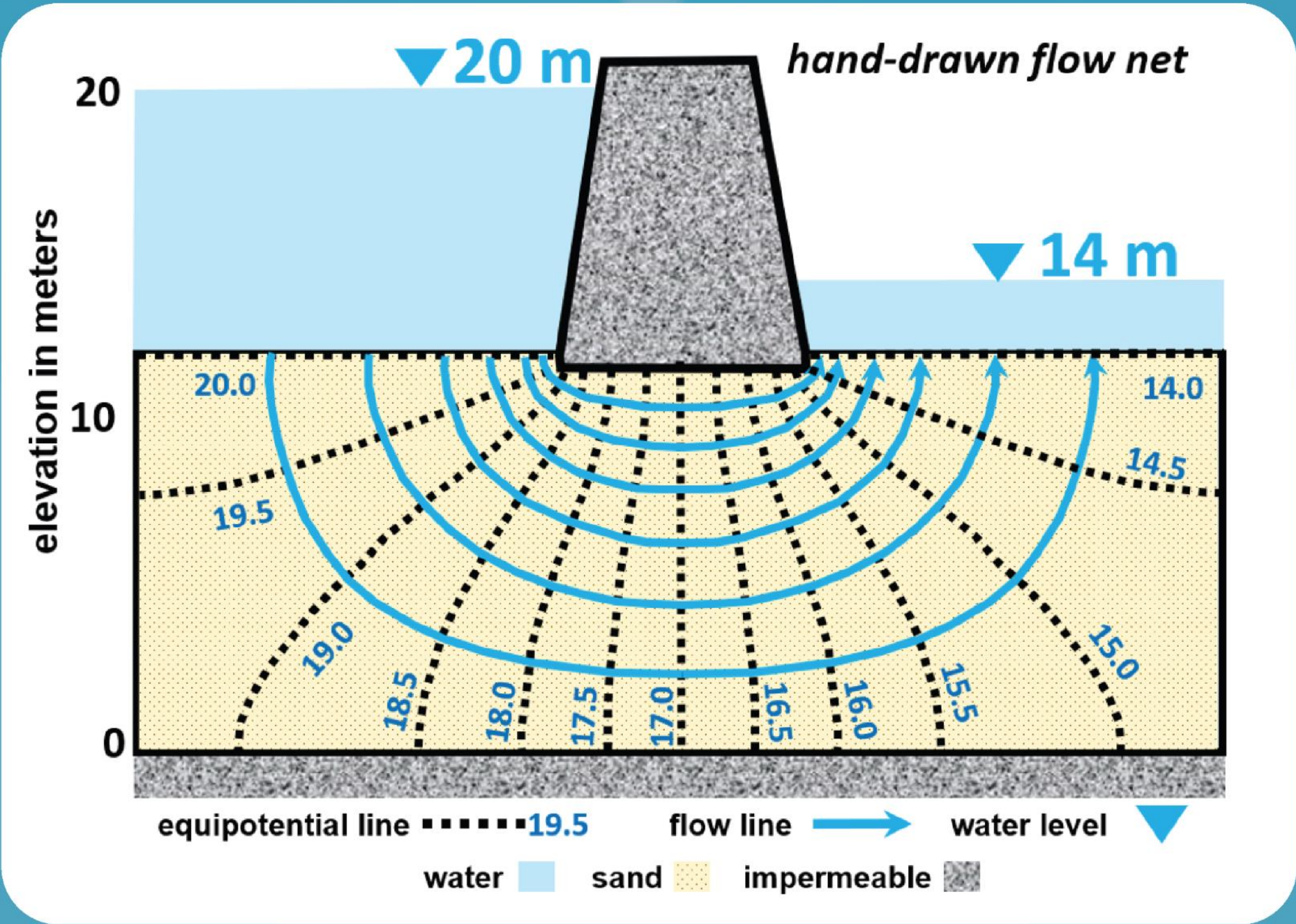


# Graphical Construction of Groundwater Flow Nets

Eileen Poeter and Paul Hsieh

THE GROUNDWATER PROJECT

# Graphical Construction of Groundwater Flow Nets

The Groundwater Project



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The Groundwater Project
Guelph, Ontario, Canada



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# The Groundwater Project Foreword

The United Nations Water Members and Partners establish their annual theme a few years in advance. The theme for World Water Day of March 22, 2022, is "Groundwater: making the invisible visible". This is most appropriate for the debut of the first Groundwater Project (GW-Project) books in 2020, which have the goal of making groundwater visible.

The GW-Project, a non-profit organization registered in Canada in 2019, is committed to contribute to advancement in education and brings a new approach to the creation and dissemination of knowledge for understanding and problem solving. The GW-Project operates the website https://gw-project.org/↗ as a global platform for the democratization of groundwater knowledge and is founded on the principle that:

*"Knowledge should be free and the best knowledge should be free knowledge." Anonymous*

The mission of the GW-Project is to provide accessible, engaging, high-quality, educational materials, free-of-charge online in many languages, to all who want to learn about groundwater and understand how groundwater relates to and sustains ecological systems and humanity. This is a new type of global educational endeavor in that it is based on volunteerism of professionals from different disciplines and includes academics, consultants and retirees. The GW-Project involves many hundreds of volunteers associated with more than 200 hundred organizations from over 14 countries and six continents, with growing participation.

The GW-Project is an on-going endeavor and will continue with hundreds of books being published online over the coming years, first in English and then in other languages, for downloading wherever the Internet is available. The GW-Project publications also include supporting materials such as videos, lectures, laboratory demonstrations, and learning tools in addition to providing, or linking to, public domain software for various groundwater applications supporting the educational process.

The GW-Project is a living entity, so subsequent editions of the books will be published from time to time. Users are invited to propose revisions.

We thank you for being part of the GW-Project Community. We hope to hear from you about your experience with using the books and related material. We welcome ideas and volunteers!

The GW-Project Steering Committee

August 2020



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# Foreword

The ability to understand and construct flow nets is an essential skill of a groundwater hydrologist. Although other books published by The Groundwater Project also discuss flow nets, the present book gives an in-depth treatment, in particular, how to graphically construct flow nets. Through this pencil-and-paper exercise, students can gain a deeper level of intuition and understanding of groundwater flow.

Many groundwater textbooks have been published in the past half century and nearly all of them draw attention to the importance of flow nets in groundwater investigations and each provides several pages about the manual method of constructing flow nets. In current professional practice, flow nets are produced quickly using readily available software after the user defines the boundary conditions and distribution of hydraulic conductivity. The challenge to those who are new to groundwater work is determining the boundary conditions and understanding the how the boundary conditions and hydraulic conductivities control the distribution of hydraulic head and flow lines. Therefore, it is important for aspiring groundwater professionals to develop an intuitive sense regarding the appearance of a flow net for a given system before generating a computer simulation of a flow net.

Intuition can be developed by creating flow nets using the "old-fashioned", pencil-and-paper method of hand-drawing flow nets. This book explains that method and presents videos showing laboratory representations of groundwater systems in Hele-Shaw parallel plates and sand box models with dye revealing the flow lines. In addition, an online interactive software tool is provided so readers have an opportunity to enhance their intuition of flow systems by creating flow nets in groundwater systems with complicated hydraulic conductivity distributions.

Eileen Poeter initiated this book and reached out to Paul Hsieh to co-author it with her. The end result is a book on creating and understanding flow nets prepared by two globally recognized experts. The book was improved through a comprehensive peer review process. Consequently, the material in this book presents the type of groundwater knowledge that supports better groundwater management and protection.

John Cherry, The Groundwater Project Leader
Guelph, Ontario, Canada, July 2020



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Eileen Poeter
Paul Hsieh



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# Graphical Construction of Groundwater Flow Nets

The Groundwater Project



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# 1 Introduction

It is not possible to see groundwater flowing through the subsurface of the earth, so the most valuable asset a groundwater hydrologist can attain is the ability to visualize the distribution of hydraulic head and flow paths in three-dimensional space. This comes easily to a few people, but many people need to immerse themselves in viewing and thinking about two-dimensional examples of steady-state flow in the form of flow nets before the visualization of flow systems becomes intuitive. The effort invested in mastering visualization of flow systems is worth the reward. Figure 1 provides a link to a video of dyed water moving through porous material. Flow nets can reveal the distribution of forces driving the flow and the flow paths.

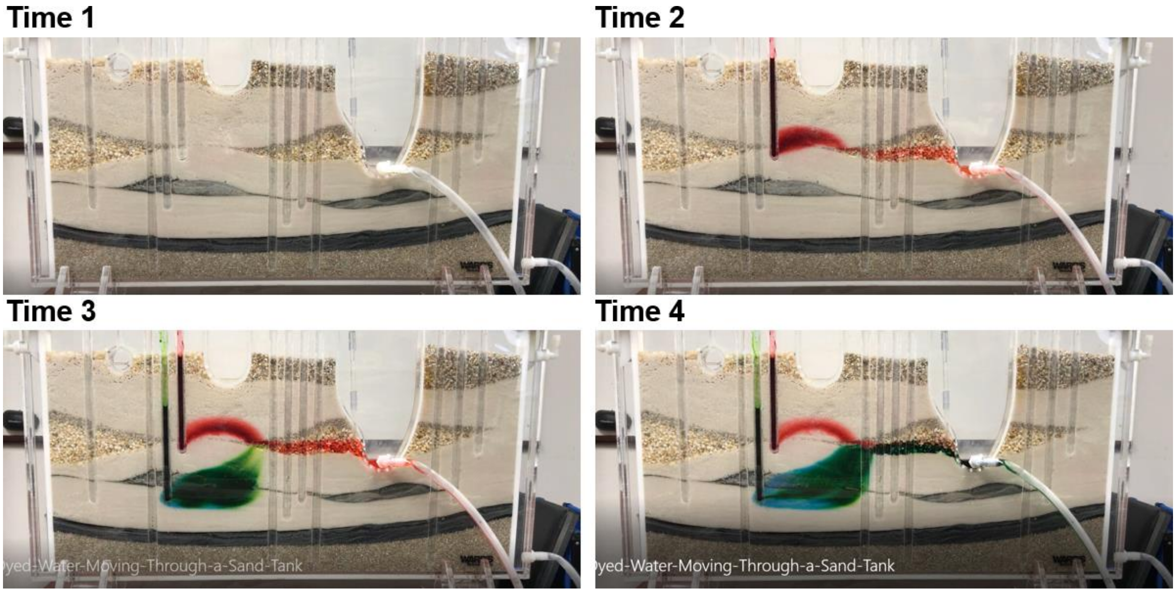


**Figure 1** - Click to view a video of dye moving through a sand tank. A few snapshots from the video are pictured here: Time 1 is before dye is introduced to the flow system, so one cannot see the flow lines; Time 2 is after red dye is injected in a shallow well and has migrated to the stream outlet; Time 3 is after green dye is injected into a deep well and has migrated upward and to the right; and, Time 4 is after the green dye has reached the gravel lens and then rapidly migrated to the stream outlet. Viewing the video elucidates the flow better than these snap shots. (Sand Tank constructed by Students at the University of Wisconsin, Stevens Point. Filmed at IGWMC of Colorado School of Mines. Edited and narrated by Eileen Poeter.)

Graphical flow net analysis is useful in every hydrogeologist's toolbox because it can: 1) provide a quick and useful understanding of flow systems when a computer is not available; 2) help a numerical model user assess their intuition regarding the results of a numerical groundwater flow model; 3) and facilitate understanding of basic concepts of groundwater flow for those new to groundwater work. However, mastery of basic numerical model construction is also a must for the modern groundwater hydrologist.

As shown in Figure 2, a flow net provides an image of equipotential lines (black dotted lines) and flow lines (blue arrows) in a groundwater flow system. This book explains graphical construction of groundwater flow nets. Before using flow nets to understand flow



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systems, it is best to begin by understanding graphical construction of two-dimensional, steady-state flow nets. It is useful to have knowledge of a few items pertinent to flow nets including: hydraulic head, boundary conditions, Darcy's Law, and the groundwater flow equations. Another Groundwater Project book↗ (Woessner and Poeter, 2020) discusses these items in detail, however brief reviews of these items are provided in this book.

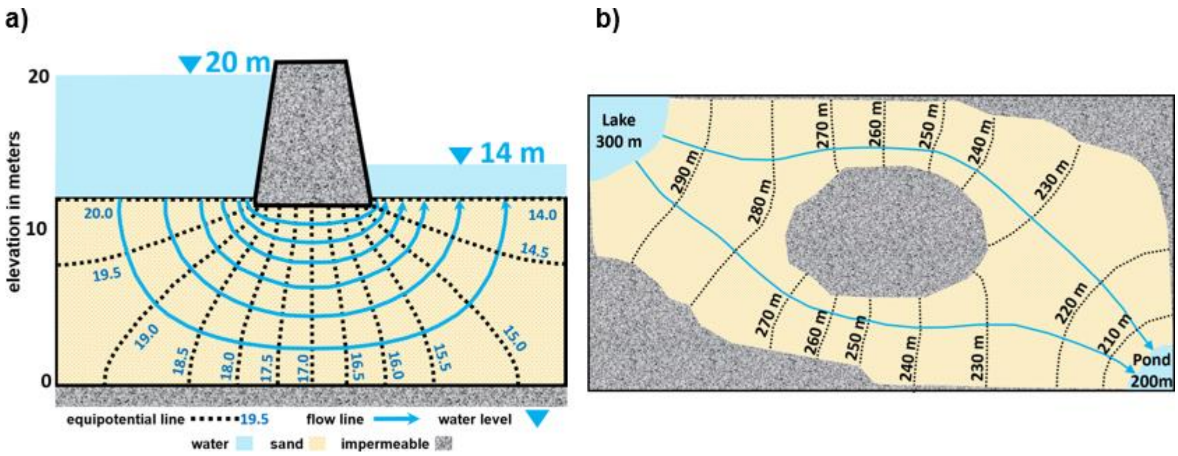


**Figure 2 -** Images of flow nets: a) groundwater moving in a vertical cross section under a dam; and, b) a plan view of groundwater flow from a lake to a pond surrounded by bedrock outcrops.

## 1.1 What is Graphical Construction of a Flow Net?

Graphical construction of a flow net is a method of using pencil and paper to obtain a solution to the steady-state, homogeneous, isotropic, groundwater flow equation. Steady flow is an equilibrium condition for which the hydraulic heads, flow rates and flow lines do not change with time, that is, inflow equals outflow. A flow net consists of two families of intersecting lines: equipotential lines, which connect locations of equal hydraulic head and flow lines that show paths of groundwater flow as shown in Figure 3. An impermeable dam is holding back a reservoir of water in Figure 3. Water seeps from the upstream reservoir into the underlying porous material then flows below the dam and seeps upward to the reservoir with a lower water level on the downstream side of the dam. The distribution of hydraulic head drives groundwater flow along groundwater flow lines. A brief review of hydraulic head is provided in Box 1↴.



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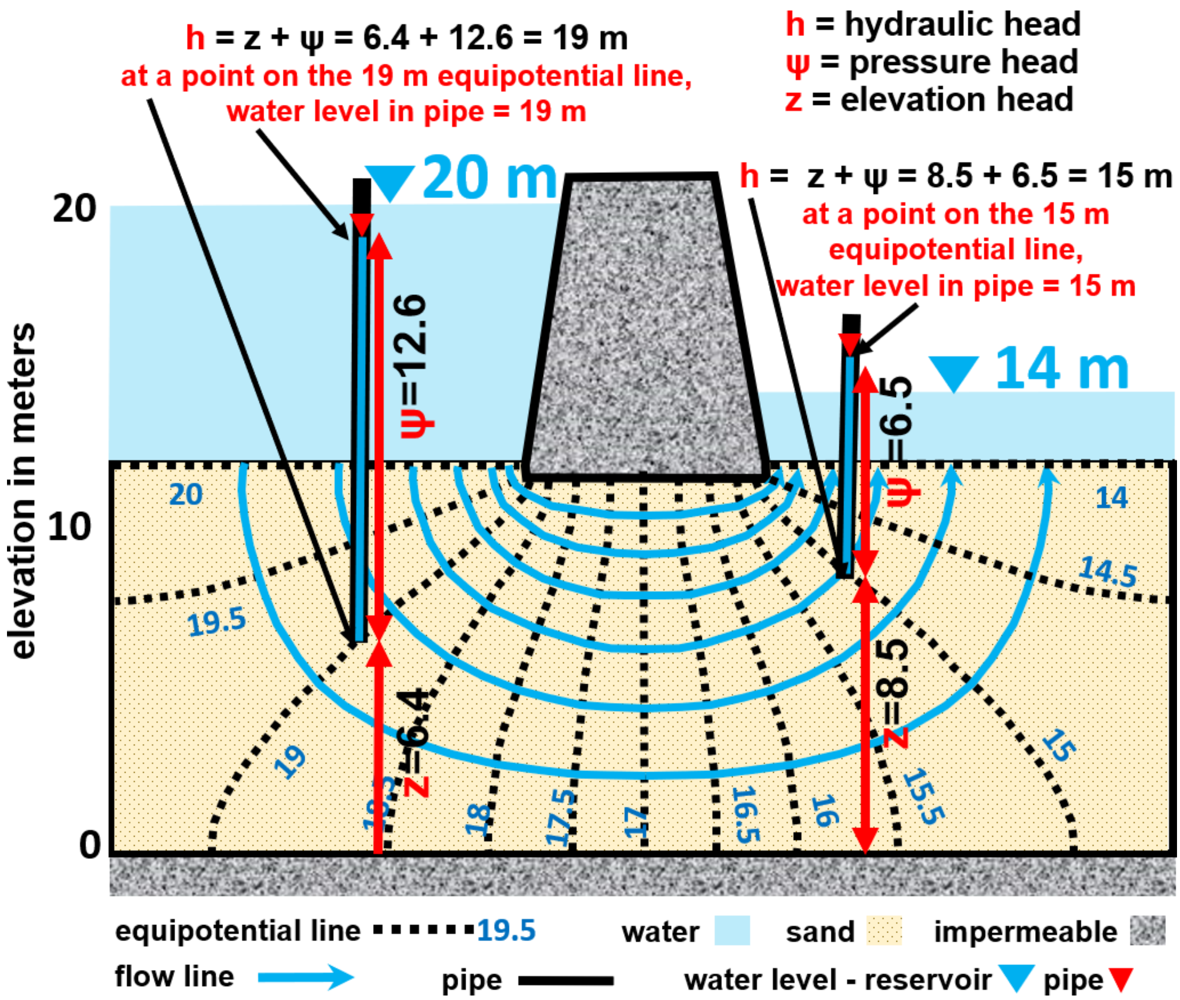


**Figure 3** - A cross-sectional view of a groundwater flow net under a dam from an upper reservoir to a lower reservoir. A flow net is comprised of two sets of lines that honor Darcy's Law and conserve mass. Equipotential lines connect points of equal hydraulic head (black dotted lines) and flow lines delineate paths of groundwater flow (blue solid arrows).

A homogeneous and isotropic groundwater system is one in which the hydraulic conductivity is the same at every location and does not vary for different directions of flow. Hydraulic conductivity is a measure of the ease with which water can pass through a material and is discussed in another Groundwater Project book↗ (Woessner and Poeter, 2020). The groundwater flow equation is based on Darcy's Law and conservation of mass. The groundwater flow equation is derived and discussed in another Groundwater Project book↗ (Woessner and Poeter, 2020). A brief overview of Darcy's Law, specific discharge, average linear groundwater velocity, and groundwater travel time are provided in Box 2↴ because the concepts are central to the material presented in this book.

Graphical construction of a flow net solves the two-dimensional, steady-state groundwater equation in a homogeneous and isotropic material with defined boundary conditions. Boundary conditions are discussed in another Groundwater Project book↗ (Woessner and Poeter, 2020). Two types of boundary conditions are used in graphical construction of two-dimensional, steady-state flow nets. The flow system domain is



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bounded by either a constant hydraulic head boundary or a no-flow boundary. It is important to remember that "no-flow" refers to no flow across the boundary, groundwater flow occurs parallel to the boundary, such that the boundary is a flow line.

Once the geometry and boundary conditions of the system are specified, the hydraulic heads throughout the system domain can be determined; if a hydraulic conductivity is given, then the rate of flow across constant head boundaries can also be determined using Darcy's Law and conservation of mass. The act of drawing the flow net must be done for a homogeneous and isotropic system; however, there is a procedure for scaling an anisotropic system so that an isotropic flow net can be drawn and then transformed into an anisotropic flow net. This is explained in Section 2.8 of this book. However, for cases in which the hydraulic conductivity is non-homogeneous (i.e., heterogeneous), constructing a flow net requires a numerical method using a computer.

Two requirements need to be kept in mind when drawing the equipotential and flow lines in order to obtain an accurate solution to the groundwater flow equation. First, the equipotential lines and the flow lines need to intersect at right angles. Second, the two sets of intersecting lines must form shapes with a constant aspect ratio (the same length to width ratio). The only realistic way for the human eye to achieve constant aspect ratios, is to draw "curvilinear squares", quadrilaterals with curved sides with an aspect ratio close to 1 (Figure 3). When these two requirements are satisfied, equipotential lines will have uniform increments (contour intervals) from one line to the next, each flow tube (a region bounded by two adjacent flow lines) will carry the same volumetric flow rate (measured, for example, in cubic meters per second). An exception to these requirements may occur near the edge of the domain where a partial (or fractional) flow tube may be drawn. This exception is discussed later in this book. Finally, it is important to remember that the value of hydraulic conductivity has no influence on the head distribution in a homogeneous system, but if the hydraulic conductivity is known, then a flow net can be used to determine the volumetric flow rate through the system.

A key difference between graphical versus numerical construction of a flow net is that the graphical method requires creating both equipotential lines and flow lines, whereas the numerical method does not. Groundwater professionals commonly use a groundwater model to compute hydraulic head, then later use a flow path tracking model (also known as a particle tracking model) to compute flow lines. A project might require computing only hydraulic head, in which case flow paths are not computed.

Typically, a numerical groundwater model computes hydraulic head for a grid (or an array) of points and, unlike a graphically constructed flow net, this gives flexibility to how equipotential lines are drawn. For example, equipotential lines need not be drawn at equal increments. Flexibility also occurs in drawing flow lines. A flow path tracking model enables one to draw a flow path starting from any location. Such flexibilities mean that numerically calculated equipotential lines and flow lines do not necessarily form shapes of



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constant aspect ratio, and flow tubes do not necessarily carry the same volumetric flow rate. Nonetheless, these computer-generated equipotential lines and flow lines form a bona fide flow net, because they satisfy the groundwater flow equation. For example, in an aquifer with homogeneous and isotropic hydraulic conductivity, computer-generated equipotential lines cross flow lines at right angles. Numerically generated flow nets are usually used to display flow patterns rather than to compute flow rates, because flow rates are calculated by numerically solving the groundwater flow equations.

A flow net can also be constructed for two-dimensional flow in a plan view. An important assumption for graphical construction of a flow net in a plan view is the absence of areally distributed recharge, such as infiltration of precipitation to the flow system. Figure 4 illustrates a plan view of a flow net between a lake and pond in an area constrained by bedrock. If the rate of recharge is trivial relative to the volumetric lateral flow from the lake to the pond, then the flow net is a sufficiently accurate for evaluating the groundwater system.

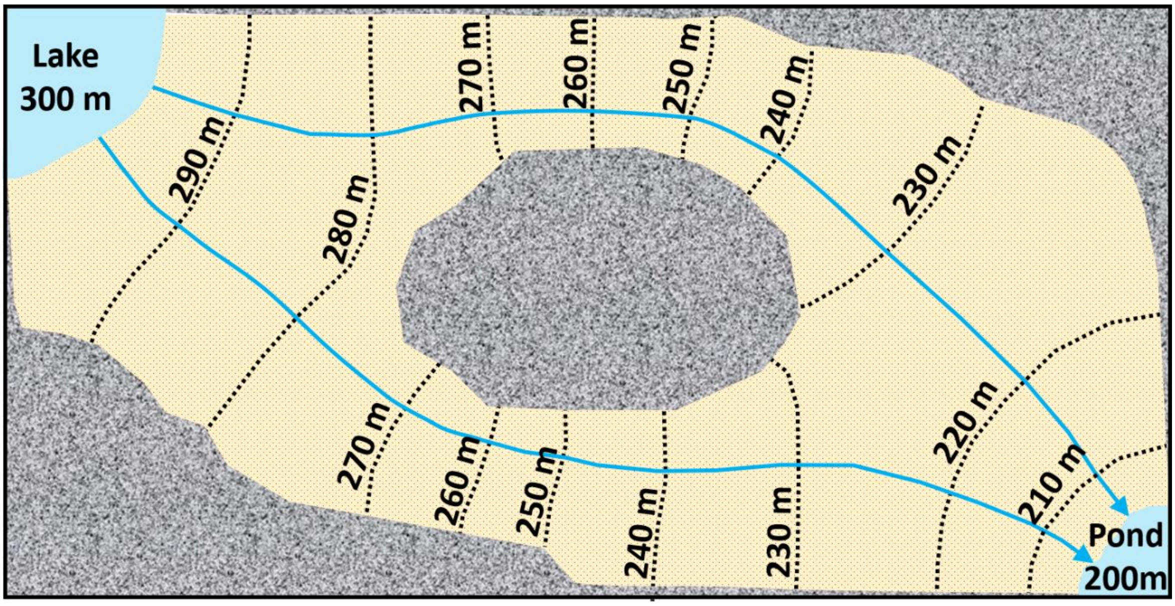


**Figure 4** - A plan view of flow in a confined aquifer penetrated by a deep lake and pond and laterally constrained by bedrock. The lake water elevation is 300 meters and the elevation of the pond surface is 200 meters. The top of the aquifer is 190 meters. Flow lines diverge on the upgradient side of the bedrock island in the middle of the aquifer and converge on the down gradient side.

# 2 Flow Net Construction

## 2.1 Approaches to Constructing Flow Nets

A flow net can be constructed using a number of approaches. All of the approaches generate equipotential and flow lines in one way or another. The graphical construction approach provides an approximate result that is often sufficient for practical purposes. Computer approaches are more versatile, but require knowledge in use of software.



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***Graphical Construction:*** This approach requires only paper and pencil, and fundamental understanding of flow net characteristics. Non-ideal conditions such as heterogeneity and anisotropy are challenging, and in such cases, numerical models are a better tool. The procedure for drawing graphical flow nets is described in Section 2.2.

***Analytical Element Approach:*** This approach can be used in more complex systems and requires a computer and software. A computer code is used to solve the groundwater flow equations for hydraulic head and stream functions to generate both equipotential line and flow lines at any location within the model domain.

