in the field. Also, all else being equal, they would predict faster migration of contaminants thus lower concentrations.

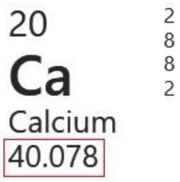
Return to Exercise 19⮥

Return to where text linked to Exercise 19⮥

Solution Exercise 20

The following table contains the results of an analysis of a water sample with the concentrations of various ions (column A) given in milligrams per liter (mg/L) in column B.

a) For column C: The atomic weight is the number at the bottom of each element cell in the interactive periodic table provided by the link; e.g., the number indicated by the red polygon on the entry for calcium provided here.



b) For column D: Divide the concentration in mg/L by the atomic weight of each ion to convert the concentration from mg/L to molarity (mmol/L).

c) For column E: Enter the valence charge for each ion.

d) For column F: Multiply the molarity by the valence charge to calculate the concentration in milliequivalents per liter (meq/L)

A	B	C	D	E	F
Solute	**Measured Concentration (mg/L)**	**Atomic weight(g)**	**Molarity (mmol/L)**	**Valence (charge)**	**meq/L**
Ca^{2+}	92	**40.08**	**2.30**	**2**	**4.59**
Mg^{2+}	34	**24.31**	**1.40**	**2**	**2.80**
Na^{+}	8.2	**23.00**	**0.36**	**1**	**0.36**
K^{+}	1.4	**39.10**	**0.036**	**1**	**0.036**
Fe(III)	0.1	**55.80**	**0.02**	**3**	**0.005**
HCO_3^-	325	**61.00**	**5.33**	**1**	**5.33**
SO_4^{2-}	84	**96.00**	**0.88**	**2**	**1.75**
Cl^-	9.6	**35.50**	**0.27**	**1**	**0.27**
NO_3^-	13	**62.00**	**0.21**	**1**	**0.21**

Return to Exercise 20↥

Return to where text linked to Exercise 20↥

Solution Exercise 21

The sum of the cation milliequivalents (i.e., positively charged ions) is 7.79 meq/L. The sum of the anion milliequivalents (i.e., negatively charged ions) is 7.56 meq/L. The charge balance is calculated as:

$$\mathrm{C.B.} = \frac{(\Sigma_{cations} - \Sigma_{anions})}{(\Sigma_{cations} + \Sigma_{anions})}(100)$$

$$\frac{\left(7.79\frac{meq}{L} - 7.56\frac{meq}{L}\right)}{\left(7.79\frac{meq}{L} + 7.56\frac{meq}{L}\right)}(100) = 0.0145(100) = 1.45\%$$

The charge balance is less than 5% and is therefore an acceptable charge balance.

Return to Exercise 21↥

Return to where text linked to Exercise 21↥

Solution Exercise 22

For column D: Divide the concentration in mg/L by the atomic weight of each ion to convert the concentration from mg/L to molarity (mmol/L).

For column F: Multiply the molarity by the valence charge to calculate the concentration in milliequivalents per liter (meq/L).

A	B	C	D	E	F
Solute	**Measured Concentration (mg/L)**	**Atomic weight(g)**	**Molarity (mmol/L)**	**Valence (charge)**	**meq/L**
Ca^{2+}	75	40.08	**1.87**	2	**3.74**
Mg^{2+}	25	24.31	**1.03**	2	**2.06**
Na^{+}	5.6	23	**0.24**	1	**0.24**
K^{+}	2.1	39.1	**0.05**	1	**0.054**
Fe(III)	0.2	55.8	**0.004**	3	**0.011**
HCO_3^-	109	61	**1.79**	1	**1.79**
SO_4^{2-}	242	96	**2.52**	2	**5.04**
Cl^-	9.3	35.5	**0.26**	1	**0.26**
NO_3^-	15	62	**0.24**	1	**0.24**

The sum of the cation milliequivalents (i.e., positively charged ions) is 6.11 meq/L. The sum of the anion milliequivalents (i.e., negatively charged ions) is 7.33 meq/L. The charge balance is calculated as:

$$\mathrm{C.B.} = \frac{(\Sigma_{cations} - \Sigma_{anions})}{(\Sigma_{cations} + \Sigma_{anions})}(100)$$

$$\frac{\left(6.11\frac{meq}{L} - 7.33\frac{meq}{L}\right)}{\left(6.11\frac{meq}{L} + 7.33\frac{meq}{L}\right)}(100) = -0.091(100) = -9.1\%$$

The absolute value of the charge balance is greater than 5%, which means that the charge balance is unacceptable. The calculated value is negative, which means that there are excess anions (or insufficient cations) represented in the analytical results.

