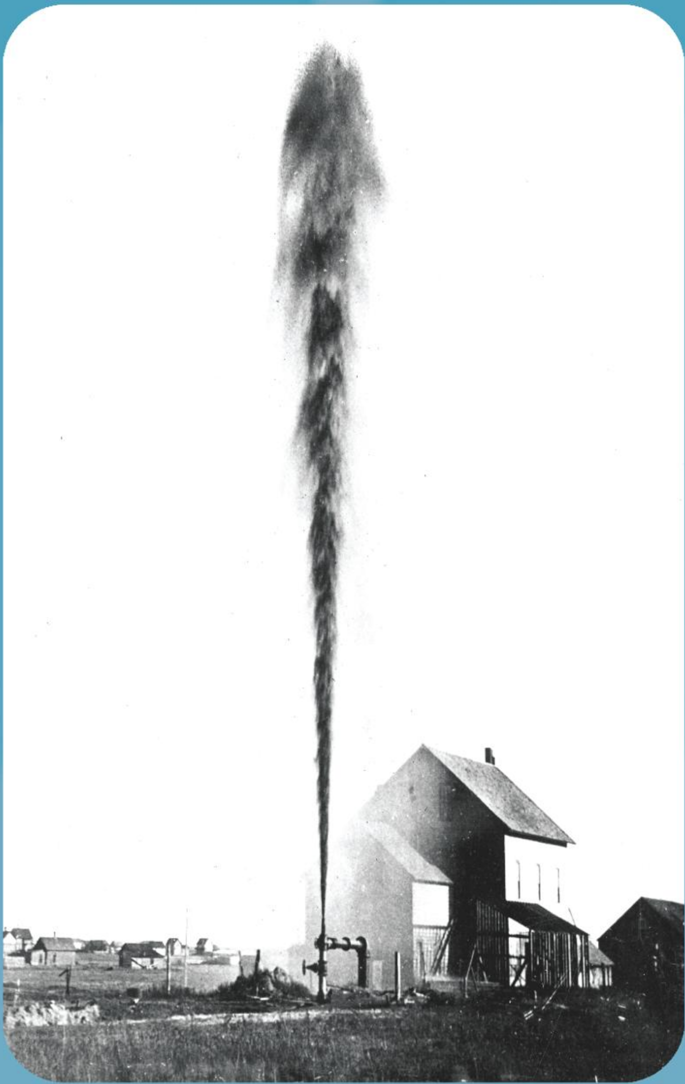

Groundwater Storage in Confined Aquifers

Herbert F. Wang

THE GROUNDWATER PROJECT

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Cover Image: Darton (1900)

Dedication

To Evan, Mia, and Noah; Melia and Chloe. Grandpa's "Saga of the Dakota Sandstone" begins around the time Laura Ingalls Wilder lived in South Dakota.

Contents

The Groundwater Project Foreword

The United Nations Water Members and Partners establish their annual theme a few years in advance. The theme for World Water Day of March 22, 2022, is "Groundwater: making the invisible visible." This is most appropriate for the debut of the first Groundwater Project (GW-Project) books in 2020, which have the goal of making groundwater visible.

The GW-Project, a non-profit organization registered in Canada in 2019, is committed to contribute to advancement in education and brings a new approach to the creation and dissemination of knowledge for understanding and problem solving. The GW-Project operates the website https://gw-project.org/↗ as a global platform for the democratization of groundwater knowledge and is founded on the principle that:

"Knowledge should be free and the best knowledge should be free knowledge." Anonymous

The mission of the GW-Project is to provide accessible, engaging, high-quality, educational materials, free-of-charge online in many languages, to all who want to learn about groundwater and understand how groundwater relates to and sustains ecological systems and humanity. This is a new type of global educational endeavor in that it is based on volunteerism of professionals from different disciplines and includes academics, consultants and retirees. The GW-Project involves many hundreds of volunteers associated with more than 200 hundred organizations from over 14 countries and six continents, with growing participation.

The GW-Project is an on-going endeavor and will continue with hundreds of books being published online over the coming years, first in English and then in other languages, for downloading wherever the Internet is available. The GW-Project publications also include supporting materials such as videos, lectures, laboratory demonstrations, and learning tools in addition to providing, or linking to, public domain software for various groundwater applications supporting the educational process.

The GW-Project is a living entity, so subsequent editions of the books will be published from time to time. Users are invited to propose revisions.

We thank you for being part of the GW-Project Community. We hope to hear from you about your experience with using the books and related material. We welcome ideas and volunteers!

The GW-Project Steering Committee

November 2020

Foreword

Groundwater science began as a modern discipline in 1856 when Henry Darcy published his "law" for the simple relationship between volumetric flow rate and the gradient of hydraulic head. Soon thereafter, this led to mathematical descriptions of steady flow to wells in homogeneous horizontal aquifers. But decades were to pass before the first mathematical descriptions for unsteady flow to wells in confined aquifers were published by C.V. Theis (1935) of the United States Geological Survey. For Theis to develop this description, he needed to understand the origin of the water pumped from confined aquifer wells. Recognition of the origin of this water was elusive because it had to be extracted from knowledge at the interface between aquifer hydraulics and geomechanics and to achieve this, scientific intuition had to evolve into quantitative thinking by the leading groundwater intellects of the time. This book: *Groundwater Storage in Confined Aquifers* by Herbert F. Wang explains that water from storage in confined granular aquifers comes from compression of the aquifer skeleton and expansion of the water as water pressure declines during pumping. Not only does this book explain the principles and processes involved, but it provides the interesting history of this discovery as a Wild West saga recounted to show how the scientific method was used to solve the mystery of the origin of confined-aquifer water.

For decades, the author of this book has conducted internationally recognized research. His teaching has focused on rock mechanics and hydrogeology and this historic record of the unfolding understanding of aquifer storativity is his favorite lecture.

John Cherry, Groundwater Project Leader

Guelph, Ontario, Canada, November, 2020

Preface

Underground aquifers are the water source for 70% of the world's irrigation use. Many of these aquifers have experienced steep drops in water pressure and are endangered from over consumption. Exporting a tomato or an almond is a transfer from a region's water bank. The invisibility of groundwater makes its management dependent on understanding the geological setting and physics of extraction. When large numbers of non-native settlers began farming in the Dakota Territory of the United States in the latter part of the 19th century, the United States Geological Survey undertook groundwater studies that would continue for nearly a century. This book provides a historical introduction to discovery of the role of aquifer deformation in response to pumping wells.

The author of this book has engaged in several decades of teaching and research in the disciplines of rock mechanics and hydrogeology. This book is based on the author's hydrogeology lecture on the "Saga of the Dakota Sandstone" in which the moral of the story is that study of the removal of groundwater from confined aquifers is where the two disciplines meet.

Acknowledgements

I greatly appreciated the opportunity to turn my lecture on groundwater storage into a book for the Groundwater Project. The manuscript was significantly improved in accuracy and readability by the following. Remaining errors and obfuscations are all mine.

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Finally, I am grateful for financial support provided by the Wisconsin Retirement System and the Social Security Administration.

Herbert Wang

1 Introduction

The subject of this book is the discovery and quantification of the concept of groundwater storage in confined aquifers. All aquifers are water-bearing and permeable, but the nature of storage is distinctly different between confined and unconfined aquifers as discussed in the GW-Project books about Groundwater in Our Water Cycle↗ by Poeter et al. (2020) and Hydrologic Properties of Earth Materials and Principles of Groundwater Flow↗ by Woessner and Poeter (2020). The upper surface of an unconfined aquifer is the water table, as discussed in both of those GW-Project books. Pumping from an unconfined aquifer drains water from its saturated void space and lowers the water table. With caveats for films of water adhering to rock grains from surface tension, the storage parameter of an unconfined aquifer at the scale of a Representative Elementary Volume (REV) (the smallest volume at which properties are the same as the properties of the whole) is the ratio of its void volume to the volume of the REV, that is, porosity. REV and porosity are discussed in the GW-Project book by Woessner and Poeter (2020). The storage parameter for an unconfined aquifer is called the specific yield. The nature of the storage mechanism is relatively straightforward.

A confined aquifer, as its name implies, is a permeable rock unit sandwiched between impermeable layers. It is recharged where it outcrops and flow is constrained to remain within the unit. The head in any cross section of a confined aquifer is the same throughout its depth and elevated above the aquifer. Pumping from a confined aquifer removes water from its void space, but the void space remains saturated. There is no water table to lower. The commonality a confined aquifer has with an unconfined aquifer is that pumping lowers the head (Figure 1). The storage in a confined aquifer resides in the compressibility of rock and of water filling the pore space in response to groundwater head or pressure changes. An interesting tipping point occurs if the head is lowered below the top of the confined aquifer because the confined aquifer then becomes unconfined and the storage parameter changes orders of magnitude from its confined to its unconfined value.