***Numerical Approach:*** This approach also requires a computer and software. Numerical approaches can be used in the most complex systems. Numerical models can be constructed quickly to create steady-state, two-dimensional flow nets. A continuous solution is not provided by numerical models because heads are calculated for discrete locations within the model domain. A very precise calculation requires closely spaced points for head calculation. The only way to know if the grid is fine enough, is to solve the same problem using a finer grid and obtain the same answer as calculated by the coarser grid. After computing the discrete distribution of hydraulic head, a flow path tracking model (also known as a particle tracking model) is used to compute flow lines in accordance with Darcy's Law.

Although computer models can be used to simulate complex flow systems, it is beneficial for groundwater professionals to acquire the ability to sketch flow nets with pencil and paper to master understanding of flow nets and to enhance understanding of the basic concepts of groundwater flow. The ability to sketch flow nets can be used for rapid initial assessment of flow systems in the field and at impromptu project meetings. However, mastery of numerical model construction is also a must for the modern groundwater hydrologist.

## 2.2 Drawing a Flow Net for a Homogeneous Isotropic System

### Basic Criteria for Drawing Graphical Flow Nets

- Darcy's Law is valid
- Hydraulic conductivity of the material is homogeneous
- Hydraulic conductivity of the material is isotropic
- The material is fully saturated
- Flow is steady
- The fluid has constant density
- The fluid has constant viscosity
- The vertical and horizontal axes are drawn to scale.

### Steps for Drawing a Flow Net Using Pencil and Paper

1. Draw the outline of the flow system to scale and label the nature of each boundary



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2. Draw equipotential lines along the boundaries where a single value of hydraulic head is specified along the boundary
3. Draw flow lines along no-flow boundaries
4. Within the flow domain, draw flow lines along paths where you envision groundwater flowing, ensuring they are perpendicular to equipotential lines on the boundaries
5. Draw equipotential lines within the flow domain, ensuring they:
    - are perpendicular to no-flow boundaries;
    - are perpendicular to flow lines; and,
    - together with flow lines, form shapes of constant aspect ratio, preferably "curvilinear squares." A curvilinear square is a four-sided shape with curved edges and an aspect ratio close to one.
6. Calculate the contour interval and label the equipotential lines
7. Calculate flow through the net using the equation for total discharge through a flow net which is presented in Section 2.4 following the description of the procedure for drawing a flow net.

## 2.3 Drawing a Flow Net for Flow Beneath an Impermeable Dam

Consider the steps for drawing a flow net through the homogeneous porous sand under an impermeable, concrete dam that is keyed into sand as shown in Figure 5. In the field, the porous geologic material below the dam extends a long distance in the upgradient and downgradient directions, but only a portion of it is illustrated here. The dam is 21 meters wide in the direction perpendicular to the figure. The water level in the reservoir contained by the dam is 10 meters above the surface of the low hydraulic conductivity material below the aquifer which is used as a datum. The water level in the reservoir below the dam is 4 meters lower than the water behind the dam and the water below the dam runs off downstream.

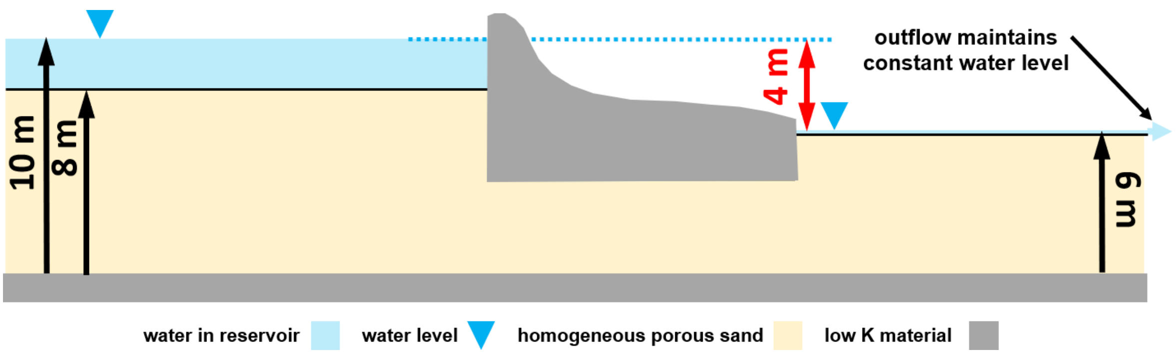


**Figure 5** - A concrete dam keyed into soil.



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We begin constructing a flow net by drawing the outline of the flow system to scale and labeling all boundary conditions (Figure 6). The low permeability concrete dam and layer underlying the sand are assumed to prevent flow from crossing those boundaries and so are labeled as no-flow boundaries (Figure 6). Any convenient level can be used as a datum for a flow net. In this case the horizontal bedrock surface below the dam provides a convenient reference for head measurements. An open body of water is hydrostatic, so the hydraulic head on the sand at the bottom of the reservoir is equal to the elevation of the reservoir water. So, these locations are constant-head boundaries with a head of 10 m on the ground surface upgradient of the dam and a head of 6 meters on the downgradient side (Figure 6). The lateral portions of the aquifer are not bounded so they must be drawn far enough from the dam so that no significant leakage occurs between the reservoirs and the underlying sand at the distant ends of the system. The highest rate of seepage into the sand will be immediately up gradient of the dam with seepage decreasing with distance up gradient. If, after constructing a flow net, it appears that the diagram is not wide enough, it can be redrawn with greater lateral extent from the dam until an acceptable flow net is obtained. Using knowledge of Darcy's Law and the fact that flow is parallel to no-flow boundaries, one flow path can be drawn along the concrete dam from the upgradient to the downgradient reservoir (Figure 6).

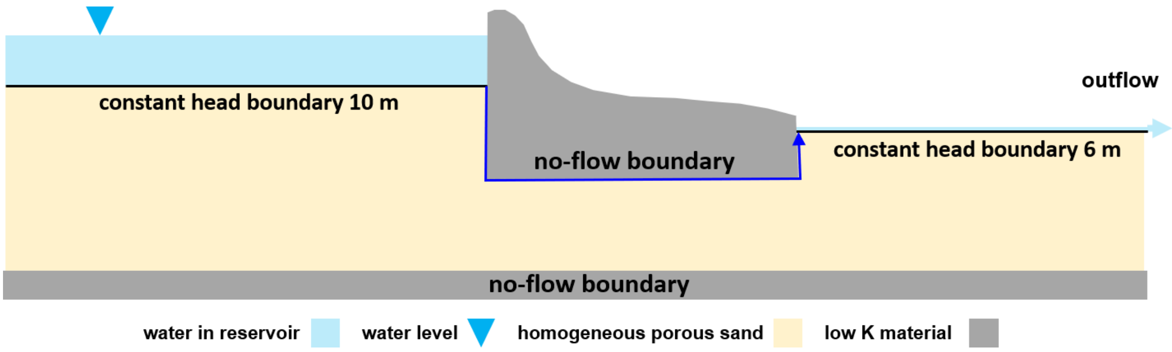


**Figure 6** - Step 1 - Draw the system to scale (no-flow boundaries are indicated by gray zones), Step 2 - Draw equipotential lines to coincide with head boundaries (black lines), Step 3 - Draw flow lines to coincide with no-flow boundaries (blue arrow following the no-flow boundary of the dam.

The next step is to envision how water is likely to move through the system and sketch some flow lines (Figure 7). The flow lines should be drawn perpendicular to the constant-head boundaries. Do not be concerned if your first attempt at sketching flow lines is not correct because errors in drawing the flow lines will show up as the equipotential lines are drawn, and can be corrected by erasing and redrawing until the flow net is correct. The first sketching of flow lines simply gets the process started.



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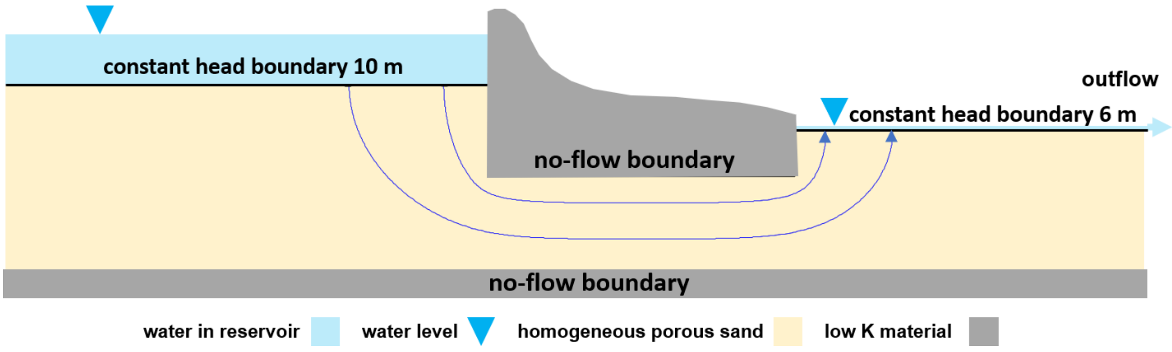


**Figure 7** - Step 4: Draw flow lines along paths where you envision groundwater flowing (blue arrows), ensuring they are perpendicular to equipotential lines on the boundaries. Do not be concerned about getting them right initially. Drawing flow nets is a trial-and-error process. As experience with flow nets grows, intuition improves and it becomes easier to place flow lines in nearly the right position on the first attempt.

Next, draw equipotential lines to show how hydraulic head varies from the constant-head boundary at the upstream reservoir to the constant-head boundary at the downstream reservoir. The equipotential lines need to be drawn perpendicular to both the no-flow boundaries and the flow lines. The equipotential lines and flow lines should intersect to form shapes with a constant aspect ratio, preferably "curvilinear squares", quadrilaterals with curved sides and having an aspect ratio close to 1. Drawing a flow net by hand is a trial-and-error process because the equipotential lines and flow lines are adjusted until curvilinear squares are formed. It is useful to sketch round shapes within and touching the boundaries of the space formed by the equipotential lines and flow lines. If the shapes are not circular, as in the first attempt to draw a flow net shown in Figure 8, then the lines should be adjusted.

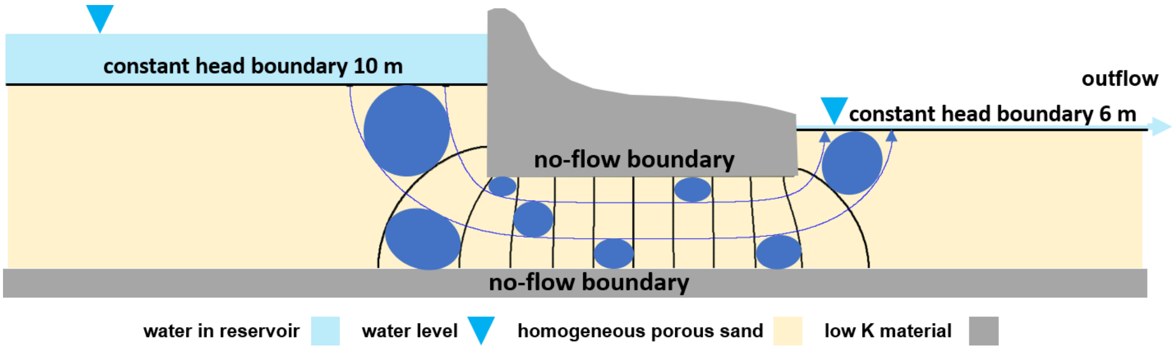


**Figure 8** - Step 5: Draw equipotential lines between lines drawn at constant head boundaries (black lines), ensuring they are perpendicular to no-flow boundaries, perpendicular to flow lines and attempting to form curvilinear squares. Drawing a flow net by hand is a trial-and-error process because the equipotential lines and flow lines are adjusted until curvilinear squares are formed. It is useful to sketch round shapes within and touching the boundaries of the space formed by the equipotential lines and flow lines. If the shapes are not circular, as in this first attempt to draw the flow net, then the lines should be adjusted.

Adjust the position of flow lines and equipotential lines until a circle fills the space between the lines fairly well as in Figure 9. If an oval is needed to fill the space then it is not a curvilinear square. A slight misfit of the circles is not important. In order to make a difference to the estimation of flow through the system, the misfits need to be large enough



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such that it is necessary to add or delete flow lines or equipotential lines in order to obtain the near-curvilinear squares because achieving the proper ratio of the number of flow lines and equipotential lines is key to drawing a valid flow net. The number of flow lines is the same in Figure 8 and Figure 9, but the number of equipotential lines differ indicating the redrawing was necessary to obtain a flow net that can be used to calculate flow through the system.

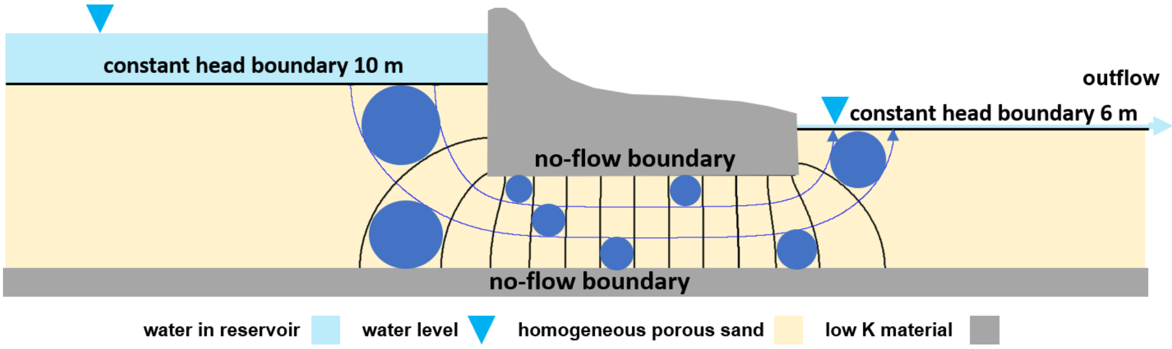


**Figure 9** - Creating shapes with a constant aspect ratio is a requirement when drawing a flow net. The best way to achieve that is by drawing curvilinear squares. Sketching a circle within the shapes can help discern whether the shapes are curvilinear squares. A slight misfit is not important. The misfits need to be large enough such that it is necessary to add or delete flow or equipotential lines in order to obtain the near-curvilinear squares as in the transition from the previous figure to this figure.

Once the flow net has an acceptable form, the next step is to calculate the values of the equipotential lines and label them. The equipotential lines represent hydraulic heads within the system between the boundary heads. The difference between the value of head on adjacent equipotential lines is called the contour interval. This interval is constant for the entire flow net. To determine the magnitude of the contour interval, first determine the total head drop across the flow net, $H$, and divide that by the number of head drops, $n_d$, in the flow net as shown in Equation 1.

$$\text{contour interval} = \frac{H}{n_d} \tag{1}$$

where:

contour interval = head difference between adjacent equipotential lines (L)

$H$ = total head drop across the flow net domain (L)

$n_d$ = number of head drops in the flow net (dimensionless)

The total head drop across the system is:

$$H = 10\text{ m} - 6\text{ m} = 4\text{ m}$$

A head drop is represented by the zone between adjacent equipotential lines. The number of head drops is not arbitrary. It is determined by drawing a flow net while adhering to the rules regarding the placement of equipotential lines. In the flow net for flow under the concrete dam there are fourteen head drops ($n_d$=14) as shown in Figure 10.



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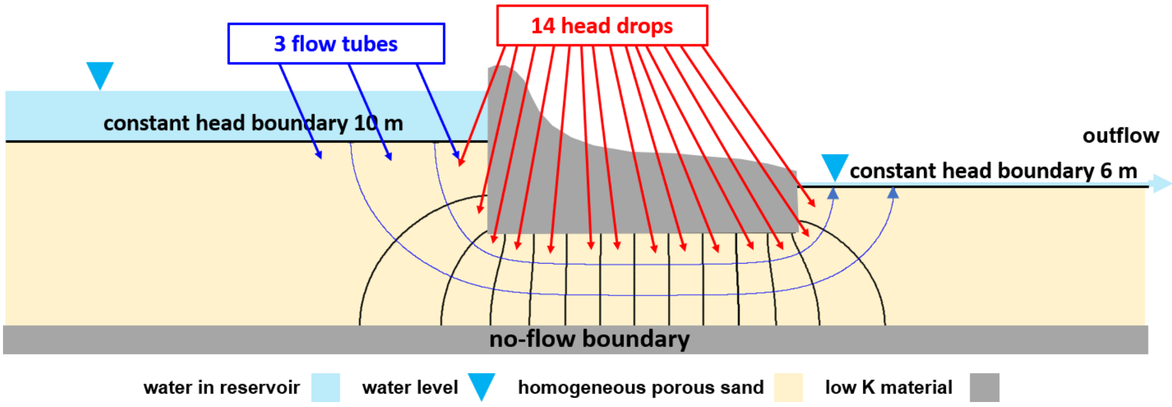


**Figure 10** - The appropriate number of head drops (spaces between equipotential lines) and flow tubes (spaces between flow lines) are determined by following the rules for drawing a flow net. This flow net has 14 head drops and 3 flow tubes.

This flow net drawing began with two internal flow lines, creating three flow tubes beneath the dam. A valid flow net can be drawn beginning with one, ten, or any number of flow tubes, as long as the appropriate number of equipotential lines are added to form shapes of constant aspect ratio, preferably curvilinear squares. No matter how many flow lines are drawn, the process of creating shapes of constant aspect ratio will yield approximately the same ratio of the number of flow tubes to the number of head drops. It is the ratio of the number of flow tubes to the number of head drops that is important. The ratio of flow tubes to head drops for the flow net of Figure 10 is 3/14 = 0.214. If the flow net is drawn with 2 flow tubes, 9 head drops will be needed to create curvilinear squares, for a ratio of 2/9 = 0.2222. If 5 flow tubes are used then 23 head drops will produce curvilinear squares, for a ratio of 5/28 = 0.217. These small differences in the ratio of flow tubes to head drops illustrate that drawing a flow net with paper and pencil yields an approximate solution.

The contour interval for the flow net is:

$$\text{contour interval} = \frac{H}{n_d} = \frac{4\text{ m}}{14} = 0.2857\text{ m}$$

The labeled equipotential lines are shown in Figure 11. The flow net does not provide precision to the 3 significant figures shown in the contour labels in the diagram. Three significant figures are shown, not because the system is known to high precision, but to adequately illustrate the difference in head between adjacent contour lines.



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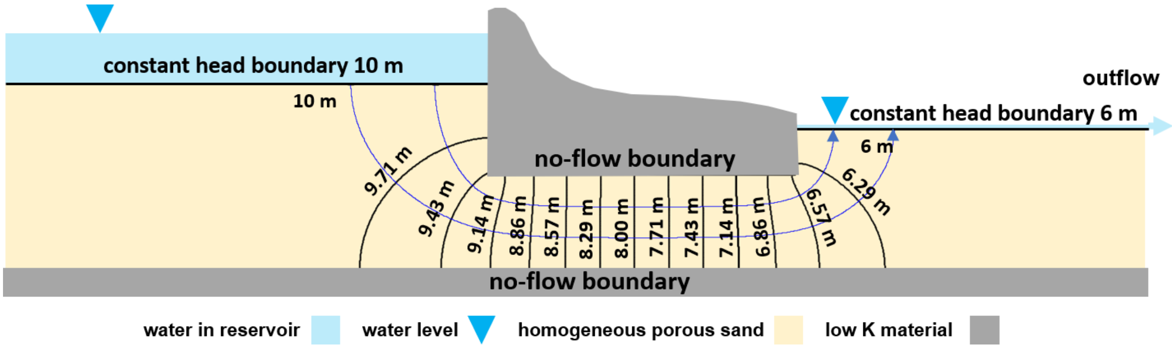


**Figure 11** - Step 6: Calculate the equipotential line contour interval and label the equipotential lines. In this case the contour interval is ~0.29 meters.

It is useful to remember that the approximate solution provided by a hand drawn flow net is sufficient for practical applications because the error is slight compared with the uncertainty associated with assuming the material is homogeneous and isotropic, and with estimating the value of hydraulic conductivity.

## 2.4 Calculating Volumetric Discharge

Volumetric discharge is the volumetric rate of water flowing through a system. This rate is reported in dimensions of volume (length cubed) over time (for example, liters per minute, or cubic meters per second).

For a flow net in which the equipotential lines and flow lines form curvilinear squares Equation 2 can be used to calculate the volumetric discharge through the flow net. The derivation of the equation for calculating flow through a flow net is provided in Box 3⤵.

$$Q_{total} = K\,H\,\frac{n_f}{n_d}w \tag{2}$$

where:

$Q_{total}$ = volumetric flow through the system ($L^3/T$)

$K$ = hydraulic conductivity of the porous medium (L/T)

$H$ = total head drop across the flow net domain (L)

$n_f$ = number of flow tubes in the flow net (dimensionless)

$n_d$ = number of head drops in the flow net (dimensionless)

$w$ = distance that the system extends into the drawing (L)

As shown by the early steps of the derivation in Box 3, Equation 2 can be adjusted to accommodate a flow net drawn with shapes of a constant aspect ratio that differs from one, as shown in Equation 3.

$$Q_{total} = K\,H\,\frac{n_f}{n_d}w\,a_r \tag{3}$$



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where:

$a_r =$ aspect ratio for one shape of constant aspect ratio in the flow net that needs to be calculated as the distance between the flow lines divided by the distance between the equipotential lines (dimensionless)

Equation 2 is applicable for the flow net of curvilinear squares shown in Figure 11. Suppose that the hydraulic conductivity of the material beneath the dam in the previous section is 0.5 m/d and the width of the dam into the image is 21 meters, then the volumetric flow rate under the dam is:

$$Q_{total} = KH\frac{n_f}{n_d}w = \left(0.5\ \frac{\text{m}}{\text{d}}\right)(4\ \text{m})\left(\frac{3}{14}\right)(21\ \text{m}) = 9\frac{\text{m}^3}{\text{d}}$$

The formula for determining the volumetric flow through a flow net does not involve the absolute dimensions of the length and height of the system. It uses only the ratio of the number of head drops to the number of flow tubes. As mentioned earlier, when drawing a flow net, it is the ratio of the number of flow tubes to the number of head drops that is important. The ratio of flow tubes to head drops for the flow net of Figure 11 is 3/14 = 0.214. If the flow net is drawn with 2 flow tubes, 9 head drops will be needed to create curvilinear squares, for a ratio of 2/9 = 0.2222. If 5 flow tubes are used then 23 head drops will produce curvilinear squares, for a ratio of 5/28 = 0.217. These small differences in the ratio of flow tubes to head drops will yield slightly different values of $Q_{total}$, illustrating that drawing a flow net with paper and pencil yields an approximate solution. When calculating $Q_{total}$, for a practical application, these slight differences are trivial compared to the uncertainty in estimating an equivalent value of homogeneous hydraulic conductivity used to calculate $Q_{total}$.

In some cases, a person may choose to start with a round number for a contour interval for equipotential lines to start drawing a flow net. If the flow net of Figure 11, is drawn with a contour interval of 0.25 m (which produces 16 head drops), then the ratio required for a valid flow net indicates that 3.43 flow tubes are necessary (that is, 3.43/16 = 0.214. Thus, the flow net has a partial flow tube to maintain the valid ratio, so one of the flow tunes has to have 0.43 of the width of the tubes that form curvilinear squares as illustrated by the deepest flow tube in Figure 12. Calculation of the volumetric flow rate through the system yields the same result because the ratio of flow tubes to head drops has not changed:

$$Q_{total} = KH\frac{n_f}{n_d}w = \left(0.5\frac{\text{m}}{\text{d}}\right)(4\ \text{m})\left(\frac{3.43}{16}\right)(21\ \text{m}) = 9\frac{\text{m}^3}{\text{d}}$$



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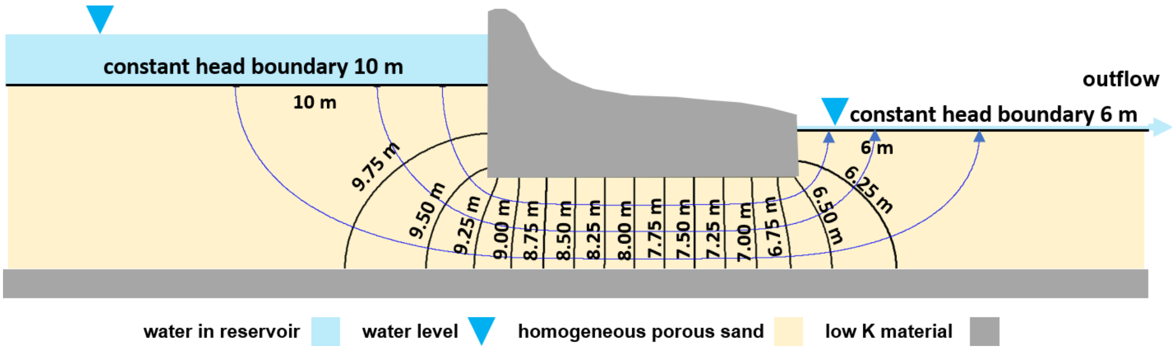


**Figure 12** - A flow net for the system illustrated in Figure 11 with a contour interval of 0.25 meters, requiring 3.43 flow tubes to achieve curvilinear squares and a valid ratio of the number of flow tubes to the number of head drops. Thus, the deepest flow tube is only ~40% of the width of the flow tubes that form curvilinear squares.

It is important to remember that drawing a flow net requires drawing the geometry of the flow system to scale. That is, the relative length and width of the system must be drawn correctly to determine the flow rate per unit length normal to the diagram. The absolute width of the system into the diagram must be known to determine the total flow through the system.

## 2.5 Drawing a Flow Net for an Unconfined System with a Water Table Boundary

Unconfined groundwater systems have a water table boundary which requires special consideration when drawing a flow net because the location of the water table boundary is not known until after the flow net construction is completed. Such systems may also have a seepage face, where groundwater seeps out along a sloping section of ground surface. The position of the water table and the length of the seepage face need to be adjusted along with the flow and equipotential lines while drawing the flow net. Because the water pressure is equal to the atmospheric pressure at the water table, the equipotential lines need to intersect the water table at the elevation equal to the value of the equipotential line label.

In the absence of recharge, the water table is itself a flow line. There is no flow across flow lines so a water table without recharge can be viewed as a no flow boundary of unknown position until after the flow net is drawn. When the water table is a flow line, equipotential lines meet the water table at right angles.