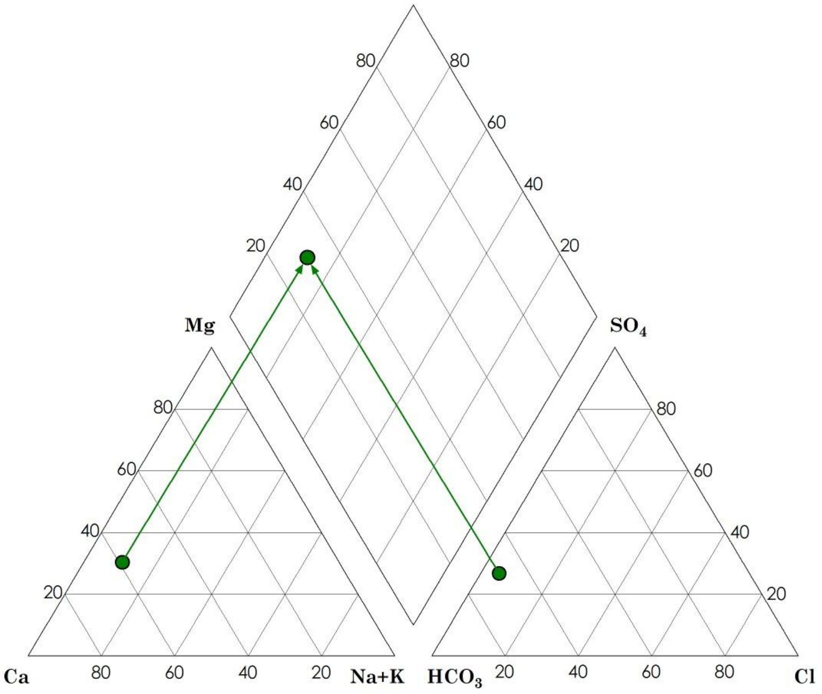
Return to Exercise 22↥

Return to where text linked to Exercise 22↥

Solution Exercise 23

Calculate the Piper Fractions:

Solutes	meq/L	Piper Fractions	
Ca^{2+}	4.59	4.59/7.78 = 0.59(100) =	**59%**
Mg^{2+}	2.80	2.8/7.78 = 0.36(100) =	**36%**
Na^{+}	0.36	(0.36+0.04)/7.78 = 0.05(100) =	**5%**
K^{+}	0.04		
	Sum cations:	**7.78 meq/L**	
HCO_3^-	5.33	5.33/7.35 = 0.73(100) =	**73%**
SO_4^{2-}	1.75	1.75/7.35 = 0.24(100) =	**24%**
Cl^-	0.27	0.27/7.35 = 0.04(100) =	**4%**
	Sum anions:	**7.35 meq/L**	



The water is a Calcium-Magnesium-Bicarbonate type as indicated in Figure 82. However, it is more appropriate to call it a Calcium-Bicarbonate water as Ca^{2+} makes up more than 50% of all the cations.

Return to Exercise 23↥

Return to where text linked to Exercise 23↥

Solution Exercise 24

Calculate the concentration for part (a) using Equation (98) and for part (b) using Equation (99).

Average linear velocity is given by Equation (93):

$$\overline{v_x} = \frac{K\frac{\partial h}{\partial x}}{\varphi_{eff}} = \frac{0.1\ \frac{\text{cm}}{\text{s}}\frac{10\ \text{cm}}{100\ \text{cm}}}{0.2} = \mathbf{0.05}\ \frac{\textbf{cm}}{\textbf{s}}$$

Dispersion coefficient is given by Equation (96):

$$D_x = \alpha_x\overline{v_x} + D_{mol} = 5\ \text{cm}\ 0.05\ \frac{\text{cm}}{\text{s}} + 1\text{x}10^{-9}\ \frac{\text{m}^2}{\text{s}}\frac{10{,}000\ \text{cm}^2}{\text{m}^2}$$

$$= \mathbf{0.25}\frac{\textbf{cm}^2}{\textbf{s}}$$

Concentration is given by Equation (98):

$$C = \frac{C_o}{2}\left[\text{erfc}\left(\frac{x-\overline{v_x}t}{2\sqrt{D_x t}}\right) + \exp\left(\frac{x\overline{v_x}}{D_x}\right)\text{erfc}\left(\frac{x+\overline{v_x}t}{2\sqrt{D_x t}}\right)\right]$$