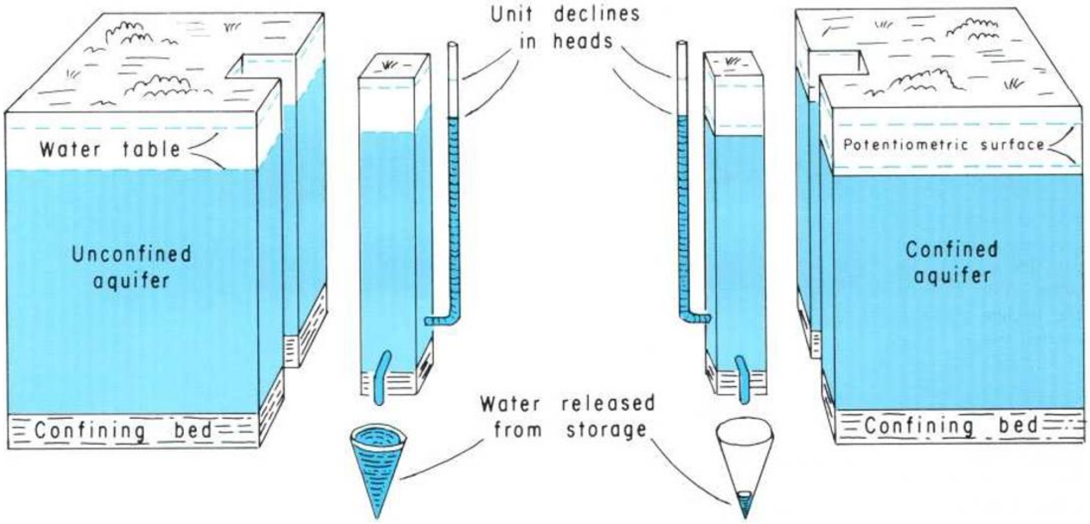

Figure 1 - Unconfined versus confined storage. The amount of water released from storage per unit head decline in an unconfined aquifer is several orders of magnitude greater than in a confined aquifer (Heath, 1983).

When a REV in a confined aquifer adds or releases water in response to a change of hydraulic head[1], this change in storage must be included in the mass balance equation for groundwater movement. In consequence, the response of the confined aquifer to pumping or other perturbations is transient and leads to the time-dependent groundwater flow equation. More detailed discussion of hydraulic head, mass balance, and the time-dependent groundwater flow equation is provided in the GW-Project book by Woessner and Poeter (2020). The property of storage is, therefore, fundamental to the understanding of groundwater availability and movement. Groundwater storage is arguably second in importance only to Darcy's law in its centrality to hydrogeology.

This book takes a historical perspective of storage in confined aquifers. Benchmark papers, which span nearly half a century, weave together threads from hydrogeology, geomechanics, and petroleum engineering with binding stitches from mathematics and physics. The goal is to appreciate the concept of storage in a deeper sense than is obtained from its mere definition. The story begins with an examination of a paradox with regard to the origin of subsurface irrigation water in the Dakota Territory in the north-central portion of the continental United States. Important milestones were the field investigation of the hydrogeologic system (Darton, 1896, 1901, 1909), establishment of the connection between aquifer deformation and pore fluid withdrawal (Meinzer, 1928), mathematical solution of the head response to a pumping well by analogy to heat transport (Theis, 1935), and finally derivation of the time-dependent governing equation for groundwater movement in terms of aquifer and water compressibility "from scratch" (Jacob, 1940).

[1] In this book, "head" means "hydraulic head."

2 Saga of the Dakota Aquifer

The Homestead Act of 1862 opened the northernmost portion of the United States' Louisiana Purchase of 1803 to settlement. Homesteaders could purchase 160 acres (0.65 km^2) of land for $18 with the obligation that it be farmed for five years. The southern part of the Dakota Territory grew from about 10,000 people in 1870 to close to 100,000 in 1880 (Fabry, 2016). High rainfall in the 1870s led to the general belief that "rain follows the plow." However, the drought of 1886-1889 led to the drilling of many wells for irrigation. The famous artesian well in Woonsocket, South Dakota, USA (Book Cover) was drilled in 1888 and had an initial wellhead pressure of 250 psi (1.7 MPa) and flowed 8000 gal/min (30 m^3/min). The drought of 1886-1889 came to the attention of the United States Congress because of its implications for the future of farming in the new states of North and South Dakota. John Wesley Powell, second director of the United States Geological Survey two years after its founding, was called to testify.[2]

> "While the Dakota sandstone is one of the most important of the known artesian reservoirs, the amount of land which can be redeemed to agriculture through its aid is yet so small that disastrous results might follow if great expectations were aroused in regard to it." (Artesian Aquifers are discussed in the GW-Project book by Woessner and Poeter (2020)).
>
> "Such is the complexity of conditions and so great is the danger of disaster through expensive exploitation in ignorance of the true conditions that the subject demands the most skillful investigation which can be bestowed."
>
> – Powell testimony (1890)

Indeed, skillful investigations were carried out by N. H. Darton[3] (1896, 1901, 1909) and continued throughout the 20th century by pre-eminent United States Geological Survey (USGS) hydrogeologists. Among these, Bredehoeft et al. gave high praise to Darton in the opening paragraph of their 1983 paper.

> "The Dakota aquifer in South Dakota is one of the classic artesian aquifers. Many modern ideas concerning artesian aquifers stem from N. H. Darton's investigation of the Dakota aquifer in the 1890s and early 1900s. This paper is based to a large extent upon Darton's data and is a tribute to Darton's ability as a hydrologist."

Bredehoeft et al. (1983) incorporated several further quotes from Darton (1909, page 60). Of two included below, the first is directly part of this book's story and the second is parenthetical. The first is Darton's conceptualization of how outcrops in the Black Hills in the western part of South Dakota provided the driving potential (as discussed in the

[2] Powell led the famed 1869 expedition of the Colorado River and Grand Canyon. Wallace Stegner's (1954) Beyond the 100th Meridian and John F. Ross' (2018) The Promise of the Grand Canyon provide full accounts.

[3] "Upon meeting another geologist at an outcrop, the other man said 'I have made my guess as to what this formation is. What is your guess?' Darton replied, 'I never guess, I find the facts and I know.'"

GW-Project book by Woessner and Poeter (2020)) for flow to the east where the thousands of irrigation wells were completed.

> "The evidence of this pressure, as found in many wells in eastern South Dakota, is conclusive that the water flows underground for many hundreds of miles. Such pressures can be explained only by the hydrostatic influence of a column of water extending to a high altitude on the west. If it were not for the outflow of the water to the east and south the initial head which the waters derived from the high lands of the intake zone would continue under the entire region, but owing to this leakage the head is not maintained, and there is a gradual diminution toward the east known as 'hydraulic grade,' a slope sustained by the friction of the water in its passage through the strata."

Darton's "evidence" was contained in a map of the potentiometric surface of the Dakota aquifer system (Figure 2). From this map and T. C. Chamberlin's (1885) elucidation of geologic and physical principles that explained artesian conditions, Darton drew a cross-sectional model of the Dakota aquifer system (Figure 3), which is often included in textbooks on hydrogeology. The second quote by Bredehoeft et al. (1983) lamented that Darton's recognition of leakage through a confining layer was forgotten for many decades.

> "Another factor which undoubtedly somewhat influences the hydraulic grade in the Great Plains region is a certain but unknown amount of general leakage through the so-called impermeable strata, especially when under great pressure."

By 1923 some 10,000 wells were drilled in South Dakota and eventually 15,000 by 1958 (Davis et al., 1961). The original pressure in the Woonsocket well had declined to 130 psi (0.9 MPa) by 1892 and its flow reduced to 1150 gal/minute (4.3 m^3/min). By 1915 the pressure had declined further to 45 psi (0.3 MPa) and by 1923 to 35 psi (0.24 MPa) (Meinzer and Hard, 1925). With the passage of 35 years, Meinzer and Hard reflected on the prescience of Powell's testimony of 1890.

> "It is exceedingly interesting and gratifying to note that in March, 1890, when the excitement over the artesian wells must have been about at its maximum, Maj. J. W. Powell, Director of the United States Geological Survey made a statement on the subject before the Committee on Irrigation of the House of Representatives which must have seemed unduly conservative at that time but which clearly indicated the temporary character of the high pressures and discharges and gave an estimate of permanent yield that appears remarkably accurate after 34 years of artesian development and decline."