The procedure for constructing a graphical flow net does not accommodate boundaries with a defined flux other than zero. However, if there is recharge across the water table and the position of the water table is known, then values of head can be defined as equal to the elevation of the water table and a flow net can be drawn that will reflect inflow at the water table. The rate of inflow can be determined if the value of hydraulic conductivity is known. This is discussed in section 2.9 which addresses topographically driven flow.



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An earth dam is used as an example for discussion of drawing flow nets in an unconfined system as illustrated in Figure 13 and described in Box 4 which can be accessed from the caption of Figure 13. In contrast to the previous section, material at the ground surface is impermeable, and earth material is brought in from nearby to construct a dam, so in this case water flows through the dam instead of under the dam. The dam surface is sealed to prevent infiltration of water into the dam structure. Consequently, the water table is a flow line.

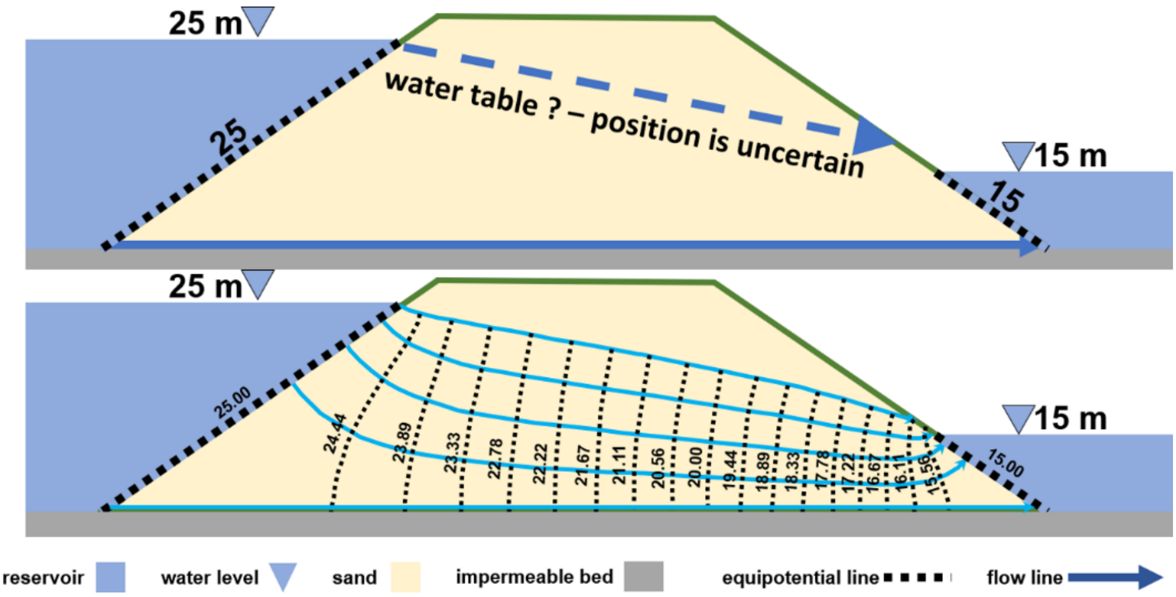


**Figure 13** – Click here to go to Box 4 which describes the procedure for drawing a flow net with a water table boundary.

## 2.6 The "Hear See Do" of Flow Nets

The Chinese philosopher Confucius (551 BC to 479 BC), stated "I hear *(read)* and I forget. I see and I remember. I do and I understand." The parenthetical "read" has been added here to adjust to our heavily text-based world 2500 years after the time of Confucius. In short, his point is that we learn more readily when we move beyond reading about something, to seeing it, and then on to doing it. Previous sections of this book provide the opportunity to "hear" (read).

One opportunity to "see" flow nets is provided using videos of flow in a Hele-Shaw cell. Like a flow net, a Hele-Shaw cell represents a planar section of a flow system. A Hele-Shaw cell consists of two parallel transparent walls with a small gap between them that is filled with a viscous liquid that moves in response to a pressure difference along the plates. The pattern of flow between the walls is equivalent to groundwater flow in a plane. A Hele-Shaw model allows us to view flow lines but does not show lines of equipotential, so one needs to envision those when viewing a Hele-Shaw model. Videos of flow in a Hele-Shaw cell can be accessed through the links in Figure 14. The first video, Figure 14a, illustrates



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flow in a system similar to the one shown in the video link provided in Figure 15 which illustrates drawing of a flow net. The second video, Figure 14b, is a flow system with recharge at the top of a land mass and discharge to both sides. The third video, Figure 14c, uses the same model as the second system but the left side is blocked at first so initially all flow exits on the right. Midway through video 14c, the right side is closed and the left side is opened so the transient shift of the flow lines can be observed.

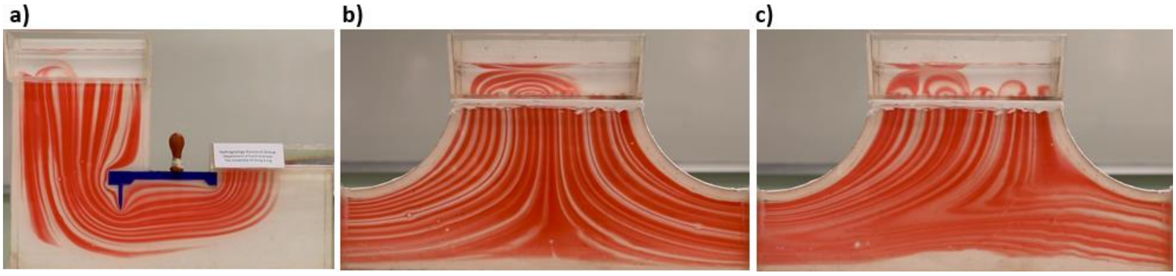


**Figure 14** - Hele-Shaw cells display flow paths for a) Click to view flow under a dam, b) Click to view flow of recharge from a ridge to a surrounding water body, c) Click to view flow of recharge from a ridge first to the lake on the right and then to the lake on the left. (- Videotaping of Hele-Shaw model simulations at the University of Hong Kong by J. Jiao and W.Z. Liang. Editing and voice over by Eileen Poeter.)

Another opportunity to "see" flow nets is provided by the video of a flow net being sketched with pencil and paper that can be accessed through the link provided in Figure 15. The video provides an opportunity to see that drawing a flow net is a process of trial-and-error, so it is useful to have an eraser available.

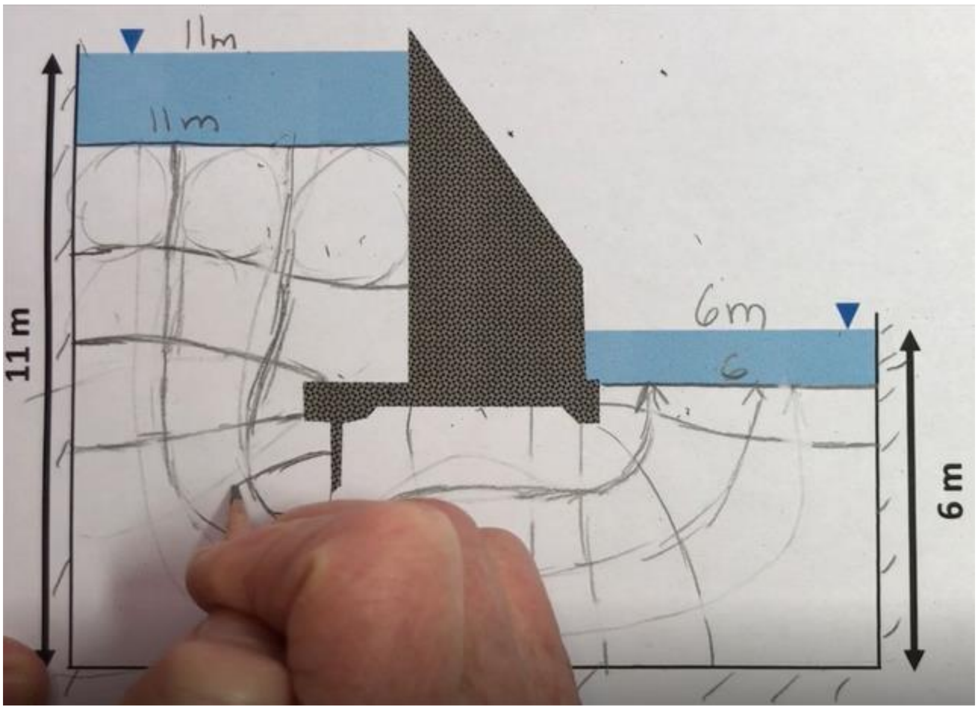


**Figure 15** - Click to view a video illustrating drawing a flow net.



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As Confucius indicated, learning is improved when we “do”. Thus, it is useful to draw flow nets when striving to understand groundwater flow. Actively drawing flow nets brings attention to aspects of flow system that may be unclear to those who are new to the study of groundwater, so the act of drawing can clarify groundwater concepts. The exercises in Section 3 provide an opportunity to draw some flow nets and then view a finished flow net.

## 2.7 Flow Nets Provide Insight into Groundwater Flow

Most everyone can envision water discharging at a given rate from an open hose, and knows that if the orifice at the opening of the hose is decreased in size by covering part of it with a finger, a high-speed jet of water will result because the discharge rate remains the same, so the velocity increases. This intuitive concept of mass conservation applies to interpreting flow tubes. Like the hose in the analogy, a flow tube carries a constant volumetric flow rate. If a flow tube becomes narrower, the specific discharge (volumetric flow rate divided by area of flow) must increase. For a homogeneous material, Darcy’s Law says that an increase in hydraulic gradient must accompany an increase in specific discharge. Therefore, in a homogeneous material, where a flow tube becomes narrower, the equipotential lines must be closer together. If the effective porosity is uniform, a higher specific discharge also implies a higher groundwater flow velocity.

The above concepts are illustrated by the two flow systems in Figure 16. In both systems, the confined aquifer is 100 m thick with the top of the aquifer an elevation of 0 meters. The hydraulic conductivity is homogeneous with a value of 0.0002 m/s. Groundwater enters the aquifer across the entire left side at a volumetric flow rate of 0.2 $m^3/s$. Groundwater exits the aquifer on the right side through an outlet, where the hydraulic head is 10 m.



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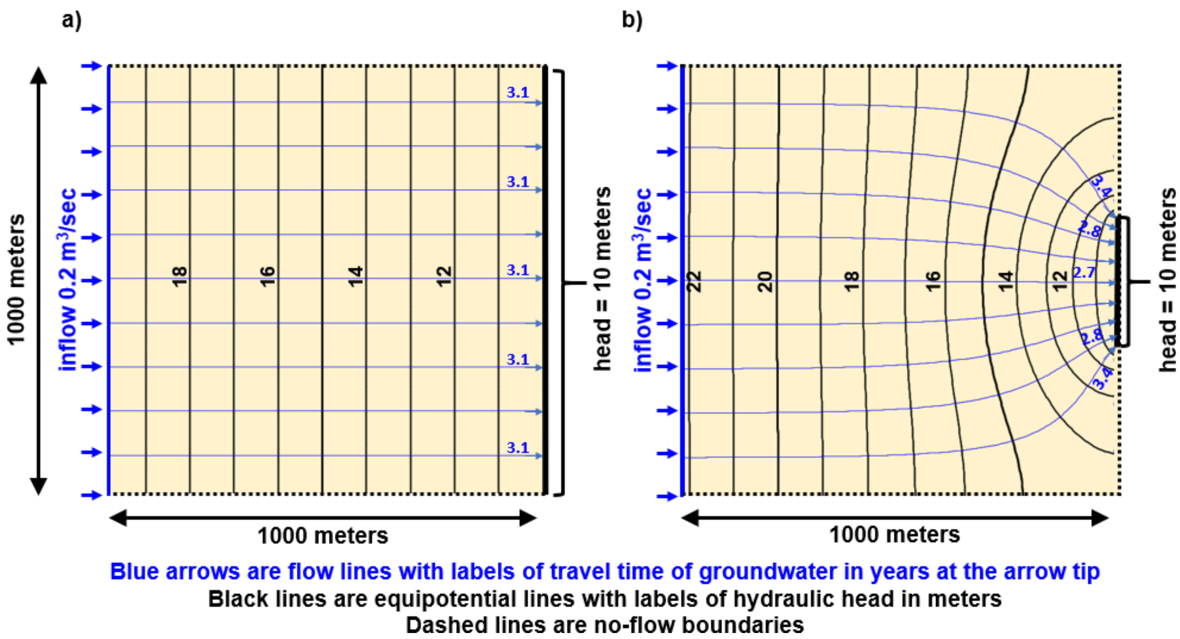


**Figure 16** - Groundwater is introduced at the same volumetric rate to the entire left side of both homogeneous systems a) and b), but the outlet on the right is narrower in the system shown in b. In order for the same volumetric rate to flow through the narrow opening in (b), the flow tubes need to be narrower and the gradient needs to be higher than in (a). This results in an increased specific discharge and velocity (specific discharge divided by porosity) near the outlet of (b). The travel times are ~3.1 years in system (a) while travel time is shorter for central flow lines in system (b) (<3 years) and longer for flow lines near the boundaries (>3 years).

In Figure 16a, the outlet spans the entire right side, resulting in straight and uniform flow tubes. In Figure 16b, the outlet is restricted to only a portion of the right side. For groundwater to exit the aquifer, flow tubes narrow near the outlet opening. The narrowing of flow tubes is accompanied by an increase in hydraulic gradient (equipotential lines closer together) in accordance with Darcy's Law. Hydraulic head contours range from 10 to 21 meters and are labeled in black.

Travel time of a packet of water traversing a flow line is indicated with blue numbers next to the arrow heads of the flow lines. The travel time is 3.1 years from entry to exit all packets traversing system with completely open boundary (Figure 16a). The same volume of water is forced through the system with the narrow outlet, but in this case the central flowlines carry packets of water across the system in a shorter time ~2.7 years, while packets near the boundaries travel more slowly, requiring 3.4 years to reach the exit (Figure 16b).

A graphically constructed flow net cannot be drawn for a heterogeneous system, so a numerical simulation of flow is used to demonstrate the impact of a lens of lower and higher hydraulic conductivity on the system of Figure 16a, as shown in Figure 17. As in Figure 16, the same volumetric flow rate enters the aquifer on the left side and exits the aquifer on the right side in Figure 17. The heterogeneous case has a lower-$K$ zone within the aquifer in Figure 17a and a higher-$K$ zone in Figure 17b.



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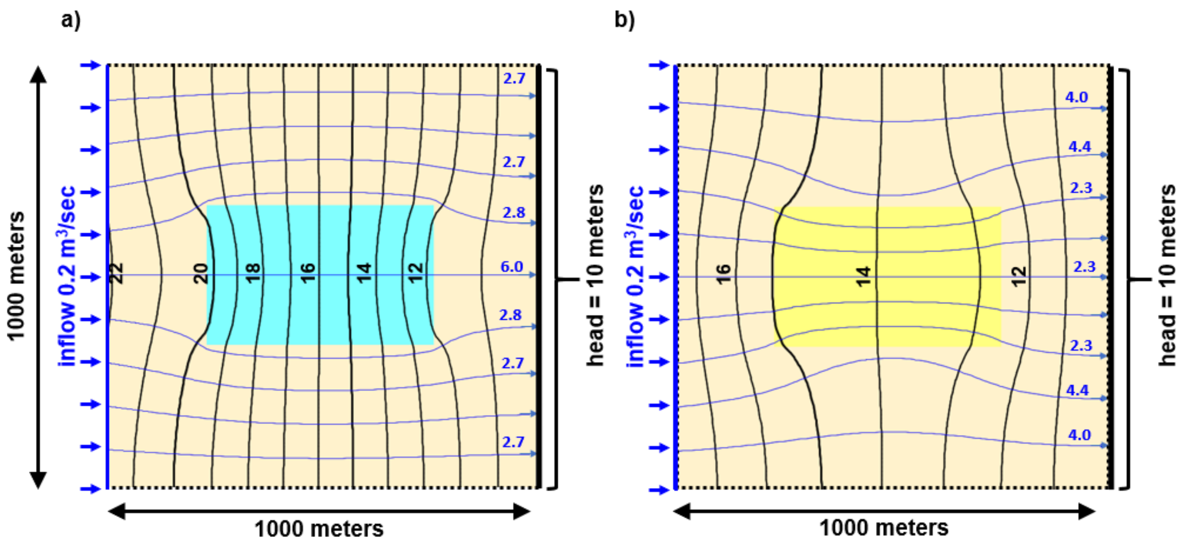


**Figure 17** - Groundwater is introduced at the same volumetric rate to these heterogeneous system as in the homogeneous system shown in Figure 16. a) Within the low-$K$ zone, flow tubes are wider and the gradient is higher (equipotential lines are closer together) than the surrounding higher-$K$ material. Packets of water require more time to travel through the low-$K$ zone. b) Within the high-$K$ zone, flow tubes are narrower and the gradient is lower (equipotential lines are further apart) than the surrounding lower-$K$ material. Packets of water require less time to travel through the high-$K$ zone.

Flow tubes carry the same volumetric flow at every location along their length, so in the case of the lower-$K$ zone (Figure 17a): (1) flow tubes widen as they enter the lower-$K$ zone and narrow as they exit the zone, and (2) equipotential lines are closer together inside the lower-$K$ zone than outside. Both of these changes are indicated by Darcy's Law because a higher gradient and larger area serve to carry the same volumetric flow as the flow tube enters the lower-$K$ zone. Overall, the system with the lower-$K$ zone requires a higher overall gradient to drive the same volumetric flow through the aquifer (notice the heads on the left side of Figure 17a are higher than those of Figure 16a). Packets of water require less time to travel across the system in the higher-$K$ zone surrounding the lower-$K$ material. This is indicated by the travel time for a packet of water traversing the central flow line being more than twice as long as the travel time along flow lines that pass around the lower-$K$ material (6.0 years versus 2.7 years). It is useful to remember that, in these examples, the same flow rate is forced to enter the systems on their left side. If instead of constant flow on the left side, the head is held constant on the left side then the system of Figure 17a will have a lower volumetric flow than the system of Figure 16a, but the shape of the flow lines of Figure 17a will be similar with flow diverging at the upgradient side of the low-$K$ zone and converging on the downgradient side.

Flow tubes carry the same volumetric flow at every location along their length, so in the case of the higher-$K$ zone (Figure 17b): (1) flow tubes narrow as they enter the higher-$K$ zone and widen as they exit the zone, and (2) equipotential lines are further apart inside the higher-$K$ zone than outside. Both of these changes are indicated by Darcy's Law



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because a lower gradient and smaller area serve to carry the same volumetric flow as the flow tube enters in the higher-$K$ zone. Overall, the system with the higher-$K$ zone requires a lower gradient to drive the same volumetric flow through the aquifer (notice the heads on the left side of Figure 17b are lower than those of Figure 16a). Packets of water require more time to traverse the system in the lower-$K$ zone surrounding the higher-$K$ material, while water packets require less time to traverse the system along the central flow line through the high-$K$ material. This is indicated by the travel time for a packet of water traversing the central flow line being about half that of the travel time along flow lines that pass around the higher-$K$ material (2.3 years versus 4.4 years). It is useful to remember that, in these examples, the same volume of water is forced to enter the systems on their left side. If instead of constant flow on the left side, the head for is held constant on the left side then the system of Figure 17b will have a higher volumetric flow than the system of Figure 16a, but the shape of the flow lines of Figure 17b will be similar with flow converging at the upgradient side of the high-$K$ zone and diverging on the downgradient side.

## 2.8 Drawing a Flow Net for a System with Anisotropic Hydraulic Conductivity

In an anisotropic system, hydraulic conductivity differs with direction. This is especially the case in groundwater basins comprised of layered sedimentary rocks of differing hydraulic conductivity. In such layered materials, groundwater flow that is roughly parallel to the layers preferentially flows through the higher hydraulic conductivity layers and thus meets less overall resistance than flow in the vertical direction, which must move through both the high and low hydraulic conductivity layers. In layered materials of differing hydraulic conductivity, the equivalent hydraulic conductivity perpendicular to the layers is lower than the hydraulic conductivity parallel to the layers. This is discussed in another Groundwater Project book↗ (Woessner and Poeter, 2020) where the procedure for calculating one equivalent hydraulic conductivity for the parallel and perpendicular directions is provided.

Graphical construction of a flow net needs to be undertaken in an isotropic material. However, a flow net can be obtained for an anisotropic material by taking the following steps: 1) transform the geometry of the system to an equivalent isotropic system; 2) draw the flow net for the isotropic system; and, 3) transform the flow net back to the anisotropic system geometry. For an anisotropic system resulting from horizontal layering, the geometric transformation consists of either scaling the vertical $z$-axis by a factor of the square root of $K_x/K_z$, as shown in Figure 18a and b, or scaling the horizontal $x$-axis by a factor of the square root of $K_z/K_x$. The step-by-step procedure for drawing a flow net in a system with anisotropic hydraulic conductivity can be accessed through the link to Box 5 that is provided in Figure 18. The transformation can be used for both cross-sectional and



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plan-view flow nets. In an anisotropic system, flow lines and equipotential lines are not at right angles to one another as shown in Figure 18c.

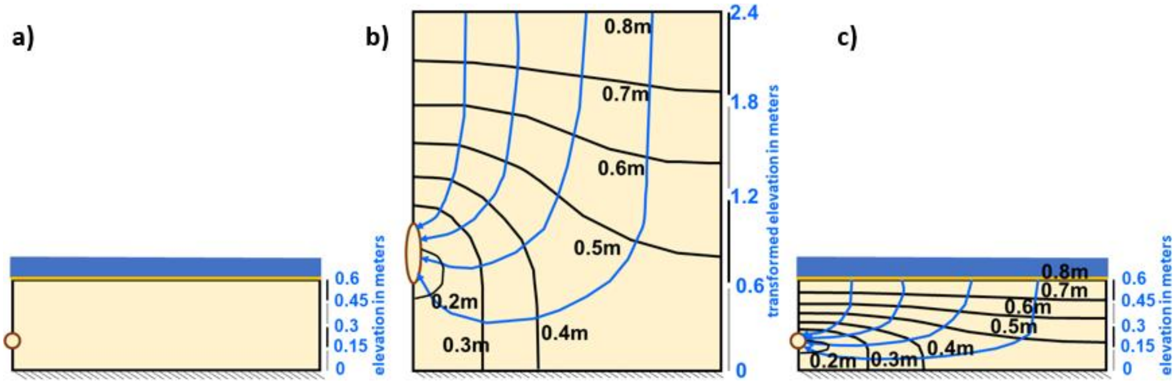


**Figure 18** - Click to view Box 5 for the step-by-step process of drawing groundwater flow nets in anisotropic systems. Drawing an anisotropic flow net: a) Flow system geometry; b) Flow system transformed with flow net drawn for isotropic conditions; c) Flow system returned to untransformed geometry with flow net for anisotropic conditions.

## 2.9 Create and Investigate Topographically Driven Flow Systems

A software tool called "TopoDrive' can be used to create and simulate topographically-driven flow systems. The flow system is a vertical cross-section with no-flow boundaries on the left, right, and bottom sides. The top boundary is the water table and is specified by the user. This flow system is different from the flow systems encountered earlier in the book in two ways. First, in the case of flow beneath an impermeable dam (Section 2.3, Figure 11), the head is uniform along the bottom of upstream reservoir (10 m) and the downstream reservoir (6 m). By contrast, in the topographically-driven flow system, the head along the water table is equal to the elevation of the water table, and therefore can vary along the boundary (unless the water table is flat). Second, in the case of flow in an unconfined system with a water table (Section 2.5, Box 4), the position of the water table is initially unknown and is determined while the flow net is constructed. Here, in the topographically-driven flow system, the position of the water table is specified. Given this specification of the water table elevation, in order to satisfy Darcy's Law and conserve mass, water must flow across the water table boundary either into or out of the system.

The TopoDrive software uses a numerical model to solve the groundwater flow equations, then draws equipotential lines at a constant contour interval throughout the system. Once the equipotential lines are drawn, you select a point(s) from which you want the program to trace a flow line(s). Because the flow lines are drawn from arbitrary starting points selected by the user, the flow tubes are not likely to carry the same groundwater discharge.



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Figure 19 provides a link to a video demonstrating the use of topodrive. For written instructions on using TopoDrive, click here to go to Box 6. The online version of TopoDrive can be accessed at http://tdpfonline.net, where readers can create their own flow systems.

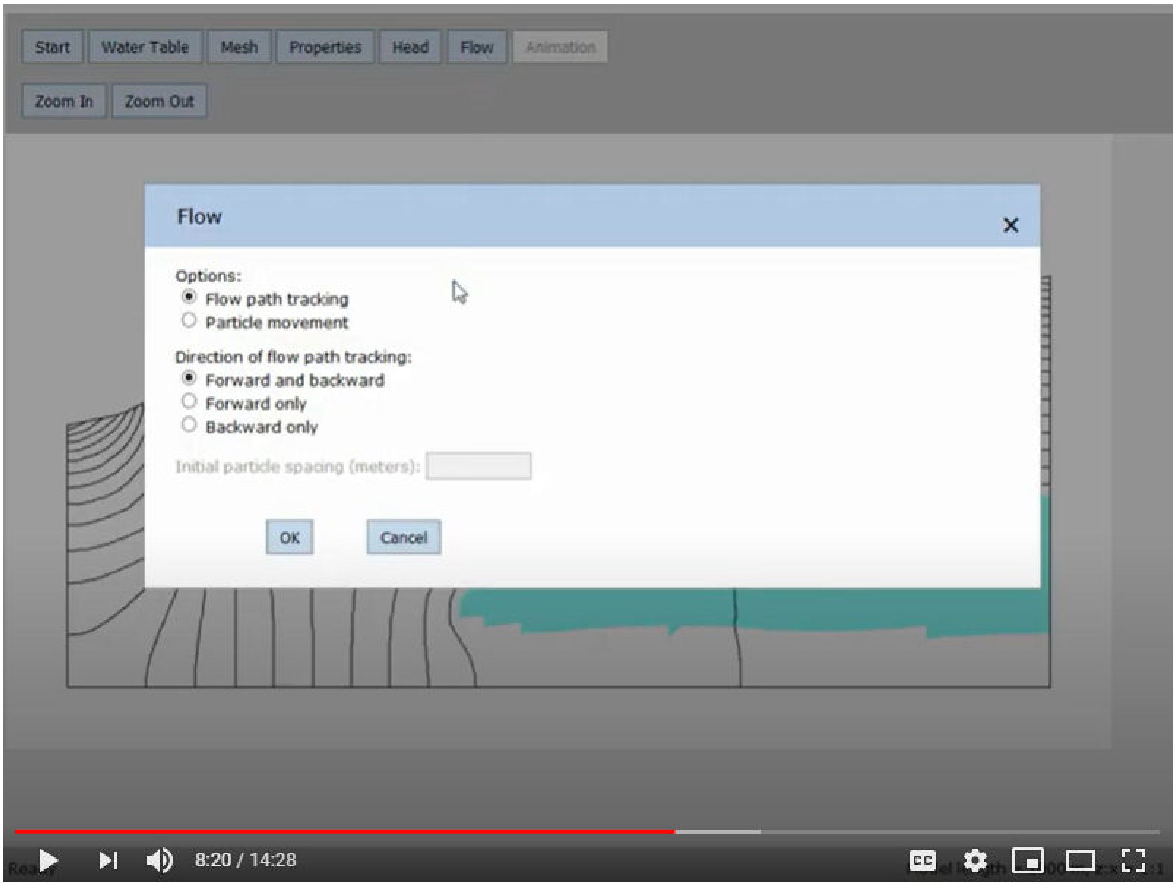


**Figure 19** - Click here for a video demonstration of using the TopoDrive software.

The TopoDrive software was created by Hsieh (2001) and updated by Hsieh (2020) to function on modern web browsers. Once one learns how to use TopoDrive, the best use of it is for checking one's groundwater system intuition. In order to do that, create a system using the instructions provided in Box 6, and before clicking the "Head" button which will draw the equipotential lines, develop a mental (or hand sketched) picture of the general appearance of the equipotential lines, then click the "Head" button to see the solution. If the head distribution is much different than what you envisioned, consider why your intuition differed from the correct solution so that you can work toward improving your intuition. Similarly, before clicking a point to initiate flow path tracking, try to visualize where the flow path will go, then click the point to see the solution. Again, if the flow path is much different than what you envisioned, consider why your intuition differs from the correct solution. In summary, rather than using the software to obtain a solution, use the software to evaluate and improve your intuition of groundwater flow systems.