<u>a) For 25 cm the concentration is</u> calculated as follows:

$$C = \frac{1000\frac{\text{mg}}{\text{L}}}{2}\left[\text{erfc}\left(\frac{25\ cm - 0.05\ \frac{\text{cm}}{\text{s}}1000\ s}{2\sqrt{0.25\frac{\text{cm}^2}{\text{s}}1000\ s}}\right)\right.$$

$$\left. + \exp\left(\frac{25\ \text{cm}\ 0.05\ \frac{\text{cm}}{\text{s}}}{0.25\frac{\text{cm}^2}{\text{s}}}\right)\text{erfc}\left(\frac{25\ \text{cm} + 0.05\ \frac{\text{cm}}{\text{s}}\ 1000\ \text{s}}{2\sqrt{0.25\frac{\text{cm}^2}{\text{s}}1000\ \text{s}}}\right)\right]$$

$$C = 500\frac{\text{mg}}{\text{L}}[\text{erfc}(-0.79057) + \exp(5)\,\text{erfc}(2.3717)]$$

$$\text{erfc}(-0.79057) = 1 - (-\text{erf}(0.79057)) = 1 + 0.7364 = 1.7364$$

$$\exp(5) = 148.4132$$

$$\text{erfc}(2.3717) = 1 - (\text{erf}(2.3717)) = 1 - 0.999204 = 0.000796$$

$$C = 500\ \frac{\text{mg}}{\text{L}}[(1.7364) + 148.4132(0.000796)]$$

$$C = 500\ \frac{\text{mg}}{\text{L}}[1.854537] = \mathbf{927}\ \frac{\textbf{mg}}{\textbf{L}}$$

Following similar steps:

<u>b) at 50 cm</u>, C=500 mg/L

<u>c) at 75 cm</u>, C=132 mg/L

d) **<u>These values make sense.</u>** We could have deduced the concentration at 50 cm because it takes 1000 seconds to reach 50 cm so the concentration would be half of the source concentration. The concentration at 25 cm should be higher than half the source concentration because it is closer than the location the contaminant would travel in 1000 seconds, while the concentration at 75 cm should be less because it is further than the average travel distance at 10000 seconds.

e) **Field systems will deviate from the conditions described by analytical solutions. Among other reasons, deviation from values predicted by analytical solutions will occur because:**

- the geologic material is heterogenous and so does no have constant hydraulic conductivity and dispersion coefficients,
- dispersivity cannot be measured in the field so it is an estimate based on the character of the material and often obtained by fitting data describing the distribution of concentration to an analytical equation,
- the rate and direction of groundwater flow in field settings varies with variations in recharge and pumping,
- source concentration varies with time,
- water samples collected in the field include water from a zone around the point of measurement and that zone is likely to have variations in concentrations,
- the sampling process requires great care so as not to alter the concentrations of a sample, and
- laboratory measurements are not precise because various aspects of the analysis process include error.

Return to Exercise 24⮥

Return to where text linked to Exercise 24⮥

Solution Exercise 25

Average linear velocity is given by Equation (93):

$$\overline{v_x} = \frac{K\frac{\partial h}{\partial x}}{\varphi_{eff}} = \frac{0.1\ \frac{\text{cm}}{\text{s}}\frac{10\ \text{cm}}{100\ \text{cm}}}{0.2} = \mathbf{0.05\ \frac{cm}{s}}$$

Dispersion coefficient is given by Equation (96):

$$\boldsymbol{D_x} = \alpha_x\overline{v_x} + D_{mol} = 5\ \text{cm}\ 0.05\ \frac{\text{cm}}{\text{s}} + 1\text{x}10^{-9}\ \frac{\text{m}^2}{\text{s}}\frac{10{,}000\ \text{cm}^2}{\text{m}^2}$$
$$= \mathbf{0.25\frac{cm^2}{s}}$$

$$\boldsymbol{D_y} = \alpha_y\overline{v_x} + D_{mol} = \frac{1}{5}5\ \text{cm}\ 0.05\ \frac{\text{cm}}{\text{s}} + 1\text{x}10^{-9}\ \frac{\text{m}^2}{\text{s}}\frac{10{,}000\ \text{cm}^2}{\text{m}^2}$$
$$= \mathbf{0.05\frac{cm^2}{s}}$$

$$\boldsymbol{D_z} = \alpha_z\overline{v_x} + D_{mol} = \frac{1}{10}5\ \text{cm}\ 0.05\ \frac{\text{cm}}{\text{s}} + 1\text{x}10^{-9}\ \frac{\text{m}^2}{\text{s}}\frac{10{,}000\ \text{cm}^2}{\text{m}^2}$$
$$= \mathbf{0.025\frac{cm^2}{s}}$$

At 1000 seconds, the center of mass of the plume is at

$$x = \overline{v_x}\text{t} = 0.05\ \frac{\text{cm}}{\text{s}}\ 1000\ s = \mathbf{50\ \boldsymbol{cm}}$$