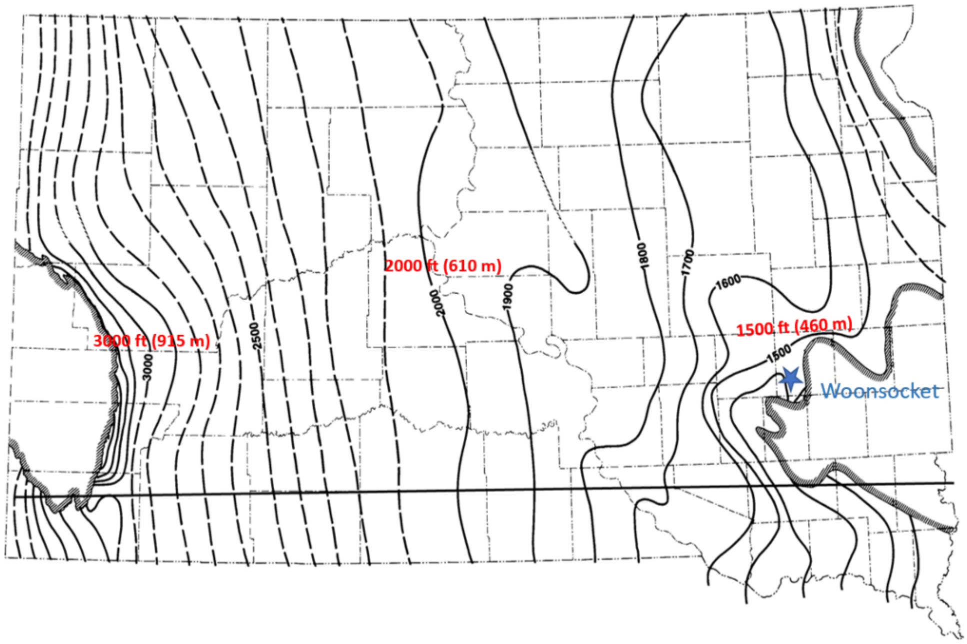

Figure 2 - Dakota aquifer's initial potentiometric surface (ft). The city of Woonsocket (elevation = 1307 ft (400 m)) is shown as a star. Shaded contours show recharge in the Black Hills in southwestern South Dakota and discharge to wells in southeastern South Dakota, respectively. The horizontal line is a profile of the cross section in Figure 2 below (reproduced from Bredehoeft et al., 1983; original from Darton, 1909).

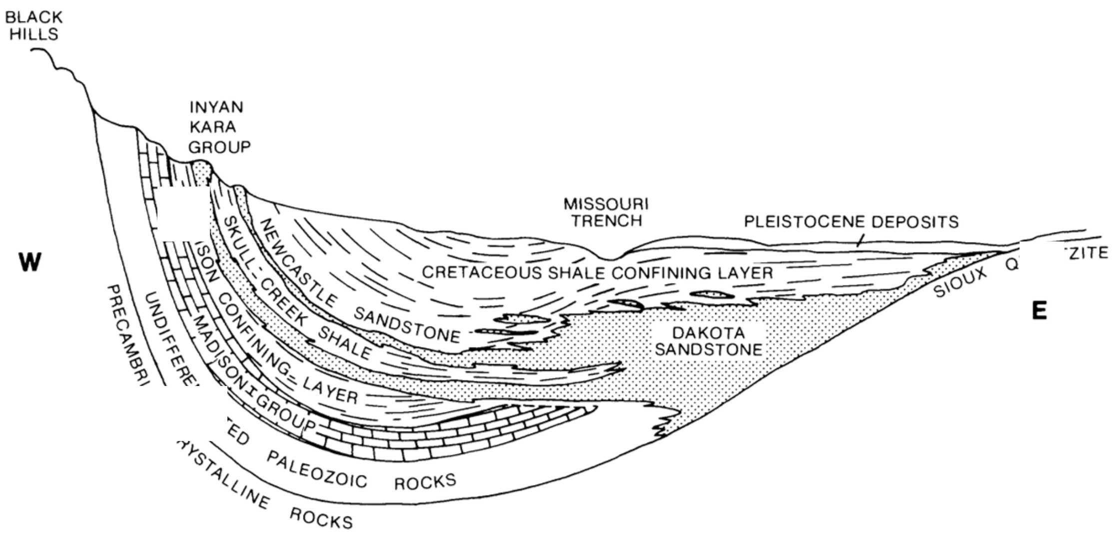

Figure 3 - West-East cross section showing the Dakota aquifer system (from Bredehoeft et al., 1983).

Although Meinzer and Hard's report primarily summarized numerous surveys and measurements of pressures and flows in irrigation wells over time, a four-page section, "Withdrawal of stored water and compression of the Dakota Sandstone," planted the seed of Meinzer's 1928 cornerstone paper "Compressibility and Elasticity of Artesian Aquifers." The difficulty Meinzer addressed in that paper was a mass balance problem.

1. Between 1886 and 1923, the average groundwater withdrawal was 3000 gallons/minute[4] (gallons/minute or gpm) (11.4 m^3/min) from a row of 18 townships (R65 – R48W), where a township is 6 miles x 6 miles.
2. But steady-state lateral flow through a representative cross section of the aquifer based on Darcy's law, $Q = KiA$, brings to the townships only 500 gal/min (1.9 m^3/min). Darcy's Law is discussed in the GW-Project book by Woessner and Poeter (2020).
   - $K = 6.25 \times 10^{-4}$ ft/s (1.9×10^{-4} m/s) (hydraulic conductivity)
   - i = 5 ft/mile (~1m/km) based on the potentiometric map (hydraulic gradient from Figure 1)
   - A = 6 miles (~10,000 m) x 60 ft (~20 m) (representative cross-sectional area of the aquifer is one township wide times thickness of the aquifer)
   - $Q = KiA = (6.25 \times 10^{-4}$ ft/s) (5 ft/ mi) (6 mi * 60 ft) (60 s/min) (7.5 gal/ft^3) = 500 gallons/minute (~2000 liters/minute) (mean west-to-east discharge transecting a north-south boundary of a township)

Calculations (1) and (2) above leave the problem of finding approximately 2500 gal/min (9.5 m^3/min), which is the difference between the extraction rate of 3000 gal/min (11.4 m^3/min) and the cross-sectional flow rate of 500 gal/min (1.9 m^3/min). Meinzer concluded that the excess production must be drawn from preexisting connate water stored in the pores of the aquifer. Meinzer drew circumstantial evidence for this behavior from a variety of hydromechanical observations:

1. F.H. King (1892, pages 67-69) wrote that "One of the surprising observations made during this study is that a heavily loaded moving train has the power of disturbing the level of water in the non-capillary spaces of the soil, but in just what manner this is brought about is not easy to see." (The water level response is shown in Figure 4 of this book following this discussion of evidence.) … "The strongest rises in the level of the water are produced by the heavily loaded trains which move rather slowly. A single engine has never been observed to leave a record, and the rapidly moving passenger trains produce only a slight movement, or none at all."
2. Terzaghi (1925) conducted laboratory experiments in which porosity (void volume divided by total volume) of a pre-compacted sand was measured in response to changes of axial stress (force per unit area) (The pressure response is shown in Figure 5 of this book following this discussion of evidence.) The sample went through several loading and unloading cycles. Although unloading produced permanent compaction, reloading followed the initial

[4] In actuality, the extraction rate declined from 10,000 gpm in 1910 to 5,000 in 1915 to 2,000 in 1920 to 1000 in 1923.

trend after the previous cycle's axial stress was reached. Relevant to Meinzer was that porosity reduction between points *a* and *b* in Figure 5 was similar to that needed to account for the missing volume of water in his calculation.

3. Schureman (1926) presented data that showed the water levels in an 800-ft deep well in Longport, New Jersey were in phase with ocean tides recorded at Atlantic City seven miles to the northeast. The two locations are on a barrier island that is about 1000 feet (~305 m) wide in the Atlantic Ocean off the east coast of the United States (The tide and well water level responses are shown in Figure 6 of this book following this discussion of evidence.) The correlation had to be due to a mechanical loading effect because 300 feet (90 m) of intervening clays meant that water could not possibly communicate directly with the deep sand (Thompson, 1926).
4. Pratt and Johnson (1926) attributed land subsidence at Goose Creek in Galveston Bay to oil, gas, and water extraction from rock pores in the underlying formations (The subsidence is shown in Figure 7 of this book following this discussion of evidence.) In the first three instances above, an applied mechanical load produced a fluid pressure response associated with the reduction of pore volume. Subsidence at Goose Creek showed the converse was also true, namely, that fluid extraction could lead to loss of pore volume.

Meinzer concluded that the difference between extraction volume and recharge volume could be resolved by a decrease in aquifer volume. The change in aquifer volume was considered to occur only through vertical consolidation, $\Delta V_{pore}/V_{pore}=\Delta b/b$, where Δ indicates the change in, or the difference between, the pore volume before and after pumping, V_{pore} is the initial pore volume, and b is the initial aquifer thickness. Limiting deformation to one dimension is an idealization of a three-dimensional problem, but frequently invoked for aquifers of large areal extent. The assumption of zero lateral strain (strain is a measure of the relative change in a length, area, or volume) reduces the problem to just the vertical dimension with vertical strain defined to be $\varepsilon_v = -\,db/b$ = 4.4 in/60 ft = 0.6 % (Meinzer, 1928, page 281), where the sign convention for ε_v is that compression is considered to be positive. The vertical strain of 0.6 % meant that the required porosity decrease was from an initial 38.2 % to 37.6 %. With these definitions, Meinzer's line of reasoning implied that water displaced by 4.4 inches (0.11 m) of aquifer compression over the 648 square miles (1680 km^2) of the 18 townships would be sufficient to yield 2500 gal/min (9.5 m^3/min) for 38 years. Thus, "compressibility of artesian aquifers" in the title of Meinzer's paper solved the mass balance problem. It provided a clear exposition of the physical basis of groundwater storage in a confined aquifer.