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## 2.10 Summary

A flow net consists of a set of equipotential lines, which illustrate the distribution of hydraulic head in a flow system, and a set of flow lines, which illustrate the paths of groundwater flow through the flow system. For construction of a flow net, the flow system needs to be in a steady state and two-dimensional in either a vertical cross-section or a horizontal plane (without areally distributed recharge). A flow net can be constructed graphically with pencil and paper according to the criteria delineated in this book, or it can be generated numerically using a computer model. In both cases, the resultant set of equipotential lines and flow lines satisfy the equation of groundwater flow.

Graphical construction of flow nets is limited to flow systems with homogeneous and isotropic hydraulic conductivity. Certain types of anisotropy can be handled by geometric transformation of the system to an isotropic equivalent. Graphical construction requires that equipotential lines and flow lines are drawn to intersect each other at right angles, and that the two sets of intersecting lines form shapes with constant aspect ratios, preferably curvilinear squares to facilitate maintaining a constant ratio and to use Equation 2 for calculating flow through the system. When these requirements are satisfied, equipotential lines have uniform increments (contour interval) from one line to the next, and all flow tubes carry the same volumetric flow rate. The exception is a partial (or fractional) flow tube, which can occur in some circumstances at the edge of a flow domain. If the hydraulic conductivity is known, the volumetric flow rate through the flow system can be calculated.

When groundwater flow is simulated by numerical computer models, equipotential lines and flow lines are used primarily to display model results rather than to calculate flow rates, which has already been done by the model. Thus, there is greater flexibility for drawing equipotential lines and flow lines. For example, equipotential lines can have irregular increments and flow tubes can have different flow rates. Nonetheless, the equipotential lines and flow lines satisfy the equation of groundwater flow and therefore, together, they constitute a flow net.

Flow nets can be used to understand some basic features of groundwater flow. For example, if a flow tube becomes narrower, the principle of mass conservation requires specific discharge in the flow tube to increase. Additionally, according to Darcy's Law, higher specific discharge indicates a higher hydraulic gradient if hydraulic conductivity is homogeneous. Therefore, narrowing of a flow tube (converging flow lines) in a homogeneous hydraulic conductivity setting is associated with equipotential lines getting closer to each other.

The flow net for a homogeneous flow field will be distorted by the presence of a zone of different hydraulic conductivity. If the zone has a lower conductivity, a flow tube gets wider as it enters the zone and becomes narrower as it exits the zone; and equipotential lines are closer to each other within the zone. The opposite occurs when the zone has a



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higher hydraulic conductivity, where a flow tube becomes narrower as it enters the zone and wider as it exits the zone; and equipotential lines are spaced further apart within the zone.

In anisotropic (but homogeneous) systems, flow lines do not meet equipotential lines at right angles as is the case in isotropic systems, but transformation of the system geometry allows for graphical construction of a flow net, which can then be transformed back to the original geometry to illustrate flow paths.

## Note on Transient Groundwater Flow Systems

For transient systems, a formal flow net cannot be drawn in the same manner as presented in this book for a steady-state system. Although equipotential lines can be drawn for a flow system at a given point in time, the time-varying flow system makes it impossible to draw flow tubes that represent a constant rate of flow along the tube. Consequently, to visualize flow for transient systems, equipotential lines can be drawn at any time in the transient evolution of the system and velocity vectors can be sketched on the equipotential line map to indicate the direction and magnitude of flow at various locations in the systems for a snap-shot of flow patterns at that time.



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# 3 Exercises

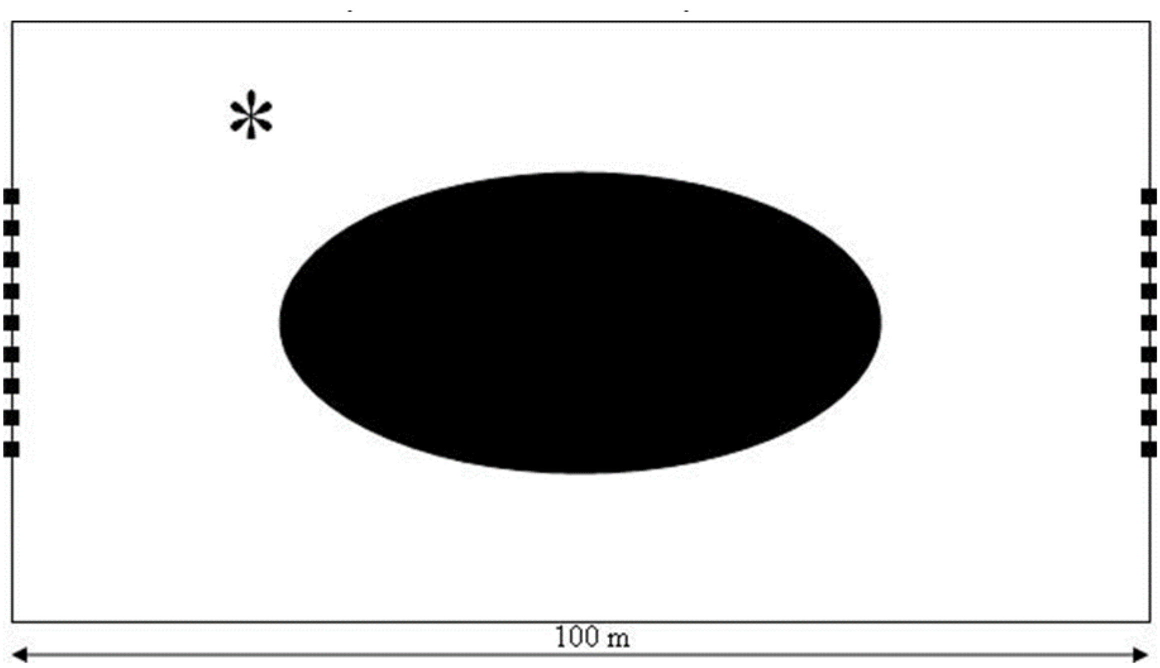
Exercise 1) A sand filter has its base at 0 meters and is 10 meters high. A plan view, to-scale diagram of it is shown below. These features are the same on every horizontal slice. There is an impermeable pillar in the center of the filter. Reservoirs on the left and right are separated from the sand by a screen that only crosses a portion of the reservoir wall. The hydraulic head in the inlet reservoir on the left is 20 m and the outlet reservoir on the right is 12 m. Hydraulic conductivity of the sand is: $K$=1×10$^{-3}$ m/s.

Draw and label a flow net.

Calculate the discharge through the system using units of cubic meters and seconds.

What is the hydraulic head in meters at the location of the * at the top of the tank?

What is the pressure head in meters at that location?

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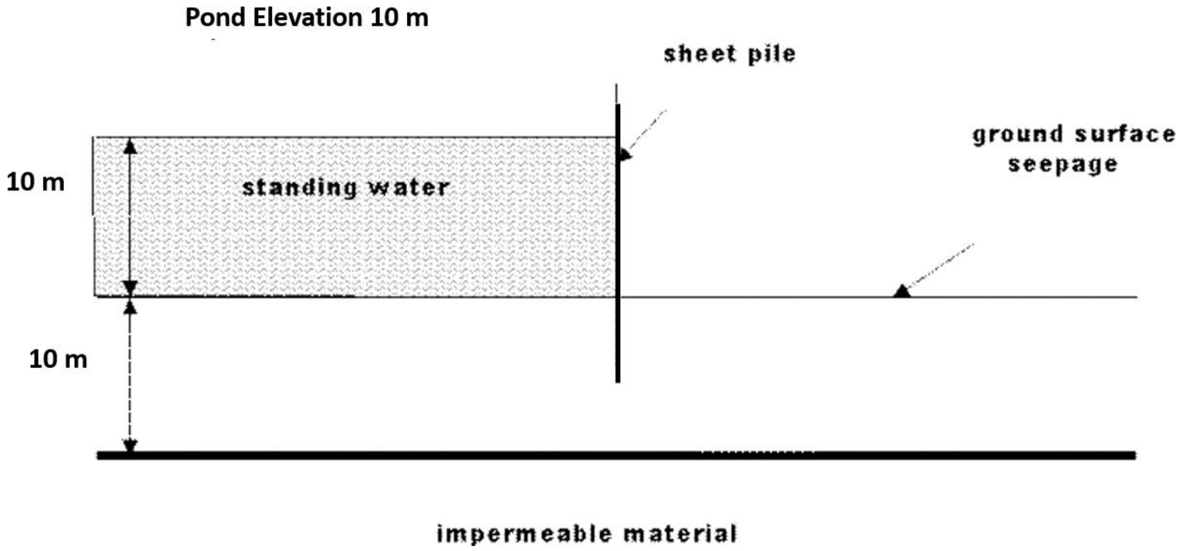
Exercise 2) An impermeable sheet pile is driven 5 m into the ground and extends 22 m in the direction perpendicular to the figure below. The sheet pile impounds 10 m of standing water on the left. Assume that the impounded water can seep through the underlying material with a hydraulic conductivity of 2 m/d to discharge at the ground surface on the right of the sheet pile.

What is the volumetric flow rate under the sheet pile wall?

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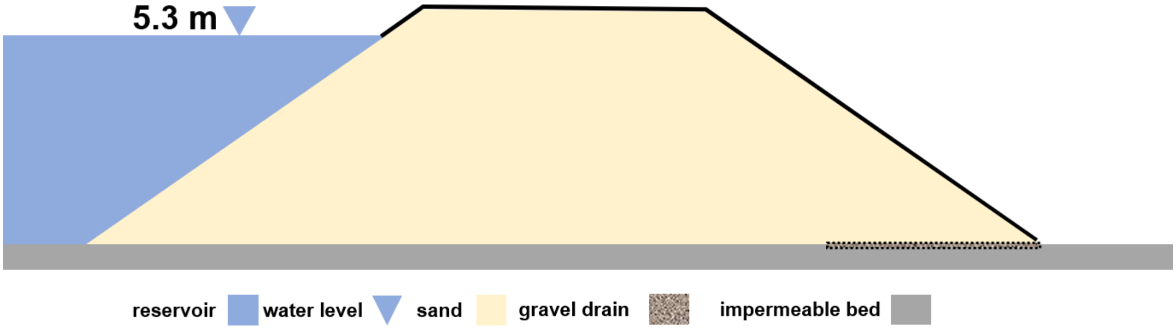
Exercise 3) An earth embankment retains water as shown in the diagram below. The earth material has a hydraulic conductivity of 0.2 m/d resting on an impermeable bed. The dam extends 28 m in the direction perpendicular to the diagram, and is capped with impermeable material such that water is prevented from infiltrating the surface of the dam. The upgradient reservoir has an elevation of 5.3 m. The gravel drain is keyed into the top of the impermeable bed on the right.

Draw and label a flow net of the system.

Calculate the discharge through the system using units of cubic meters and seconds.

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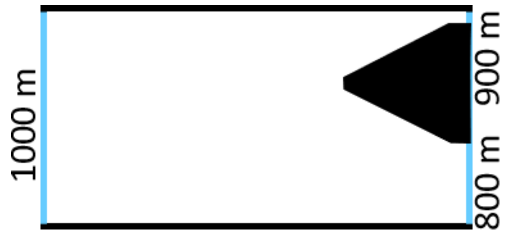
Exercise 4) A portion of an anisotropic ($K_x$=225 m/d and $K_y$ = 9 m/d), 100 m thick, confined aquifer is shown in map-view in the diagram below. The area is 281 km long and 144 km wide, and bounded by impermeable boundaries on the north and south. An impermeable material cuts through the aquifer as shown by the black zone. Head is measured at 1000 m on the west boundary, 900 m on the northeast boundary and 800 m on the southeast boundary.

Draw and label a flow net of the system.

Calculate the discharge through the system using units of cubic meters and seconds.

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# 4 References

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# 5 Boxes

## Box 1 – Review of Hydraulic Head

Mechanical energy is the primary driving force of groundwater flow. In other words, groundwater flows from locations of higher to lower mechanical energy. In some places, flow is driven by thermal or chemical differences. However, in many cases these situations are not important when drawing a flow net in a shallow groundwater system. Energy is force times distance. However, for groundwater evaluation it is convenient to express energy in terms of hydraulic head, defined as the mechanical energy per unit weight of water. Hydraulic head ($h$) is composed of two components: potential energy from the water's elevation in the gravitational field and energy from the fluid pressure distribution. To have the same dimension as elevation (that is, length), we replace pressure with pressure head, which is pressure divided by the product of water density and the acceleration of gravity. Pressure head is the height of a column of water required to cause a given pressure. For example, a one-meter high column of water will produce a pressure of about 9800 pascals, or 1.4 pounds per square inch. Therefore, a pressure head of 1 meter is equivalent to a pressure of 9800 pascals.

Hydraulic head in a groundwater system is the sum of elevation head and pressure head as shown in Equation Box 1-1.

$$h = z + \frac{p}{\rho g} = z + \psi \qquad \text{(Box 1-1)}$$

where:

$h$ = hydraulic head at a point in a groundwater system (L)

$z$ = elevation of the point (L)

$p$ = pressure at the point ($F/L^2$, Force/Area)

$\psi$ = $p/\rho g$ is pressure head (L)

$\rho$ = density of water ($M/L^3$)

$g$ = acceleration of gravity ($L/T^2$)

The components of hydraulic head in an open body of water are illustrated in Figure Box 1-1. A horizontal plane is chosen as a datum for elevation measurements. Sea level is often used as a datum. In these figures, the horizontal bedrock surface is used as the datum. For a hollow pipe that is open on both ends, as indicated by the solid vertical parallel lines in Figure Box 1-1, hydraulic head is being measured at the bottom of the pipe. The elevation of the measuring point, $z$, is the elevation of the bottom of the pipe above the datum; the pressure head, $\psi$, is the height of the column of water in the pipe above the



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measurement point; and, the hydraulic head, *h*, at the measurement point is the sum of the elevation head and the pressure head. The sum of the elevation and pressure head equals the elevation of the water level in the pipe. Figure Box 1-1 shows *z*, $\psi$, and *h* for both deep and shallow pipes in a static body of water. A reservoir is under hydrostatic conditions meaning that the water is motionless. In such a situation, the head is the same at all locations in the reservoir. One could observe this by placing a tube in a bathtub and noticing that the elevation of the water in the tube is the same no matter where the tube is located.

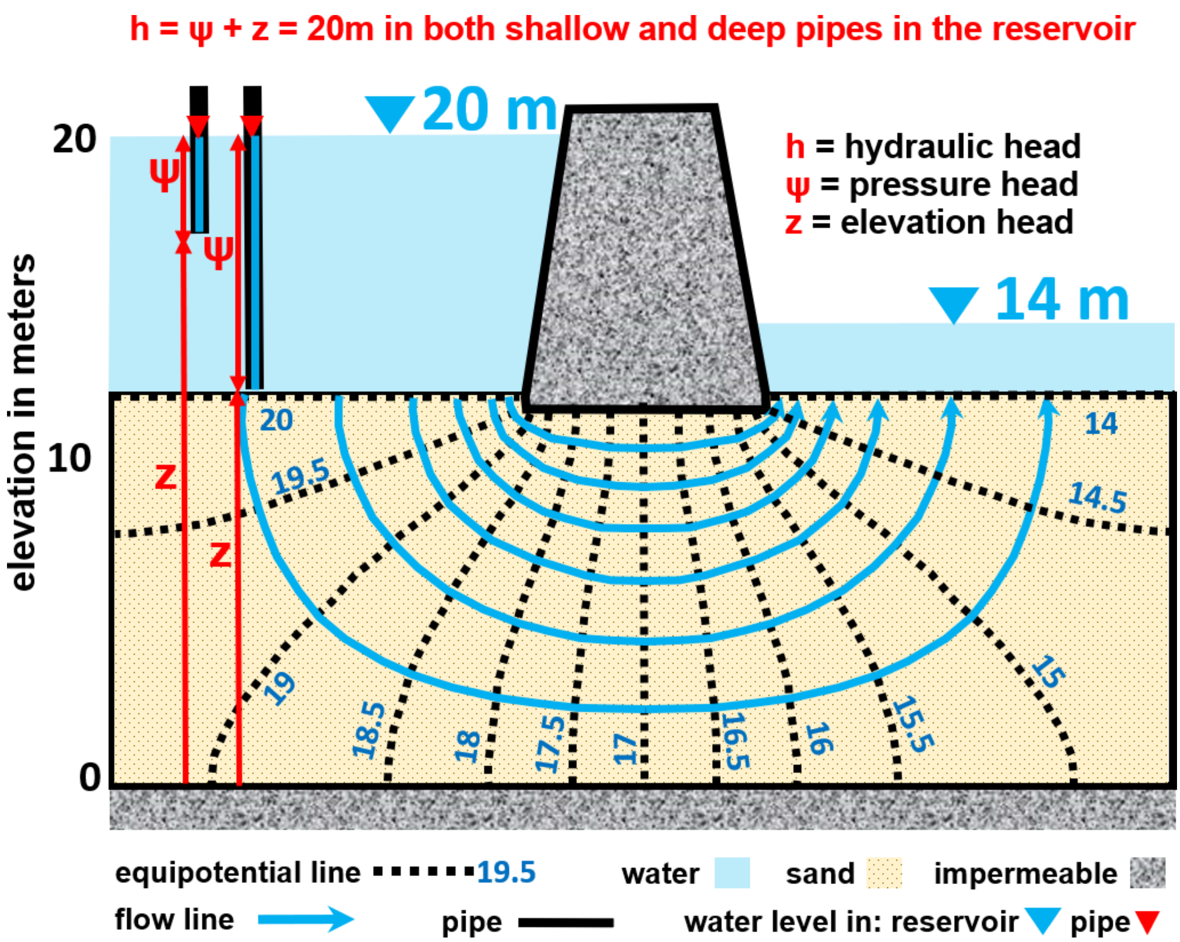


**Figure Box 1-1** - Schematic illustrating components of hydraulic head in an open body of water upgradient of a concrete dam. Hydraulic head is equal at all locations within a static body of water.

In a groundwater system below a dam with reservoirs at different elevations, the groundwater is in motion. It flows from the upper to the lower reservoir through the porous material below the dam. Figure Box 1-2 shows *z*, $\psi$, and *h* for hollow pipes (monitor wells) in a groundwater system. Again, the water level in each well indicates the hydraulic head at the location of the bottom of the pipe. Because groundwater flows from a region of higher head to a region of lower head, flow is from the left side of the figure to the right side. Head is "lost" as water flows through a porous medium because mechanical energy is converted to thermal energy, but the change in temperature is too small to measure. As water flows



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from the upper reservoir to the first piezometer on the left, 1 m of head is lost. An additional 4 m of head is lost by the time the water reaches the location of the piezometer on the right.

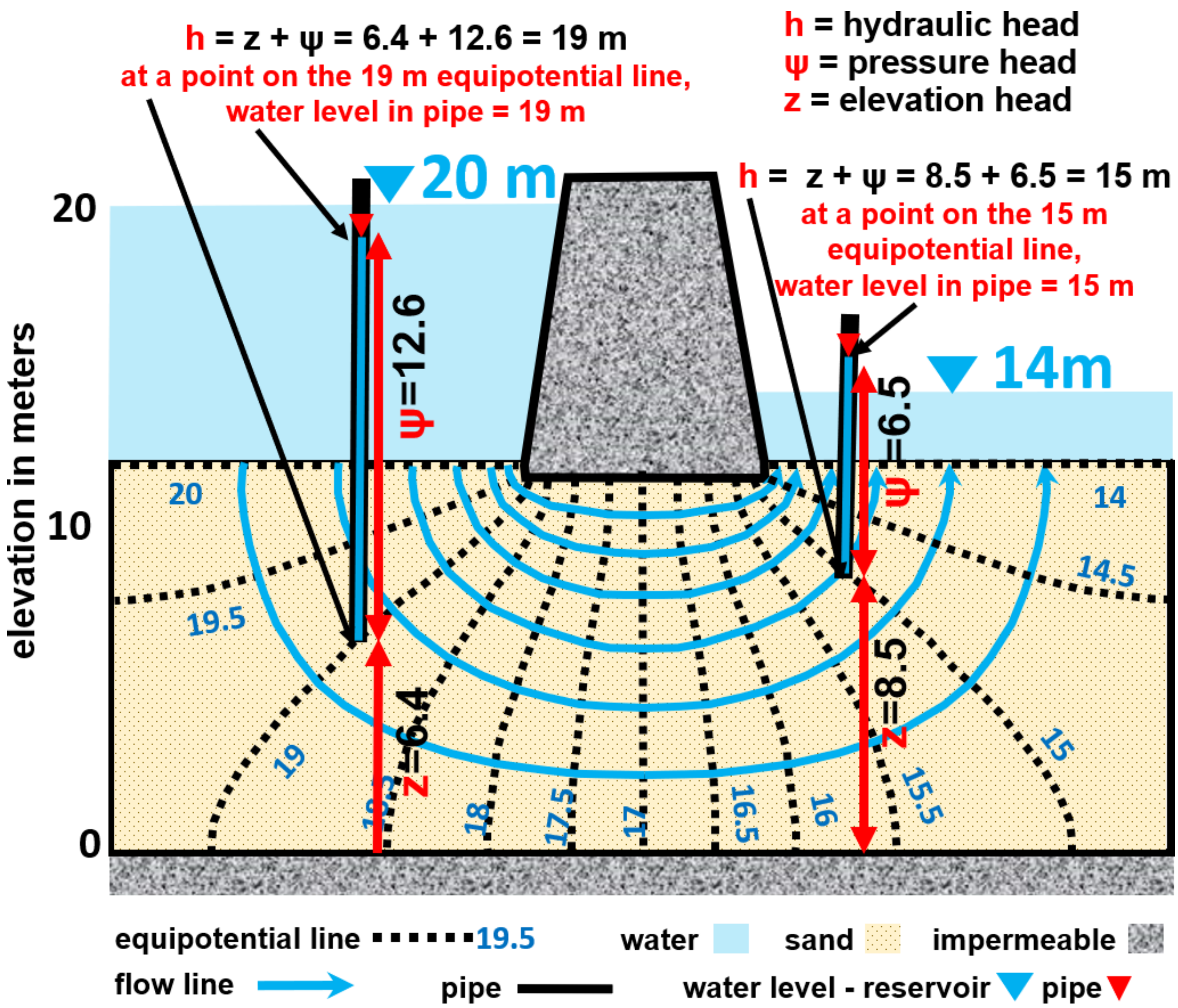


**Figure Box 1-2** - Schematic illustrating components of hydraulic head in the groundwater system for flow under a concrete dam.