Concentration relative to the center of mass is given by Equation (99):

$$C = \frac{M}{8(\pi t)^{\frac{3}{2}}\sqrt{D_xD_yD_z}}\exp\left(-\frac{X^2}{4D_xt} - \frac{Y^2}{4D_yt} - \frac{Z^2}{4D_zt}\right)$$

<u>a) For C at x=25 cm,</u> X is -25 cm because the point of interest is 25 cm behind the center of mass, while Y=0 and Z=0 because the point is on the centerline and not to the side nor below nor above**:**

$$C = \frac{500\ \frac{\text{mg}}{\text{L}}}{8(\pi\ 1000\ s)^{\frac{3}{2}}\sqrt{\left(0.25\frac{\text{cm}^2}{\text{s}}\right)\left(0.05\frac{\text{cm}^2}{\text{s}}\right)\left(0.02\frac{\text{cm}^2}{\text{s}}\right)}}$$
$$\exp\left(-\frac{(-25)^2}{4\left(0.25\frac{\text{cm}^2}{\text{s}}\right)1000\ \text{s}} - \frac{(0)^2}{4\left(0.05\frac{\text{cm}^2}{\text{s}}\right)1000\ \text{s}}\right.$$
$$\left. - \frac{(0)^2}{4\left(0.02\frac{\text{cm}^2}{\text{s}}\right)1000\ \text{s}}\right) = \mathbf{6\frac{mg}{L}}$$

b) Concentration at the center of mass (50 cm directly down gradient of the point source with no deviation up, down, or to the sides) after 1000 sec, X is 0 cm, Y is 0 cm, and Z= 0 cm.

$$C = \frac{500\ \frac{\text{mg}}{\text{L}}}{8(\pi\ 1000\ s)^{\frac{3}{2}}\sqrt{\left(0.25\frac{\text{cm}^2}{\text{s}}\right)\left(0.05\frac{\text{cm}^2}{\text{s}}\right)\left(0.02\frac{\text{cm}^2}{\text{s}}\right)}}$$

$$\exp\left(-\frac{(-0)^2}{4\left(0.25\frac{\text{cm}^2}{\text{s}}\right)1000\ \text{s}}-\frac{(0)^2}{4\left(0.05\frac{\text{cm}^2}{\text{s}}\right)1000\ \text{s}}\right.$$

$$\left.-\frac{(0)^2}{4\left(0.02\frac{\text{cm}^2}{\text{s}}\right)1000\ \text{s}}\right)=\mathbf{11}\frac{\textbf{mg}}{\textbf{L}}$$

c) Concentration at 10 cm to the left and 10 below the center of mass of the plume after 1000 sec, X is 0 cm, Y is 10 cm, and Z is -5 cm.

$$C = \frac{500\ \frac{\text{mg}}{\text{L}}}{8(\pi\ 1000\ s)^{\frac{3}{2}}\sqrt{\left(0.25\frac{\text{cm}^2}{\text{s}}\right)\left(0.05\frac{\text{cm}^2}{\text{s}}\right)\left(0.02\frac{\text{cm}^2}{\text{s}}\right)}}$$

$$\exp\left(-\frac{(-0)^2}{4\left(0.25\frac{\text{cm}^2}{\text{s}}\right)1000\ \text{s}}-\frac{(-10)^2}{4\left(0.05\frac{\text{cm}^2}{\text{s}}\right)1000\ \text{s}}\right.$$

$$\left.-\frac{(-5)^2}{4\left(0.02\frac{\text{cm}^2}{\text{s}}\right)1000\ \text{s}}\right)=\mathbf{5}\frac{\textbf{mg}}{\textbf{L}}$$

d) Although the aquifer properties and gradient are the same for both cases, for part (a) there is a continuous input of contaminant at a concentration of 500 mg/L while for part (b) there is only one slug of 500 mg introduced to the groundwater. To compound that significant difference in mass, the mass in the system of part (b) spread not only in the direction of flow but also to the side and down.

e) **Field systems will deviate from the conditions described by analytical solutions. Among other reasons, deviation from values predicted by analytical solutions will occur because:**

- the geologic material is heterogenous and so does no have constant hydraulic conductivity and dispersion coefficients,
- dispersivity cannot be measured in the field so it is an estimate based on the character of the material and often obtained by fitting data describing the distribution of concentration to an analytical equation,

- the rate and direction of groundwater flow in field settings varies with variations in recharge and pumping,
- source concentration varies with time,
- water samples collected in the field include water from a zone around the point of measurement and that zone is likely to have variations in concentrations,
- the sampling process requires great care so as not to alter the concentrations of a sample, and
- laboratory measurements are not precise because various aspects of the analysis process include error.