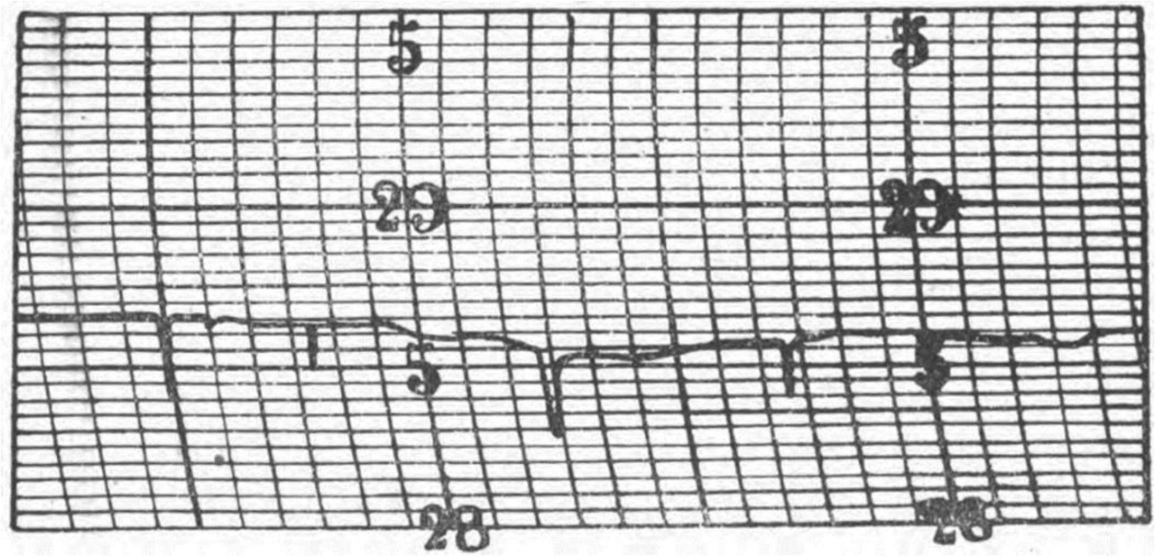

Figure 4 - Changes in water level (at a point about 40-ft (12-m) deep and 140 ft (43 m) from train tracks) induced by moving trains as measured by a float attached to a lever arm with a pen that rested on a vertically oriented cylinder which rotated with time at the top of the well to record a rise in water level as downward motion of the pen (King, 1892).

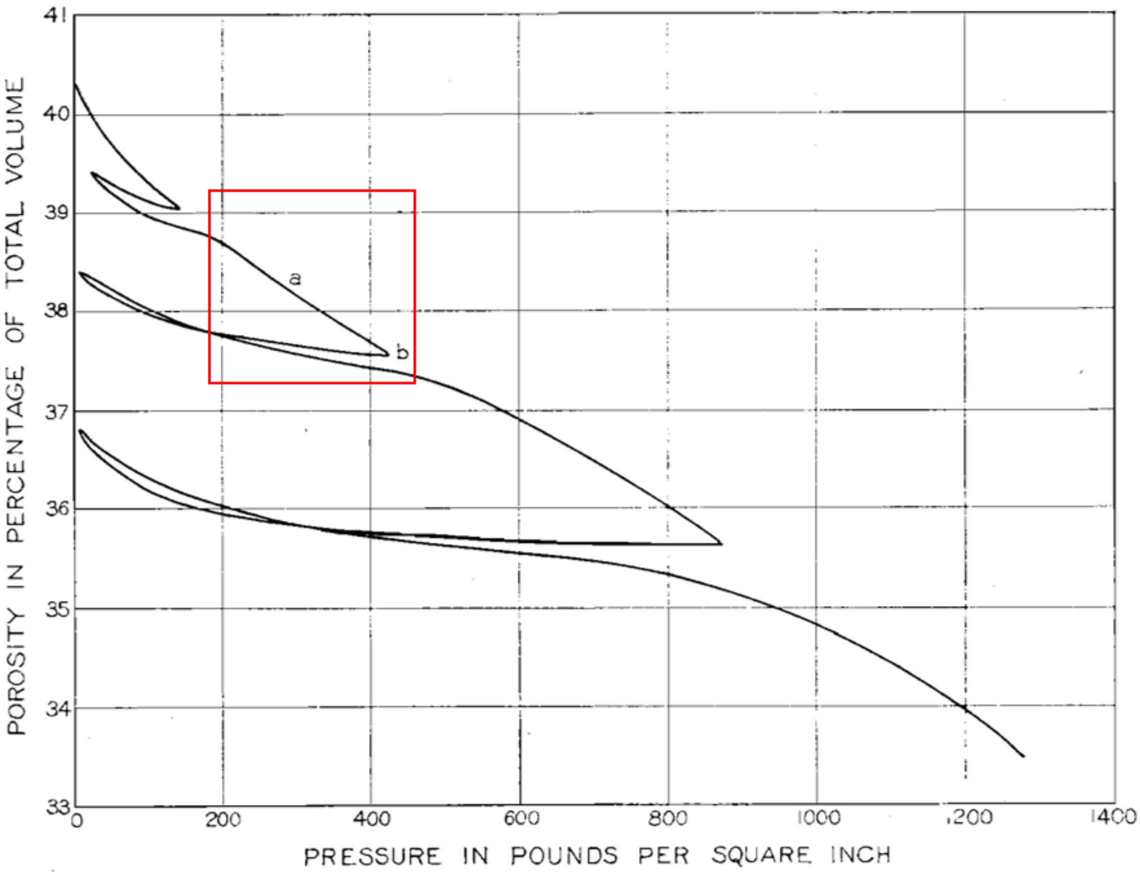

Figure 5 - Porosity reduction with increasing vertical stress ('pressure' in the label for the horizontal axis). The boxed area between points *a* and *b* corresponds approximately to the decline in fluid pressure in the Dakota aquifer (redrawn by Meinzer (1928) from Terzaghi (1925)).

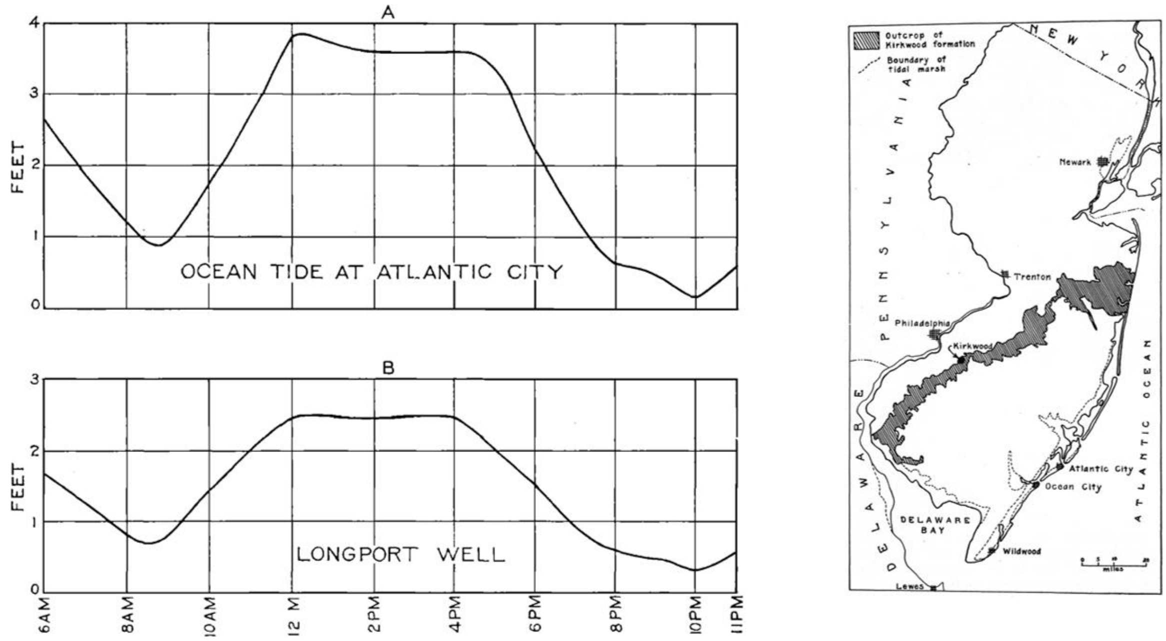

Figure 6 - Left: Ocean tides (Graph A) on January 22, 1926 measured at Atlantic City, New Jersey, 7 miles (11 km) northeast of where water levels (Graph B) were recorded in an 800-ft deep well at Longport, New Jersey (redrawn by Meinzer (1928) from Schureman (1926)). The in-phase response of well levels to ocean tides is evidence of mechanical loading because a lag time would be evident if the water levels were responding to fluid flow. Right: Map from Thompson (1926). The shaded band is the outcrop of the Kirkwood formation that was believed to include the sand at 800-ft depth in the Longport well.