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# Box 2 – Review of Darcy's Law

Darcy's Law (Equation Box 2-1) states that the volumetric flow rate (discharge) of groundwater in a porous material is 1) directly proportional to the difference in hydraulic head between two locations, 2) indirectly proportional to the length of the flow path between those locations, and 3) directly proportional to the area through which flow occurs. The proportionality is converted to an exact equation by including a proportionality constant, in this case, the hydraulic conductivity. Darcy's Law is illustrated in Figure Box 2-1. Darcy's Law dictates that heads will decline linearly in a homogeneous material with a uniform flow area as shown in Figure Box 2-2.

$$Q = \text{hydraulic conductivity} \frac{\text{head difference}}{\text{distance between heads}} \text{ area} = KiA \qquad \text{(Box 2-1)}$$

where:

$Q$ = volumetric flow rate ($L^3/T$)

$K$ = hydraulic conductivity of the porous medium (L/T)

$i$ = hydraulic gradient in the direction of flow, which is $h_2 - h_1$ of Figure Box 2-1 divided by the distance between the heads (dimensionless L/L)

$A$ = area perpendicular to the direction of flow ($L^2$)

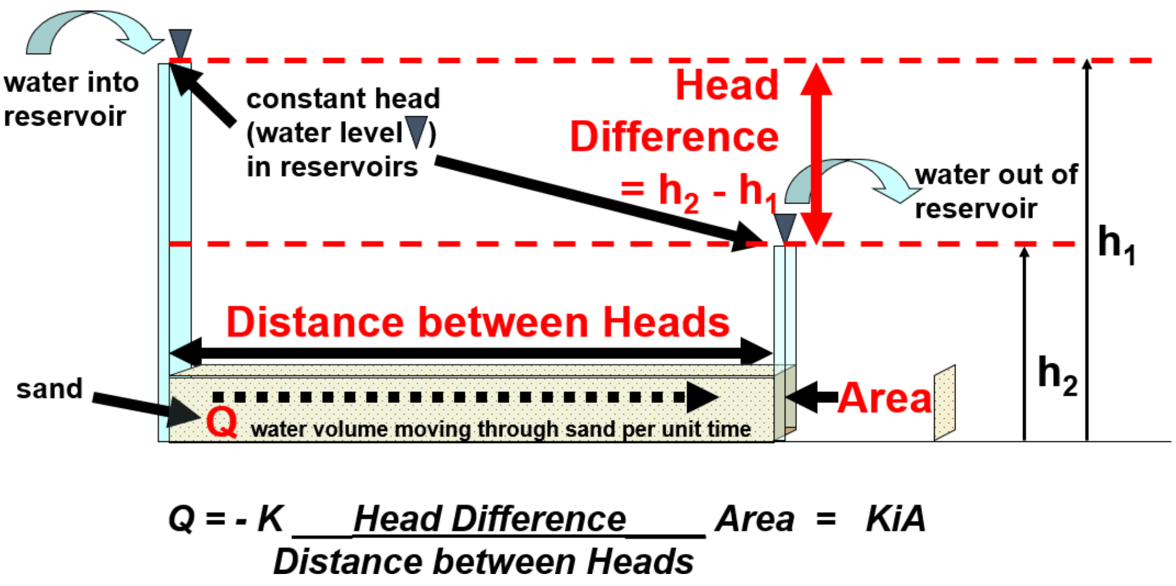


**Figure Box 2-1** - Illustrated parameters of Darcy's Law.



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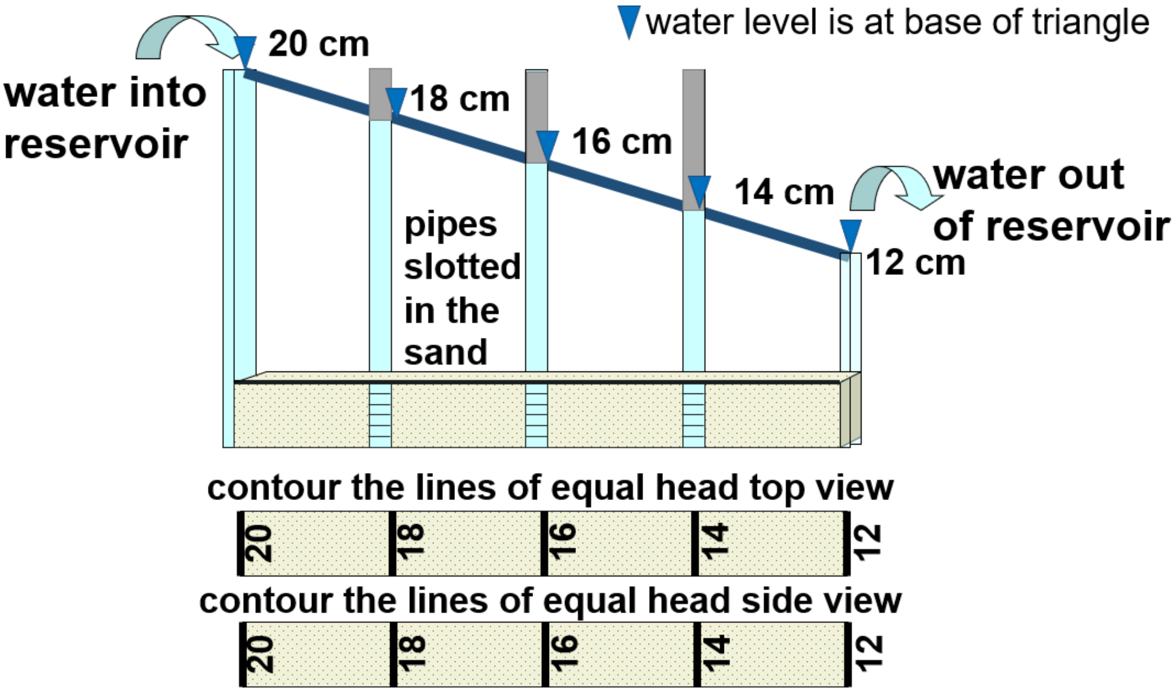


**Figure Box 2-2** - According to Darcy's Law, hydraulic head declines linearly in a homogeneous material with a constant flow area.

Specific discharge is the discharge (volumetric flow rate) divided by the flow area as shown in Equation Box 2-2. And, because the dimension of specific discharge is length over time (L/T), like the dimension for velocity, specific discharge is also referred to as Darcy velocity. However, this term can be confusing because it does not refer to the actual velocity of the groundwater (Figure Box 2-3a). Therefore, specific discharge, not Darcy velocity, is the preferred terminology used in this book.

$$q = \frac{Q}{A} \quad \text{also it can be determined as} \quad q = Ki \qquad \text{(Box 2-2)}$$

where:

$q$ = specific discharge (L/T)

At the microscopic scale, the detailed movements and velocities of groundwater flow in the void space between solid grains are exceedingly complex and are essentially impossible to accurately describe. However, Darcy's Law and groundwater hydrology deal with flow at a macroscopic scale, which ignores the complex twists and turns of flow at the microscopic scale. At the macroscopic scale (the scale at which Darcy's Law applies), we can define an "average linear groundwater velocity" as the specific discharge divided by the effective porosity, as shown in Equation Box 2-3. Effective porosity is the volume of void space that contains flowing water divided by the total volume of the porous medium (the combined volume of void space that contains flowing water and solid grains), as shown in Figure Box 2-3c. The term "linear" refers to the conceptualization that the water travels in a direct path between two points in the direction of the maximum gradient. The



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term “average”, however, does not refer to the average of the velocities at the microscopic scale. Instead, it is better to think of “average” as meaning “macroscopic.” To avoid writing the entire term “average linear groundwater velocity, the shorter form, “groundwater velocity”, is used in the rest of this box.

$$\bar{v} = \frac{q}{n_e} = \frac{Q}{An_e} \qquad \text{(Box 2-3)}$$

where:

$\bar{v}$ = average linear groundwater velocity (L/T)

$n_e$ = effective porosity, which is the quotient of the volume of interconnected pore space and the total volume of the material (dimensionless)

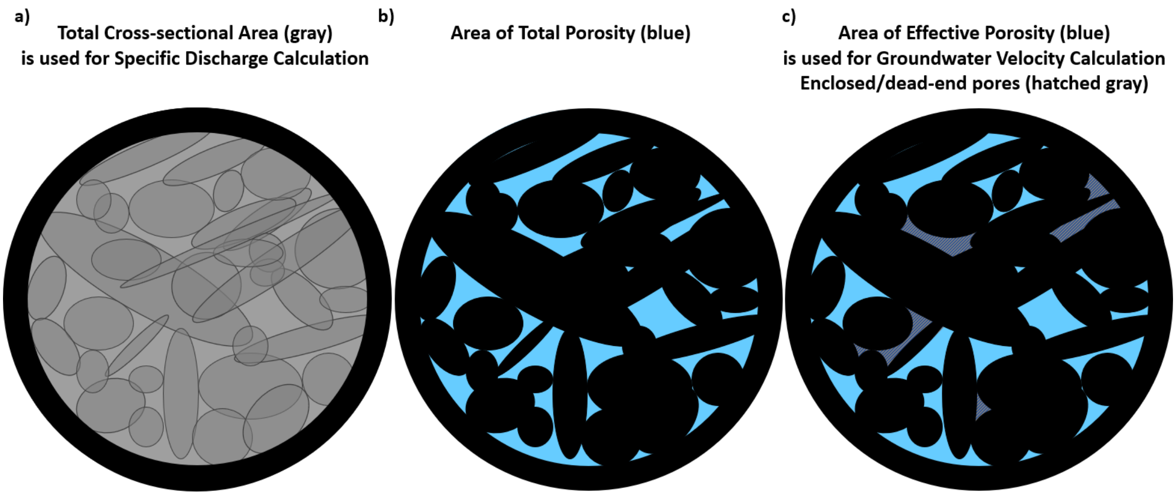


**Figure Box 2-3** - Areas used to calculate specific discharge and average linear velocity: a) specific discharge is defined as the volumetric flow rate divided by the total cross-section area (shown in gray); b) porosity includes all pore spaces as shown in blue; c) average linear velocity is higher than specific discharge because it accounts for only the area of groundwater flow through connected and non-dead-end pore spaces (blue area).

When considering groundwater travel time, we use the concept of a “packet of water” to represent a small amount of water that flows together without splitting up into smaller portions. Sometimes the term “particle of water” is used to mean a packet of water. If the groundwater velocity is uniform along a flow path, the time required for a packet of water to move from one location to another along the flow path is computed by dividing the distance of travel by the groundwater velocity. This is shown in Equation Box 2-4. For a flow field with constant velocity, the travel time from point A to B is distance divide by the velocity as would be done for travel in a car. For a flow field with non-uniform groundwater velocity, calculating the travel time along a flow path from point A to point B requires dividing the flow path into small segments and computing the travel time in each segment. The total travel time from A to B is then the sum of the travel times through



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all the segments. In practical applications, this calculation is usually done by a computer software that tracks packets of water through a simulated groundwater flow system.

$$t_t = \frac{\text{distance}}{\bar{v}} \quad \text{(Box 2-4)}$$

where:

$t_t$ = groundwater travel time along a flow path between two locations (T)

distance = distance along the groundwater flow path (L)

$\bar{v}$ = groundwater velocity (L/T) assumed to be uniform along the flow path

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# Box 3 – Derivation: Formula for Volumetric Flow Rate through a Flow Net

Volumetric flow rate (discharge) is the volume of water flowing through a system per unit of time. It is reported in a dimension of volume, or length cubed, over time (for example, liters per minute, or cubic meters per second).

Flow through a homogeneous sand with a hydraulic conductivity of 0.4 cm/s in a rectangular tank is used to derive the formula for volumetric flow rate through a flow net. A diagram of the rectangular tank is shown in Figure Box 3-1. Within the tank, sand fills the volume of a box, with all sides impermeable except where water enters and exits along the dashed black lines through a screen that contains the sand. The box is 33 cm tall, 66 cm long and 50 cm wide (that is, it extends 50 cm "into" the picture). Any cross-section cutting through the box from left to right is identical to every other left-right cross-section, so only one cross section of the box is shown Figure Box 3-1. The water level on the left side is maintained at 50 cm and the water level on the right side is maintained at 44 cm. Flow is from left to right.

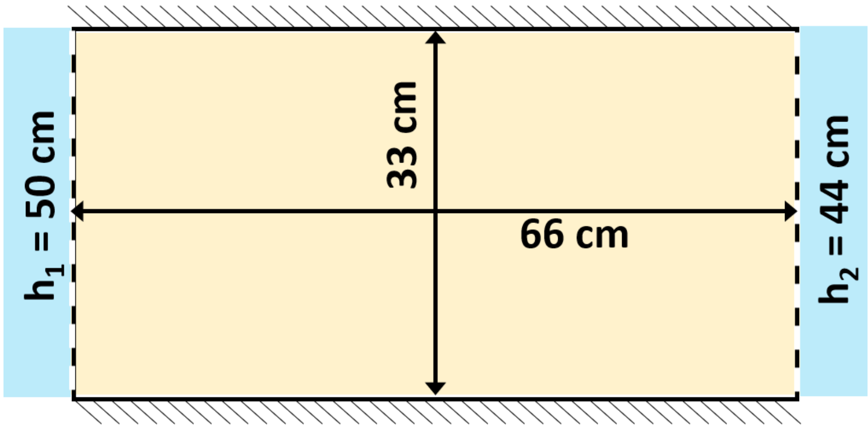


**Figure Box 3-1** - A rectangular box filled with sand with constant head reservoirs on each side.

For this rectangular tank, we do not need to construct a flow net and then apply a formula to determine the volumetric flow rate. Given the simple geometry of this flow tank, the volumetric flow rate can be determined directly by substituting values into Darcy's Law (Equation Box 3-1) as follows:

$$Q = -KiA \qquad \text{(Box 3-1)}$$

where:

$Q$ = volumetric flow rate ($L^3/T$)

$i$ = hydraulic gradient (dimensionless)

$A$ = area perpendicular to the direction of flow ($L^2$)



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$$Q = -\left(0.4\frac{\text{cm}}{\text{s}}\right)\left(\frac{44\text{ cm} - 50\text{ cm}}{66\text{ cm}}\right)(33\text{ cm})\,(50\text{ cm}) = 60\ \frac{\text{cm}^3}{s}$$

$$Q = \left(60\ \frac{\text{cm}^3}{s}\right)\left(60\frac{\text{s}}{\text{min}}\right)\left(\frac{\text{liter}}{1000\text{ cm}^3}\right) = 3.6\frac{\text{liters}}{\text{min}}$$

Although it is easy to use Darcy's Law to calculate the volumetric flow rate for a simple flow geometry, it is not easy for flow with complex geometry. To determine the volumetric flow rate for the case of complex geometry, it is necessary to either (1) graphically construct a flow net and then apply a formula to calculate the flow rate, or (2) use a numerical computer model to calculate the flow rate. This box addresses the first method and derives the necessary formula. The flow tank is used as an example, but the derived formula is valid for complex cases.

To develop the formula for the flow tank, a graphical flow net is constructed as is shown in Figure Box 3-2. For this simple flow geometry, the equipotential lines are straight and vertical, while the flow lines are straight and horizontal. The two sets of lines intersect at right angles to form squares. Every flow tube carries the same volumetric flow rate. Darcy's Law can be used to calculate the volumetric flow in an individual flow tube by calculating flow across the width $y$ for a square in the flow net (for example, the one highlighted in black in Figure Box 3-2).

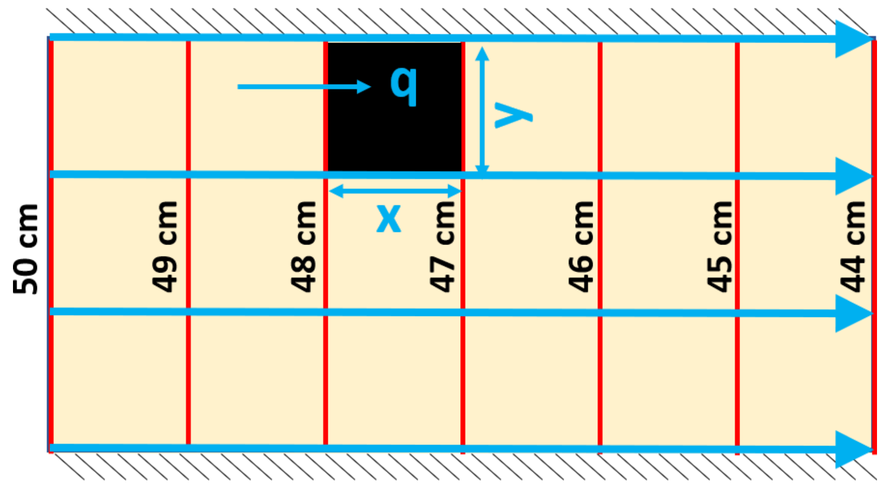


**Figure Box 3-2** - Diagram for deriving equation to determine volumetric flow rate of a flow net per unit width into the diagram through one square of a flow net.

The head difference between the equipotential lines in a flow net (Equation Box 3-2) is determined as the quotient of the total head drop across the flow net, $H$ ($H$ = 6 cm in this case), and the number of head drops, $n_d$ ($n_d$= 6 in this case, as shown in Figure Box 3-3).

$$h_{diff} = \text{contour interval} = \frac{H}{n_d} \qquad \text{(Box 3-2)}$$

where:



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contour interval = head difference between adjacent equipotential lines (L)

$H$ = total head drop across the flow net domain (L)

$n_d$ = number of head drops in the flow net (dimensionless)

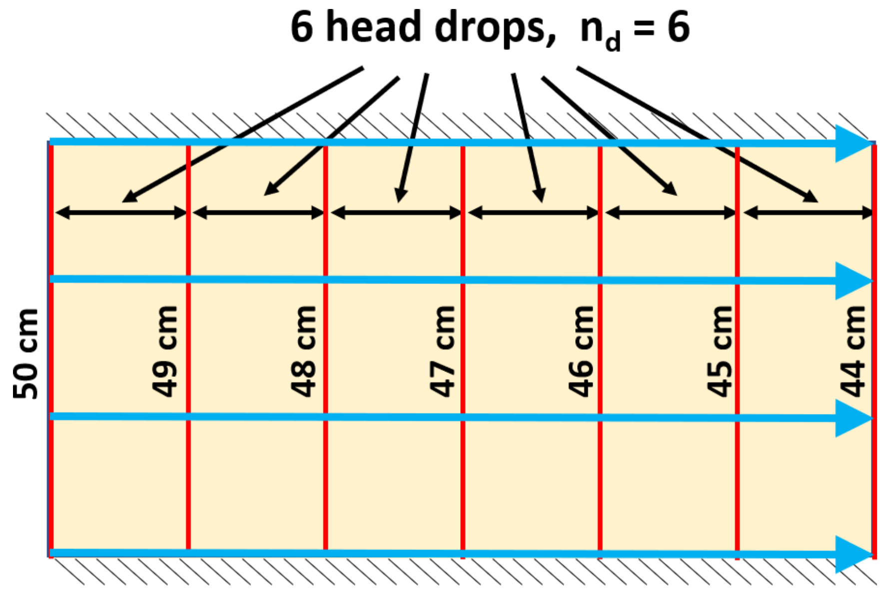


**Figure Box 3-3** - Number of head drops in the example flow net.

For this flow net, $h_{diff}$= ((6 cm) / (6 head drops)), or 1 cm as is already indicated by the contour line labels in Figure Box 3-2 and Figure Box 3-3.

The volumetric flow rate through one flow tube per unit width normal to the diagram is determined using Darcy's Law as the product of hydraulic conductivity ($K$), the gradient ($h_{diff}/x$) and the width of the flow tube in the plane of the flow net (Equation Box 3-3).

$$Q'_{tube} = K\frac{h_{diff}}{x}y \qquad \text{(Box 3-3)}$$

where:

$Q'_{tube}$ = volumetric flow rate through one flow tube per unit width perpendicular to the diagram ($L^2/T$)

$x$ = distance between the equipotential lines (L)

$y$ = width of the flow tube in the plane of the flow net (L)

For a square in the flow net (such as the one highlighted in black in Figure Box 3-2), $x$=$y$, and the equation can be simplified by canceling $y$ with $x$, leaving the formula for the flow through a flow tube per unit width into the diagram as shown in Equation Box 3-4.



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$$Q'_{tube} = K\, h_{diff} = K\, \frac{H}{n_d} \qquad \text{(Box 3-4)}$$

For flow nets with more complex geometry, the equipotential lines and flow lines intersect to form curvilinear squares rather than exact squares. In this case, the ratio of $x$ to $y$ can be considered an aspect ratio. For a curvilinear square, the aspect ratio is 1, so that the simplification from Equation Box 3-3 to Equation Box 3-4 is still valid.

Because $Q'_{tube}$ is the same for all flow tubes, the volumetric flow rate through the entire system per unit width perpendicular to the diagram can be obtained by multiplying $Q'_{tube}$ by the number of flow tubes as in Equation Box 3-5.

$$Q' = Q'_{tube} n_f = K \frac{H}{n_d} n_f \qquad \text{(Box 3-5)}$$

where:

$n_f$ = number of flow tubes in the flow net

$Q'$ = volumetric flow rate through the entire system per unit width perpendicular to the diagram ($L^2/T$)

This flow net has 6 head drops ($n_d$ = 6) and 3 flow tubes ($n_f$ = 3) as shown in Figure Box 3-4.

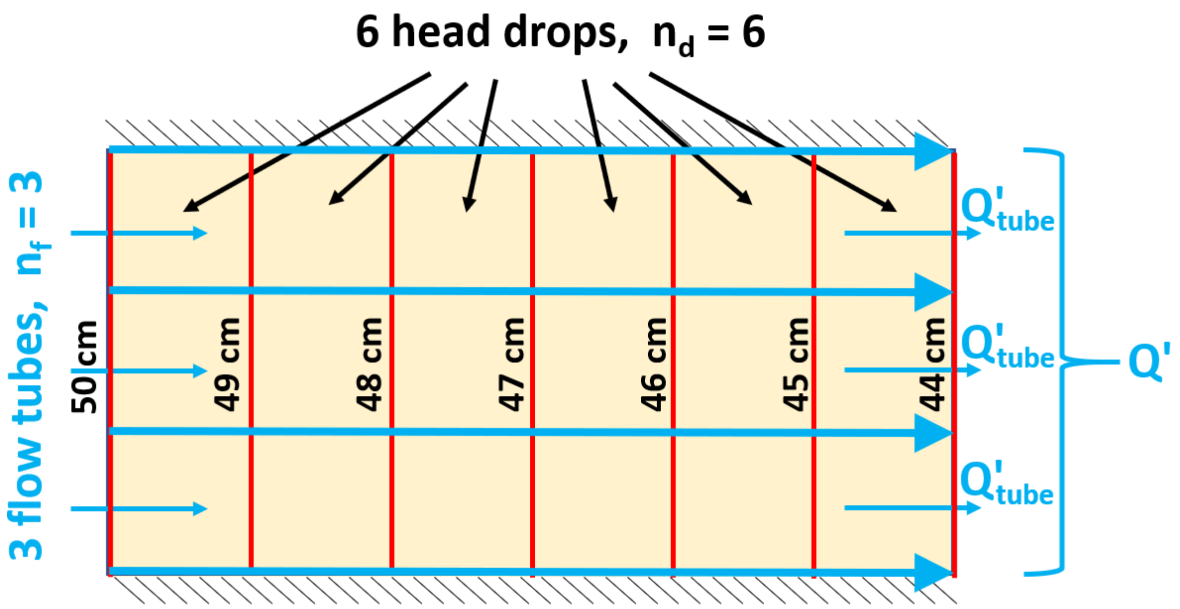


**Figure Box 3-4** - Number of flow tubes and head drops in the example flow net.

The parameter $Q'$ has a dimension of length squared over time, $L^2/T$, because it represents the volumetric discharge rate for one unit-width into the diagram (Equation Box 3-6).



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$$Q' = K\frac{H}{n_d}n_f \quad = KH\frac{n_f}{n_d} \quad : \quad \frac{\mathrm{L}}{\mathrm{T}}\,\mathrm{L}\frac{-}{-} \quad : \quad \frac{\mathrm{L}^2}{\mathrm{T}} \qquad \text{(Box 3-6)}$$

where:

$-$ = Indicates a parameter is dimensionless

To calculate the total volumetric flow rate through the box, $Q_{total}$, we multiply $Q'$ by the distance the box extends "into" the figure, producing a dimension of length cubed over time, $\mathrm{L}^3/\mathrm{T}$ as in Equation Box 3-7.

$$Q_{total} = K\,H\frac{n_f}{n_d}\,w \,:\, \frac{\mathrm{L}^3}{\mathrm{T}} \qquad \text{(Box 3-7)}$$

where:

$w$ = distance that the flow net domain extends in the direction normal to the plane of the diagram (L)

The formula can be used to calculate volumetric flow through graphically constructed flow nets with complex geometry. The formula for determining the volumetric flow through a flow net does not involve the absolute dimensions of the length and height of the system. It uses only relative values of the length and height of the box, so the drawing must be to scale. That is, the relative length and width of the drawing must be correct. Also, the distance that the system extends normal to the diagram must be known to determine the total flow through the system.

By substituting the appropriate values of $n_f$, $n_d$, $H$, $K$ and $w$ for the rectangular box into the equation for volumetric discharge through a flow net, the calculated flow equals that calculated by Darcy's Law at the beginning of this box.

$$Q_{total} = \left(0.4\ \frac{\mathrm{cm}}{sec}\right)\ (6\ \mathrm{cm})\ \left(\frac{3}{6}\right)(50\ \mathrm{cm}) = 60\ \frac{\mathrm{cm}^3}{\mathrm{s}}$$

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# Box 4 – Drawing a Flow Net for an Unconfined System with a Water Table Boundary

Unconfined groundwater systems have a water table boundary which requires special consideration when drawing a flow net because the location of the water table boundary is not known until an acceptable flow net has been drawn.

To illustrate inclusion of a water table in a flow net, consider flow through an earthen dam with a hydraulic conductivity of 0.2 m/d resting on an impermeable base (Figure Box 4-1). The dam extends 55 m in the direction perpendicular to the diagram, and is capped with impermeable material such that water is prevented from infiltrating the surface of the dam. Water enters from the up-gradient reservoir on the left and exits to the downstream reservoir on the right.

Begin the steps for drawing a flow net as described in section 2.3 and shown in Figure Box 4-1. The position of the water table is not known until the flow system is revealed by following the rules for drawing a flow net, so the initial sketch indicates this uncertainty by using a dashed line with "?".

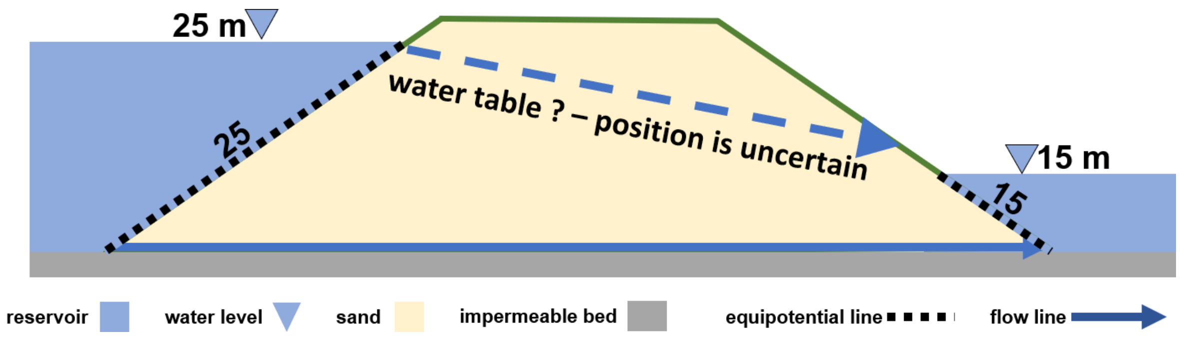


**Figure Box 4-1** - Step 1 - Draw the system to scale, Step 2 – Draw equipotential lines to coincide with head boundaries, Step 3 - Draw flow lines to coincide with no-flow boundaries.

Drawing a flow net is a trial-and-error process because equipotential and flow lines are adjusted until curvilinear squares are formed. The added complication when drawing an unconfined flow net is that the position of the upper boundary (the water table) and length of the seepage face are also adjusted while working to create curvilinear squares.