Return to Exercise 25⮥

Return to where text linked to Exercise 25⮥

Solution Exercise 26

a) The c**<u>apture zone reaches a maximum width</u>** at which the regional flow is equal to the discharge of the well. The maximum width can be calculated using Equation (101).

$$B = 2y_{max} = \frac{Q}{Kib} = \frac{0.005\ \frac{\mathrm{m}^3}{\mathrm{s}}}{5.5\mathrm{x}10^{-3}\ \frac{\mathrm{m}}{\mathrm{s}}\ 2.5\mathrm{x}10^{-4}\ \frac{\mathrm{m}}{\mathrm{m}} 20\ \mathrm{m}} = \mathbf{182\ m}$$

So, the maximum half width of the capture zone is 91 m.

b) The **<u>stagnation point location</u>** is given by Equation (100).

$$x_o = \frac{Q}{2\pi Kib} = \frac{0.005\ \frac{m^3}{s}}{2\ \pi\ 5.5x10^{-3}\ \frac{m}{s}\ 2.5x10^{-4}\ \frac{m}{m} 20\ m} = \boldsymbol{28.9\ m}$$

downgradient of the well.

c) If the spill is 100 meters to the northwest of the well then it is at a location that is, by Pythagorean's theorem at coordinates (-70.7 m, 70.7 m) relative to the well. That is, x = square root of half the hypotenuse squared. Or the square root ((100 m) (100 m) /2) = 70.7 m.

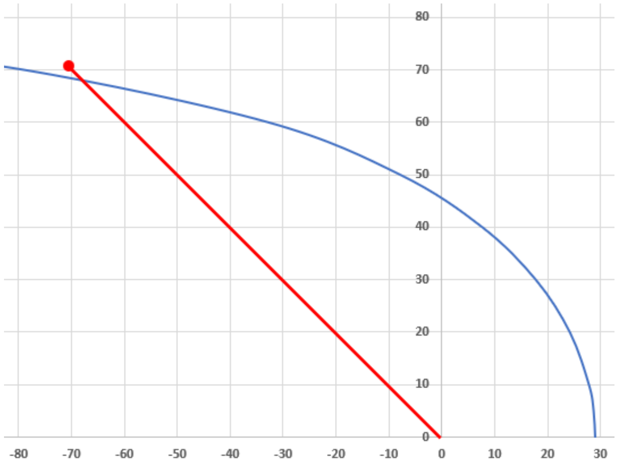
The relationship between the position x relative to the well and the half width, y, of the plume is given by Equation (102).

$$x = \frac{-y}{tan\left(\frac{2\pi Kiby}{Q}\right)}$$

The x location where the plume is 70 m wide is

$$x = \frac{-70.7\ m}{tan\left(\frac{2\ \pi\ 5.5x10^{-3}\ \frac{m}{s}\ 2.5x10^{-4}\ \frac{m}{m} 20\ m\ 70.7\ m}{0.005\ \frac{m^3}{s}}\right)} = -84.3\ m$$

<u>Consequently, the current capture zone does not lie beneath the spill so no adjustment of the pumping rate is needed, however given that it is so close to the capture zone, pumping might be slightly reduced to provide a factor pf safety. A sketch of the capture zone with a 100-meter line extending to the northwest showing the pint of the spill is shown here.</u>



Return to Exercise 26⮥

Return to where text linked to Exercise 26⮥

Solution Exercise 27

Abutting and crossing fracture terminations are, by definition, connected to other fractures. Blind fracture terminations end within the matrix and do not connect to other fractures. Diagram (a) has a larger distribution of abutting and crossing fracture terminations, which suggests greater connectivity of the fracture networks and, therefore, greater overall permeability that diagram (b). The answer, therefore, is the aquifer represented in diagram (a).

Return to Exercise 27⮥

Return to where text linked to Exercise 27⮥

23 Thought Questions Answered

Thought Question 1 Answer: The primary source of energy driving the hydrologic cycle is the sun. A secondary source of energy is the internal heat of the Earth, which is primarily sourced from the radioactive decay of unstable elements in the Earth's mantle and core.

Thought Question 2 Answer: Porosity is a scalar quantity, as it does not vary with respect to direction. Effective porosity is also considered to be a scalar quantity; however, some modelling studies have shown that it can vary slightly with respect to the direction of flow depending on the orientation of the pore connections. In practice, both quantities are generally considered to be scalar.