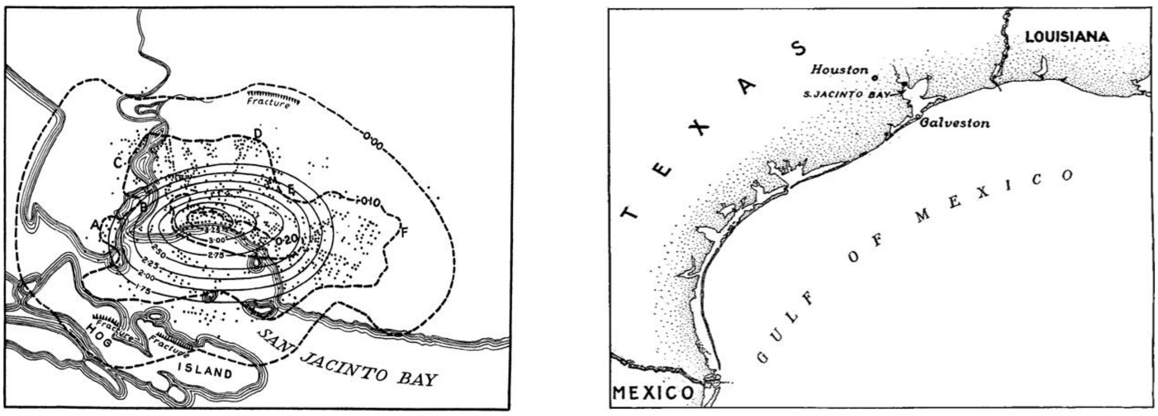

Figure 7 - Several feet of subsidence (a decrease in land elevation) occurred as a result of extraction of oil, gas, water, and sand from the Goose Creek Oil Field on Galveston Bay (Pratt and Johnson, 1926). Left: Closely-spaced, elliptically-shaped contours are for 8-year period; dashed, irregularly-shaped contours are for one-year period. Individual wells are shown as dots. The State of Texas tried to claim ownership after the field was submerged because the State held title to land under water. Right: Map also from Pratt and Johnson (1926).

3 Theis Solution

The concept of storage is most frequently encountered in hydrogeology as the "*S*" parameter in Theis' (1935) solution. Theis conceptualized the well-drawdown problem in heat conduction terms, which he expressed in correspondence to his former college classmate, Clarence Lubin, who had become a mathematics professor at the University of Cincinnati (Freeze, 1985).

> "The flow of ground water has many analogies to the flow of heat by conduction. We have exact analogies in ground water theory for thermal gradient, thermal conductivity, and specific heat. I think a close approach to the solution of some of our problems are [sic] probably already worked out in the theory of heat conduction. Is this problem in radial flow worked out?: Given a plate of given constant thickness and with constant thermal characteristics at a uniform initial temperature to compute the temperatures thruout [sic] the plate at any time after the introduction of a sink kept at 0 temperature? And a more valuable one from our standpoint: Given the same plate under the same conditions to compute the temperatures after the introduction of a sink into which heat flows at a uniform rate? I forgot to say that the plate may be considered to have infinite areal extent."

Lubin provided Theis with the solution from Carslaw[5] (1921) according to Banks (2015) which Theis duly noted in his paper. Theis prominently stated the heat flow analogy, as he did in his earlier letter to Lubin.

> "Darcy's law is analogous to the law of the flow of heat by conduction, hydraulic pressure being analogous to temperature, pressure-gradient to thermal gradient, permeability to thermal conductivity, and *specific yield to specific heat* [italics added]. Therefore, the mathematical theory of heat-conduction developed by Fourier and subsequent writers is largely applicable to hydraulic theory. This analogy has been recognized, at least since the work of Slichter[6], but apparently no attempt has been made to introduce the function of time into the mathematics of ground-water hydrology."

> "In heat-conduction a specific amount of heat is lost concomitantly and instantaneously with fall in temperature. It appears probable, analogously, that *in elastic artesian aquifers a specific amount of water is discharged instantaneously from storage as the pressure falls* [italics added]."

For groundwater storage, Theis used the variable S, which he originally called the "specific yield" in the quote above and not to be confused with "specific yield" of an unconfined aquifer. Tellingly, the analogy to specific heat[7] is the only attribute Theis provided for S, in contrast to a short description of the physical meaning of the coefficient of transmissibility T. Theis in his 1935 paper provided no mechanistic insight to groundwater storage; it was a property inferred from the heat flow analogy. However, Theis later elaborated on S in an Author's Note added to a 1952 United States Geological Society (USGS) reprint of his 1935 paper.

> "The factor S in the equations given is called 'specific yield' in the text of the paper. Later consideration has shown it advisable to call this term the "coefficient of storage" of the aquifer and to define it as the quantity of water in cubic feet that is discharged from each

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[5] H.S. Carslaw and J. C. Jaeger's book "Conduction of Heat in Solids" (second edition) published by Clarendon Press in 1959, is a rich source of theory and analytical solutions applicable to quantitative hydrogeology.

[6] Slichter's history was summarized in 1987 by H. F. Wang in the article "Charles Sumner Slichter: An engineer in mathematician's clothing" *in* History of Geophysics, volume 3, The History of Hydrology, edited by Edward R. Landa and Simon Ince, pages 103-112.

[7] Theis later in his paper uses the term "specific heat per unit-volume," which is the analogue of "specific storage" defined as storage capacity per unit volume. Because "specific" generally refers to the per unit mass value of a quantity, "specific heat per unit-volume" is called "volumetric heat capacity."

> vertical prism of the aquifer with basal area equal to 1 square foot and height equal to that of the aquifer when the water level falls 1 foot."

Here, Theis backs away from using the term "specific yield" for a confined aquifer. Today, "storativity" is used synonymously with "coefficient of storage," and Theis' verbal definition is the one commonly provided in textbooks and illustrated for a confined aquifer in Figure 1.

Theis' definition of coefficient of storage is for two-dimensional radial flow. In three-dimensions, the analog of specific heat is specific storage, in which the amount of water removed from a representative elementary volume REV per unit head decline is normalized by the volume of the REV. For precision, the verbal definition will be translated into an equation, although most hydrogeology textbooks follow Theis and omit doing so. A key quantity to define is the increment of fluid content, ζ (Equation 1), which has its origin in soil mechanics and the theory of poroelasticity (Biot, 1941; Wang, 2000).

$$\zeta = \frac{\Delta V_w}{V} \tag{1}$$

where:

ΔV_W = volume of water added to or removed from storage in an REV (L^3)

V = volume of representative elemental volume (L^3)

The quantity ΔV_w is positive when water is added to the aquifer and negative when water is removed from the aquifer. The quantity ΔV_w represents a volume of water transported to or from an external source at a reference pressure, usually atmospheric pressure, because pressure gages typically measure the difference between the absolute pressure and atmospheric pressure ("gage pressure"). Thus, ΔV_w is an increment of water volume added to or removed from the aquifer, much as money might be added to or withdrawn from a bank account. The amount of water added to or removed from the aquifer is normalized by the volume V of the REV. Then translating Theis' words into an equation gives a mathematical definition for specific storage as shown in Equation 2a and 2b.

$$S_S = \frac{\zeta}{\Delta h} \tag{2a}$$

Specific storage S_S has units of inverse height, such as inverse meters, 1/m. The definition can also be expressed for a change in fluid pressure because $\Delta p = \rho_w g \Delta h$. Head ($h$) is defined and discussed in the GW-Project book by Woessner and Poeter (2020).

$$S_S = \rho_w g \frac{\zeta}{\Delta p} \tag{2b}$$

where:

ρ_w = water density (M/L^3)

g = acceleration due to gravity (L/T^2)

In two-dimensional radial flow, the storativity, S, for a confined aquifer of thickness b, is specific storage times aquifer thickness, $S=S_s b$, because storativity per Theis' definition is the amount of water "discharged from each vertical prism of the aquifer" of unit area per unit decline in head. Storativity is dimensionless as the dimension of S_s is inverse length.