The next step is to draw flow lines along paths where you envision groundwater will flow, ensuring they are perpendicular to equipotential lines on the boundaries (Figure Box 4-2). There is no recharge from the impermeable upper boundary, so the uppermost flow line forms the water table. The flow line should meet the 25 m equipotential line (constant head boundary) at a right angle. The downstream end of the water table should meet the dam surface at an elevation higher than the surface of the downstream reservoir (forming a seepage face as shown in Figure Box 4-2), and at a slope equal to the slope of the dam face. In the same way that the initial position of the water table is unknown until after the flow net is drawn, the length of the seepage face is unknown until after a valid flow net



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is drawn. Hydraulic head along a seepage face is equal to the elevation of the ground surface because the gage pressure along the seepage face is zero. Unlike the water table, the location of the seepage face boundary is known because it will be on the downgradient face of the dam, only its length is unknown before sketching the flow net.

Although, for expediency, we have drawn the flow lines in the correct position here, it is likely the first attempt to draw flow lines will require adjustment when drawing a flow net. It is expected that the flow net lines will be erased and redrawn as needed until the criteria for a valid flow net are met.

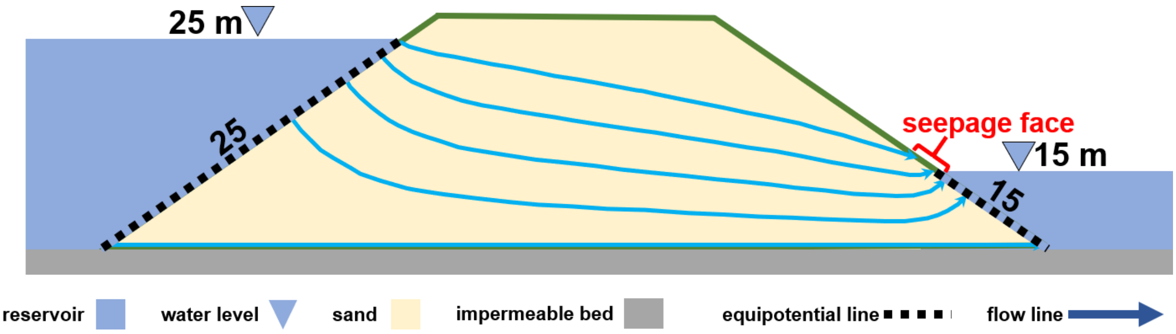


**Figure Box 4-2** - Step 4: Draw flow lines along paths where you envision groundwater flowing, ensuring they are perpendicular to equipotential lines on the boundaries. It is likely the first attempt to draw flow lines will require adjustment. Drawing flow nets is a trial-and-error process.

As one draws the flow net, it is important to remember that the hydraulic head value of an equipotential line is equal to the elevation at which it meets the water table (or the seepage face) because hydraulic head is the sum of pressure head (in terms of a height of a column of water) and elevation. Gage pressure is typically used for quantifying pressure, with atmospheric pressure being equivalent to zero gage pressure. At the water table and along the seepage face, the gage pressure is zero so the hydraulic head is equal to the elevation. For this reason, it is useful to add lines of equal elevation before sketching the equipotential lines (Figure Box 4-3) as a guide for drawing the flow net.

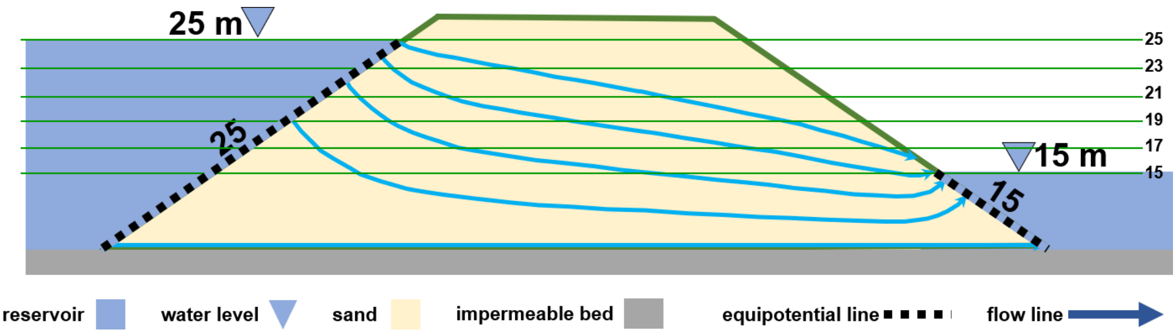


**Figure Box 4-3** - Add lines of equal elevation before sketching the equipotential lines, because the value of an equipotential line is equal to elevation where it intersects the water table and seepage face, so the elevation lines provide a guide for placing the equipotential lines.



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Once the elevation grid is drawn, proceed to sketch the equipotential lines at right angles to no-flow boundaries and flow lines. Make sure that equipotential lines meet the water table and the seepage face at an elevation that is the same as the hydraulic head of the equipotential line. The equipotential lines and flow lines should intersect to form curvilinear squares. As before, one way to decide if you are creating curvilinear squares is to draw a circle between the intersecting lines. If the circle fits roughly within the shapes, then they are approximately curvilinear squares (Figure Box 4-4).

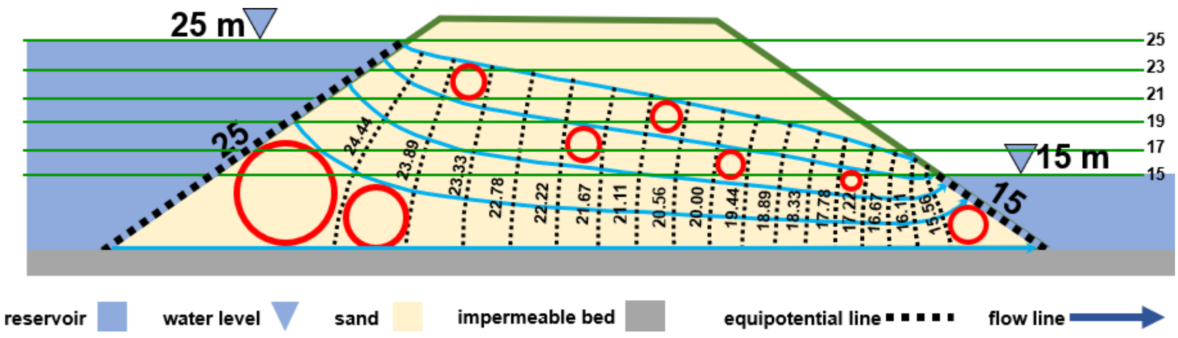


**Figure Box 4-4** - The shapes are curvilinear squares if circles fit approximately within them, but some flow nets may include partial flow tubes as shown here by the narrow flow tube at the bottom of the flow net. The flow net has 18 head drops and 4 flow tubes.

A rough sketch of the equipotential lines provides an estimate of the number of head drops that will create curvilinear squares. In this case, it is determined that 18 head drops create curvilinear squares. So, the contour interval is determined by dividing the total head drop of 10 m by 18 to obtain a value of 0.56 m head drop between each pair of equipotential lines. This establishes the values of the contour lines and knowing that they must meet the water table at the elevation equal to their values further constrains the position of the lines. For example, the first equipotential line to the right of the upper reservoir will have a value of 24.44 m and so the intersection of the equipotential line and the water table should be at that elevation.

Often, both equipotential lines and flow lines need to be erased and redrawn repeatedly before achieving curvilinear squares with equipotential lines meeting the water table at right angles and at an elevation equal to their value. Even after adjustment, a hand drawn flow net is only an approximate solution to the flow equations. For the purpose of this book, a fairly precise flow net is shown as Figure Box 4-5. The flow net does not provide precision to the 3 significant figures shown in the contour labels in the diagram. Three significant figures are shown, not because the system is known to high precision, but to adequately illustrate the difference in head between adjacent contour lines.



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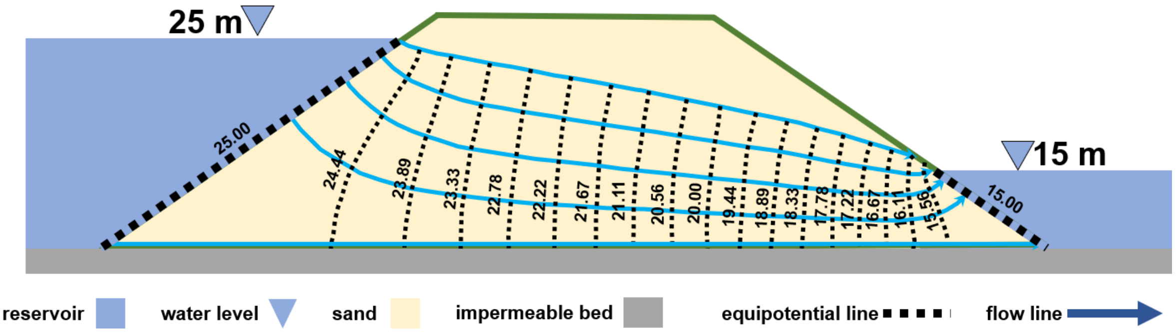


**Figure Box 4-5** - Draw equipotential lines for the unconfined flow net, ensuring that 1) their value is equal to the elevation of the water table where they meet the water table and to the elevation of the ground surface where they meet the seepage face, 2) that they meet flow lines at right angles, and 3) that the intersecting lines form curvilinear squares. This result was obtained after sketching and erasing until all of the criteria were met.

Given that the dam extends 55 m into the image and the hydraulic conductivity of the earth material was 0.2 m/d, the volumetric flow rate through the dam is:

$$Q_{total} = KH\frac{n_f}{n_d}w = \left(0.2\,\frac{\mathrm{m}}{\mathrm{d}}\right)\,(10\ \mathrm{m})\,\left(\frac{4}{18}\right)\,(55\ \mathrm{m}) = \ 24.44\,\frac{\mathrm{m}^3}{\mathrm{d}} \sim 25\,\frac{\mathrm{m}^3}{\mathrm{d}}$$

which is about 125 oil drums full of water each day, and would take about 100 days to fill an Olympic-size swimming pool. As noted earlier, it is important to recognize that the volumetric flow rate determined from a flow net is an approximate value.

If we had started by sketching equipotential lines with a round number contour interval, for example, a 1-m interval, there would be 10 head drops ([25-15]/1 = 10). In that case, the flow net would have fewer flow tubes and it would not be possible to form curvilinear squares for all tubes because to obtain the ratio of 4/18=0.222 with 10 head drops, 2.22 flow tubes are needed. Consequently, one of the flow tubes needs to be a 0.22 portion of a curvilinear square as shown by the deepest flow tube of Figure Box 4-6. In that case, the volumetric flow rate through the dam is:

$$Q_{total} = KH\frac{n_f}{n_d}w = \left(0.2\,\frac{\mathrm{m}}{\mathrm{d}}\right)\,(10\ \mathrm{m})\,\left(\frac{2.22}{10}\right)\,(55\ \mathrm{m}) = \ 24.42\,\frac{\mathrm{m}^3}{\mathrm{d}} \sim 25\,\frac{\mathrm{m}^3}{\mathrm{d}}$$

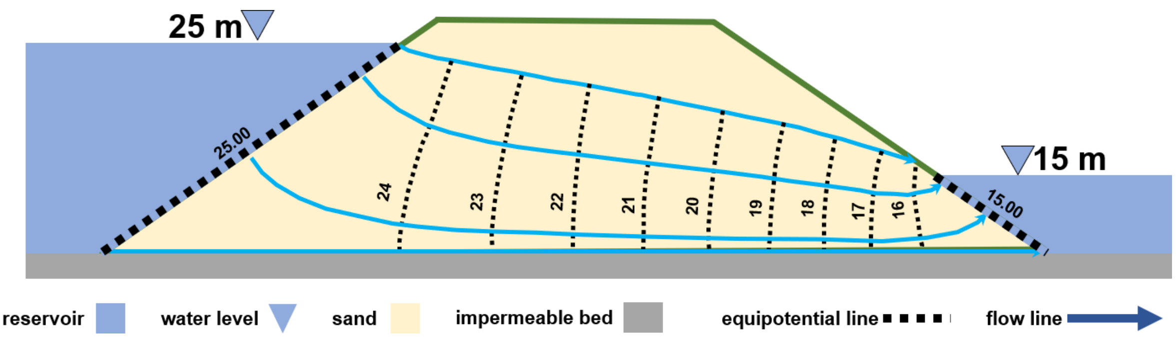


**Figure Box 4-6** - Some flow nets may include partial flow tubes as shown here by the narrow flow tube at the bottom of the flow net. The flow net has 10 head drops and 2.22 flow tubes.

Return to where text links to Box 4↥



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# Box 5 – Drawing Flow Nets for Anisotropic Systems

Hydraulic conductivity in an anisotropic aquifer varies with direction. In that case, the flow lines and equipotential lines of a flow net will not meet at right angles. Nonetheless, flow nets can still be graphically constructed if the manner in which the hydraulic conductivity varies with direction is the same everywhere in the flow system (that is, if the hydraulic conductivity is "homogeneously anisotropic"). Under this circumstance, a flow net can be graphically constructed by (1) transforming the geometry of the system to an isotropic system, (2) drawing a flow net for the isotropic system, and (3) transforming the flow net back to the original anisotropic system.

To understand the rationale behind the geometric transformation, it is useful to consider how hydraulic conductivity varies with direction. If we extract a core sample from a porous medium along a given direction and measure the hydraulic conductivity along the longitudinal axis of the core, we obtain the directional hydraulic conductivity ($K_d$) in that direction. If hydraulic conductivity is isotropic, then $K_d$ is the same in all directions. If hydraulic conductivity is anisotropic, then $K_d$ varies with direction. The direction in which $K_d$ attains its maximum value is known as the maximum principal direction. The $K_d$ in this direction is denoted as $K_{max}$. Perpendicular to maximum principal direction is the minimum principal direction, along which $K_d$ attains its minimum value, denoted as $K_{min}$. Development and explanation of the hydraulic conductivity ellipse is provided in Groundwater Project book↗ (Woessner and Poeter, 2020).

A polar-coordinates plot is a useful way to illustrate $K_d$. In such a plot, the square root of $K_d$ is plotted in all directions as distance from the origin. In the isotropic case, the result is a circle. In the anisotropic case, the result is an ellipse, known as the hydraulic conductivity ellipse (Figure Box 5-1a). The major and minor axes of the ellipse are aligned respectively in the maximum and minimum principal directions of $K_d$. In the general case, the ellipse can be in any orientation. Figure Box 5-1b, shows the case in which the ellipse's major and minor axes are aligned with a rectangular ($x$-$y$) coordinate system. In this case, $K_{max}$ and $K_{min}$ can be written as $K_x$ and $K_y$ (if $K_{max}$ is associated with the $x$ direction) or $K_y$ and $K_x$ (if $K_{max}$ is associated with the $y$ direction). It is for this setting that we will illustrate the geometric transformation procedure below.



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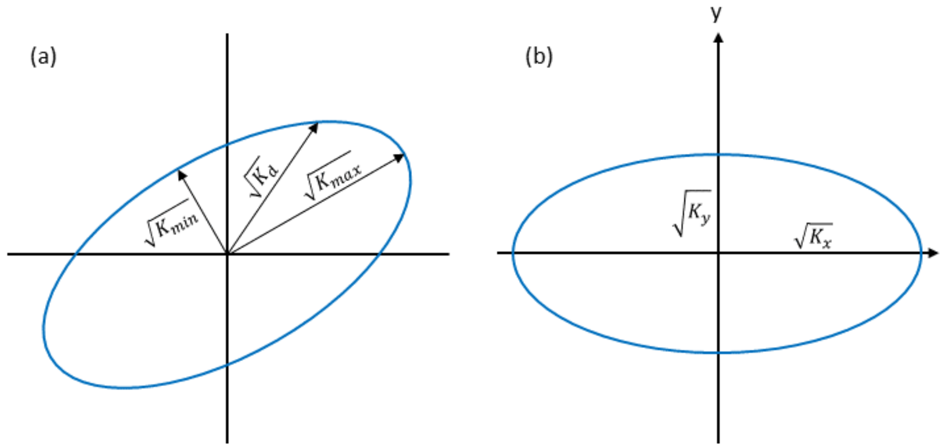


**Figure Box 5-1** - Hydraulic conductivity ellipse (a) in general orientation and (b) with major and minor axes aligned with the rectangular (*x*-*y*) coordinate system.

The geometric transformation from an anisotropic system to an isotropic system can be viewed as transforming the hydraulic conductivity ellipse into a circle. This can be done by transforming either the $y$ axis or the $x$ axis. When transforming the $y$ axis, we multiply the $y$-coordinates of the ellipse by the ratio $\sqrt{K_x}/\sqrt{K_y}$. That is, any point ($x$,$y$) in the original coordinate system will be moved to a point ($x$,$Y$) in the transformed coordinate system where, $Y$ is defined in Equation Box 5-1.

$$Y = y\frac{\sqrt{K_x}}{\sqrt{K_y}} \quad \text{(Box 5-1)}$$

If $K_y$ is less than $K_x$, the circle will be larger than the original ellipse (and circumscribe it), whereas if $K_y$ is greater than $K_x$, the circle will be smaller than the original ellipse and the ellipse will circumscribe the circle.

Alternatively, we could multiply the $x$-coordinates of the ellipse by the ratio $\sqrt{K_y}/\sqrt{K_x}$. That is, any point ($x$,$y$) in the original coordinate system will be moved to a point ($X$,$y$) in the transformed coordinate system where, $X$ is defined by Equation Box 5-2.

$$X = x\frac{\sqrt{K_y}}{\sqrt{K_x}} \quad \text{(Box 5-2)}$$

Either transform results in an acceptable isotropic geometry for the system as shown in Figure Box 5-2.



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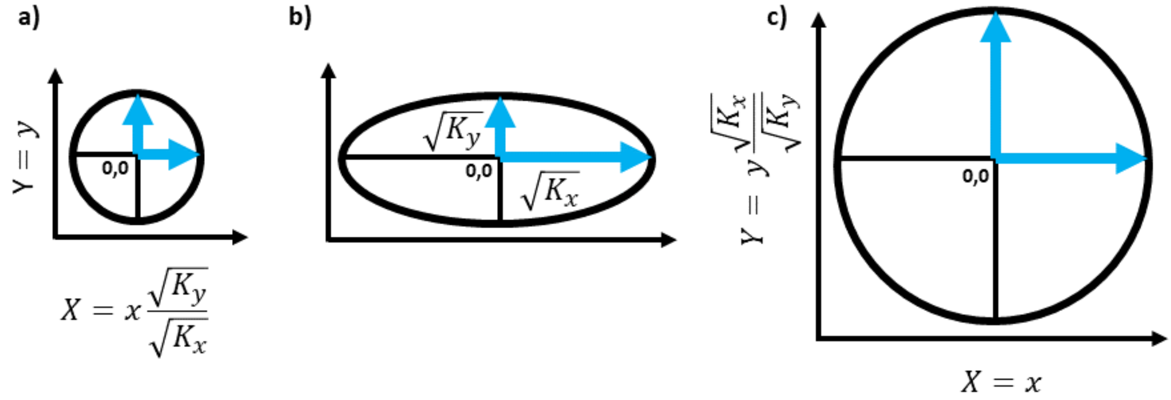


**Figure Box 5-2** - Transformation of an anisotropic hydraulic conductivity ellipse (center) into an isotropic ellipse (circle) by either transforming the *x*-axis (left) or the *y*-axis (right).

An example is provided to make the process clear. Suppose you want to draw a flow net for an irrigated field with many parallel drains as shown in Figure Box 5-3. In this system, the drains are 100 m long. The horizontal hydraulic conductivity, $K_x$, is 0.16 m/d and the vertical hydraulic conductivity, $K_y$, is 0.01 m/d. The ground surface elevation is 0.6 m above bedrock, and the centers of the 0.1 m-diameter circular drains are 0.2 m above bedrock (so the bottom of each drain is at 0.15 m and the top is at 0.25 m). If the bedrock is the datum, then its elevation is 0.0 m. Now, suppose the field is flooded to a water elevation 0.8 m above bedrock.

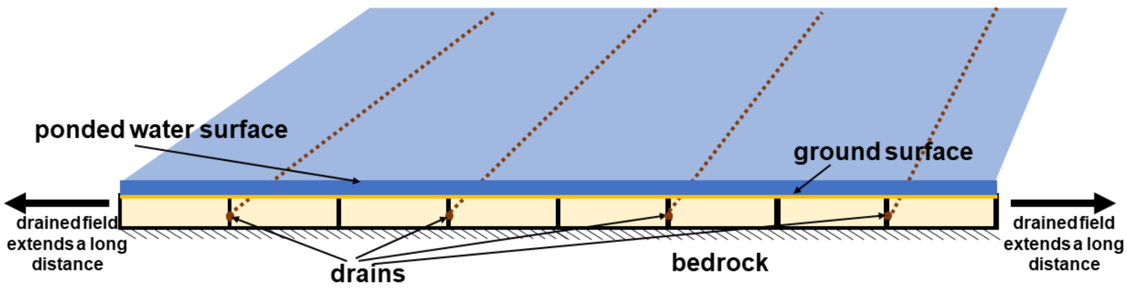


**Figure Box 5-3** - A groundwater flow system beneath an irrigated field with long, parallel drains.

Because the same flow pattern repeats itself in alternating mirror images throughout the field, only a small portion of the drained field needs to be drawn to develop a flow net (Figure Box 5-4).



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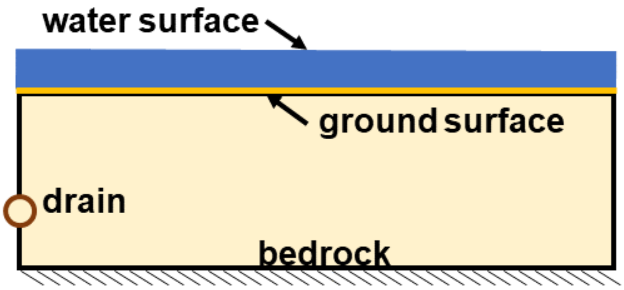


**Figure Box 5-4** Only a small portion of the field with parallel drains needs to be drawn to develop a flow net. A groundwater divide (no-flow boundary) occurs halfway between two adjacent drains. In addition, flow to the drain from the left is a mirror image of flow from the right. This symmetry enables us to consider the flow to the drain from only one side. The flow net developed for the above figure will repeat across the many drains of the field in alternating mirror images.

The horizontal hydraulic conductivity is 16 times higher than the vertical, $K_x = 16 K_y$, so the ratio of the semi-axes of the ellipse is $\sqrt{K_x}/\sqrt{K_y} = \sqrt{0.16}/\sqrt{0.01} = 4$. To convert the anisotropic system to an isotropic system, we can stretch the system vertically by a factor of 4. We leave the $x$-coordinates as they are and multiply the $y$-coordinates by 4. The ground surface is at 0.6 m so it is increased to 2.4 m; the top of the drain is at 0.25 m so it is increased to 1.0 m; the bottom is at 0.15 m so it is increased to 0.6 m, while the bedrock remains at 0 m (4×0 = 0). The transformed system has the same width, but is 4 times taller than the anisotropic system. The shape of the circular drain becomes an oval (Figure Box 5-5).

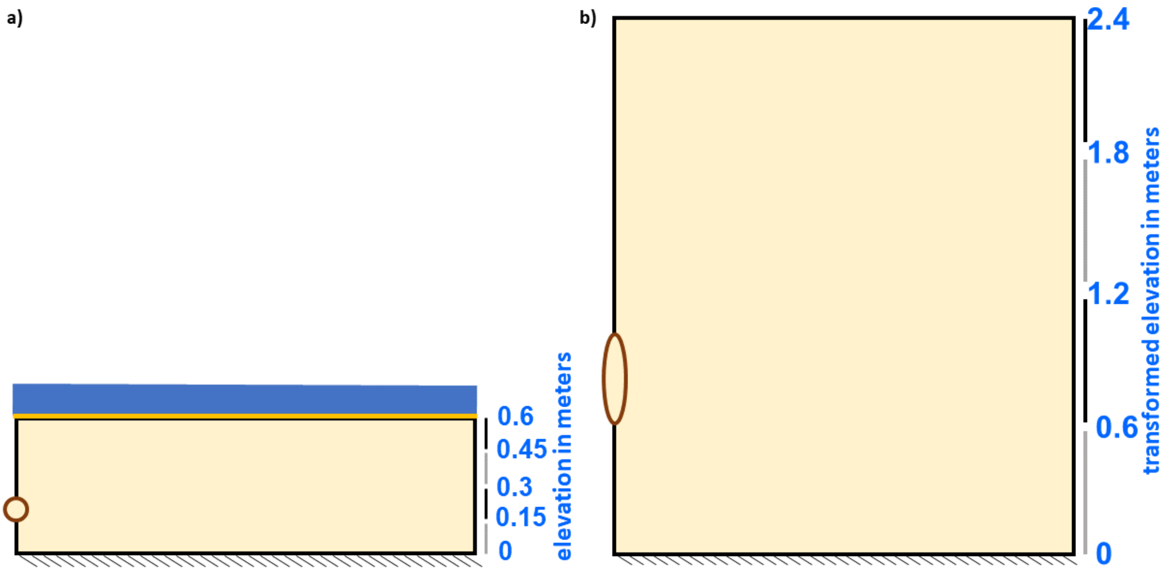


**Figure Box 5-5** - Geometric transformation of the (a) anisotropic system on the left to an (b) isotropic system on the right by transforming the vertical axis. The transformed geometry on the right is 4 times taller and as wide as the left one. The shape of the circular drain becomes an oval.

A flow net is drawn in the transformed section (Figure Box 5-6) according to the steps of flow net construction under isotropic conditions as described in section 2.2 of this



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book. We know the hydraulic head at the ground surface is equal to the elevation of the ponded water (0.8 m). We assume the pressure is atmospheric in the drain (that is, the water flowing to the drain discharges at the end of the drain without backing up water in the drain). Hydraulic head is the sum of pressure head in terms of a height of a column of water and elevation. Atmospheric pressure is used as the zero-reference point for quantifying pressure so, at the drain, the pressure is zero and the hydraulic head is equal to the elevation. Consequently, the hydraulic head at the top of the drain is 0.25 m, at the midpoint it is 0.2 m and at the bottom it is 0.15 m. We expect water to flow from the ground surface to the drain, so we sketch in flow lines and equipotential lines and continue to adjust until they form curvilinear squares (Figure Box 5-6).