Thought Question 3 Answer: $S_y\ and\ \phi_{\mathrm{eff}}$ have essentially the same value. S_y reflects the fractional volume of interconnected pores that drain when water levels is lowered. ϕ_{eff} reflects the fractional volume of interconnected pores that are part of the active flow in a porous medium.

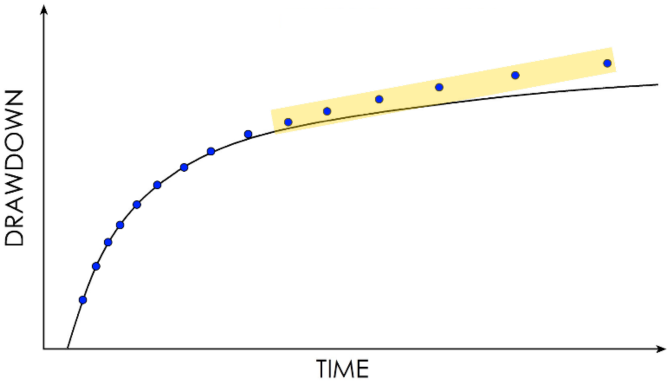
Thought Question 4 Answer: The limits of Darcy's Law may be exceeded in karst or fractured media because high velocity flow in conduit or fractures may be turbulent. Its limits may be exceeded in tight, fine-grained, materials in which the hydraulic gradient is not sufficient to overcome viscous fluid forces and thus cannot force groundwater through pores.

Thought Question 5 Answer: The average linear velocity considers both high and low groundwater velocities so some contaminants will arrive at a downgradient receptor sooner than predicted by the average linear velocity and some will arrive later.

Thought Question 6 Answer: Specific storage is the volume of water released from a unit volume of aquifer for a unit decline of hydraulic head, so it is volume per volume per length (i.e., L^3 per L^3 per L, which is $L^3L^{-3}L^{-1}$) thus has units of L^{-1}. Storativity is the product of specific storage and aquifer thickness (i.e., $L^{-1}L$) so it is dimensionless.

Thought Question 7 Answer: Storage in a confined aquifer is controlled by the elastic response of the aquifer, and storativity values for typical confined aquifers range from 1 x 10^{-6} to 1 x 10^{-4}. Storage in an unconfined aquifer is controlled by drainage of the pore space, therefore, the storativity will essentially be equivalent to the specific yield, which typically ranges from 0.01 to 0.5. If all other factors are equal (thickness, permeability, pumping rates, etc.), then a greater storativity results in less drawdown. Since confined aquifers tend to have much lower storativity than unconfined aquifers, the confined aquifer will experience greater drawdown.

Thought Question 8 Answer: A sketch of the situation is shown below. The highlighted points plot above the type curve, which indicates that the observation well is experiencing more drawdown than expected from an ideal confined aquifer. A deviation from the Theis curve toward more drawdown in the later time indicates a no-flow boundary in the aquifer is decreasing the aquifer area available to provide flow from storage to the well.



Thought Question 9 Answer: Transmissivity is underestimated when determined using head in a pumping well that is experiencing well losses because the lower head would suggest a steeper gradient is needed to deliver the rate of water discharged by the well then exists in the formation just beyond the well bore. Thus, the estimated T would be less than estimated using data from an observation well.

Thought Question 10 Answer: Hydrogeologists might study a flow system that supports an ecosystem that is valuable to humans for reasons that do not include a water supply – for example, a sensitive groundwater-supported wetland that provides a critical part of the surface water system and contains endangered or critical species. Another example is a flow system near a contaminated site that is not sufficiently productive to be considered an aquifer but is still capable of transmitting contamination to receptors.

Thought Question 11 Answer: As indicated in Section 3.3.3, permeability is a function of the properties of the fluid as well as the properties of the media through which the fluid flows; more specifically, the density and viscosity of the fluid. A high concentration of contaminant could potentially increase the density of the groundwater enough to alter the hydraulic conductivity in the vicinity of the higher density and to alter the force driving the flow. The same could happen if there is sufficient contaminant present to affect the viscosity of the water.

Thought Question 12 Answer: Dispersivity spreads contaminants out so a setting with a high dispersivity would have a breakthrough curve with earlier first arrivals, a lower peak and longer tail.

Thought Question 13 Answer: The stage-discharge curve is dependent on the stream channel morphology, so anything that changes the profile of the stream channel, e.g., significant erosion, deposition of sediment, or a landslide into a stream channel, would likely change the stage-discharge curve. Also, changes within the watershed or catchment area supplying water to the gaging station can result in stream morphology changes. For example, extensive urbanization within the watershed results in a lot of paved surfaces, which in turn increases runoff and peak flows that can increase erosion throughout the stream channel.