This recounting of Theis' 1935 solution for drawdown due to a pumping well in a horizontal aquifer demonstrates the important role that the heat conduction analogy played in developing the concept of hydrogeologic storage. This analogy is both physical and mathematical (Table 1). Head/temperature are the driving potentials for flow of water/flow of heat according to Darcy's/Fourier's law. Specific storage (volume of water discharged from storage per unit aquifer volume per unit decline in head) is the analog of specific heat per unit-volume (addition of heat required to raise the temperature of a unit volume of material one degree). An interesting contrast between this analog pair is that the definition of specific storage is usually expressed as a double negative (*discharge per decline*) given the importance of extracting water from an aquifer, whereas the definition of specific heat is expressed as a double positive (*addition to raise*). Table 1 is not yet complete as the paired governing equations will be discussed in Section 5 on the diffusion equation.

Table 1 - Analogous quantities in groundwater flow and heat flow (Wang and Anderson, 1982; Wang, 2000; Anderson, 2007).

Groundwater Flow	Heat Flow
Fluid head, $h = p/(\rho_w g) + z$ [m]	Temperature, T [°K]
Water added to storage, ΔV_w [m^3]	Heat, ΔQ [J]
Groundwater flux, [m^3/m^2/s = m/s]	Heat flux, q [J/(m^{2}s) = W/m^2]
Hydraulic Conductivity, K [m/s]	Thermal Conductivity, K [J/(m °K s) = W/(m °K)]
Specific Storage, $S_s = S/b$ [1/m]	Specific heat capacity (per unit volume), ρc [J/(°K m^3)]
Hydraulic diffusivity, K/S_s [m^2/s]	Thermal diffusivity, $K/(\rho c)$ [m^2/s] (here K is thermal conductivity, see above)
Darcy's law, $q = -K\,(dh/dx)$	Fourier's law, $q = -K\,(dT/dx)$

4 Jacob's Compressibility Formula for Aquifer Storage

The next major advance in the understanding of hydrogeological storage was made by C. E. Jacob (Titus, 1973) in a 1940 paper in which he linked Theis' concept of storage as a property akin to heat capacity to Meinzer's analysis of water stored in the Dakota aquifer as being due to aquifer compressibility. As his acknowledgment[8] makes clear, Jacob shared ideas with his colleague, C. V. Theis, at the United States Geological Society (USGS). Jacob's goal was to derive the groundwater equivalent of the partial differential equation for

[8] "*The writer is greatly indebted to his colleagues in the Geological Survey for timely suggestions and criticism, and especially to C. V. Theis, as many of the ideas expressed herein have been taken from his personal communications to the writer.*"

time-dependent heat flow, and thereby place the mathematical description of groundwater flow on firmer physical ground than a plausible analogy.

> "The writer proposes to derive "from scratch" the fundamental differential equation governing the flow of water in an elastic artesian aquifer, considering in turn each of the assumptions that are necessary to the derivation of the equation."

Jacob computed the mass balance in a REV over a time step in which there was a change in pressure Δp. His derivation was based on three physical principles: (1) A fluid pressure decline equates to an effective increase in vertical stress. (2) A fluid pressure decline expels a water volume equal to the loss of porosity associated with aquifer compressibility. (3) A fluid pressure decline leads to volume expansion of water in the pores due to the compressibility of water itself.

1. The title of Meinzer's 1928 paper called attention to the role of "compressibility and elasticity of artesian aquifers." As evidence, he presented Terzaghi's experiment in which a vertical stress caused porosity loss in sandstone (Figure 5). The effect of lowering pore pressure is pictured to have the same effect as an equal increase in vertical stress (Figure 8). This concept is formally stated as the "Law of Effective Stress" as shown in Equation 3.

$$\varepsilon_v = \beta_v(\sigma_v - p) = \beta_v \, \sigma_e \quad (3)$$

where:

$\beta_v = \varepsilon_v/\sigma_v$ = vertical compressibility for $p = 0$ (drained conditions)

$\sigma_e = \sigma_v - p$ = effective vertical stress defined to be the difference between the vertical stress and the pore pressure

The sign convention is that compressive stress is positive.

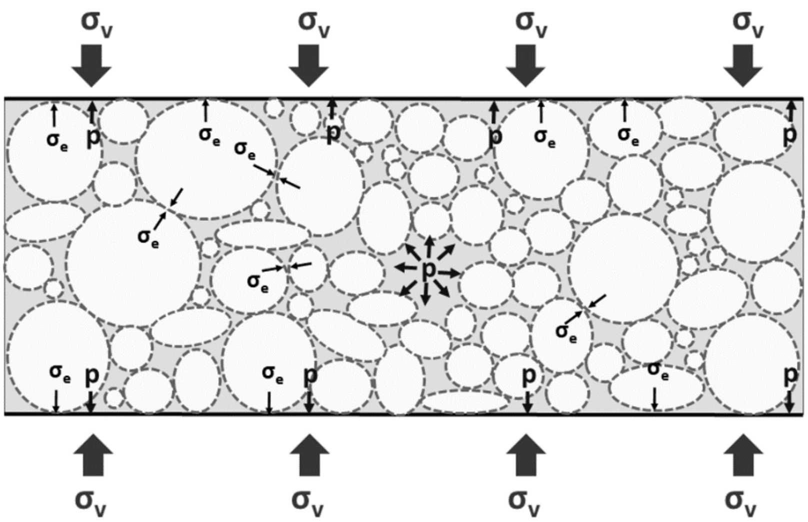

Figure 8 - The law of effective stress says that the effective vertical stress $\sigma_e = \sigma_v - p$ controls the vertical strain (after Atkinson, 2000).

2. Jacob reasoned that extraction of water from an aquifer does not change the total vertical stress. With total vertical stress constant, an increase in effective stress is equal but opposite to the decrease in fluid pressure, that is, $\Delta\sigma_e = -\Delta p$, so Equation 3 becomes Equation 4.

$$\Delta\varepsilon_v = -\beta_v \Delta p \tag{4}$$

The sign convention is that vertical strain and vertical stress are positive in compression. Given the assumption of zero lateral strain, the vertical strain is equal to the decrease in porosity, with the caveat that the compressibility of the solid grains is negligible. In other words, $\Delta\varepsilon_v$ is equal to the volume of water per unit volume of a REV that is removed from storage due to aquifer compressibility.

3. In addition, the expansion of the pore water volume in response to a fluid pressure decrease must be included in the water budget for a REV (Equation 5).

$$\frac{\Delta V_w}{V_w} = -\beta_w \Delta p \tag{5}$$

where:

β_w = Compressibility of water (4.5×10^{-10} Pa^{-1})
V_w = nV, where n is porosity and V is volume of the REV

By adding the two sources of water released from storage for a pore pressure decrease of $-\Delta p$, Jacob obtained an expression for coefficient of storage, S, that includes aquifer compressibility and water compressibility as well as aquifer thickness and porosity as shown in Equations 6a and 6b.

$$S = \rho_w g b(\beta_v + n\beta_w) \tag{6a}$$

Dividing by aquifer thickness gives specific storage.

$$S_s = \rho_w g(\beta_v + n\beta_w) \tag{6b}$$

The international system of units (SI) units of S_s are 1/m as indicated for Equation 2.

Jacob then obtained from mass conservation the partial differential equation for radial flow in an aquifer of thickness b that was identical in form to what Theis inferred by analogy with heat conduction. The difference was that the coefficient of storage S was expressed in terms of compressibility rather than as a quantity defined by analogy.

Terzaghi's experiment (Figure 5 and Equation 6b) can be used to estimate the specific storage of loose sand. The vertical compressibility βv between points a and b is calculated from the slope to be 7×10^{-10} Pa^{-1} after converting from English units. The term $n\beta_w = 2 \times 10^{-10}$ Pa^{-1} for a porosity of 38 %. Adding the terms and multiplying by the factor $\rho_w g$ gives a specific storage of 2.3×10^{-4} m^{-1} and the ratio of sand-to-water compressibility is 3.5. Values of specific storage are dependent on rock type as well as variability within a lithology. With those caveats, Table 2 provides order-of-magnitude values for a small set of geologic materials. The amount of water obtained for irrigation from the Dakota aquifer makes clear that large volumes of fluid can be stored in highly compressible earth materials. Nevertheless, the specific storage values in Table 2 are orders of magnitude smaller than specific yield of unconfined aquifers whose values are the aquifer porosity.

Table 2 - Rock compressibility and specific storage of a few geologic materials (Palciauskas and Domenico, 1989). The rock compressibility and specific storage values are for isotropic confining stress, not vertical stress. The values come from a variety of sources. The limestone measurements are for barometric or tidal loadings. The other values came from handbooks and were calculated with assumptions. Specific storage in terms of head was computed from its value in terms of pressure using $\rho_w = 1000$ kg/m^3 and $g = 9.8$ m/s^2. For comparison, compressibility of water $\beta_w = 4.5 \times 10^{-10}$ Pa^{-1}.