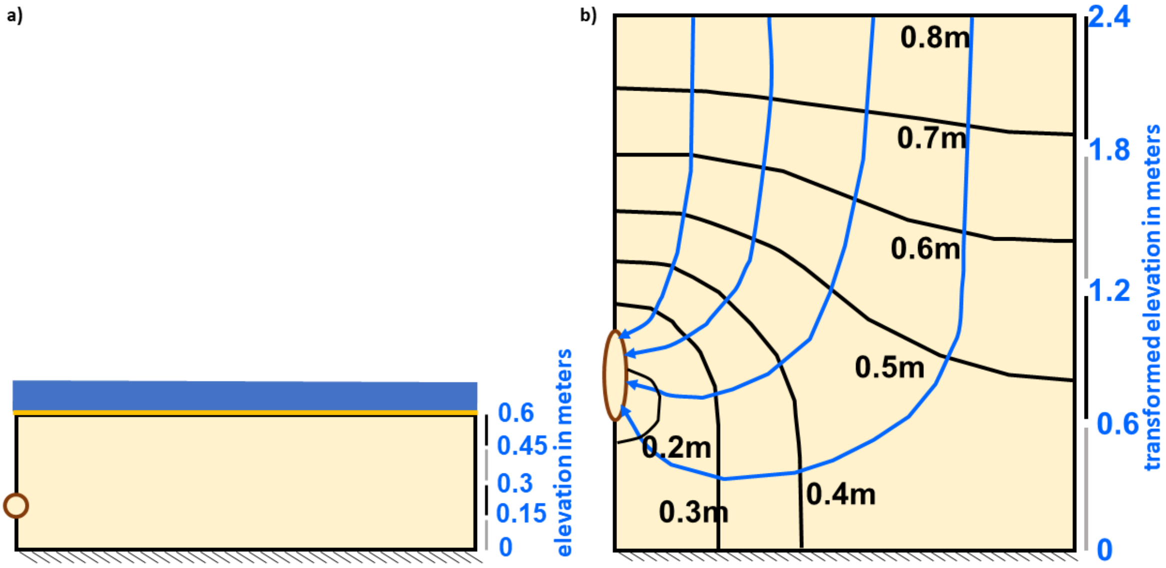


**Figure Box 5-6** - An isotropic flow net is drawn in the transformed isotropic system (b) on the right.

Next, we transform the section back to its original coordinate system (Figure Box 5-7). This is done by shifting each $x$ and $y$ coordinate where the equipotential lines and flow lines intersect, dividing the $y$ coordinate by 4, and plotting the intersection point at the original $x$ location and the reduced $y$ location on the original system geometry. Once the intersection points are plotted, the lines can be connected. The transformed equipotential lines and flow lines do not meet at right angles and the intersecting lines do not form squares in the anisotropic system geometry (Figure Box 5-7).



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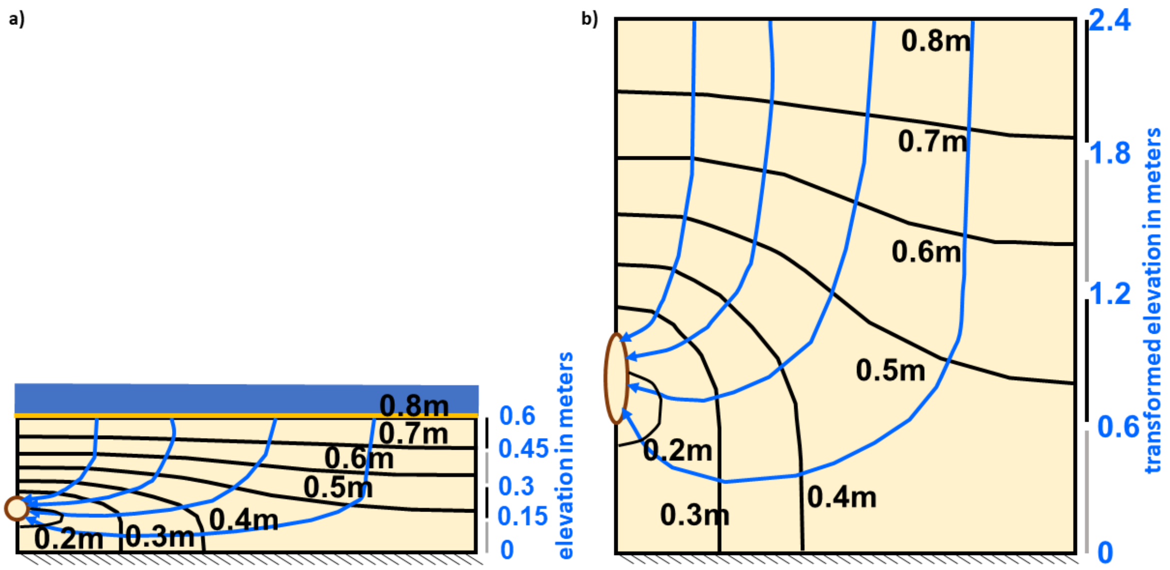


**Figure Box 5-7** - The (b) isotropic flow net on the right is transformed back to the (a) anisotropic geometry on the left.

Finally, we estimate the volumetric flow through the system. To accomplish this, we use the formula presented here as Equation Box 5-3. It is discussed and derived in Box 4 of this book.

$$Q_{total} = K\,H\frac{n_f}{n_d}\,w \qquad \text{(Box 5-3)}$$

where:

$Q_{total}$ = volumetric flow rate through the system ($L^3/T$)

$K$ = hydraulic conductivity of the porous medium (L/T)

$H$ = head difference across the flow net (L)

$n_f$ = number of flow tubes in the flow net (dimensionless)

$n_d$ = number of head drops in the flow net (dimensionless)

$w$ = distance that the system extends into the drawing (L)

When Equation Box 5-3 is applied to an anisotropic system, an equivalent hydraulic conductivity is used to account for the differing values in the horizontal and vertical direction. Equivalent hydraulic conductivity for an anisotropic system is calculated as shown in Equation Box 5-4.

$$K_{equivalent} = \sqrt{K_x K_y} \qquad \text{(Box 5-4)}$$

Using Equation Box 5-4, the equivalent hydraulic conductivity for the flow net shown in Figure Box 5-7a is:

$$K_{equivalent} = \sqrt{K_x K_y} = \sqrt{\left(0.16\ \frac{\text{m}}{\text{d}}\right)\left(0.01\ \frac{m}{d}\right)} = 0.04\ \frac{\text{m}}{\text{d}}$$



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The total head drop, $H$, is estimated as 0.6 m (that is, the difference between the 0.8 m head at the ground surface and the average 0.2 m head along the drain). The average head along the drain is estimated as 0.2 m because the head at the top of the drain is 0.25 m, the center is 0.2 m, and the bottom is 0.1 m.

The total flow to one side of the drain is calculated using Equation Box 5-3 with an equivalent hydraulic conductivity of 0.04 m/d, a total head drop H of 0.6 m, 5 flow tubes ($n_f$), 7 head drops ($n_d$), and a length of drain, $w$, of 100 m:

$$Q_{total} = K\,H\,\frac{n_f}{n_d}\,w$$

$$Q_{total} = \left(0.04\,\frac{\mathrm{m}}{\mathrm{d}}\right)\,(0.6\ \mathrm{m})\,\left(\frac{5}{7}\right)\,(100\ \mathrm{m}) \sim \left(1.7\,\frac{\mathrm{m}^3}{\mathrm{d}}\right)\left(1000\,\frac{\mathrm{liters}}{\mathrm{m}^3}\right)\left(\frac{1}{1440}\,\frac{\mathrm{d}}{\mathrm{min}}\right) \sim 1.2\,\frac{\mathrm{liters}}{\mathrm{min}}$$

The flow needs to be doubled to account for drainage from both sides of the drain, so approximately 2.4 liters per minute.

$$\text{Discharge from both sides to the drain} = Q_{total-both-sides} = 2\,Q_{total} \sim 2.4\,\frac{\mathrm{liters}}{\mathrm{min}}$$

A 0.3 m diameter pipe can transport 50 liters per minute without backing up so we made a reasonable assumption when we decided that the drain would be at atmospheric pressure.

Anisotropy can occur in a horizontal flow net as well as in a vertical one. Anisotropy in the horizontal plane is generally the result of a directional fabric in the material such as fracture planes. The process of creating the flow net is similar. However, the principal directions for flow in the plan view might not be as obvious as for flow in a vertical cross section (as above example). The principal directions in a vertical cross section are often (but not always) taken to be horizontal and vertical because many subsurface settings consist of horizontal layers. By contrast, the principal directions for flow in a plan view are generally not in east-west/north-south directions. For an anisotropic system in a plan view, it is necessary to know the principal directions and align the $x$-$y$ coordinate system to these directions. The geometric transformation can then be carried out for flow net construction.

Return to where text links to Box 5↥



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# Box 6 – Create and Investigate Topographically-driven Flow Systems

## Introduction

The online version of TopoDrive is designed to run in a web browser and does not require any plug-ins.

IF YOU ARE READY TO GO DIRECTLY TO INFORMATION ON HOW TO USE THE TOPODRIVE MODEL, PROCEED TO Running the Model↴

A topographically-driven flow system is one in which ground water flows from higher-elevation recharge areas (where hydraulic head is higher) to lower-elevation discharge areas (where hydraulic head is lower). The boundaries of the flow domain are as follows (Figure Box 6-1):

- The top boundary (AB) is the water table, which is assumed to lie close to land surface.
- The two vertical boundaries (BC and AD) are no flow boundaries.
- The bottom boundary (CD) is also a no-flow boundary.

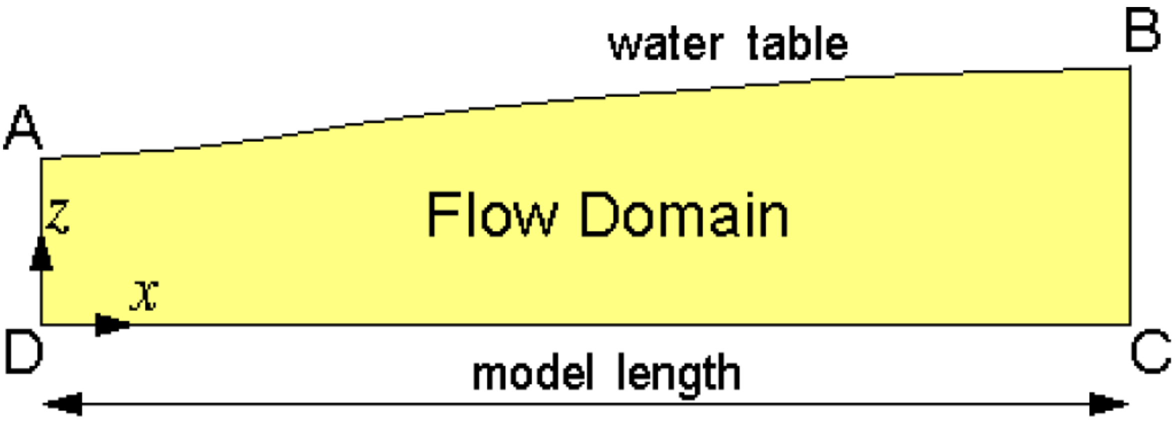


**Figure Box 6-1** - Diagram of the TopoDrive flow system.

The no-flow boundary A-D-C-B might represent low-permeability bedrock that bounds the basin. Alternatively, the vertical BC boundary might represent a groundwater flow divide on a ridge. Also, point A may represent the center of a river, thus groundwater on both sides of the river flows towards the river forming a no-flow boundary along AD. **Important Note:** By specifying the position of the water table, it is assumed that the pattern of recharge and discharge is such that the water table is maintained at steady state.



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## Governing Equation

The steady-state ground-water flow equation to be solved is:

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right) = 0$$

where $h$ is hydraulic head, and $K_x$ and $K_z$ are the principal values of the hydraulic conductivity ellipse. The principal directions are assumed to be parallel to the $x$ and $z$ axes.

## Boundary Conditions

Assuming we know the position of the water table, the boundary condition along the water table (AB) is

$$h = z$$

where $z$ is the elevation of the water table.

Along the vertical boundaries BC and AD, the no-flow boundary condition is

$$\frac{\partial h}{\partial x} = 0$$

Along bottom boundary CD, the no-flow boundary condition is

$$\frac{\partial h}{\partial z} = 0$$

After solving for hydraulic head $h$, the $x$ and $z$ components of the linear velocity vector are computed by

$$v_x = \frac{K_x}{n}\frac{\partial h}{\partial x} \qquad v_z = \frac{K_z}{n}\frac{\partial h}{\partial z}$$

Where $n$ is porosity. The velocity vectors are used for calculating flow paths and the advective movement of fluid particles.

## Running the Model

Open the model by linking to https://tdpfonline.net↗ and click the "Launch TopoDrive" button to start the TopoDrive software. TopoDrive will appear in a new browser window (Figure Box 6-2).



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**Figure Box 6-2** - TopoDrive software window.

Running the model involves 7 steps. To begin each step, click the corresponding button at the top of the window (Figure Box 6-3). A dialog box appears for you to enter the necessary input data. The three buttons on the second row allow you to zoom in and zoom out. To quit the model, simply close the browser window.


**Figure Box 6-3** - TopoDrive software buttons.

Step 1: Start -- Specify model dimension
Step 2: Water Table -- Specify the position of the water table
Step 3: Mesh -- Specify the dimension of the model mesh
Step 4: Properties -- Specify hydraulic conductivity and porosity
Step 5: Head -- Compute hydraulic head

After hydraulic head is computed, two options are available. You may proceed to 6a/7a or 6b/7b



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Step 6a: Flow (Path) -- Track flow paths from selected points
Step 7a: Animation -- Animate the evolution of flow paths

Or:

Step 6b: Flow (Particle) -- Set up initial distribution of fluid particles
Step 7b: Animation -- Animate the advective movement of fluid particles

Additional buttons can be used to zoom in and zoom out. The web browser's "Print" command can be used to print the image in the window. Closing the browser window terminates the program.

**If you are feeling uncertain about how to proceed, here are some suggested inputs:**

## Example 1

*Step 1: Click the **Start** button then input: Domain length: **1000** Vertical Exaggeration: **1***

*Step 2: Click **Water Table** button then place the cursor to the left of the left axis fairly low and click, move the cursor to the right within the model area and click, continue to move the cursor to the right within the model area and click creating a shape for the water table, finally move the cursor to the right of the right axis click to complete the upper boundary of the model. The upper portion of the vertical boundary on each side will be clipped to terminate at the line you drew. Your line now defines the water table and thus the heads at the top of the model.*

*Step 3: Click the **Mesh** button then input: Number of columns: **60** Number of Rows: **30***

*Step 4: Click the **Properties** button: Initially the entire mesh is set at the medium value of hydraulic conductivity (indicated by white). Click on the blue rectangle, OK. Place the cursor within the mesh and draw a polygon by clicking at the vertices. Double clicking the last vertex completes the polygon. You now have a high hydraulic conductivity zone in the model. Next, click the **Properties** button again and click on the pink rectangle. Draw another polygon on the mesh to define a zone of low hydraulic conductivity. An alternative way to finish the polygon is to single-click the last vertex and then click the **Done Polygon** button. Notice you can change the values of hydraulic conductivity for each color, you can choose anisotropic and specify different hydraulic conductivity in the horizontal and vertical directions. You can also change the porosity which will not affect the flow lines but will affect the velocity of flow.*

*Step 5: Click the **Head** button then input: Number of contour intervals: **40** Then click **Compute***

*Step 6: Click the **Flow** button then choose: **●Flow Path Tracking** **●Forward and Backward** **OK***

*Step 7: **Click locations within the flow field** and the flow path will be drawn in both the forward and backward directions from that locations.*



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*One possible example of a finished product is shown in Figure Box 6-4. The blue zone has 100 times the hydraulic conductivity of the white zone. The pink zone has 1/100 times the hydraulic conductivity of the white zone.*

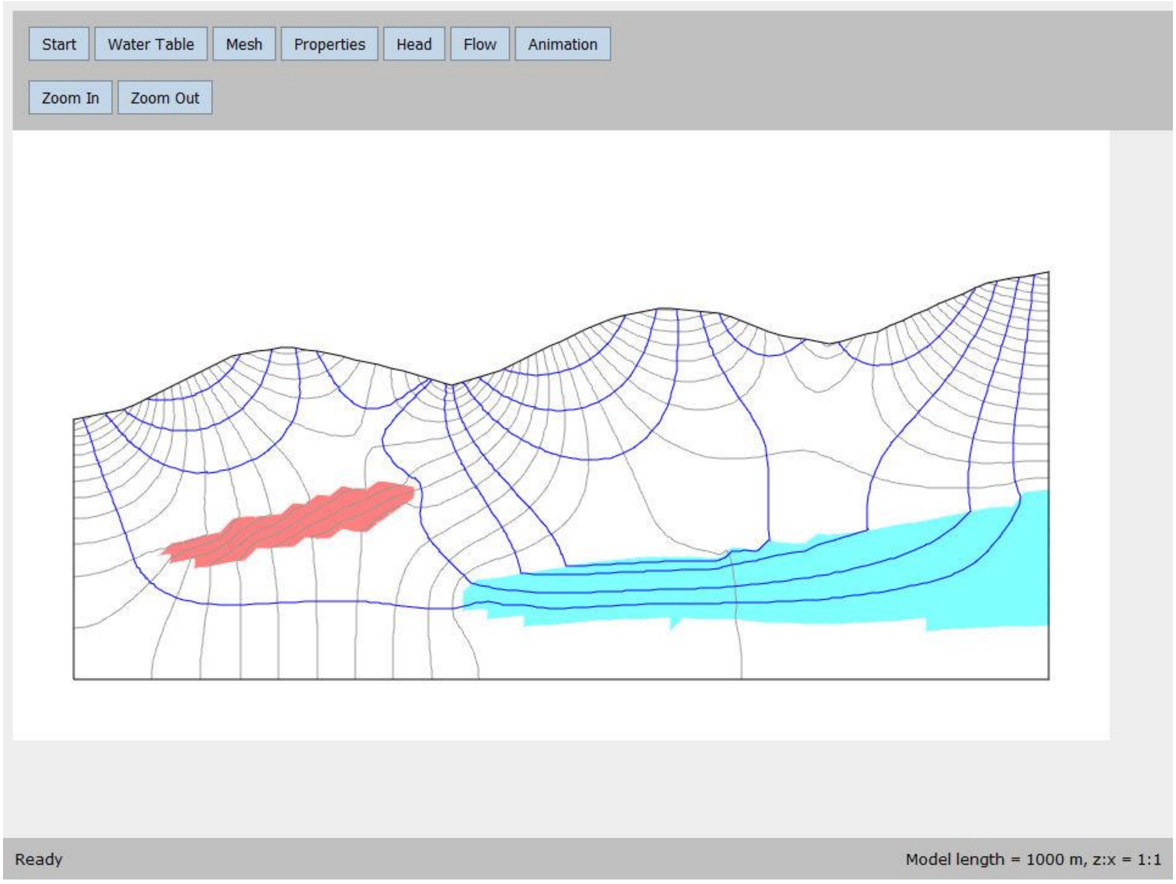


**Figure Box 6-4** - Example of one final product of a TopoDrive simulation.

## Example 2

Follow steps 1 through 5 for example 1. Alternatively, if you still have the TopoDrive Window open, you can go back to step 6 and choose different options as follows:

*Step 6: Click the **Flow** button then input: •**Particle Movement** Initial particle spacing: **5 m OK***

*Now use the cursor to draw a polygon anywhere in the model and double click when you have completed the shape. You will see dots in the shape that are 5 m apart (Figure Box 6-5).*



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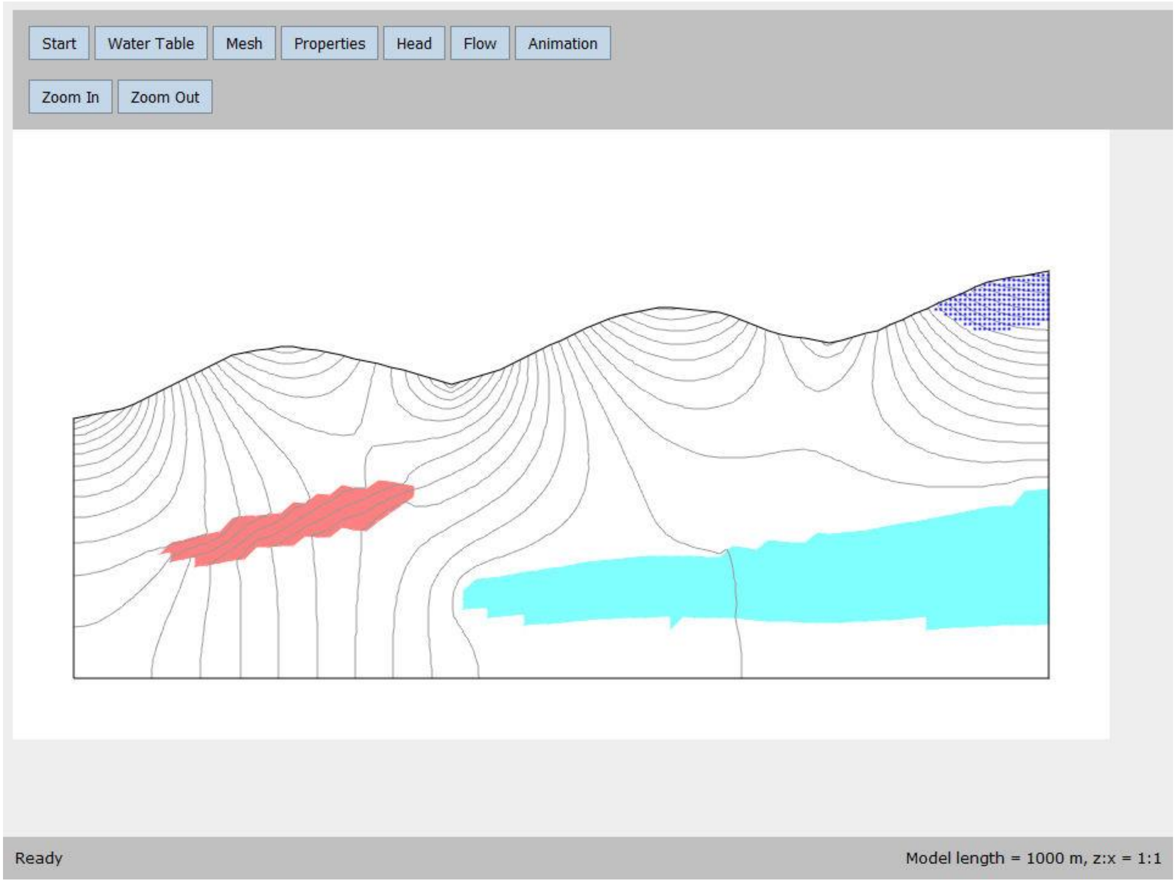


**Figure Box 6-5** – Starting positions of a group of particles within the TopoDrive model.

*Step 7: Click on the Animation button then input:*

> *1 sec of animation time=:* ***50*** *days* *animation smoothness=**10** frames per sec* ***OK***
>
> *Click anywhere within the model and the particles will begin to move, if you click within the model again the particles will pause, then click again to continue and so on (Figure Box 6-6).*
>
> *If you setup different properties then your particles may move too fast or slow. If this is the case, adjust the amount of time represented by 1 second of animation.*

*The particle locations for example 2 are shown after 1460 days (four years) in Figure Box 6-7. Note that the elapsed time is shown in the bottom of the TopoDrive window.*



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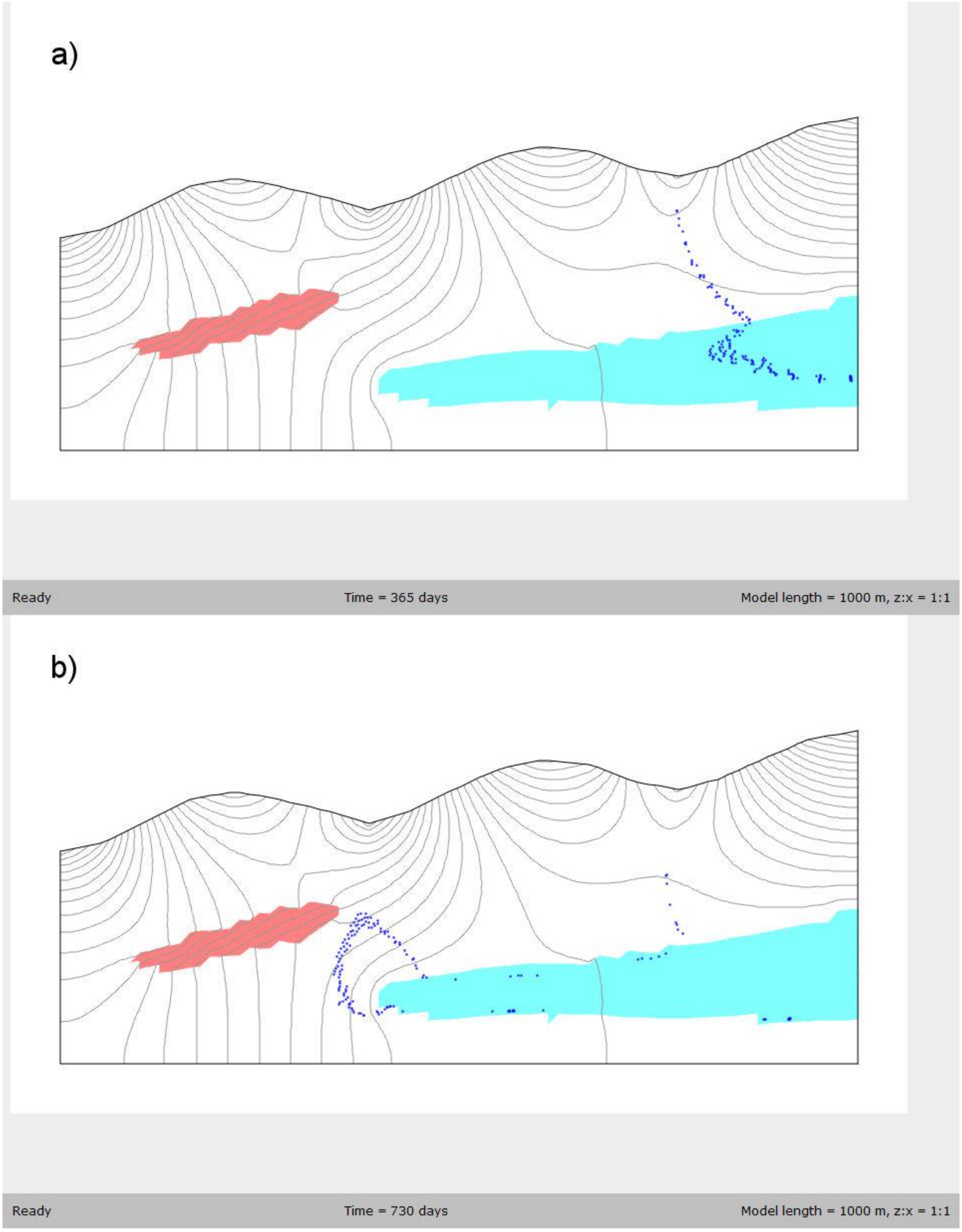


**Figure Box 6-6** - Particle positions at two times during the animation: a) 365 days (1 year); and, b) 730 days (2 years).