Thought Question 14 Answer: The stream runoff peak occurs earlier than the baseflow peak because surface runoff is rapid—on the order of days or weeks—while baseflow discharge is slow, typically occurring over weeks or months.

Thought Question 15 Answer: The shape of the runoff and baseflow portions of a storm discharge curve depend on the size and topography of the basin as well as the character of the storm and the nature of the groundwater system.

Thought Question 16 Answer: The thickness of the capillary fringe depends on the size of pore openings. Its variability within and between aquifers depends on the variations in pore size distributions in the vicinity of the water table. The water table fluctuates and so the thickness of the fringe can vary with time as the water table occurs in materials of differing pore sizes.

Thought Question 17 Answer: If effective stress drops to zero in unconsolidated sediments near the surface, a *quick* condition occurs, that is the material losses its solid behavior and instead behaves as a liquid, i.e. it becomes quicksand. If the effective stress drops to zero in consolidated rock beneath the surface, the pore fluid pressure will cause the rock to fracture. This is the process involved in *fracking*, or hydraulically fracturing a formation to increase permeability and fluid recovery.

Thought Question 18 Answer: Geologic examples of the four types of fractured systems:

- purely fractured, a very dense, fractured, igneous rock with no matrix porosity;

- fractured formation, a shale;
- double porosity, a fractured sandstone;
- heterogeneous system, a fractured sandstone in which mineral deposition has clogged the fractures.

Thought Question 19 Answer: Discharge is directly proportional to density because hydraulic conductivity increases as density of the fluid increases, but it is inversely proportional to viscosity because hydraulic conductivity decreases as viscosity increases. The properties of the fluid are needed to calculate hydraulic conductivity from permeability, and density is in the numerator while viscosity is in the denominator.

24 Notations

A = cross-sectional area (L^{2})

α = compressibility of the aquifer matrix (LT^{2}M^{-1}) , with the exception in the following row

α = in section 9.2.4, angle of refraction, also defined in the row above this row

b = saturated thickness of the aquifer (L), with the exception in the following row

b = in Sections 16 and 17, aperture (distance between walls of fracture) (L), also defined in the row above this row

β = compressibility of water (LT^{2}M^{-1})

d = number of flow tubes, areas bounded by adjacent pairs of flow lines (:)

$\Delta\sigma$ = change in effective stress (MLT^{-2})

$\Delta L/L$ = change in length (thickness) over the total length (L)

$\Delta Storage$ = change in storage over time in a system that balances input and output

dh/dl = hydraulic gradient or change in hydraulic head (:)

$\partial h/\partial x$ = hydraulic gradient with respect to x (:)

e = void ratio (:)

f = number of equipotential drops across a flow net (:)

g = gravitational acceleration (LT^{-2})

h = hydraulic head (L)

H = head of the backwater above the weir crest in ft

$\mathrm{H_a}$ = heat energy added (L) expressed as height of column of water

H_L = mechanical energy lost (L) expressed as height of column of water

H_E = heat energy extracted (L) expressed as height of column of water

i = hydraulic gradient (dh/dl) (:)

$I(t)$ = input to a system over time

k = intrinsic permeability (L^{2})

K = hydraulic conductivity (LT^{-1})

L = length of the weir crest in ft

μ_{fluid} = viscosity of the fluid (ML^{-1}T^{-1})

μ = viscosity (ML^{-1}T^{-1})

n = porosity (:), with the exception in the following row
n = in section 13.5.2, Manning roughness coefficient (:), also defined in the row above
ω = gravimetric moisture content (:)
$O(t)$ = output from a system over time
p = pressure ($ML^{-1}T^{-2}$)
P_f = pore fluid pressure ($ML^{-1}T^{-2}$)
P_h = hydrostatic pressure; hydrostatic pore fluid pressure ($ML^{-1}T^{-2}$)
ϕ = porosity (:)
ϕ_{eff} = effective porosity (:)
ψ = matric potential (or negative pressure head) (L)
Q = volumetric discharge (L^3T^{-1})
q = specific discharge of groundwater flow (LT^{-1})
q' = flow per unit width of aquifer (L^2T^{-1})
r = radius (distance from pumping well) (L)
R = hydraulic radius (L)
ρ_{bulk} = bulk density (ML^{-3})
ρ_{fluid} = density of the fluid (ML^{-3})
ρ_f = fluid density (ML^{-3})
s = drawdown (L)
S = storativity, $S = S_s b$ (:)
S_d = degree of saturation (:)
σ = total vertical stress; also called the normal stress ($ML^{-1}T^{-2}$)
σ' = effective stress, or “grain to grain” stress in the rock ($ML^{-1}T^{-2}$)
σ_v = vertical stress ($ML^{-1}T^{-2}$)
S_r = specific retention (:)
S_s = specific storage (L^{-1})
S_y = specific yield (:)
θ = volumetric moisture content (L^3L^{-3})
T = aquifer transmissivity, $T = Kb$ (L^2T^{-1})
t = time (T)
u = excess fluid pressure

v = velocity (LT^{-1})

v_{avg} = average linear velocity (LT^{-1})