Geologic material	**Rock compressibility** β, 10^{-10} Pa^{-1}	**Specific storage in terms of pressure** $S_S/(\rho_W g)$, 10^{-10} Pa^{-1}	**Specific storage in terms of head** S_S, 10^{-6} m^{-1}
Clay	160	162	159
Mudstone	4.6	5.4	5.3
Kayenta Sandstone	1.1	1.2	1.2
Limestone	0.3	0.95	0.93
Hanford Basalt	0.22	0.44	0.43

In hydrogeology, aquifer compressibility is typically more significant than water compressibility. On the other hand, petroleum engineers generally considered oil reservoirs to be incompressible because they are at greater depth. Jacob examined this assumption for the East Texas oilfield described by Muskat (1937) in his classic treatise *Flow of Homogeneous Fluids through Porous Media*. The field produced 500 million barrels (80 million m^3) of oil with a pressure drop of 375 psi (2.6 MPa). By assuming a rigid reservoir, Muskat required that the oil in place had to contain sufficient dissolved gas to increase the fluid compressibility by a factor of 20 in order to produce this quantity from fluid compressibility alone, even though the oil was undersaturated. Jacob suggested instead that compressibility of the reservoir's Woodbine sand and associated clay beds would more likely account for the production.

It should be emphasized that the storage coefficients S and S_S in Equation 6a or 6b can be measured directly in the field from a pumping test or in the laboratory by adhering to its definition as the ratio of water volume removed from storage due to pore pressure change (Figure 9). However, this direct measurement is difficult to perform accurately because fluid storage in tubing connected to the rock sample's pore volume must be included in the accounting.

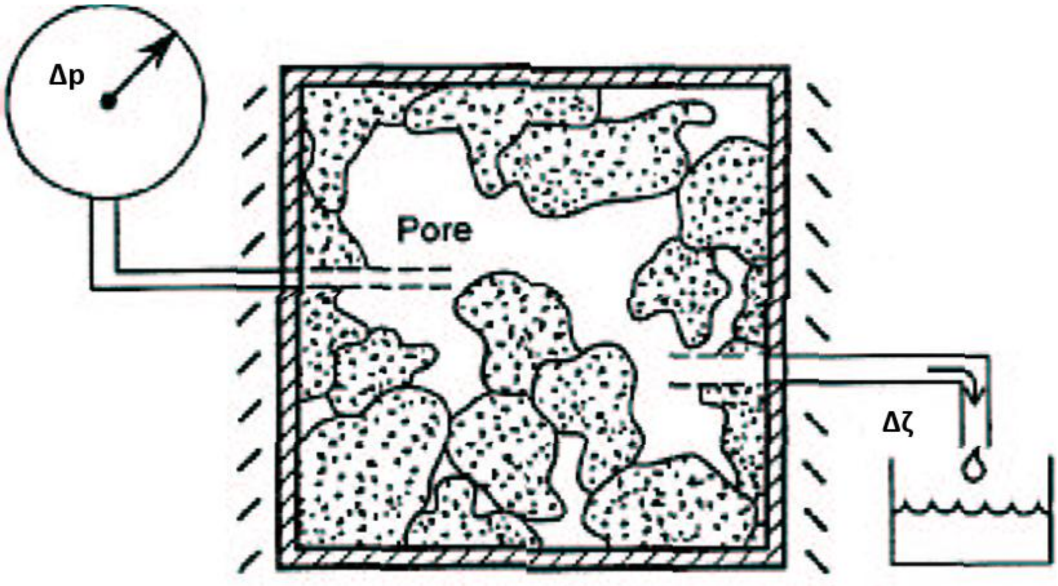

Figure 9 - Measurement of specific storage in terms of change in fluid pressure Equation 2b (Wang, 2000).

In addition to Equation 6a, Jacob derived equations for the water-level response to aquifer loading by water tides or changes in barometric pressure, also in terms of aquifer and water compressibility, which could provide an indirect measurement of coefficient of storage. The assumed large horizontal extent of the loading induces a fluid pressure in the aquifer because the fluid cannot escape as it is being loaded. In soil mechanics, the aquifer is said to be *undrained*. In tidal or barometric loading, the vertical stress is decidedly *not constant*. Water can be added to or removed from storage when vertical stress is applied in addition to when pore pressure changes (compare with Equation 2b). Therefore, in general, the increment of fluid content must be expressed in terms of changes in both vertical stress and pore pressure. The simplest form for an equation is to consider increment of fluid content to be a linear function of both variables.

$$\zeta = -\beta_v \Delta\sigma_v + \rho_w g S_s \Delta p \qquad (7a)$$

The sign convention is that ζ is positive when water is added to the REV and $\Delta\sigma_v$ is positive when the REV is compressed. The first term on the right in Equation 7a is the increment of fluid content associated with a change in vertical stress *when there is no change in fluid pressure.* Jacob made the assumption that the volume of water in storage in a REV decreases by the same amount as does the volume of the REV itself, that is, the change of water in storage is the negative of the strain, which is why the coefficient of $\Delta\sigma_v$ in Equation 7a is β_v. The second term on the right in Equation 7a is the increment of fluid content associated with a change in fluid pressure *when there is no change in vertical stress*, which is precisely the definition of specific storage (compare with Equation 2).

Equation 7a is one of two basic constitutive equations of poroelasticity for the special case of areally extensive, vertical loading (Wang, 2000). The other constitutive equation linearly relates vertical strain to changes in vertical stress and pore pressure.

$$\Delta\varepsilon_v = \beta_v \Delta\sigma_v - \beta_v \Delta p \tag{7b}$$

Equation 7b gives the vertical strain as the sum of two terms. The first term states that the vertical strain is the vertical strain due to a change in vertical stress *when there is no change in fluid pressure* and the second term is the vertical strain associated with a change in fluid pressure *when there is no change in vertical stress*. The coefficient of proportionality is β_v for both terms by the law of effective stress, i.e., Equation 7b is simply a restatement of Equation 3.

The in-phase response of water levels in an aquifer to ocean tides (Figure 6) was cited by Meinzer as evidence of aquifer elasticity. The ratio of the increase in water level in a well to the increase in ocean level, is called tidal efficiency, T.E. This ratio is equal to the ratio of $\Delta p/\Delta\sigma_v$. Assuming no water enters or leaves the aquifer by virtue of the large areal extent of the loading and inserting the undrained condition, $\zeta=0$, in Equation 7a gives the undrained pore pressure response to be $\Delta p=\beta_v\sigma_v/(\rho_w g S_S)$. Then substituting Equation 6b for S_S yields Equation 8.

$$T.E. = \frac{\beta_v}{\beta_v + n\beta_w} \tag{8}$$

Barometric efficiency, B.E., is similarly defined to be the ratio of the increase in water level to the increase in vertical stress (expressed as an equivalent head increase). A difference from the tidal efficiency in terms of well response is that a change Δp in atmospheric pressure directly changes the water level in the well by $-\Delta p/\rho_w g$. This must be added to the amount induced in the aquifer by the atmosphere loading as in the tidal loading case. Thus, $B.E. = 1 - T.E.$ and can be expressed as Equation 9.

$$B.E. = \frac{n\beta_w}{\beta_v + n\beta_w} \tag{9}$$

The expressions for tidal efficiency and barometric efficiency contain the same aquifer properties as specific storage, that is, vertical compressibility, water compressibility, and porosity. Measuring *T.E.* or *B.E.* and assuming n and β_W to be knowns means that β_V, and hence, S_S can be obtained from Equation 8 or 9 and the ratio of the contribution from water compressibility, $n\beta_W$, to aquifer compressibility, β_V, can be calculated.

Jacob (1941) used tidal efficiency to compute indirectly the coefficient of storage because it was more easily determined than the barometric efficiency. He compared the relative contributions of aquifer elasticity and water compressibility from tidal responses (e.g., Figure 6) with those from pumping tests at depths between 715 and 800 ft in the Lloyd sand on Long Island of the United States. The ratio of aquifer compressibility to water compressibility obtained from tidal efficiency was 1.7 whereas it was 2.8 from pumping test analysis.

5 Diffusion Equation

The study of heat diffusion has a long and storied history in mathematical physics beginning with Fourier in 1822. The rate of heat transport through a material depends, of course, on thermal conductivity, but also storage, because the rate of temperature change in a REV depends on how much heat must be transported in order to change its temperature. For the same thermal conductivity, heat diffusion is slower for a high heat capacity than for a low heat capacity. Thus, heat capacity is something of a foil to thermal conductivity. This inverse role of storage to conductivity in determining the rate of diffusion is captured by taking their ratio. In heat transport, it is called thermal diffusivity, $\kappa = K/(\rho c)$, where K is thermal conductivity, ρ is density, c is the specific heat capacity, and ρc is specific heat per unit volume. The groundwater analog is that $\kappa = T/S$ for two-dimensional, horizontal flow; and $\kappa = K/S_S$ for one-dimensional linear flow. All diffusivities have units of m^2/s. The definitions of hydraulic and thermal diffusivity are shown in Table 3, which is a continuation of Table 1. Theis did not include the diffusion equation in his 1935 paper, probably because it was so well known in mathematical physics, and so it is included in Table 3. Theis did, however, present Lubin's derivation of the thermal solution analogous to a pumping well. The derivation started with the solution for the "instantaneous line-source coinciding with the axis of z of strength Q"[9] (Table 3). The analogous solution for head, h, is the same if Q is interpreted to be a slug of water added to a well per unit length.