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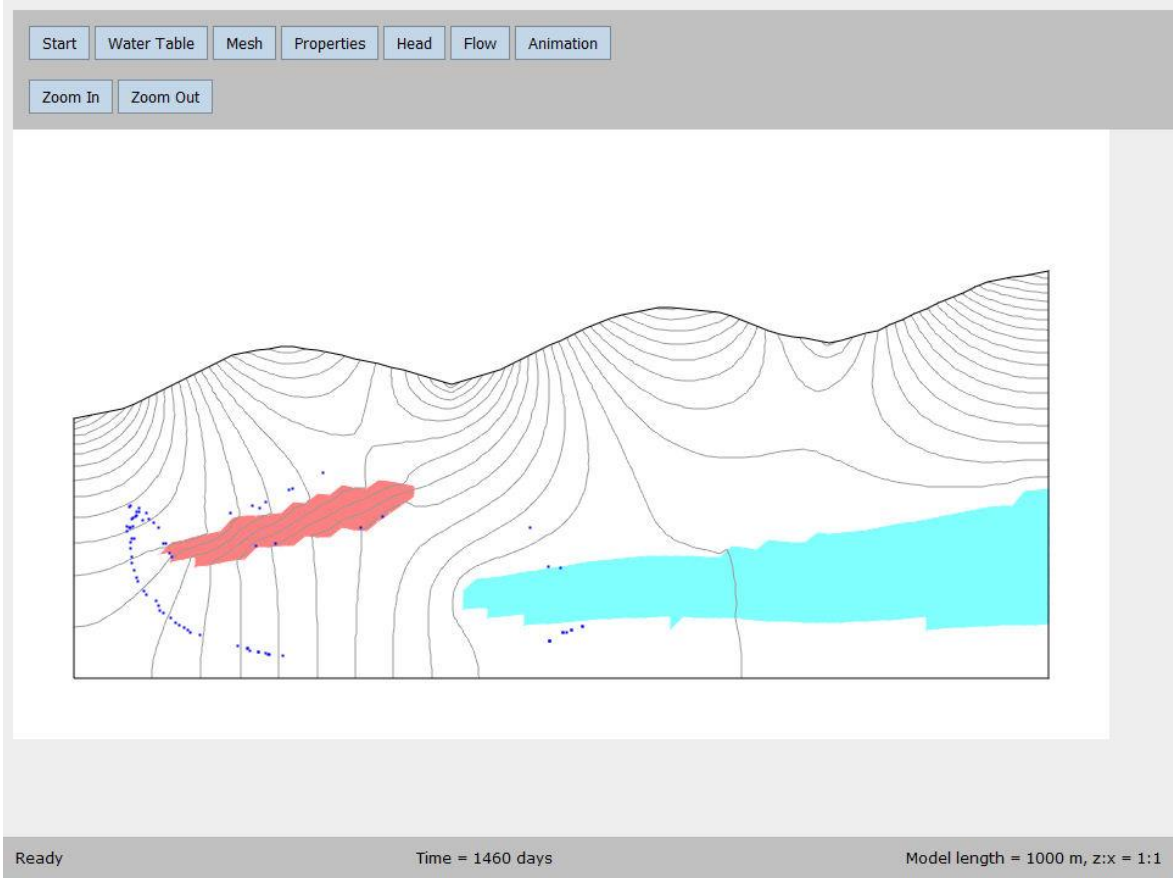


**Figure Box 6-7** - Particle positions after 1460 days (4 years). Many particles have exited the aquifer. A strand of particles is moving around the left side of the low hydraulic conductivity zone (pink region) and a few particles have moved into the low hydraulic conductivity zone.

Return to where text links to Box 6



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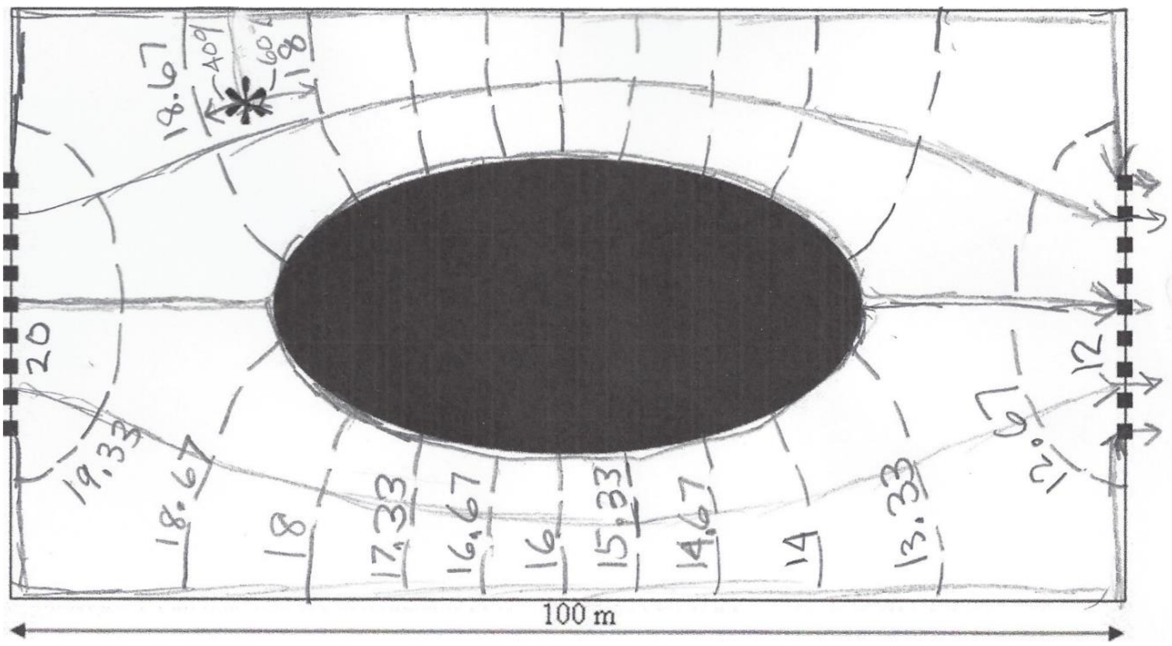
# 6 Exercise Solutions

## Exercise 1 - Solution




$$\text{contour interval} = \frac{8\,\text{m}}{12\ \text{head drops}} = 0.6\overline{6}$$

$$Q_{total} = K\,H\,\frac{n_f}{n_d}\,w = 1\times10^{-3}\,\frac{\text{m}}{\text{s}}\ 8\,\text{m}\ \frac{4}{12}\ 10\,\text{m} = 0.2\overline{6}\ \frac{\text{m}^3}{\text{s}}$$

$$\text{hydraulic head @ } * \approx 18 + 0.6 \times 0.6\overline{6} \approx 18.4\,\text{m}$$

$$\text{pressure head} = \text{hydraulic head} - \text{elevation} = 18.4\,\text{m} - 10\,\text{m} = 8.4\,\text{m}$$

Comment: You might be wondering: what happens along the flow line that begins exactly at the midpoint of the inlet screen? The answer to this question addresses the relation between a mathematical characterization of a process and what happens in reality. From a mathematical standpoint, the flow line that begins exactly at the midpoint of the screen flows directly towards the pillar. When the flow line meets the pillar, mathematical analysis shows that there are two possible solutions. The flow line can either go around either side of the pillar. When both possibilities are drawn in the flow net, there is the appearance that the flow line splits at the pillar into two branches, one branch going around the left side of the pillar, and the other branch going around the right side. However, this is a misleading conceptualization. Instead, it is better to think of the flow line as representing two separate flow lines, one starting at a minute distance to the left, and the other starting at a minute distance to the right (Figure below). In this conceptualization,



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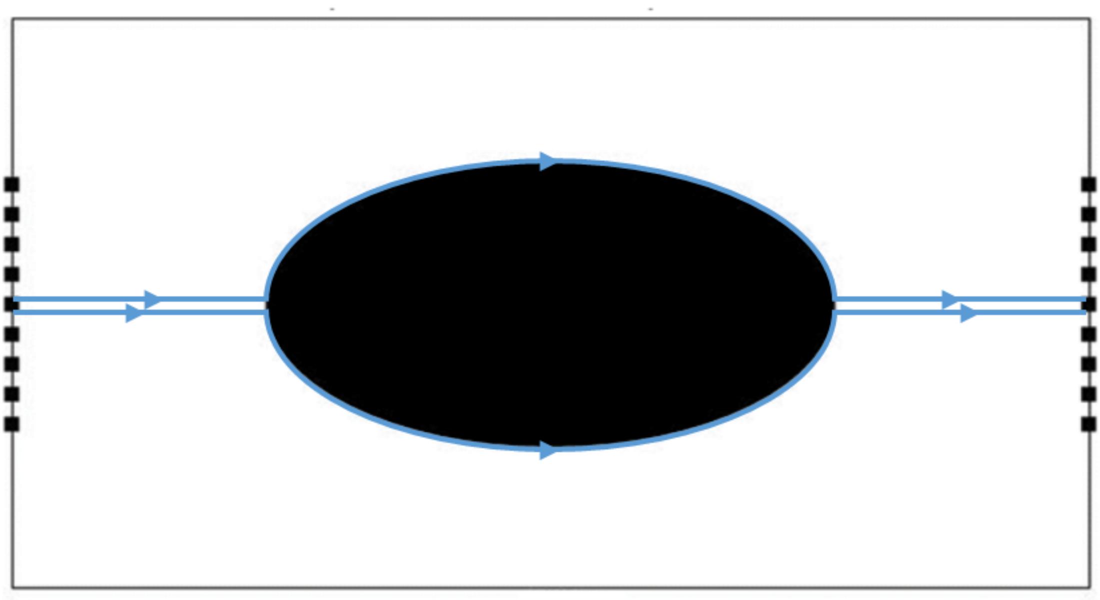
there is no notion of a flow line splitting. In reality, it is not possible to define a flow line starting at the midpoint of the inlet screen with mathematical exactness, because real objects do not possess exact geometric shapes. The most reasonable statement that can be made about a flow line that starts near the midpoint of the inlet screen is that it is equally likely to go around the left side of the pillar as around the right side.



Return to Exercise 1↥



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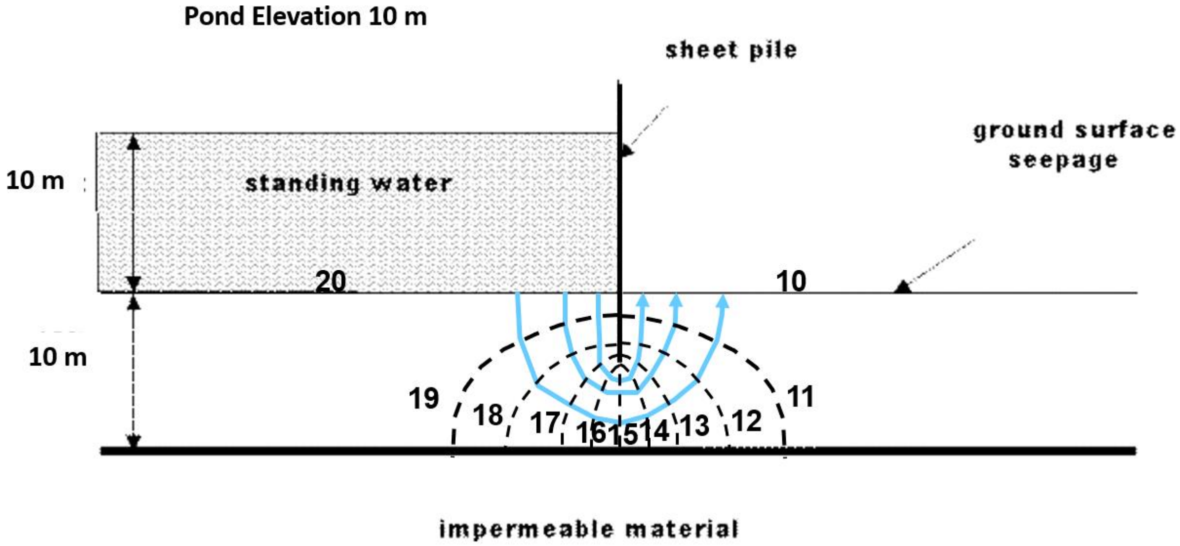
# Exercise 2 - Solution




**$Q_{total}$ = (2 m/day) (10 m) (4 flow tubes) / (10 head drops) (22 m) = 176 m$^3$/day**

**Contour interval = (10 m) / (10 head drops) = 1 m**

Return to Exercise 2



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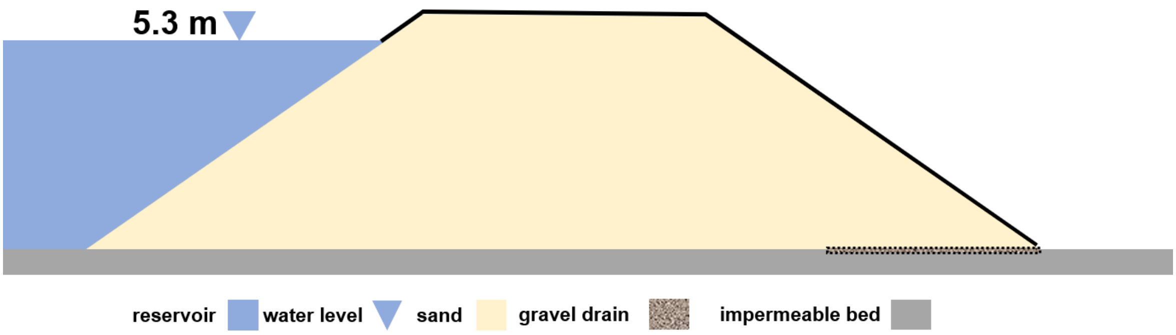
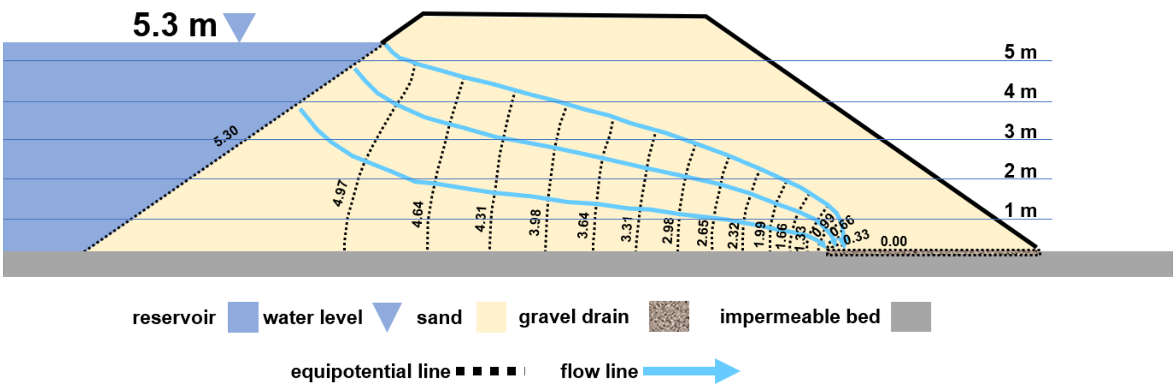
# Exercise 3 - Solution




Contour interval = (5.3 m) / (16 head drops) = 0.33125 m

$$Q_{total} = K\ H\ w\ \frac{n_f}{n_d}\ a_r$$

The aspect ratio is one, so $a_r = 1$

$Q_{total}$ = (0.2 m/d) (5.3 m) (28 m) (3 flow tubes) / (16 head drops) = 5.6 m$^3$/d

Return to Exercise 3↥



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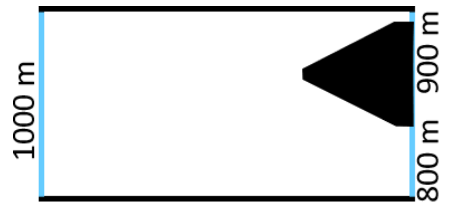
# Exercise 4 - Solution

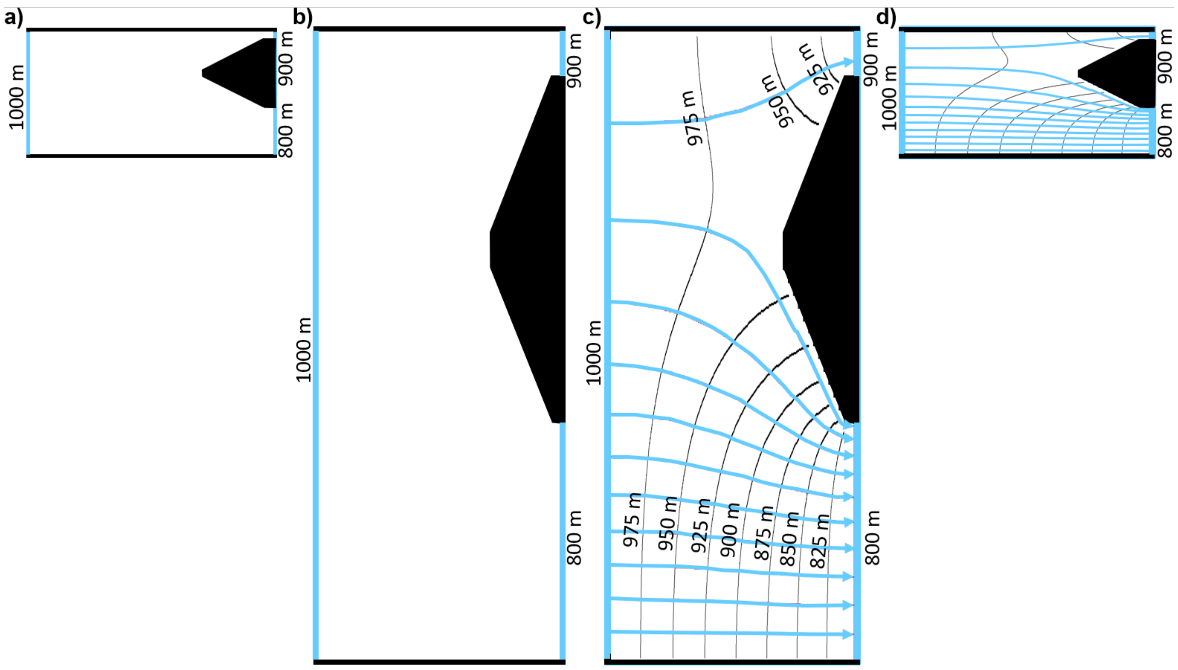



a) Original system

b) The system can be stretched by a factor of 5 in the $y$ direction to account for the anisotropy because $225^{0.5}/9^{0.5}$ = 15 / 3 = 5

c) A flow net is sketched in the transformed image as if the system is isotropic

d) The system is transformed back to 1/5 the width to reveal the flow lines in the anisotropic system




Contour interval = (200 m) / (8 head drops) = 25 m

$$Q_{total} = K\ H\ w\ \frac{n_f}{n_d}\ a_r$$

The aspect ratio is one, so $a_r = 1$

$K_{equivalent\ for\ an\ anisotropic\ flow\ net} = (K_x K_y)^{0.5}$ = ((225 m/d)(9 m/d))$^{0.5}$ = 45 m/d

$Q_{total}$ = (45 m/d) (200 m) (100 m) (12 flow tubes) / (8 head drops) = 1,350,000 m³/d

Return to Exercise 4↥



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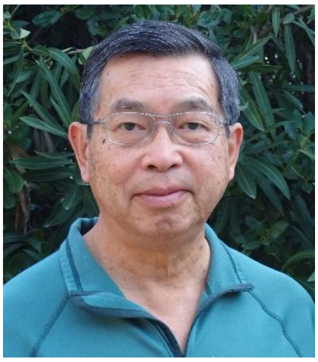
# 7 About the Authors

**Dr. Eileen Poeter** is an Emeritus Professor of Geological Engineering at Colorado School of Mines, where she taught groundwater courses and advised more than 40 graduate students who worked with her on groundwater system investigations and modeling research projects. She is also Past Director of the Integrated Groundwater Modeling Center; and retired President of Poeter Engineering. With 40 years of experience modeling groundwater systems, she has consulted to attorneys, industries, engineering companies, government agencies, research labs, and citizen groups on groundwater modeling projects for: aquifer storage and recovery; slurry wall performance; drainage at proposed nuclear power plant facilities; regional groundwater management, large-scale regional pumping, dam seepage, contaminant migration, impacts of dewatering, and stream-aquifer interaction. Dr. Poeter is an author of groundwater modeling software including evaluation of model sensitivity, assessment of data needs, model calibration, selection and ranking of models, and evaluation of predictive uncertainty. She was the National Groundwater Association (NGWA) Darcy Lecturer in 2006 and received their M. King Hubbert award in 2017 as well as being an NGWA Fellow and Life Member.



**Dr. Paul Hsieh** is an independent groundwater hydrologist, having retired in 2018 from the U.S. Geological Survey after 41 years of service as a research hydrologist. He received his B.S.E. from Princeton University in Civil Engineering, and M.S. and Ph.D. from the University of Arizona in Hydrology and Water Resources. His research at the USGS spanned over diverse topics that included groundwater flow and solute transport in fractured rocks, development and application of computer simulation models, interaction between groundwater and earthquakes, and volcano hydrology. During the 2010 Deepwater Horizon oil spill, he served on the federal government's science team on oil spill response. He is a Fellow of the Geological Society of America and the American Geophysical Union, and received the Service to America medal in 2011 from the Partnership for Public Service.



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# Modifications from original release

**General changes:**

A number of minor typographical errors were corrected such as deletion of extra blank spaces.

Where units were “sec” or “day”, they were changed to “s” and “d”.

Units that were in italics were changed to not italics.

Words in equations were changed to not italics.

Equation variables that were not in italics were changed to italics.

Parentheses were placed around certain quantities to improve clarity of equations.

References to (Poeter and Hsieh, 2020, gw.project.org) were removed to be consistent with the latest GW Project formatting. This citation was removed from the reference list.

Minor changes to formatting of reference list to be consistent with GW-Project formats.

Table of Contents was updated to ensure page numbers are correct given these revisions.

**Specific changes:**

page i, ii, Removed small caps font.

page iii, Removed keywords.

page iii, Added link to Groundwater Project email sign-up.

page iii, Added citation information.

page iii, Changed spelling of Steven Moran to Stephen Moran.

page v-vii, Table of Contents updated.



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page 1, Replaced “Edited and narrated by Poeter 2020, gw-project.org” with “Edited and narrated by Eileen Poeter.”

page 12, Deleted “(Equation 2).”

page 16, Figure 14 caption. Replaced “Simulations by Jiao and Liang, 2019;” by “Videotaping of Hele-Shaw model simulations at the University of Hong Kong by J. Jiao and W.Z. Liang.”

page 28, Removed “What is the specific discharge?”

page 28, Removed “What is the average linear velocity is the effective porosity is 30%?”

page 29, Removed the reference to Jiao and Liang, 2019.

page 29, Reference for Woessner and Poeter 2020. Changed to Hydrogeologic Properties of Earth Materials and Principles of Groundwater Flow. The Groundwater Project, Guelph, Ontario, Canada, https://gw-project.org/books/hydrogeologic-properties-of-earth-materials-and-principles-of-groundwater-flow/

pages 30-31, Replaced $\Psi$ with $\psi$.

page 39, Replaced “flow area per unit width perpendicular to the direction of flow” with “width of the flow tube in the plane of the flow net”.

page 39, The variable $q_{tube}$ is replaced by $Q'_{tube}$. Changed the units for $Q'_{tube}$ from $L^3/T$ to $L^2/T$.

page 39, Equation Box 3-3. Removed "$1 unit\_width\_into\_the\_diagram$".

page 39, Definitions of variables after Equation Box 3-3. Added the definition of the variable $y$.

page 40, Deleted “and omitting the value of one for the unit thickness into the diagram”.

page 40, The variable $q_{tube}$ is replaced by $Q'_{tube}$, including in Figure Box 3-4.

pages 40-41, The variable $q$ is replaced by $Q'$, defined as “volumetric flow rate through the entire system per unit width perpendicular to the diagram ($L^2/T$).”



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page 41, Replaced “flux-based flow net” by “flow net”.

pages 42-43, Replaced “Figure Box4-4” with “Figure Box 4-2”.

page 43, Replaced “Figure Box 4-5” with “Figure Box 4-3”.

page 43, Replaced “Figure Box 4-6” with “Figure Box 4-4”.

pages 43-44, Replaced “Figure Box 4-7” with “Figure Box 4-5”.

page 45, Replaced “Figure Box 4-8” with “Figure Box 4-6”.

pages 46-47, Replaced “Figure Box5-9” with “Figure Box 5-1”.

page 47, Replaced equation number “Box 5-8” with “Box 5-1”.

page 47, Replaced equation number “Box 5-9” with “Box 5-2”.

page 47, Replaced “any point ($x$,$y$) in the original coordinate system will be moved to a point ($X$,$Y$) in the transformed coordinate system” with “any point ($x$,$y$) in the original coordinate system will be moved to a point ($X$,$y$) in the transformed coordinate system.”

pages 47-48, Replaced “Figure Box 5-10” with “Figure Box 5-2”.

page 48, Replaced “Figure Box5-11” with “Figure Box 5-3”.

pages 48-49, Replaced “Figure Box5-12” with “Figure Box 5-4”.

page 49, Replaced “Figure Box5-13” with “Figure Box 5-5”.

pages 49-50, Replaced “Figure Box5-14” with “Figure Box 5-6”.

pages 50-51, Replaced “Figure Box5-15” with “Figure Box 5-7”.

pages 51-52, Replaced equation number “Box 5-10” with “Box 5-3”.

page 51, Replaced equation number “Box 5-11” with “Box 5-4”.

page 53, Replaced “Figure Box 6-16” with “Figure Box 6-1”.

page 54, Replaced the variables $K_{xx}$ and $K_{zz}$ with $K_x$ and $K_z$, respectively.



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page 57, Replaced “Figure Box 6-44” with “Figure Box 6-4”.

Page 58, Replaced “(Figure Box6-)” with “*(Figure Box 6-6)*”

Page 65, Replaced “$5^{0.5}$” with “$9^{0.5}$”

page 65, Removed the calculations for *q* and *v*.



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