V_{solids} = volume of the solids (L^3)

V_{total} = total volume of the sample ((L^3)

V_{voids} = volume of the voids (L^3)

w = rate of vertical leakage or recharge to the aquifer (LT^{-1})

x = some distance from h_1 (L)

x, y = Cartesian coordinates (L)

x, y, z = lengths in the principal directions in our coordinate systems

z = elevation head (L) , with the exceptions in the two following rows

z = in section 15.3, equation 97, thickness of the layer of material (L), also defined in the row above and below

z = in section 15.3, equations 99 and 100, thickness of the overlying layer of water (L), also defined in the two rows above

25 About the Author

Matthew M. Uliana, PhD, PG, is a geologist with teaching, research, and professional expertise in hydrogeology, aqueous geochemistry, environmental geology, surface water hydrology, and general earth science. He is a registered professional geoscientist (Texas, USA) with over thirty years of industry experience providing geology, hydrogeology, and environmental consulting services to clients in mining, water resources, waste disposal, manufacturing, and construction. He also has over seven years of post-graduate experience at the university level conducting academic research projects and teaching classes ranging from introductory/general education courses to upper division major courses to graduate courses for master's and doctoral students.

He was born and raised in the suburbs of eastern Pennsylvania, USA, where close proximity to the Appalachian Mountains and an associated love of the outdoors led to an early interest in the earth sciences. He attended James Madison University (JMU), Virginia, USA, on a gridiron football scholarship and ultimately received a BS degree from JMU with a double major in geology and anthropology. He then relocated to Austin, Texas, USA, to attend graduate school at the University of Texas at Austin. While in Austin, he successfully completed master's and doctoral degrees in geological sciences while simultaneously founding one of the most infamously mediocre heavy metal bands in Central Texas history. He spent several years wallowing in the murky halls of academia before fleeing the ivory tower and establishing a full-time practice as a consulting hydrogeologist.

After more than a decade of self-employment, part-time work for established firms, and the occasional adjunct teaching appointment at the local university, he accepted a full-time position with INTERA Incorporated in Austin as a principal hydrogeologist. In 2022, he relocated to Western Australia to work in the INTERA Perth office. Since arriving in Perth, he has spent his time leading and supporting industrial water supply, mine closure, and groundwater modeling projects, sampling the wares from the local breweries, and searching for a new band worthy of his exceptionally amateur talents as a rock bass player.

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Modifications to Original Release

Changes from the Original Version to Version 2

Original Version: January 13, 2025, Version 2: January 19, 2025

General: updated the Table of contents to include this modification section

Page numbers refer to the original PDF.

page ii, added Version 2

page 154, included 2nd term in the 1D solute transport equation that is needed in some cases to accommodate low Peclet numbers

page 154, changed "The full version of this equation" to "This equation" because the full version is now included in this book

page 268, included 2nd term in solution of Exercise 24 which influence the calculated value of concentration at the x= 25 cm location

page 283, now followed by page A, to include this modification section

Changes from the Version 2 to Version 3

Version 2: January 19, 2025, Version 3: March 18, 2025

Page numbers refer to the version 2 PDF.

page iii, change version number and date

page 9, Equation 2, added negative sign to Darcy's Law and a sentence at the end of the subsequent paragraph stating "The negative sign indicates that flow is in the opposite direction of the gradient."

page 24, Equation 23, added negative sign to Darcy's Law

page 25, Equation 24, added negative sign to Darcy's Law

page 35, Equation 33, added negative sign to Darcy's Law

page 39, Equation 35, added negative sign to Darcy's Law

page 41, Equation 38, added negative sign to Darcy's Law page 51, Equation 44, added negative sign to Darcy’s Law

page 57, Equation 59, added negative sign to Darcy’s Law

page 118, Equation 89, added negative sign to Darcy’s Law

page 118, Equation 90, added negative sign to Ohm’s Law

page 118, Equation 90, change to continuous form to be more parallel to Equation 89

page 145, Equation 93, added negative sign to Darcy’s Law

page 248, Solution Exercise 3, added statements indicating the direction of the gradients

page 250, Solution Exercise 5, added statement indicating the direction of the velocity