Table 3 - Diffusion equation and Theis' solution for groundwater flow and heat flow.

	Groundwater Flow	**Heat Flow**
Diffusivity (κ)	Hydraulic diffusivity K/S_S [m^2/s]	Thermal diffusivity $K/(\rho c)$ [m^2/s]
Governing equation for planar flow	$\kappa\left(\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}\right)=\frac{\partial h}{\partial t}$	$\kappa\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)=\frac{\partial T}{\partial t}$
Line source solution	$h=\frac{Q}{4\pi St}e^{-(x^2+y^2)/4\kappa t}$	$T=\frac{Q}{4\pi Kt}e^{-(x^2+y^2)/4\kappa t}$
Line source definition	Q is the volume of water per unit length added to the aquifer in a line at the origin	Q is the quantity of heat per unit length added to the plate in a line at the origin
Approximate propagation time to x	$t = x^2/(2\kappa)$	

[9] The "Q" in Table 3 differs from Theis' because his definition "Q" included dividing by the specific heat per unit-volume rather than incorporating it explicitly as is done here. Also, Theis, following Carslaw, used "v" instead of "T" for temperature.

The one-to-one correspondence between columns in Tables 1 and 3 is the physical and mathematical analogy Theis exploited. Both temperature and head are governed by the diffusion equation. In two-dimensional radial flow, a high transmissivity means that fluid is transported quickly in space. On the other hand, high storativity delays the movement because more fluid must be moved into or out of storage to effect a given head change. At an extreme, if an aquifer system has zero storage, then any change of internal or external boundary conditions in flow or pressure will be rapidly accommodated by a new steady state. Hydraulic diffusivity determines how fast a disturbance, such as suddenly injecting a slug into a well or starting to pump a well or changing a boundary condition, will propagate through an aquifer. The nature of diffusion is that an initial spike in head spreads out and diminishes in amplitude. Diffusion "flattens the gradient" so that the system's approach to steady state slows with time (Figure 10a). Meanwhile, at any point away from the source, the disturbance rises with time, peaks, and then decays (Figure 10b). The time of arrival of the peak occurs at $t = x^2/(2\kappa)$ in the case of one-dimensional linear flow. This value of t is a characteristic time for how long it takes for a sudden change at the origin to decay and spread out to a distance x. In the example in Figure 10b, it takes about one hour for the peak to arrive at 200-meters distance for a diffusivity of 6.8 m^2/s.

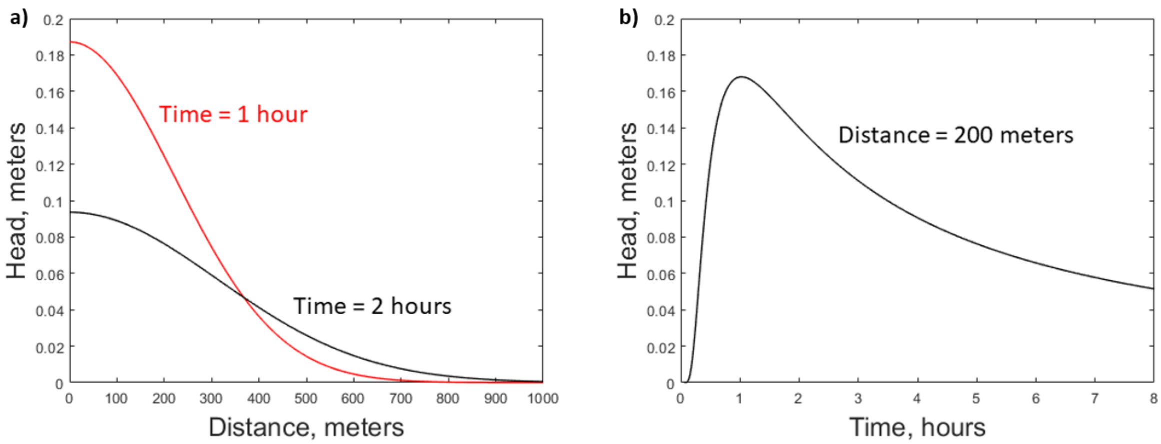

Figure 10 - Solutions for an instantaneous slug injected into a well for an aquifer with hydraulic diffusivity of 6.8 m^2/s. a) Profiles at different times showing how hydraulic diffusion flattens the gradient. b) Time history showing a delay of approximately one hour for the peak head to arrive at 200-meter distance.

6 Summary

The recognition of aquifer storage and its physical basis was key to solving two problems in the understanding of the origin and movement of groundwater. Meinzer (1928) demonstrated that aquifer compressibility and loss of pore volume from pore pressure decline accounted for about 80 % of the water consumed by irrigation in South Dakota over a 40-year period around the turn of the 20th century. Theis (1935) invoked the property of storage by analogy with heat transport to solve the problem of time-dependent change in head due to pumping a well. Jacob (1940) derived the groundwater flow equation from scratch by explicitly considering the physical basis for storage in terms of aquifer compressibility and water compressibility. This narrative in three acts transitioned the understanding of groundwater flow from steady state to "non-equilibrium," transient behavior.

7 Exercises

Exercise 1

Singha (2008) provides an active learning exercise↗ for introducing the concept of groundwater extraction from confined aquifers. This exercise uses a juice container as a simple analog for a confined pore space, and demonstrates how a decrease in fluid pressure from pumping causes an increase in effective stress assuming a constant total stress.

In brief, a bag of juice is used to represent a pore in a confined aquifer. The experimenter is asked to apply a downward force on the top of the juice container to represent the weight of the overburden material and maintain this constant force as they proceed. This force represents the total stress. The experimenter thinks about: 1) why the container does not collapse in response to the force; and 2) what will happen with a straw is inserted in the container. Then the experimenter inserts a straw (equivalent to drilling the confined aquifer). Next, while maintaining the force, they "pump" the aquifer by sipping juice from the straw. The experimenter will notice that the volume of the juice container decreases and is asked to consider what is happening in the "aquifer." The experimenter will see that the container does not collapse prior to pumping when the overburden force is applied because the effective stress of the aquifer (the container), and the fluid pressure in the pore (the juice) push against weight of the overburden (their hand). Depending on the magnitude of the overburden force and the length of the straw, it may be that juice squirts out when the straw is inserted which is analogous to a flowing artesian well. If the force is not as large, the juice will rise in the straw above the container, but will not flow out of the straw. Singha (2008) provides more details.

Exercise 2

The goal of this experiment is to measure the specific storage S_S of a balloon, which simulates aquifer elasticity. The experiment is designed to provide observational meaning to the variable, increment of fluid content, and the influence of the state of stress on the specific storage.

In short, a balloon is stretched over the end of a burette clamped to a meter stick and attached to a tall ring stand. The balloon and burette are filled with a known volume of water to an arbitrary height on the meter stick such that the volume in the balloon can be determined. Then a measured volume of water is added and, assuming the compressibility of water and the burette are small, the volume of water must go into either the balloon or the burette. The volume entering the balloon can be determined by knowing the volume that went into the burette as determined from the water level in the burette. The three-dimensional storage properties of the balloon are then determined by adding water and noting the head change in the burette. This can be repeated a number of times to determine if the storage value is constant. Water can be removed and the process repeated to determine if the balloon is elastic. Finally, the experiment can be repeated with the balloon laterally confined in a plexiglass tube to obtain the one-dimensional specific storage (S_s) of the balloon which can be compared with the measurement in the case of three-dimensional expansion of the balloon.

The details of the exercise (including diagrams) as well, as information about using the exercise to teach the concept, are available on the Science Education Resource Center at Carleton College website↗. A pdf of the laboratory exercise titled "Aquifer Elasticity and Specific Storage is available here↗. The underlying theory is discussed in this book as well as in a textbook chapter by Herb F. Wang that is available on the same website (textbook chapter↗).

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About the Author

Dr. Herb Wang is Professor Emeritus of Geoscience at the University of Wisconsin-Madison, where he joined the faculty in 1972. His research in poroelasticity deals with the interaction between stress and pore fluid pressure. His recent research in this area has been in the context of Enhanced Geothermal Systems (EGS). His other recent research is in Distributed Acoustic Sensing (DAS) in which fiber-optic cable is the sensor for seismic waves. Dr. Wang's teaching has included hydrogeology, groundwater modeling, tectonophysics, rock mechanics, and environmental justice. He has been faculty advisor to 15 Ph.D. students, and he has published books on groundwater modeling and poroelasticity (see References above) and approximately 100 research articles. He received the 2003 Distinguished Teaching award from Wisconsin's Phi Beta Kappa alpha chapter and the 2004 Distinguished Faculty award from the Department of Geoscience Board of Visitors.

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